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Revised hyperbolic method for predicting settlement of soft clay ground based on weighted nonlinear regression

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Abstract. The hyperbolic method is a method of predicting the future settlement from the measured settlement based on the assumption that the speed of settlement decreases in the form of a hyperbola over time. However, this method forces artificial linearity during linear regression analysis because both the independent variable (time) and dependent variables (time/settlement) contain time. Thus, to overcome this problem, this study proposed the nonlinear and weighted nonlinear regression hyperbolic methods to supplement the statistical incompleteness of the original hyperbolic method based on linear regression. A hyperbolic method that applied nonlinear regression analysis and a weighted nonlinear regression hyperbolic method that applied high weight linearly over time were proposed as alternatives to the existing linear regression hyperbolic method. The accuracies of the original, nonlinear, and weighted nonlinear regression hyperbolic methods were analyzed and compared based on the time-settlement data measured from six sites with thick soft clay layers in Busan New Port. The proposed methods yielded superior results compared to the original hyperbolic method. Further research on the forms of various weight functions can lead to the formulation of an optimal weighted nonlinear regression hyperbolic method.

Keywords: consolidation; field monitoring; regression; settlement prediction; soft clay site

1. Introduction

Construction of structures on soft soil frequently leads to unpredictable and excessive settlement, which complicates the prediction of geotechnical and structural responses. As a result, cracking and differential settlement in structures may occur in many cases. Notably, coastal regions, including those in Japan, Singapore, Norway, and South Korea, have reported numerous cases of substantial settlement in soft ground over extended periods (Jun *et al.* 2021, Marjadi and Jang 2025, Olsson 2010, Watebe *et al.* 2012, Zhang *et al.* 2017). The construction of ports and airports in these areas has highlighted the need for effective solutions to address long-term settlement challenges. In the context of South Korea, where smart port and airport projects on soft ground are being actively promoted, concerns regarding soft ground settlement have become increasingly prominent.

When a load is applied to the ground, as pore water cannot drained promptly, the excessive pore water pressure develops and resists the external load at the beginning. Over time, the pore water is drained and the volume of the soil changes (compression), resulting in the settlement of the ground. This phenomenon is referred to as consolidation.

The consolidation can be roughly divided into primary compression, which is caused by the discharge of pore water, and secondary compression, which is generated by the rearrangement of the soil structure (Bjerrum 1967, Buisman 1936, Haan 1992, 1996, Liu *et al.* 2019, Taylor and Merchant 1940, Terzaghi 1943). For sand, primary compression rapidly occurs because water is discharged instantaneously owing to high permeability. However, in clay ground with low water permeability, primary and secondary compressions occur slowly over a long period, causing the long-term settlement problem.

In general, soil properties (e.g., strength parameter and compression index) are calculated via ground surveys to predict the consolidation settlement in the soft ground. Theories, such as the one-dimensional consolidation theory (Terzaghi 1943) and the consolidation theory (Barron 1948), are applied using the soil properties. However, the one-dimensional consolidation theory involves various assumptions and cannot accurately predict the actual settlement in the field owing to the uncertainty of ground information. Dalgıç and Şimşek (2002) compared the settlement predicted based on the consolidation theory with the actually measured settlement, and obtained a consistency of approximately 88%. To address this problem, the surface settlement plate is installed at the site in the actual construction stage to continuously measure the settlement of the ground over time. Consequently, the measured data are used to inductively predict the future settlement (Asaoka 1978, Chung *et al.* 1998, Hoshino 1962, Lei *et al.* 2019, Monden 1963, Quang and Giao 2014, Tan *et al.* 1991, Yoo and Kim 2000).

The hyperbolic method is a method of predicting the

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future settlement from the measured settlement based on the assumption that the speed of settlement decreases in the form of a hyperbola over time. It is the most representative method to inductively predict the future settlement (Al-Shamrani 2005, Arulrajah *et al.* 2003, 2004, Choo *et al.* 2010, Tan *et al.* 1991, Tan 1993, 1995, Tan and Chew 1996). Statistically, a hyperbola is a representative curve for which linear regression analysis is difficult. However, the hyperbolic method calculates the hyperbolic coefficient by conducting linear regression analysis after setting the time (t) and time/settlement (t/S) as the independent and dependent variables, respectively. A hyperbolic formula is presented based on the calculated hyperbolic coefficient, and the future settlement is predicted using the formula.

The Hoshino, root S, and log S methods predict the future settlement in a manner similar to that of the hyperbolic method. These methods conduct linear regression analysis by substituting the square of the settlement (S^2), square root of the settlement ($S^{1/2}$), and log of the settlement ($\log S$), respectively, instead of the settlement of the hyperbolic method (Chung *et al.* 1998, Hoshino 1962, Yoo and Kim 2000). They are similar to the hyperbolic method in that they predict the future settlement using the coefficient calculated via linear regression analysis.

The Asaoka method is a future settlement prediction method that applies the time-settlement measurement data constructed at equal intervals based on the consolidation theory proposed by Mikasa (1963). Methods to predict the final settlement using graphical approaches have also been proposed (Asaoka 1978, Arulrajah *et al.* 2003, 2004). Monden (1963) proposed a process for confirming the optimal future settlement prediction method by applying the trial and error method, wherein the process of analyzing the settlement was repeated by assuming consolidation.

Arulrajah *et al.* (2003, 2004) examined the accuracy of the hyperbolic and Asaoka methods through the measurement data of the Changi East land reclamation project of Singapore, and proposed the optimal time interval (Δt) of the Asaoka method. Guo and Chu (2017) proposed a formula to predict the settlement in a manner similar to that of the Asaoka method by applying a new fitting method for Oedometer Consolidation test results. Further, they examined its accuracy compared to the existing Asaoka and hyperbolic methods based on the measurement data of Changi East land reclamation project of Singapore and the southeast coast of the Kingdom of Saudi Arabia. Kwak *et al.* (2022) examined the accuracy of settlement prediction methods by applying the hyperbolic, root S, log S, and Asaoka methods to the data measured from new port in Busan, South Korea. The analysis results presented in several studies have suggested that the hyperbolic and Asaoka methods exhibit the highest accuracy among the existing measurement data-based settlement prediction methods.

Recently, machine learning (ML) and deep learning (DL) have been explored for settlement prediction. These approaches utilize field measurements to learn hidden patters in settlement behavior without relying on simplifying assumptions or predefined regression equations. Studies such as Hong *et al.* (2024), Hong *et al.* (2025a), and Chen *et al.* (2023) have demonstrated that ML and DL models, including artificial neural networks (ANN), long short-term memory (LSTM) networks, and gated recurrent units (GRU), can provide accurate predictions compared with conventional methods. While these models show promising results, the models often require extensive amount of data and computational resources (Thompson *et al.* 2020) which can hinder their application in real-world construction projects. In addition, they are often sensitive to data noise and uncertainty, and may even produce physically inconsistent results (Hong *et al.* 2025b). Consequently, conventional data-based settlement prediction methods (e.g., hyperbolic and Asaoka methods) remain widely used in practice, especially for projects on deep soft deposits.

The hyperbolic and Asaoka methods, however, have limitations from the statistical and artificial interpolation perspectives, respectively. The hyperbolic method forces artificial linearity during linear regression analysis because both the independent variable (time) and dependent variables (time/settlement) contain time. Whereas, the Asaoka method requires the supplement of data through linear interpolation owing to the requirement of measurement data at the same time interval. Furthermore, there has been limited evaluation of the underlying causes of underestimation or overestimation that may arise when these methods are applied to actual field monitoring data, indicating a need to more critically assess their interpretational limitations and develop improved approaches (Chung and Kweon 2021, 2022). The recently proposed settlement prediction methods based on cutting-edge analysis techniques require costly equipment and complex operations, rendering their direct application in the field difficult. Thus, study attempted to improve the existing settlement prediction method by proposing a method that was easy to apply in the field and did not require complex operations.

To supplement the statistical incompleteness of the hyperbolic method, which is the most representative method among the existing measurement-based settlement prediction methods, 1) a hyperbolic method that applies nonlinear regression analysis and 2) a weighted nonlinear regression hyperbolic method that applies high weight linearly over time were proposed instead of the existing linear regression hyperbolic method. In addition, the accuracies of the existing linear, nonlinear, and weighted nonlinear regression hyperbolic methods were compared and analyzed using the surface settlement plate data measured from six sites of Busan New Port.

The remainder of the paper is organized as follows. Section 2 presents the original hyperbolic method and the hyperbolic methods that apply nonlinear regression and weighted nonlinear regression analysis. Section 3 describes how settlement prediction was performed using the original, nonlinear, and weighted nonlinear hyperbolic methods to analyze the applicability and accuracy of each method. Section 4 summarizes the study and presents conclusions.

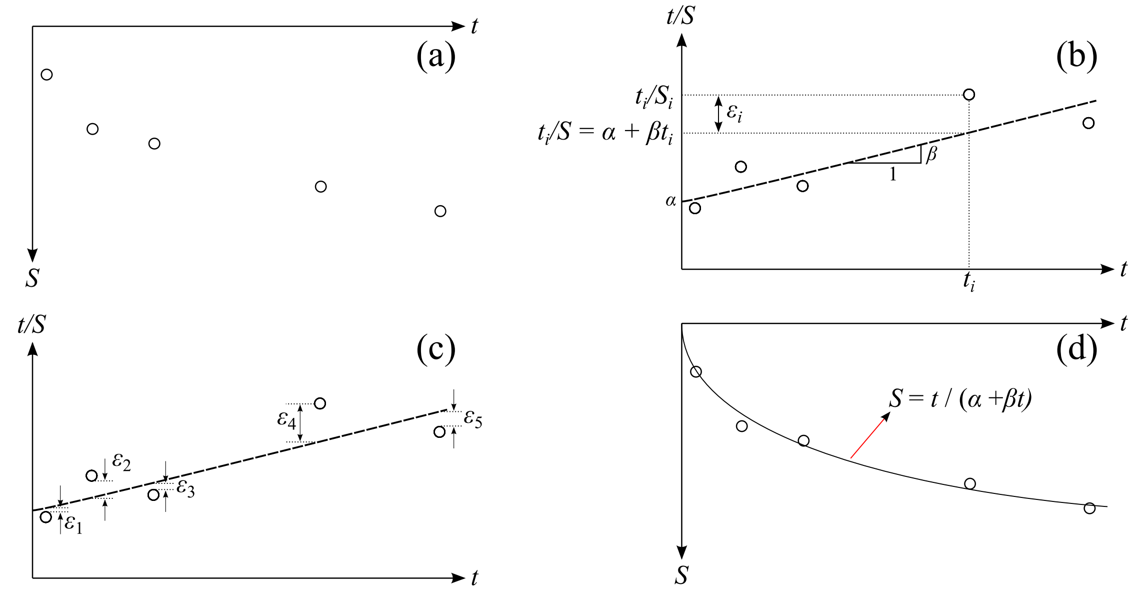


Fig. 1 Application steps of the hyperbolic method (Tan *et al.* 1991): (a) settlement monitoring data, (b) axes transformation, (c) definition of error and (d) construction of a time-settlement curve

2. Hyperbolic methods to predict consolidation settlement of clay ground

2.1 Original hyperbolic method (Tan et al. 1991)

The hyperbolic method predicts the future settlement using the initially measured settlement data under the assumption that the settlement is generated in the form of a hyperbola over time. Tan *et al.* (1991) proposed the first prediction formula in the form of Eq. (1). Thereafter, many researchers conducted research on future settlement prediction by applying actual field data based on the proposed hyperbolic method (Tan *et al.* 1991, Tan 1993, 1995, Tan and Chew 1996, Arulrajah *et al.* 2003, 2004, Al-Shamrani 2005, Choo *et al*. 2010).

$$S = \frac{t}{\alpha + \beta t} \tag{1}$$

where S is the settlement, t is the time, and α and β are the hyperbolic coefficients. From the perspective of regression analysis, a hyperbola is a function for which linear regression analysis is difficult. To overcome this issue and conduct linear regression for Eq. (1), Tan *et al.* (1991) deformed Eq. (2) as follows.

$$\frac{t}{S} = \alpha + \beta t \tag{2}$$

Fig. 1 shows the linear regression method proposed by Tan *et al.* (1991) using Eq. (2). The time-settlement data measured in the field, as shown in Fig. 1(a), were converted into the data that have t as independent variable (x-axis) and t/S as dependent variable (y-axis), as shown in Fig. 1(b) where the residual (or error) ε_i at the i^{th} measurement point is calculated as

$$\varepsilon_i = (\alpha + \beta t_i) - \left(\frac{t_i}{S_i}\right) \tag{3}$$

After calculation of residuals at every point as shown in Fig. 1(c), the sum of the squared residual ρ_h can be estimated as

$$\rho_h = \sum_{i=1}^{n} (\varepsilon_i)^2 \tag{4}$$

Then, the linear regression was performed to minimize ρ_h to determine hyperbolic coefficients α and β; consequently, the straight regression line (dot line in Fig. 1(c)) can be determined.

The slope and intercept of this regression line correspond to the hyperbolic coefficients α and β, respectively. By substituting the calculated α and β into Eq. (1), the settlement prediction equation in the form of a hyperbola at the settlement measurement point can be determined, as shown in Fig. 1(d). Consequently, the future settlement can be predicted.

2 2 Application of nonlinear regression

As examined above, in the original hyperbolic method, the hyperbolic coefficients are determined by conducting linear regression analysis after setting time t and t/S as independent and dependent variables, respectively through axes transformation. This method may raise the multicollinearity problem in that artificial linearity is forced in regression analysis because both the independent and dependent variables contain time t. Multicollinearity has a negative impact on data analysis owing to the correlation between the independent and dependent variables used for regression analysis. This has been examined by many researchers since it was first proposed by Frisch (1934) in the field of statistics (Farrar and Glauber 1967, Daoud 2017, Shrestha 2020, Thompson *et al.* 2017). The cause of the multicollinearity problem with the original hyperbolic method lies in axes transformation. Therefore, in this study,

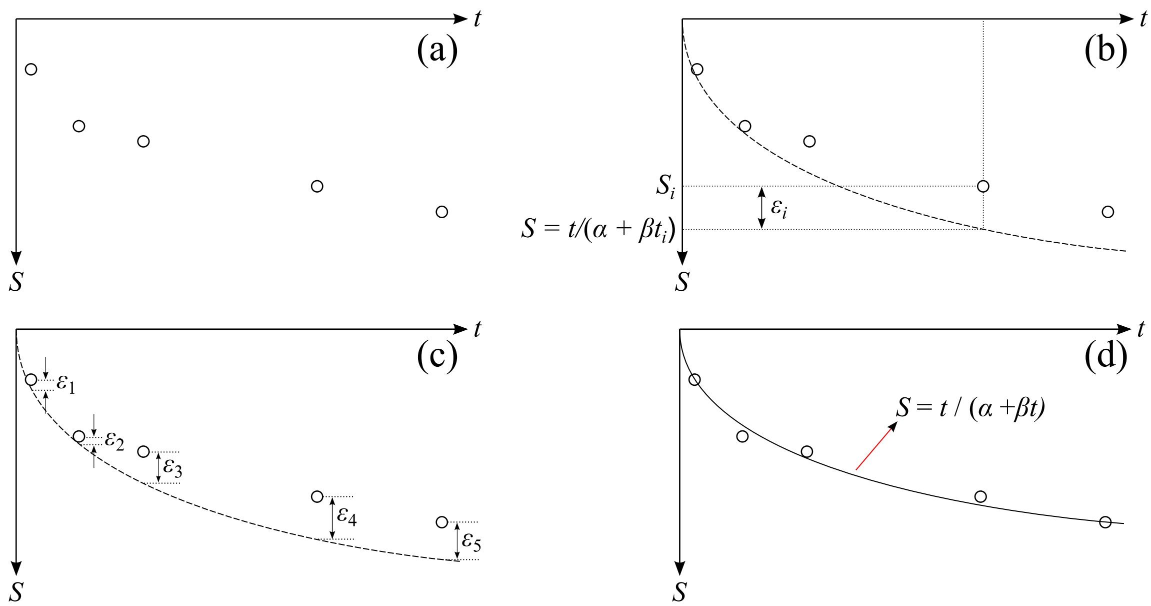


Fig. 2 Application steps of the hyperbolic method based on the nonlinear regression: (a) settlement monitoring data, (b) calculation of error at a single point, (c) definition of errors at multiple points and (d) construction of a time-settlement curve

axes transformation was not performed, and a method of finding the hyperbolic coefficients by conducting nonlinear regression analysis was proposed.

Fig. 2 illustrates the application of the nonlinear regression method in the hyperbolic method. In Fig. 2(a), the settlement measurements are marked as symbols and the dashed line is the hyperbolic regression curve. When the i^{th} measurement point is assumed as t_i and the corresponding measured settlement as S_i after final embankment loading, the residual ε_i between the regression curve (Eq. (1)) and the measured settlement is expressed as follows (Fig. 2(b)).

$$\varepsilon_i = \frac{t_i}{\alpha + \beta t_i} - S_i \tag{5}$$

When the number of all measurement data is n, the sum of the squared residual ρ_n is as follows (Fig. 2(c)).

$$\rho_n = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[\frac{t_i}{\alpha + \beta t_i} - S_i\right]^2 \tag{6}$$

In this study, the hyperbolic coefficients α and β that minimize ρ_n were selected using the least square method for nonlinear regression analysis. The calculated hyperbolic coefficients α and β were then substituted into Eq. (1) to construct the settlement prediction equation, and the future settlement was calculated.

2.3 Application of weighted nonlinear regression

According to Choo *et al*. (2010), at a soft clay ground stabilization site under the application of the preloading method, the groundwater level rises rapidly at the beginning of embankment and it descends to the previous position over time. Owing to the rise of the groundwater level and the stabilization process at the beginning of embankment, initial settlement measurement data contain many errors, thereby decreasing data reliability. To address this problem, the nonlinear regression hyperbolic method and the weighted regression analysis concept, which assigns higher weight to data, were proposed in this study.

As the most recently obtained settlement measurement data contain more important information than the settlement measurement data obtained at the beginning of loading, weight values were assigned during nonlinear regression analysis such that the latest data could have higher weigh.

Fig. 3 schematically shows the application weight in the nonlinear regression. In the weight nonlinear regression, the procedure (Fig. 3(a) to 3(c)) to the estimation of residual ε_i at each measurement point is identical to the nonlinear regression case (Fig. 2(a) to 2(c)). Then, the weight of each measurement was calculated as follows (Fig. 3(d)).

$$w_j = \frac{t_j}{\sum_{i=1}^{n} t_i} \tag{7}$$

which make the sum of the weights of all data equal to 1 and the more recent data has the greater weight linearly. The weighted residual ε_i^* can be calculated by multiplying the weight set in Eq. (7) by the residual (Eq. (5)) at each measurement time as follows.

$$\varepsilon_i^* = w_i \varepsilon_i = w_i \left[\frac{t_i}{\alpha + \beta t_i} - S_i\right] \tag{8}$$

With the weighted residual, the sum of the squared weighted residual ρ_w is as follows.

$$\rho_w = \sum_{i=1}^{n} (\varepsilon_i^*)^2 = \sum_{i=1}^{n} \left[w_i \left(\frac{t_i}{\alpha + \beta t_i} - S_i\right)\right]^2 \tag{9}$$

In the weighted nonlinear regression hyperbolic method, when nonlinear regression analysis is conducted by applying the least square method, the coefficients α and β that minimize ρ_w (Eq. (9)) were determined. The determined

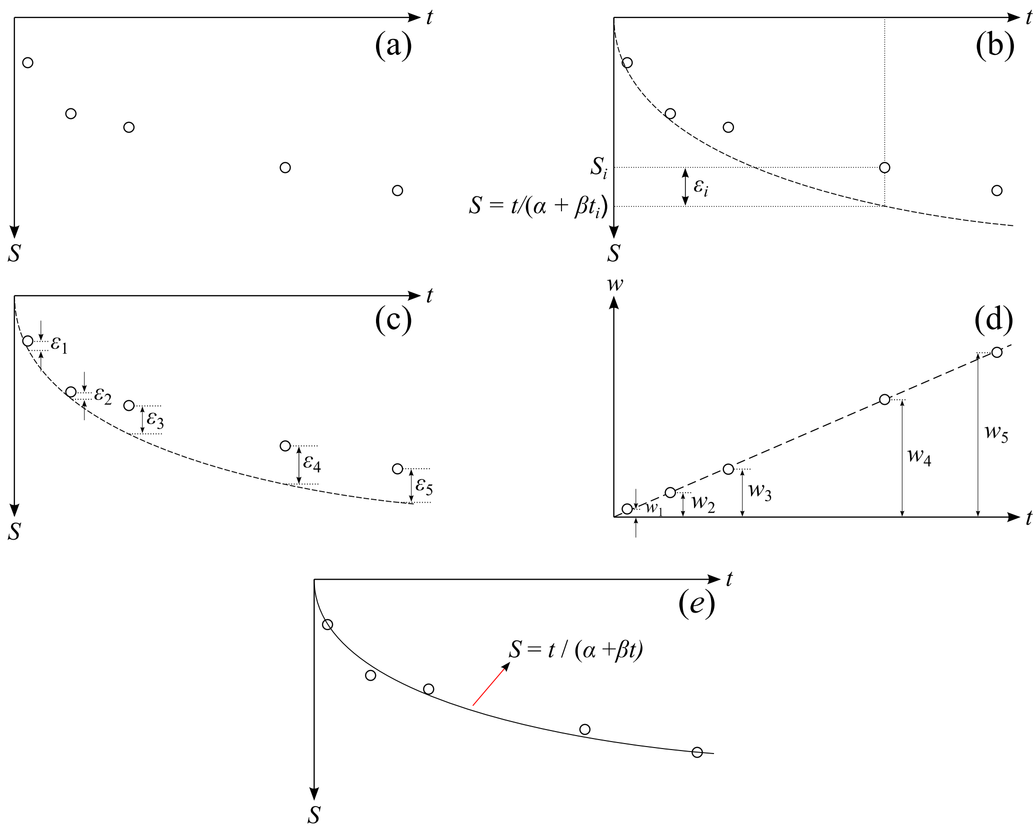


Fig. 3 Application steps of the hyperbolic method based on the weighted nonlinear regression: (a) settlement monitoring data, (b) calculation of error at a single point, (c) definition of errors at multiple points, (d) estimation of weight for each point, and (e) construction of a time-settlement prediction curve

Fig. 4 The geographical location of Busan New Port

coefficients α and β were substituted into Eq. (1) to construct the hyperbolic settlement prediction equation (Fig. 3(e)).

3. Settlement prediction

3.1 Geological properties of the target field

The present study focuses on Busan New Port which locates in the southeast part of the Korean Peninsula (Fig. 4). As shown in Fig. 4, Busan New Port is in the west of Busan across Nakdong River. Geologically, the ground in Busan New Port mainly consists of the sedimentation of deltaic deposits composed of various types of geo-materials from Nakdong River (Park *et al*. 2015). In this area, the bedrock is characterized as Cretaceous Ansan volcanic rocks and various types of hornblende granite in the Mesozoic era. Over the weathered rock and soil layers originated from the bedrock, thick layers have been formed by fine silty clay particles transported by Nakdong River. According Chung *et al.* (2002), the predominant clay mineral of the alluvial soil is illite.

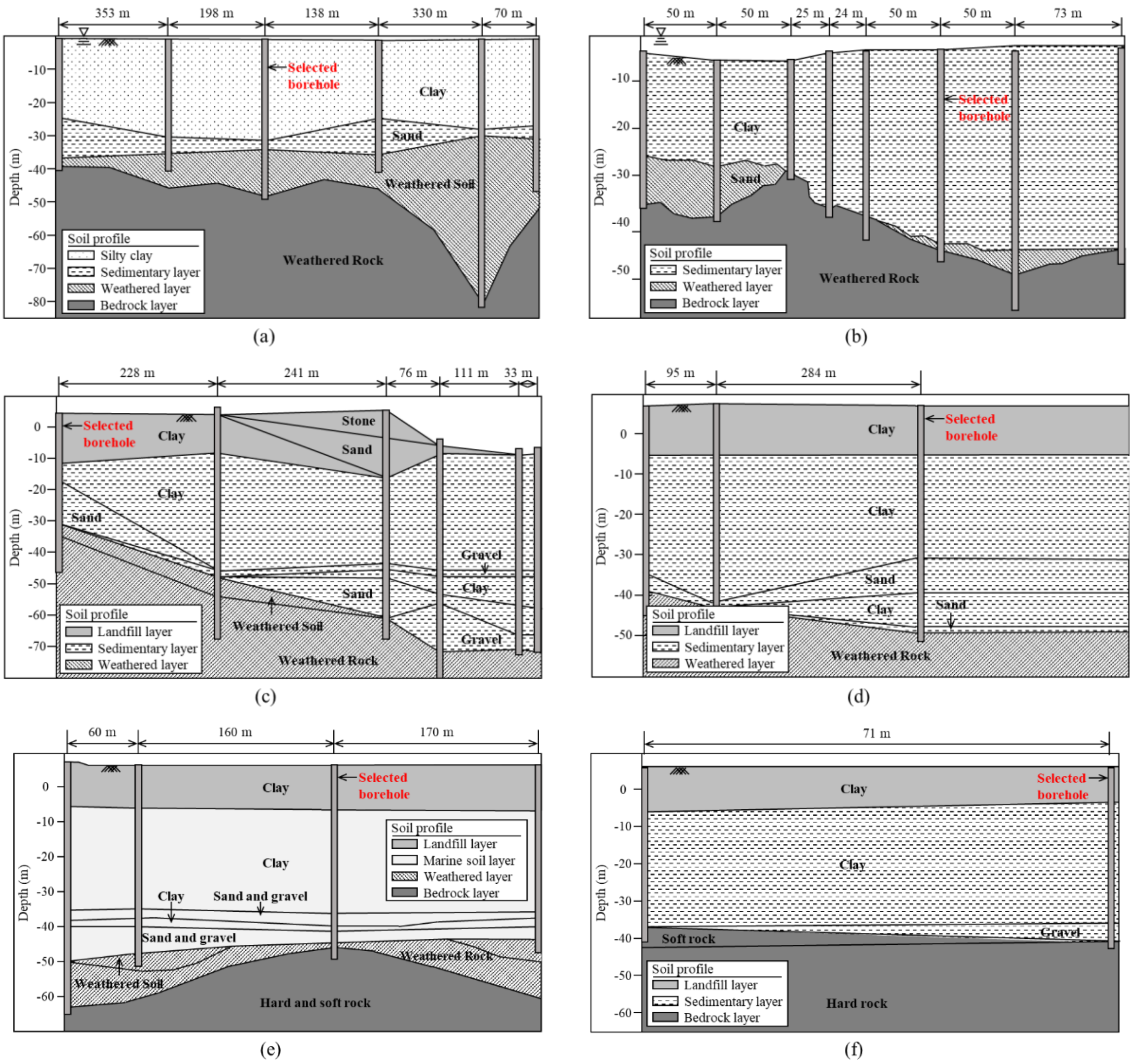


Fig. 5 Sectional view of soil layers of the selected sites in Busan New Port: (a) *A* site, (b) *B* site, (c) *C* site, (d) *D* site, (e) *E* site, and (f) *F* site

Within Busan New Port area, this study selected six sites (site *A* to *F*) where the site investigation and field settlement monitoring data were collected. Fig. 5 schematically illustrates the sectional views of soil layers at six selected sites. As shown in Fig. 5, sites *A* and *B* consist of bedrock, weathered layer, and sedimentary layer from bottom to top, while sites *C* to *F* have a landfill layer at the top. Fig. 5 also reveals that the soft clay layer, which is very compressible, has thickness between 20 to 50 meters. The site investigation reports that the soft clay layers in all six sites are predominantly composed of silty clay. These silty clay soils are primarily classified as *CL* and *CH* soils according to the Unified Soil Classification System (USCS). This site investigation results have aligns with the geological and geotechnical previous studies for the Busan region (Chung *et al*. 2002, Chung *et al*. 2005, Chu *et al*. 2016). Fig. 6 shows boring logs for all selected sites with corresponding Standard Penetration Test (SPT) *N* value versus depth. Fig. 6 definitely shows that there exist thick (more than 30 meters) very soft layers where the *N* value is mostly 0/30 for all sites.

To examine the settlement at each site in the construction stage, settlement plates were installed at 29 (site *A*), 31(site *B*), 51 (site *C*), 53 (site *D*), 115 (site *E*), and 69 (site *F*) locations originally. For each settlement plate, a rod was attached to the bottom plate made of a stiff material. The height of the rod was set considering the fill height. In addition, a protection casing was installed outside the rod to minimize the influence of the external environment and the height change at the top of the rod was periodically measured using a total station.

3.2 Procedures

As for the surface settlement plate measurement data used in the analysis, the data measured for more than 170 days after the final completion of embankment were selected. The settlement plates with abnormal data tendencies were excluded from the analysis such as cases identified as clear measurement errors. Consequently, 17, 16, 21, 24, 21, and 16 settlement plates were selected at *A* *B*, *C*, *D*, *E*, and *F* sites, respectively, and the settlement was

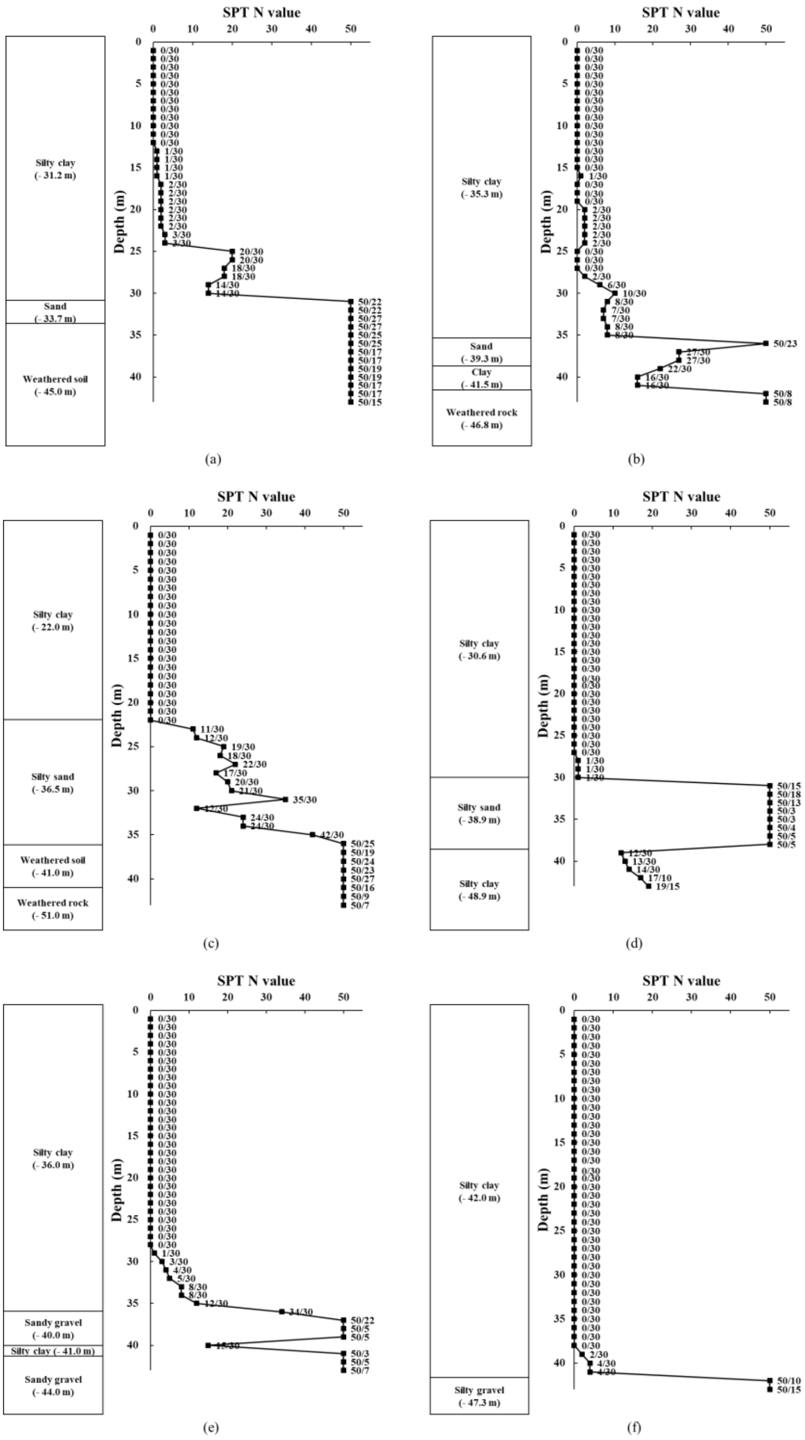


Fig. 6 Soil profile and Standard Penetration Test (SPT) *N* values of the selected boreholes in the target sites: (a) *A* site, (b) *B* site, (c) *C* site, (d) *D* site, (e) *E* site, and (f) *F* site

predicted from a total of 115 settlement plates. For all time-settlement monitoring data, the units of time and settlement were days and centimeters, respectively.

In this study, to analyze the feasibility and accuracy of the three settlement prediction methods (original hyperbolic method, nonlinear regression hyperbolic method, and weighted nonlinear regression hyperbolic method) for a given data set, a Python program was developed to run regression analysis based on the least square method.

Fig. 7 illustrates the analysis procedure for a single prediction method. First, this study set the data usage range t_D, within which monitoring data were used in the settlement prediction and error estimation ranges t_E. Monitoring data were used to quantify the prediction accuracy. Using time-settlement data (black symbols in Fig. 7) within t_D, this study constructed the predicted time-settlement curve (dashed line in Fig. 7) after running the regression analysis. Based on difference (ε_i in Fig. 7)

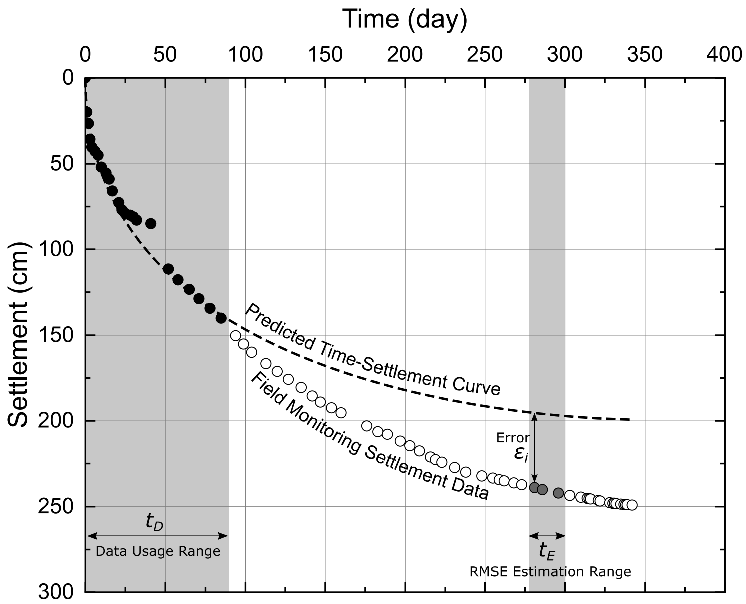


Fig. 7 Field monitoring settlement data and predicted time-settlement curve with the definitions of data usage range t_D, RMSE estimation range t_E, and error ε_i for the regression analysis of the hyperbolic methods

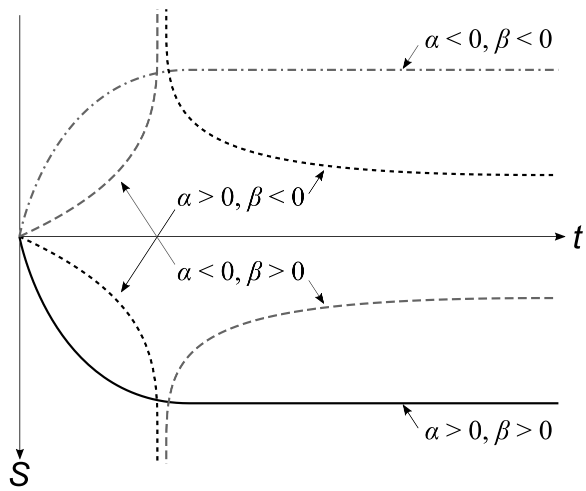


Fig. 8 Shape of hyperbolic curves with respect to signs of hyperbolic coefficients α and β

between the predicted time-settlement curve and monitoring data (gray symbols in Fig. 7) within t_E, to quantify the averaged accuracy within t_E, this study calculated root mean square error (RMSE), which can be mathematically expressed as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \varepsilon_i^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \left(S_i - S_p(t_i)\right)^2} \tag{10}$$

where n is the number of monitoring data within t_E, t_i is the time when i^{th} data was monitored within t_E, S_i is i^{th} monitored settlement within t_E, S_p is the predicted time-settlement curve, and ε_i is i^{th} error (= $S_i - S_p(t_i)$). Thus, the unit of RMSE is identical to that of settlement and RMSE represents the averaged error between the prediction and monitored data within t_E. This study considered t_D that increased by 30 days starting from 60 days and t_E ranges of 140–160, 280–300, and 420–440.

3.3 Prediction results

3.3.1 Divergence check

This study considered the original, nonlinear, and weighted nonlinear hyperbolic methods; all methods produced a hyperbolic time-deformation curve (Eq. (1)) from regression analyses on the monitored ground deformation data; thus, there are two coefficients (α and β) of the hyperbolic curve. In the regression analyses, there was no constraint on signs of coefficients; the regression analysis could produce negative hyperbolic coefficients.

Fig. 8 illustrates the shape of hyperbolic curves with respect to signs of coefficients α and β. As evident, when both coefficients α and β were positive, the time-

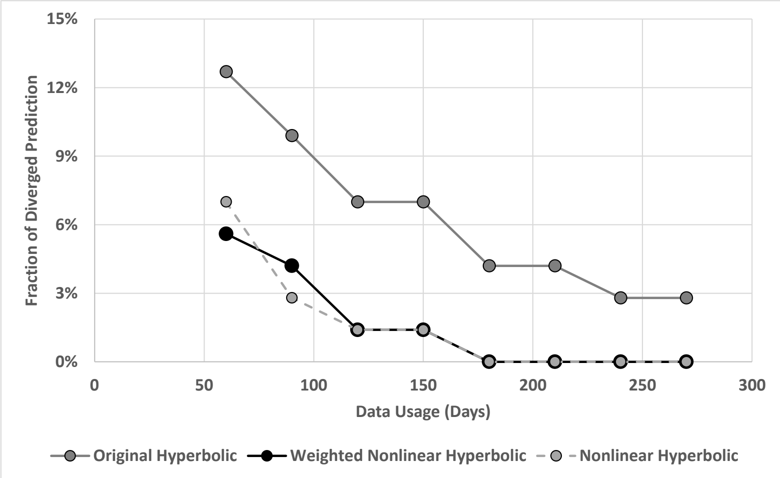


Fig. 9 Fraction of diverged prediction vs. data usage when original, nonlinear, and weighted nonlinear hyperbolic methods are used for the settlement prediction

deformation curve represented the ground settlement. Whereas, it described the ground heaving when both α and β were negative. When $\alpha\beta < 0$, the time-deformation curve diverged, which is not physically acceptable.

For given 115 monitoring data, this study constructed time-settlement prediction curves and checked signs of coefficients α and β by increasing the data usage range t_D by 30 days starting from 30 days. Fig. 9 presents a plot of the fraction of diverged prediction (when $\alpha\beta < 0$) versus the data usage range. As evident, the original hyperbolic method produced the diverged time-settlement prediction curves more frequently than the nonlinear and weighted nonlinear hyperbolic methods, particularly when the data usage range was small. In the view of convergence in Fig. 9, for data usage range greater than 180 days, both the nonlinear and weighted nonlinear hyperbolic methods did not produce diverged time-settlement prediction curves. However, the original hyperbolic method still generated diverged time-settlement prediction curves in approximately 5% of the total cases. This result implies that nonlinear and weighted nonlinear hyperbolic methods offer better advantages than the original hyperbolic method to secure the feasibility (or stability) of the settlement prediction.

3.3.2 Prediction quality

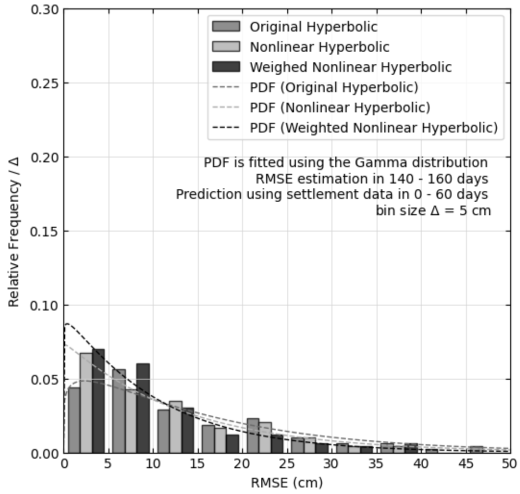
To analyze the quality of settlement predictions from three hyperbolic methods by avoiding the trap of averages, this study first excluded cases where the diverged time-settlement curves were generated after the regression analysis. Considering the settlement monitoring period, when t_E = 140–160, 280–300, and 420–440, a total of 101, 59, and 14 settlement monitoring datapoints were used in the analysis, respectively. For each t_E, to quantify the prediction quality, this study constructed the distribution of RMSE and estimated the statistics such as mean, median, standard deviation, and 90% percentile from the distribution.

Fig. 10 shows of histograms of RMSEs between the monitored data and settlement prediction from the original, nonlinear, and weighted nonlinear hyperbolic methods when RMSE estimation period was 140–160 days. In the histogram, the bin size Δ was set as 5 cm and vertical axis represents the relative frequency divided by Δ, which represent probability density. Figs. 10(a) to 10(c) represent the distributions of RMSE within t_E = 140–160 days when t_D = 0 to 60, 90, and 120 days, respectively. Fig. 10 also shows the corresponding probability density functions (PDF) based on the Gamma distribution, which is mathematically expressed as

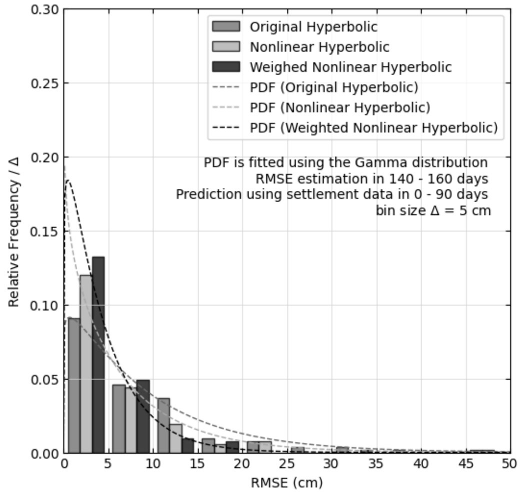
$$f(x) = \frac{x^{k-1} e^{\frac{-x}{\theta}}}{\theta^k \Gamma(k)} \tag{11}$$

where x is a probability variable, e is the Euler's number (or natural exponential), k is the shape parameter, θ is the scale parameter, and Γ is the Gamma function (for an arbitrary positive integer k, $\Gamma(k) = (k-1)!$). The parameters k and θ of the Gamma distribution were fitted from the corresponding histogram. In Fig. 10, for all hyperbolic methods, as t_D increased, the distributions of RSME generally became more positively skewed. This implied that the prediction accuracy increased as more data were used in the regression analysis. When t_D = 0 to 60 days (Fig. 9(a)), probability densities for weighted nonlinear hyperbolic, nonlinear hyperbolic, and original hyperbolic methods were 0.071, 0.068, and 0.045 respectively. Whereas, for t_D = 0 to 120 days (Fig. 9(c)), probability densities for weighted nonlinear hyperbolic, nonlinear hyperbolic, and original hyperbolic methods increased to 0.188, 0.163, and 0.124, respectively. Thus, the distributions evolved to more positively skewed shape in the order of the weighted nonlinear hyperbolic, nonlinear hyperbolic, and original hyperbolic methods with increase in t_D.

Fig. 11 shows the (a) mean, (b) median (or 50% percentile), (c) standard deviation, and (d) 90% percentile of RMSEs versus data usage range t_D when t_E = 140–160 days. As shown, the weighted nonlinear hyperbolic method always produced time-settlement prediction curves with the lowest mean, median, standard deviation, and 90% percentile of RMSE; and the nonlinear hyperbolic method generated time-settlement prediction curves with smaller



(a)



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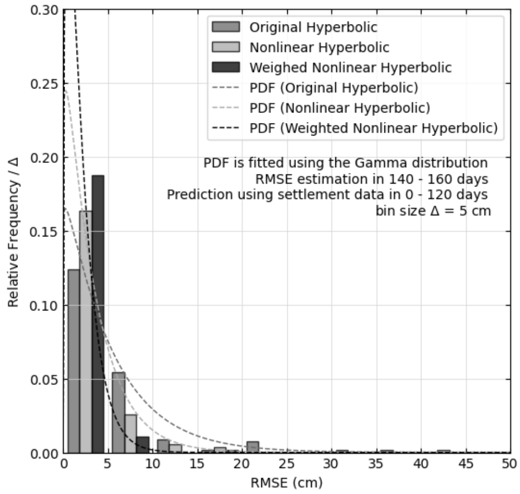


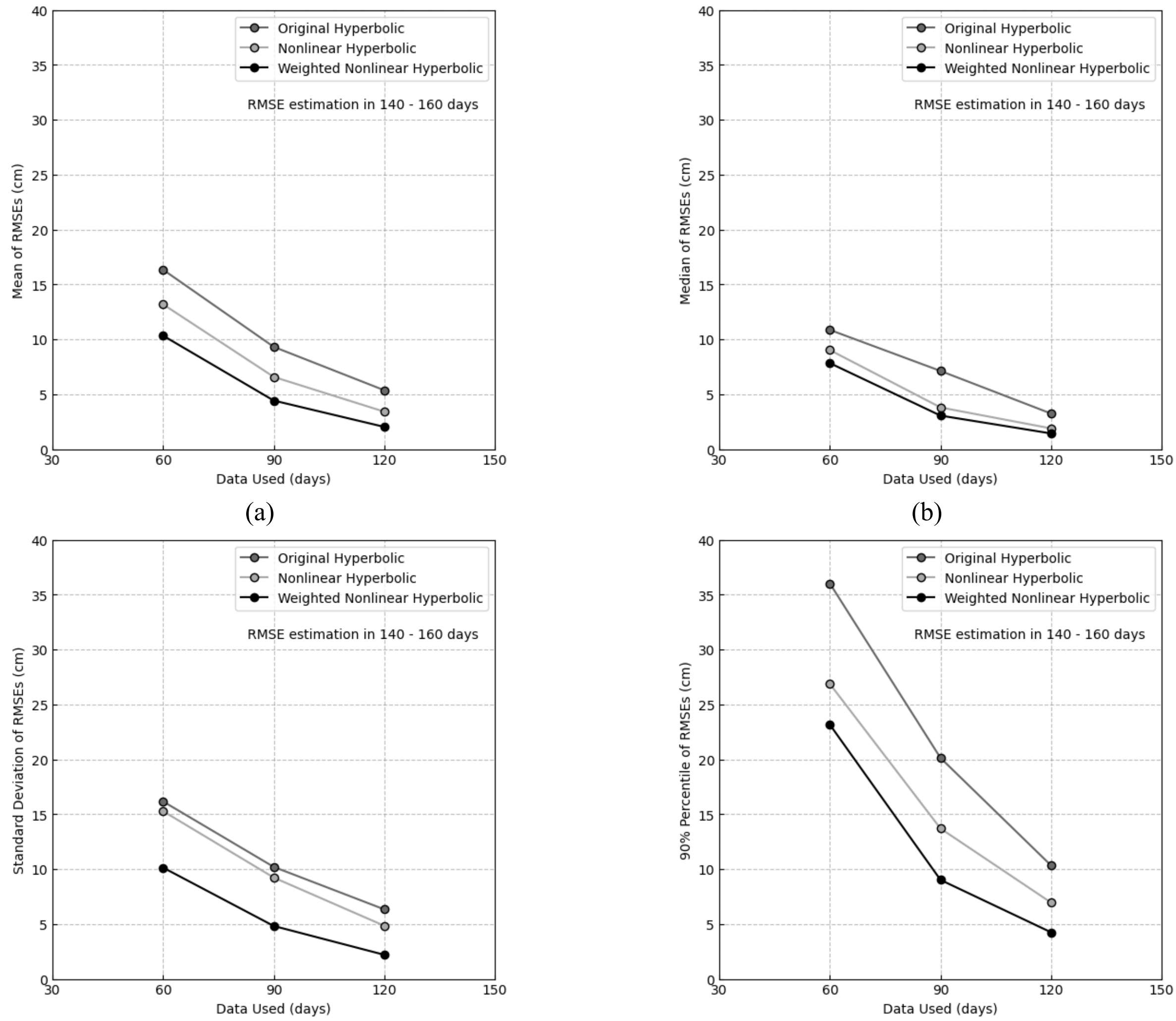
(c)

Fig. 10 Histograms and probability density functions (PDF) based on the Gamma distribution of RMSEs between th monitored settlement data and predicted settlement data from the original, nonlinear, and weighted nonlinear hyperboli methods when RMSE estimation period is 140–160 days: settlement data used (a) 0–60 days, (b) 0–90 days, and (c) 0–12 days

mean, median, standard deviation, and 90% percentile of RMSE than the original hyperbolic method. Thus, the accuracy and stability of settlement prediction increases in the order of the weighted nonlinear, nonlinear, and original hyperbolic methods was obtained. In particular, the mean of RMSE estimation in 140–160 days decreased by up to 36.6% compared to the original hyperbolic method when the weighted nonlinear hyperbolic method was applied. Considering that the standard of residual settlement in South Korea is 10 cm, the settlement prediction results can be applied to the field when the mean of RMSE is approximately 10 cm. According to Fig. 11, t_D = 0–60 and 0–90 days were found for the weighted nonlinear and original hyperbolic methods, respectively, indicating that the weighted nonlinear hyperbolic method could make predictions approximately 30 days earlier than the original hyperbolic method.

Fig. 12 shows the histograms of RMSEs between the monitored data and settlement prediction from the original, nonlinear, and weighted nonlinear hyperbolic methods when t_E = 280–300 days. For Figs. 12(a) to 12(d), t_D was identical to 0 to 90, 150, 210 and 270 days, respectively. Fig. 12 shows the corresponding PDF based on the Gamma distribution. Similar to case of t_E = 140–160 days (Fig. 10), as t_D increased, the distributions evolved to more a positively skewed shape in the order of the weighted nonlinear hyperbolic, nonlinear hyperbolic, and original hyperbolic methods.

Fig. 13 shows the (a) mean, (b) median (or 50% percentile), (c) standard deviation, and (d) 90% percentile of RMSEs versus data usage range t_D when t_E = 280–300 days. In this case, the weighted nonlinear hyperbolic method also always produced time-settlement prediction curves with the lowest mean, median, standard deviation, and 90% percentile of RMSE. In the case applied to Fig. 12, t_D that exhibited approximately 10 cm for the mean of RMSE was observed for 0–150 days and 0–210 days for the weighted nonlinear and original hyperbolic methods. This



(a) (b)

(c) (d)

Fig. 11 Evolution of (a) mean, (b) median, (c) standard deviation, and (d) 90% percentile of RMSEs with respect to prediction data usage when RMSE estimation period is 140–160 days

confirmed that it is possible to make predictions approximately 60 days earlier.

Fig. 14 shows the histograms of RMSEs between the monitored data and settlement prediction from the original, nonlinear, and weighted nonlinear hyperbolic methods when t_E = 420–440 days. In Figs. 14(a) to 14(d), t_D is identical to 0 to 120, 210, 300 and 390 days, respectively. Fig. 14 also shows the corresponding PDF based on the Gamma distribution. When t_D = 0–120 (Fig. 14(a)), which is approximately 28% of t_E = 420–440, the distributions of RMSE exhibited almost uniform shapes although the variability was the smallest in the weighted nonlinear hyperbolic method. Thus, the prediction quality could not be secured. Similar to Figs. 10 and 12, as t_D increased, the distributions evolved to a more positively skewed shape in the order of the weighted nonlinear hyperbolic, nonlinear hyperbolic, and original hyperbolic methods.

Fig. 15 shows the (a) mean, (b) median (or 50% percentile), (c) standard deviation, and (d) 90% percentile of RMSEs versus data usage range t_D when t_E = 420–440 days. In Fig. 15(a), the weighted nonlinear hyperbolic method also always produced time-settlement prediction curves with the lowest mean of RMSE. In Fig. 15(b), generally the median of RMSE was the least for the weighted nonlinear hyperbolic method except t_D = 90 days, where the median from the weighted nonlinear hyperbolic method was 10 cm greater than that from the nonlinear hyperbolic. In Fig. 15(c), generally the standard deviation of RMSE was also the smallest for the weighted nonlinear hyperbolic method except t_D = 120 days, where the standard deviation from the weighted nonlinear hyperbolic method was slightly greater than other methods. In Fig. 15(d), generally 90% percentile of RMSE was also the smallest for the weighted nonlinear hyperbolic method except t_D = 60 days, which is very early stage compared to t_E = 420–440 days. Notably, when t_D > 120 days and t_D < 210 days, the 90% percentile from nonlinear hyperbolic method was slightly greater than that from the original hyperbolic method.

Fig. 16 shows of histograms of error ε_f between the final monitored settlement data and corresponding settlement prediction from the original, nonlinear, and weighted

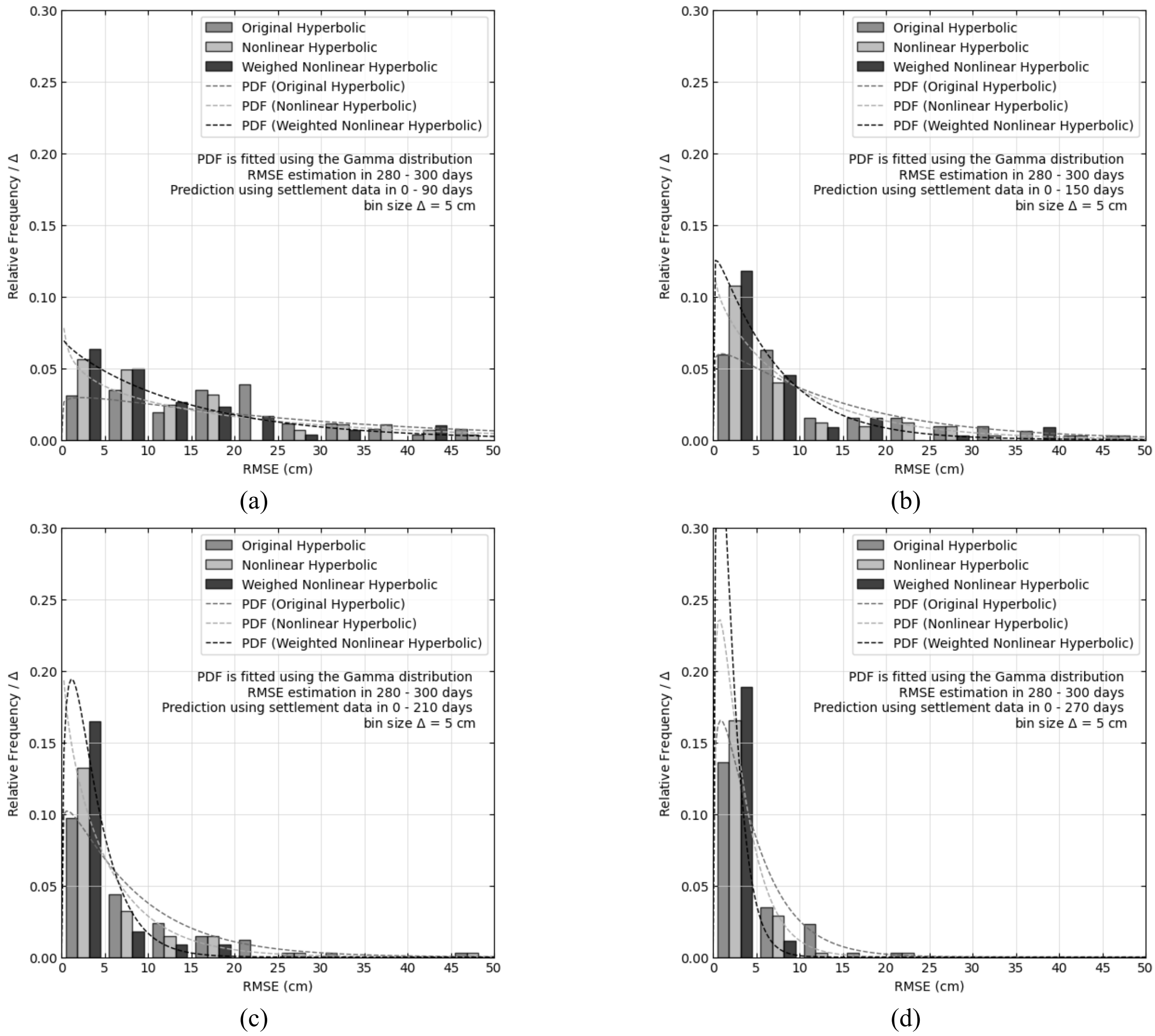


Fig. 12 Histograms and probability density functions (PDF) based on the Gamma distribution of RMSEs between the monitored settlement data and predicted settlement data from the original, nonlinear, and weighted nonlinear hyperbolic methods when RMSE estimation period is 280–300 days: settlement data used (a) 0–90 days, (b) 0–150 days, (c) 0–210 days, and (d) 0–270 days

nonlinear hyperbolic methods. In the histogram, the bin size Δ was set as 2 cm and vertical axis represents the relative frequency divided by Δ, which represent probability density. Fig. 16(a), (b), and (c) represent the distributions of the errors when t_D = 0 to 150, 270, and 390 days, respectively. Fig. 16 also shows the corresponding probability density functions (PDF) based on the normal distribution, which is mathematically expressed as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)}{2\sigma^2}} \tag{12}$$

where x is a probability variable, e is the Euler's number (or natural exponential), σ^2 is the variance, and μ is the mean. The variance and mean of the normal distribution were fitted from the corresponding histogram. In Fig. 16, for all hyperbolic methods, as t_D increased, the distributions of ε_f generally have smaller variance, which implies that the prediction accuracy generally increased as more data were used in the regression analysis. From the fitted normal distributions in Fig. 16, the dispersion of ε_f from the nonlinear hyperbolic method is similar to it from the original hyperbolic. However, the dispersion of ε_f from the weighted nonlinear hyperbolic method is significantly smaller than them from the nonlinear and original hyperbolic methods. This trend the originated from the weight function (Eq. (7)) that provides the greater weight value for the more recent monitored settlement.

3.4 Discussion

As both the nonlinear regression hyperbolic method and weighted nonlinear regression hyperbolic method, which were newly proposed in this study, were improved based on the hyperbolic method, certain of the limitations of the hyperbolic method still remain. In the case of the hyperbolic method, two coefficients were still used despite the application of nonlinear regression owing to the assumption that settlement occurred over time in the form of a

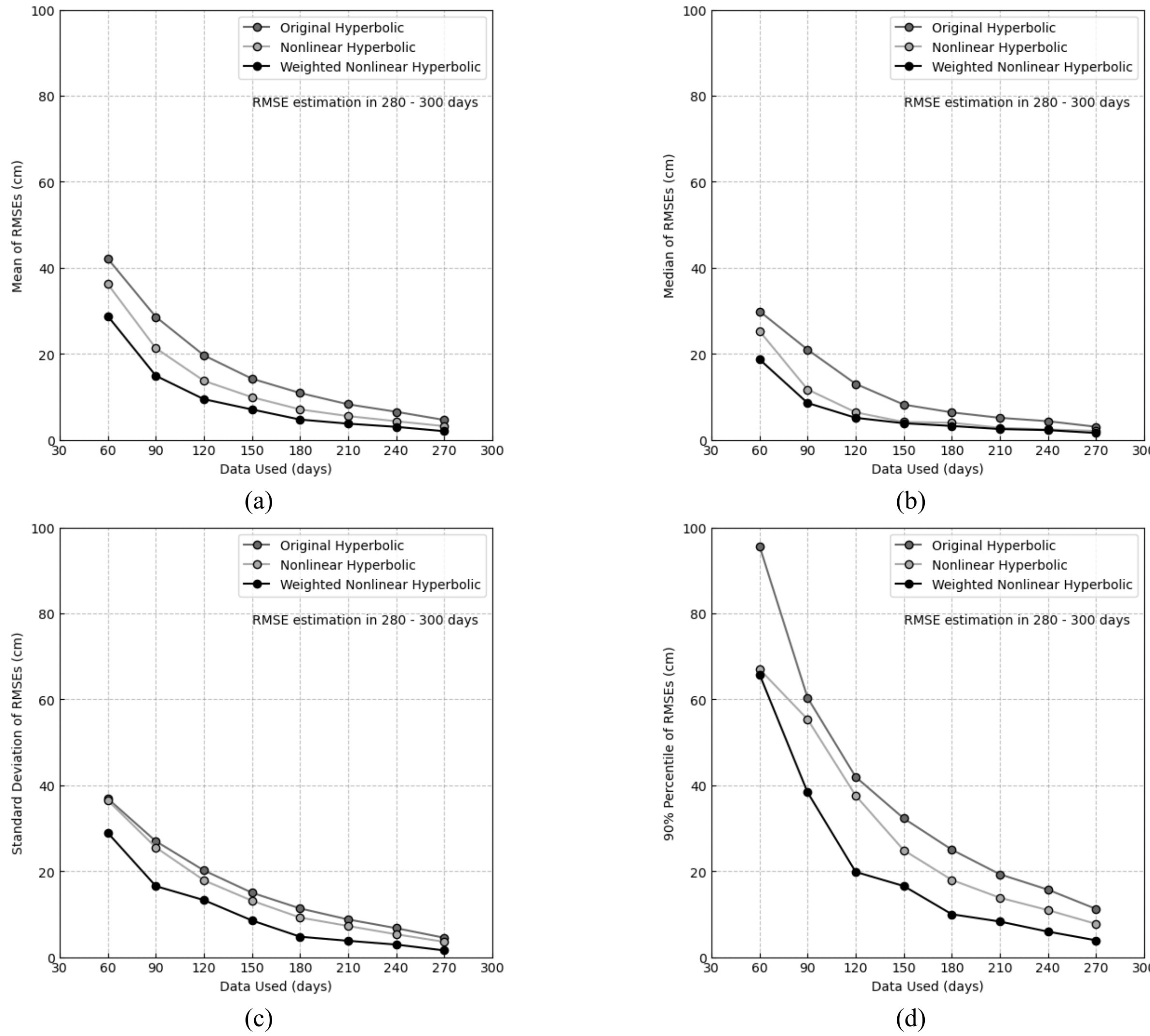


Fig. 13 Evolution of (a) mean, (b) median, (c) standard deviation, and (d) 90% percentile of RMSEs with respect to prediction data usage when RMSE estimation period is 280–300 days

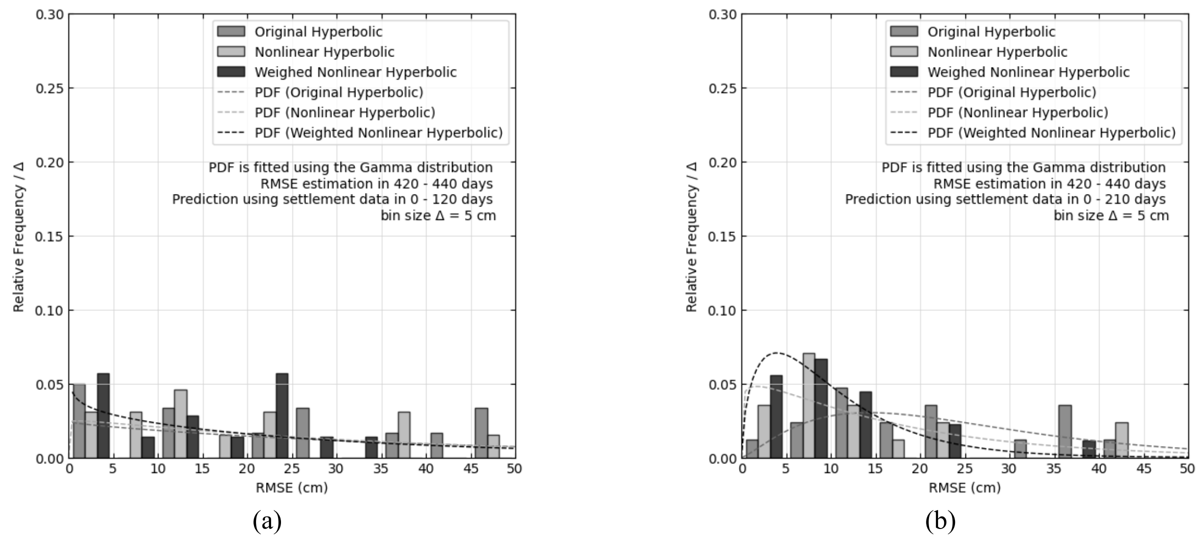
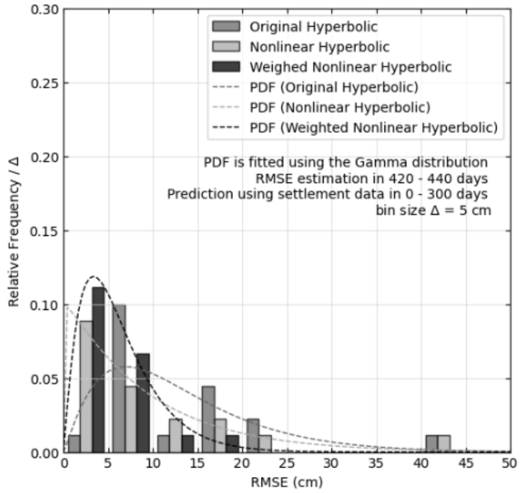
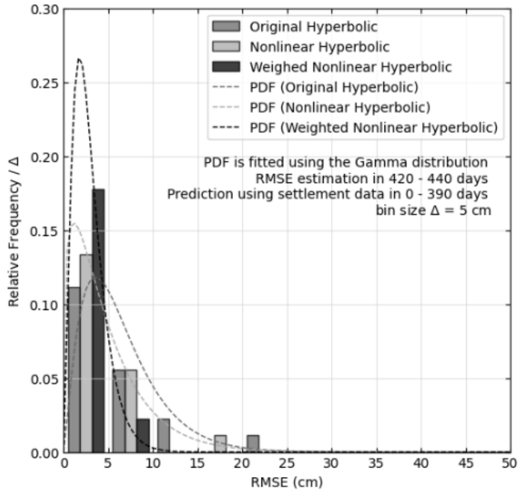


Fig. 14 Histograms and probability density functions (PDF) based on the Gamma distribution of RMSEs between the monitored settlement data and predicted settlement data from the original, nonlinear, and weighted nonlinear hyperbolic methods when RMSE estimation period is 420–440 days: settlement data used (a) 0–120 days, (b) 0–210 days, (c) 0–300 days, and (d) 0–390 days

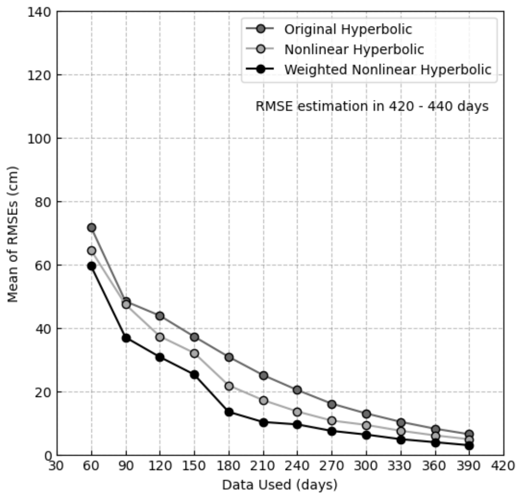


(c)

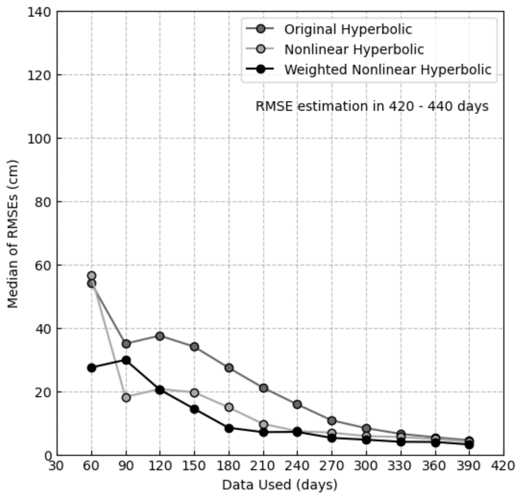


(d)

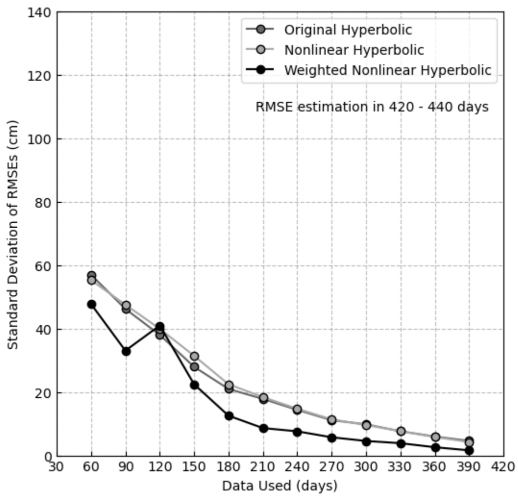
Fig. 14 Continued-



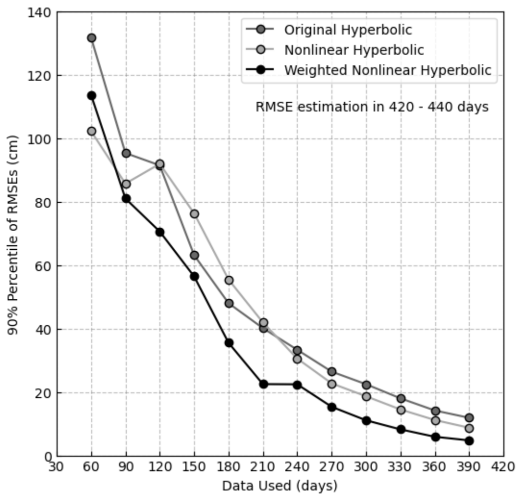
(a)



(b)

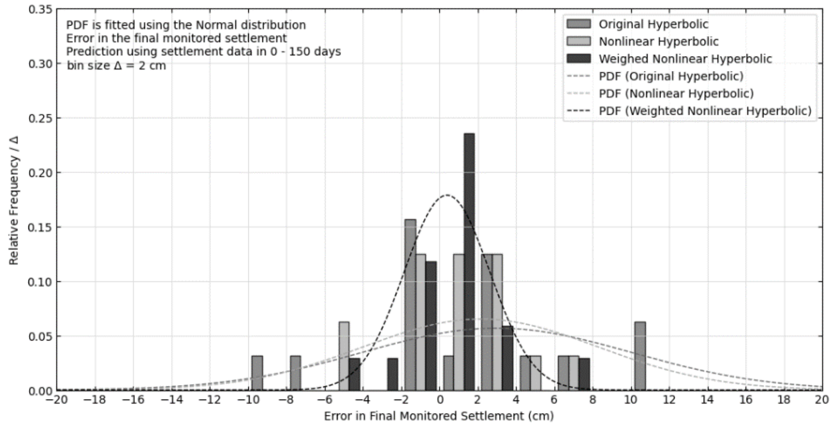


(c)

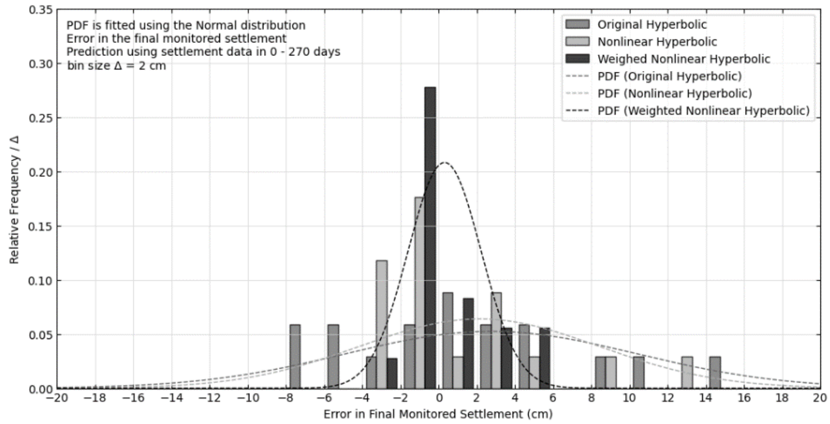


(d)

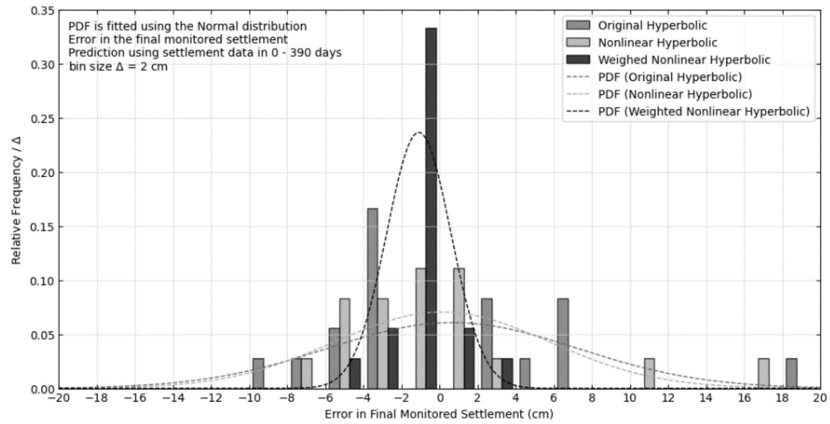
Fig. 15 Evolution of (a) mean, (b) median, (c) standard deviation, and (d) 90% percentile of RMSEs with respect to prediction data usage when RMSE estimation period is 420–440 days



(a)



(b)



(c)

Fig. 16 Histograms and probability density functions (PDF) based on the normal distribution of error ε_f between the final monitored settlement data and predicted settlement data from the original, nonlinear, and weighted nonlinear hyperbolic methods: settlement data used (a) 0–150 days, (b) 0–270 days, and (c) 0–390 days

hyperbola. This accompanies the limitations in terms of the degree of freedom when nonlinear regression is performed.

As mentioned in the *Procedures* section, settlement plates with abnormal data tendencies were excluded from the analysis. Such tendencies include abrupt changes in settlement rate or even sudden heaving, which were regarded as abnormal and thus outside the applicable boundary conditions of the proposed methods.

The weighted nonlinear regression hyperbolic method applies the weight function to assign weight to recent data. In this study, the weight linearly increased over time. The weight function can be applied in various forms, and it is expected that the nonlinear regression results will vary depending on the form. Therefore, if further research is conducted on the influence of various weight functions, it will be possible to propose the optimal weighted nonlinear regression hyperbolic method.

In addition, this study focused on the marine clay ground in the southeastern part of Korea. If analysis is conducted for various other ground conditions, it will be possible to derive significant improvement measures, such as optimal settlement prediction methods for each ground condition.

Previous studies have further suggested that the regression parameters of the hyperbolic method may incorporate geotechnical significance. In particular, β has been interpreted as inversely related to the final settlement, thereby reflecting soil compressibility (C_c), while α has been associated with the initial settlement rate and influenced by drainage and permeability conditions (Tan *et al*. 1991, Yoo and Kim 2000, Al-Shamrani 2005, Chung and Kweon 2021). In this study, α and β were primarily treated as regression coefficients for curve fitting; however, future research should more systematically investigate their correlations with fundamental geotechnical properties.

4. Conclusions

This study proposed the nonlinear and weighted nonlinear regression hyperbolic methods to supplement the statistical incompleteness of the original hyperbolic method based on linear regression. The accuracy of the original, nonlinear, and weighted nonlinear regression hyperbolic methods were analyzed using the time-settlement data measured from six sites with thick soft soil layers in Busan New Port. Based on this, the conclusions were drawn as follows.

- This study derived settlement prediction curves by setting section t_D to perform regression in various ways, and the prediction accuracy was quantified through t_E, which is the settlement prediction and error estimation ranges. As t_D increased, the distributions evolve to more positively skewed shape rapidly in the order of the weighted nonlinear hyperbolic, nonlinear hyperbolic, and original hyperbolic methods.
- The mean of RMSE estimation result decreased by up to 36.6% compared to the original hyperbolic method when the weighted nonlinear hyperbolic method was applied. Thus, the weighted nonlinear regression hyperbolic method was confirmed to exhibit significantly higher accuracy of settlement prediction than the original hyperbolic method.
- As standard of residual settlement in South Korea is 10 cm, the settlement prediction results can be applied to the field when the mean of RMSE is approximately 10 cm. Considering this, prediction accuracy was compared according to various regression analysis sections. It was found that the weighted nonlinear hyperbolic method could make predictions approximately 30–60 days earlier than the original hyperbolic method.
- However, the proposed weighted nonlinear regression hyperbolic method has certain limitations in terms of the degree of freedom, and research on the form of the weight function could not be conducted. If further research is conducted on the forms of various weight functions, it will be possible to propose the optimal weighted nonlinear regression hyperbolic method.

Further research is needed to evaluate the sensitivity of settlement predictions to different forms of weight distributions in the weighted nonlinear regression hyperbolic method, which will help establish more reliable approaches for optimizing weight selection under various ground conditions.

Acknowledgements

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References

Al-Shamrani, M. A. (2005), “Applying the hyperbolic method and Cα/Cc concept for settlement prediction of complex organic-rich soil formations”, *Eng. Geol.*, **77**(1-2), 17-34. https://doi.org/10.1016/j.enggeo.2004.07.004.

Arulrajah, A., Nikraz, H. and Bo, M.W. (2003), “Factors affecting field settlement assessment and back-analysis by the Asaoka and hyperbolic methods”, *J. Aust. Geomech. Soc.*, **38**(2), 29-37. https://doi.org/10.25916/sut.26250881.v1.

Arulrajah, A., Nikraz, H. and Bo, M.W. (2004), “Factors affecting field instrumentation assessment of marine clay treated with prefabricated vertical drains”, *Geotext. Geomembranes.*, **22**(5), 415-437. https://doi.org/10.1016/j.geotexmem.2003.09.001.

Asaoka, A. (1978), “Observational procedure of settlement prediction”, *Soils Found.*, **18**(4), 87-101. https://doi.org/10.3208/sandf1972.18.4_87.

Barron, R.A. (1948), “Consolidation of fine-grained soils by drain wells by drain wells”, *Trans. Am. Soc. Civ. Eng.*, **113**(1), 718-742. https://doi.org/10.1061/TACEAT.0006098.

Bjerrum, L. (1967), “Engineering geology of Norwegian normally-consolidated marine clays as related to settlements of buildings”, *Géotechnique*, **17**(2), 83-118. https://doi.org/10.1680/geot.1967.17.2.83.

Buisman, A.S. (1936), “Results of long duration settlement tests”, *Proc. 1st ICSMFE*, **1**, 103-107.

Chen, X.X., Yang, J., He, G.F. and Huang, L.C. (2023), “Development of an LSTM-based model for predicting the long-term settlement of land reclamation and a GUI-based tool”, *Acta Geotech.*, **18**(7), 3849-3862. https://doi.org/10.1007/s11440-022-01749-5.

Choo, H., Lee, W., Hong, S.J. and Lee, C. (2016), “Application of the dilatometer test for estimating undrained shear strength of Busan New Port clay”, *Ocean Eng.*, **115**, 39-47. https://doi.org/10.1016/j.oceaneng.2015.11.017.

Choo, Y.S., Kim, J.H., Hwang, S.H. and Chung, C.K. (2010), “The optimization of hyperbolic settlement prediction method with the field data for preloading on the soft ground”, *J. Korean Geotech. Soc.*, **26**(7), 147-159. https://doi.org/10.7843/kgs.2010.26.7.147.

Chung, S.G., Choi, G.H., Choi, H.K. and Cho, K.Y. (1998), “Root-s method for consolidation analysis”, *J. Korean Geotech. Soc.*, **14**(2), 41-53.

Chung, S.G., Giao, P.H., Kim, G.J. and Leroueil, S. (2002), "Geotechnical properties of Pusan clays", *Can. Geotech. J.*, **39**(5), 1050-1060. https://doi.org/10.1139/t02-05.

Chung, S.G., Ryu, C.K., Jo, K.Y. and Huh, D.Y. (2005), "Geological and geotechnical characteristics of marine clays at the Busan new port", *Mar. Georesour. Geotechnol.*, **23**(3), 235-251. https://doi.org/10.1080/10641190500225712.

Chung, S.G. and Kweon, H.J. (2021), "Prediction of radial consolidation settlement with consideration of sampling range effect: Updated observational methods", *J. Geotech. Geoenviron. Eng.*, **147**(12), 04021151. https://doi.org/10.1061/(ASCE)GT.1943-5606.000270.

Chung, S.G. and Kweon, H.J. (2022), "Prediction of three-dimensional consolidation settlement:observational method and its applicability", *Int. J. Geomech.*, **23**(3), 04022308. https://doi.org/10.1061/IJGNAI.GMENG-7918.

Dalgıç, S. and Şimşek, O. (2002), "Settlement predictions in the Anatolian motorway, Turkey", *Eng. Geol.*, **67**(1-2), 185-199. https://doi.org/10.1016/S0013-7952(02)00154-0.

Daoud, J.I. (2017), "Multicollinearity and regression analysis", *J. Phys.: Conf. Ser.*, **949**(1), 012009. https://doi.org/10.1088/1742-6596/949/1/012009.

Farrar, D.E. and Glauber, R.R. (1967), "Multicollinearity in regression analysis: The problem revisited", *Rev. Econ. Stat.*, **49**(1), 92-107. https://doi.org/10.2307/1937887.

Frisch, R. (1997), *Statistical Confluence Analysis by Means of Complete Regression Systems (1934), in The Foundations of Econometric Analysis*, Cambridge University Press..

Guo, W. and Chu, J. (2017), "New observational method for prediction of one-dimensional consolidation settlement", *Géotechnique*, **67**(6), 516-522. https://doi.org/10.1680/jgeot.16.P.089.

Haan, E.D. (1992), "The formulation of virgin compression of soils", *Géotechnique*, **42**(3), 465-483. https://doi.org/10.1680/geot.1992.42.3.465.

Haan, E.D. (1996), "A compression model for non-brittle soft clays and peat", *Géotechnique*, **46**(1), 1-16. https://doi.org/10.1680/geot.1996.46.1.1.

Hong, S., Ko, S.J., Woo, S.I., Kwak, T.Y. and Kim, S.R. (2024), "Time-series forecasting of consolidation settlement using LSTM network", *Appl. Intell.*, **54**(2), 1386–1404. https://doi.org/10.1007/s10489-023-05219-7.

Hong, S., Lee, M.H., Yoo, B.S., Kwak, T.Y. and Kim, S.R. (2025a), "Application of deep learning algorithms for predicting consolidation settlement", *KSCE J. Civ. Eng.*, **29**(1), 100072. https://doi.org/10.1016/j.kscej.2024.100072.

Hong, S., Chung, T.K., Yoo, B.S and Kim, S.R. (2025b), "Innovative LSTM–transformer hybrid models for predicting consolidation settlement", *Acta Geotech.*, 1-27. https://doi.org/10.1007/s11440-025-02752-2.

Hoshino, S. (1962), "Problems of foundations in recent years", *J. Soc. Civ. Eng.*, **47**(7), 63-67.

Jun, S.H., Lee, J.H., Park, B.S. and Kwon, H.J. (2021), "Design charts for consolidation settlement of marine clays using finite strain consolidation theory", *Geomech. Eng.*, **24**(3), 295-305. https://doi.org/10.12989/gae.2021.24.3.295.

Kwak, T.Y., Hong, S., Lee, J.H. and Woo, S.I. (2022), "Analysis of the limitations of the existing subsidence prediction method based on the subsidence measurement data and suggestions for improvement method through weighted nonlinear regression analysis", *J. Korean Geotech. Soc.*, **38**(12), 103-112. https://doi.org/10.7843/kgs.2022.38.12.103.

Lei, H., Feng, S., Wang, L. and Jin, Y. (2019), "Field instrumentation and settlement prediction of ground treated with straight-line vacuum preloading", *Geomech. Eng.*, **19**(5), 447-462. https://doi.org/10.12989/gae.2019.19.5.447.

Liu, W., Shi, Z., Zhang, J. and Zhang, D. (2019), "One-dimensional nonlinear consolidation behavior of structured soft clay under time-dependent loading", *Geomech. Eng.*, **18**(3), 299-313. https://doi.org/10.12989/gae.2019.18.3.299.

Marjadi, H.K. and Jang, J. (2025), "Compressional behavior of fine-grained soils: Clastic and biogenic silty soils", *Geomech. Eng.*, **41**(1), 71-82. https://doi.org/10.12989/gae.2025.41.1.071.

Mikasa, M. (1963), *Consolidation of Soft Clay: New Consolidation Theory and Its Application*, Kajima Shuppan-kai.

Monden, H. (1963), "A new time-fitting method for the settlement analysis of foundation on soft clays", *Mem. Fac. Eng. Hiroshima Univ.*, **2**(1).

Olsson, M.A.T.S. (2010), "Calculating long-term settlement in soft clays", Licentiate Thesis, Chalmers University of Technology, Gothenburg.

Park, H.I., Kim, K.S. and Kim, H.Y. (2015), "Field performance of a genetic algorithm in the settlement prediction of a thick soft clay deposit in the southern part of the Korean peninsula", *Eng. Geol.*, **196**, 150-157. https://doi.org/10.1016/j.enggeo.2015.07.012.

Quang, N.D. and Giao, P.H. (2014). "Improvement of soft clay at a site in the Mekong Delta by vacuum preloading", *Geomech. Eng.*, **6**(5), 419-436. http://dx.doi.org/10.12989/gae.2014.6.4.419.

Shrestha, N. (2020). "Detecting multicollinearity in regression analysis", *Am. J. Appl. Math. Stat.*, **8**(2), 39-42. https://doi.org/10.12691/ajams-8-2-1.

Tan, S.A. (1993), "Ultimate settlement by hyperbolic plot for clays with vertical drains", *J. Geotech. Eng.*, **119**(5), 950-956. https://doi.org/10.1061/(ASCE)0733-9410(1993)119:5(950).

Tan, S.A. (1995), "Validation of hyperbolic method for settlement in clays with vertical drains", *Soils Found.*, **35**(1), 101-113. https://doi.org/10.3208/sandf1972.35.101.

Tan, S.A. and Chew, S.H. (1996), "Comparison of the hyperbolic and Asaoka observational method of monitoring consolidation with vertical drains", *Soils Found.*, **36**(3), 31-42. https://doi.org/10.3208/sandf.36.3_31.

Tan, T.S., Inoue, T. and Lee, S.L. (1991), "Hyperbolic method for consolidation analysis", *J. Geotech. Eng.*, **117**(11), 1723-1737. https://doi.org/10.1061/(ASCE)0733-9410(1991)117:11(1723).

Taylor, D.W. and Merchant, W. (1940), "A theory of clay consolidation accounting for secondary compression", *J. Math. Phys.*, **19**(1-4), 167-185. https://doi.org/10.1002/sapm1940191167.

Terzaghi, K. (1943), *Theoretical Soil Mechanics*, Wiley, New York, NY, USA.

Thompson, C.G., Kim, R.S., Aloe, A.M. and Becker, B.J. (2017), "Extracting the variance inflation factor and other multicollinearity diagnostics from typical regression results", *Basic Appl. Soc. Psychol.*, **39**(2), 81-90. https://doi.org/10.1080/01973533.2016.1277529.

Thompson, N.C., Greenewald, K., Lee, K. and Manso, G.F. (2020), "The computational limits of deep learning", *arXiv preprint* arXiv:2007.05558. https://doi.org/10.48550/arXiv.2007.05558.

Watabe, Y., Udaka, K., Nakatani, Y. and Leroueil, S. (2012), "Long-term consolidation behavior interpreted with isotache concept for worldwide clays", *Soils Found.*, **52**(3), 449-464. https://doi.org/10.1016/j.sandf.2012.05.005.

Yoo, H.K. and Kim, J.H. (2000), "A study on the prediction of long-term settlement by the modified hyperbolic method", *J. Korean Geotech. Soc.*, **16**(3), 163-172.

Zhang, Y.D., Liao, B.K., Lam, P.W., Jurong, S. and Leung, C.F. (2017), "Geotechnical aspects of container port development", *Proceedings of the GeoSS 10th Anni. Conf.*, Singapore.

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