

Prediction of the safety factor and the failure surface for slope stability analysis using Jaya optimization method

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Abstract. In geotechnical engineering, the limit equilibrium technique is known as a widely used method in slope stability analyses. There are many different methods in the literature for this technique to be used in slope stability analyses. Among these methods, the Morgenstern-Price method provides more realistic results compared to other methods since it considers equilibrium conditions of both force and moment and thus provides solutions. Therefore, this study integrates the Morgenstern-Price (M-P) method with the Jaya optimization algorithm to determine critical failure surfaces and safety factors in slope stability analysis. Since finding the minimum safety factor and the associated critical failure surface is essentially an optimization problem, Jaya algorithm, which is one of the many algorithms for solving this optimization problem and also works faster and reaches the result in a shorter time due to its simple structure, was used. For the purpose of examining the performance and currency of this optimization method, two case studies are analyzed and compared with existing optimization methods. The results demonstrate that the Jaya algorithm achieves comparable or improved performance in identifying critical failure surfaces while maintaining computational efficiency.

Keywords: critical failure surface; jaya optimization; morgenstern-price method; safety factor; slope stability

1. Introduction

Slope stability has long been a critical focus for civil engineers in geotechnical engineering. Initially, the emphasis was on understanding natural slope failures; however, with the growing human population and increasing demands, engineers began to examine the stability of artificial slopes created for various purposes. In the early to mid-1900s, the surge in engineering and mining activities necessitated extensive excavations that were becoming deeper and the elevation differences between the base and the top were increasing. These developments resulted in the formation of steep surfaces known as slopes. Over time, stability issues emerged in these artificially created slopes, leading to significant accidents (Keskin 2008). Consequently, the stability challenges of artificial slopes have gained prominence alongside those of natural slopes, spurring researchers to investigate these issues further.

‘Stability analysis’, which investigates the equilibrium positions of soil-rock masses shaped by slopes under given project duration and loading conditions is one of the basic subjects of soil and rock mechanics. The stability analysis’s primarily scope is to obtain the safety factor (F) of the designed slope mass for given operating and loading conditions. The fundamental approach that is constantly repeated for the slope stability analysis is to obtain the minimum safety factor by investigating the critical failure

surface (Arioğlu and Tokgöz 2005). Therefore, researchers have developed various numerical and analytical approaches such as limit analysis method (LAM), limit equilibrium method (LEM), distinct element method (DEM), finite difference method (FDM) or finite element method (FEM) to accurately evaluate the slopes’ stability and obtain the minimum safety factor from past to present. Various methods, such as Janbu (1954), Bishop (1955), Morgenstern and Price (1965), Spencer (1967)’s approaches, have been developed to obtain the safety factor with the use of limit equilibrium approaches for a circular failure surface. In the slope stability analysis by employing of LEM, there are two main steps. The first step involves formulating an equation to determine the safety factor. The second step entails generating and assessing trial failure surfaces to identify the critical failure surface. Limit equilibrium approaches take into account only forces, only moments, or both forces and moments equilibrium associated with a failure surface that is supposed to pass through a soil mass (Fredlund *et al.* 1981). The methods such as Simplified Bishop, Simplified Janbu and Ordinary Fellenius are among the first group of limit equilibrium approaches that solely regard the force or moment equation of equilibrium. The methods like Generalized (Rigorous) Janbu method of slices, Morgenstern-Price and Spencer are the second group of limit equilibrium techniques that regard the moment and force equations of equilibrium at the same time. It is typical to achieve a comparatively higher safety factor value when using limit equilibrium approaches that regard both the force and moment equations of equilibrium, compared to methods that only consider either the force or the moment equation of equilibrium. But although the values obtained from the methods in the second group are higher than the

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other group, they give more realistic results (Solati and Habibagahi 2006). For a slope in a homogeneous soil layer, a circular failure surface method is adequate. However, for a slope in a heterogeneous multi-soil layer, a non-circular failure surface method should be taken into account. Because, circular methods can overestimate the safety factor. Limit equilibrium has additionally been employed to analyze non-circular failure surfaces (Bardet and Kapuskar 1989, Zolfaghari *et al.* 2005, Zhu *et al.* 2005, Khajehzadeh *et al.* 2011, Himanshu *et al.* 2021).

The Morgenstern-Price (M-P) Method is a widely accepted approach and relies on a limit equilibrium analysis, which focuses on the balance of forces and moments applied to a possible failure surface for a slope. This method was designed by engineers J. Morgenstern and W. Price in 1965. The primary reason for utilizing the M-P Method in this article is that it offers an exact solution in terms of achieving both force and moment balance, provided certain assumptions are met. As a result, it is possible to say that the safety factor obtained from both moment and force equilibrium is closer to reality those obtained only from either moment or force equilibrium (Cheng *et al.* 2008). Additionally, the method yields realistic results for surfaces that exhibit considerable internal distortion of the sliding soil mass.

Finding the minimum value of safety factor and the associated critical failure surface is fundamentally an optimization problem. Previously many researchers have reported on the adoption of traditional optimization methods in the slope's analysis issues (Baker and Garber 1978, Baker 1980, Michalowski 1995, Greco 1996, Yamagami and Jiang 1997, Zhu 2001, Zhu *et al.* 2001, Sarma and Tan 2006, Zhou *et al.* 2023). Traditional optimization methods have several limitations and may not perform effectively. As a result, in slope stability analysis, engineers have recently begun applying meta-heuristic algorithms (McCombie and Wilkinson 2002, Kahatadeniya *et al.* 2009, Zhu and Chen 2014, Gandomi *et al.* 2015, Himanshu *et al.* 2021, Kaveh and Seddighian 2021, Xie *et al.* 2022, Kumar *et al.* 2023, Alok *et al.* 2024, He *et al.* 2024, Singh *et al.* 2024). Ant Colony Optimization, Tabu Search Algorithm, Particle Swarm Optimization, Artificial Bee Colony Algorithm, Firefly Algorithm, Harris Hawks Optimization, Cuckoo Search, Grey Wolf Optimization, Fish Swarm Optimization and Genetic Algorithm etc. can be given as examples of these meta-heuristic methods. When examining the studies of some of these authors, Kumar *et al.* (2023) utilized Scoops3D conduct a slope stability analysis, followed by a probabilistic evaluation employing a hybrid computational model that integrates an artificial neural network with the firefly algorithm. Zhang *et al.* (2022) presented a novel generalized artificial intelligence model based on a deep multiple layer perceptron neural network and the Harris Hawks Optimization algorithm to estimate the friction angle of clays from various regions for the evaluation of slope stability. Asteris *et al.* (2022) examined the application of tree-based models, specifically decision tree, random forest, and AdaBoost, in the classification of slope stability under seismic loading conditions.

The focus of this paper is to examine the stability analysis of homogeneous soil slopes, assessing them based on the assumption of a non-circular failure surface. The limit equilibrium method is used to identify the critical failure surface using the M-P Method for obtaining the safety factor in this research. As a result, a computer code has been created in Matlab using Jaya optimization method to identify the critical failure surface and the corresponding minimum safety factor. Consequently, two sloped soil examples were solved with this algorithm to achieve the goal and the results were compared with previous studies. The numerical values of these examples were sourced from the article of Yamagami and Ueta (1988) and Zolfaghari *et al.* (2005).

This study consists of seven sections, starting with this Introduction section. The remainder of this paper is organized as follows. Section 2 emphasizes significance of this research. Section 3 states in details the formulation of the slope stability problem solved with the M-P Method used for determining the safety factor. In Section 4, the Jaya optimization algorithm is developed to determine the safety factor and the objective function based on this algorithm is presented. In Section 5, the application of the proposed method on benchmark slope stability problems have been exemplified. In Section 6, the main conclusions and discussion of those studies are provided. Finally, information is given about the limitations of the proposed model and future work in Section 7.

2. Research significance

In this study, the Morgenstern-Price (M-P) method integrated with the Jaya algorithm to identify critical failure surfaces and corresponding minimum safety factors. For this aim, the proposed model is able to make an effective search to find global best solution.

3. Equations for slope stability analysis using Morgenstern-Price Method

This study presents general failure surface of a slope with using Morgenstern-Price Method (Zhu *et al.* 2005, Zolfaghari *et al.* 2005, Kahatadeniya *et al.* 2009, Himanshu *et al.* 2021, Ouyang *et al.* 2022) that takes into account a number of vertical slices divided into finite widths as shown in Fig. 1. To facilitate the analysis, the moving soil mass is segmented to vertical slices' limited number, with each slice's base aligned at the failure surface. It is thought that the section of the failure surface within each slice is linear, leading to a piecewise linear illustration of the failure surface. The slice width can be considered either constant or variable. In this study, the slice width is taken as constant to facilitate the calculation processes.

As illustrated in Fig. 2, each vertical slice is in equilibrium under the influence of internal and external forces. Considering the i th slice in Fig. 2, the point G_i is the slice's gravity center; W_i is the slice weight; $W_i a_h$ and $W_i a_v$ are the horizontal and vertical pseudo-static seismic forces; a_h and a_v are the horizontal and vertical pseudo-static

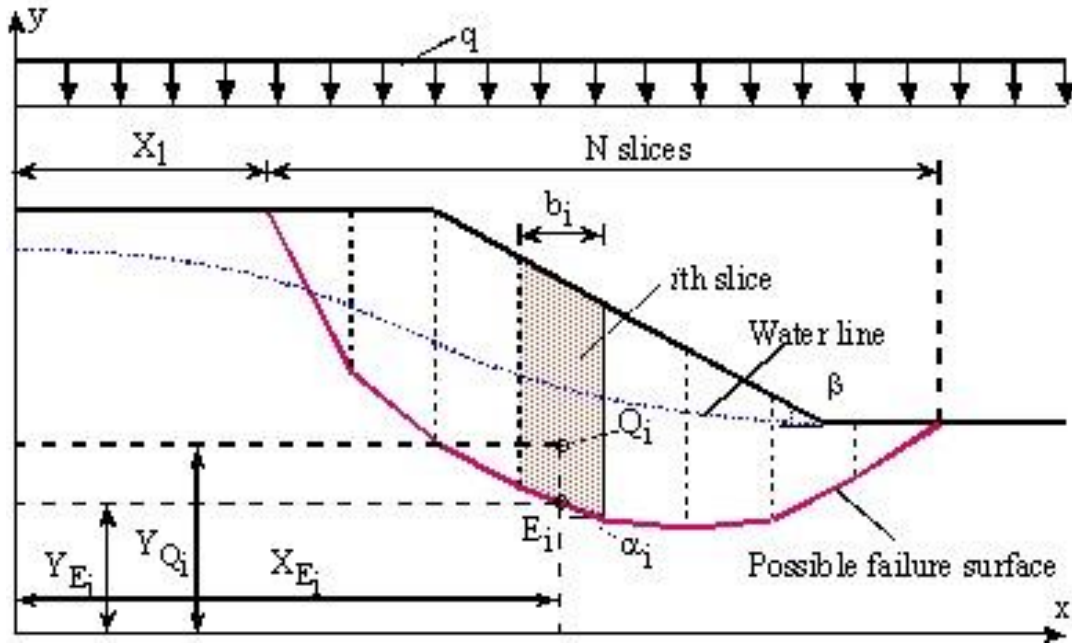


Fig. 1 Possible failure surface of a slope

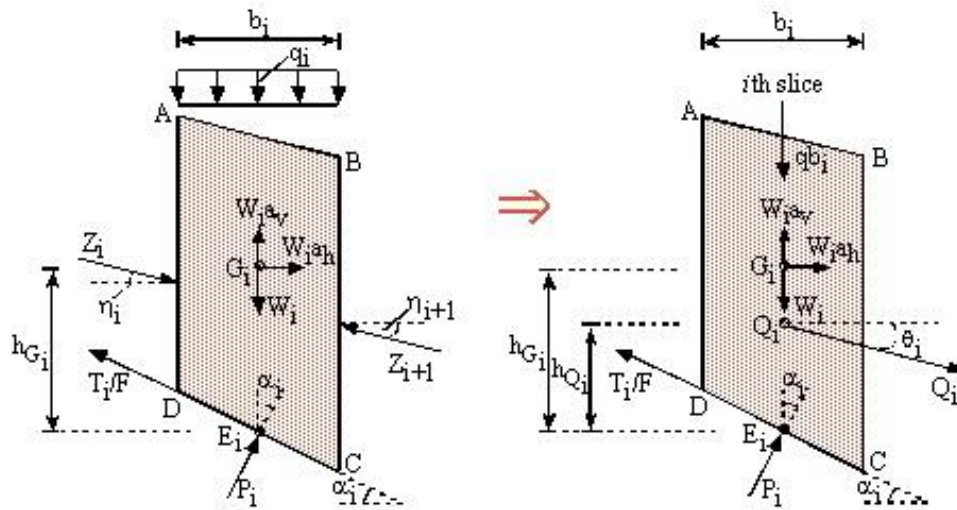


Fig. 2 Representation of forces in the slice *i*

$$Q_i = \frac{c' b_i \sec \alpha_i + \frac{\tan \phi'}{F} (W_i \cos \alpha_i - W_i a_v \cos \alpha_i - W_i a_h \sin \alpha_i - u b_i \sec \alpha_i + q b_i \cos \alpha_i) - W_i \sin \alpha_i + W_i a_v \sin \alpha_i - W_i a_h \cos \alpha_i - q b_i \sin \alpha_i}{\cos(\alpha_i - \theta_i) \left(1 + \tan(\alpha_i - \theta_i) \frac{\tan \phi'}{F} \right)} \quad (1)$$

seismic coefficients and $q b_i$ is the uniformly distributed load acting to the slice's part. Additionally, at the point Q_i , the resultant force between the slices Q_i , which arise from the forces at both the slice's vertical boundary surfaces, is located. At the failure surface at the slice's base, the resultant normal force P_i and the resultant shear force T_i/F are present, where T_i represents the soil's shear resistance force and F denotes the safety factor.

The resultant force between slices, Q_i obtained from the force balance in the i th slice in Fig. 2 can be expressed as in Eq. (1) (Zolfaghari *et al.* 2005).

In Eq. (1); c' , ϕ' and u_i represent the effective cohesion, the effective internal friction angle and the pore water pressure respectively.

From the moment balance of the i th slice, the moments' sum should be zero at the point E_i , which is approximately the i th slice base's midpoint.

$$\sum_{i=1}^N M_{E_i} = Q_i \cos \theta_i h_{Q_i} - W_i a_h h_{G_i} = 0 \quad (2)$$

In Eq. (2), N represents slices' number.

For i th slice, firstly Q_i is calculated in Eq. (1). Then the calculated Q_i is substituted into Eq. (2) to calculate h_{Q_i} .

Y_{Q_i} , the y coordinate of the Q_i point in Fig. 2, is calculated using Eq. (3)

$$Y_{Q_i} = Y_{E_i} + h_{Q_i} \quad (3)$$

The sum of the resultant force (Q_i)'s horizontal and vertical components between slices obtained from all slices should be zero.

$$\sum_{i=1}^N Q_{xi} = \sum_{i=1}^N (Q_i \cos \theta_i) = 0 \quad (4)$$

$$\sum_{i=1}^N Q_{yi} = \sum_{i=1}^N (Q_i \sin \theta_i) = 0 \quad (5)$$

When the moments of Q_i from all slices are taken with respect to any point, this total moment value should be zero. If the origin is chosen as any point, Eq. (6) is written.

$$\sum_{i=1}^N (M) = \sum_{i=1}^N (Q_i \cos \theta_i Y_{Q_i} + Q_i \sin \theta_i X_{Q_i}) = 0 \quad (6)$$

The ratio obtained by dividing the vertical force between slices by the horizontal force between slices has a definition in terms of $\lambda f(x_i)$ in the M-P Method.

$$\frac{Q_i \sin \theta_i}{Q_i \cos \theta_i} = \tan \theta_i = \lambda f(x_{E_i}) \quad (7)$$

As can be seen in Eq. (7), this ratio depends on a scaling factor λ and a prescribed function $f(x)$, which can be taken as constant, half-sine, trapezoidal, or a user-defined function which represents the interslice forces' direction. In the Spencer method, where the resultant forces between slices are in the same direction along all slices, a constant ratio is assumed between the normal and shear forces at the interslice interfaces for all slices. In this study, it is considered that $f(x)$ function equals 1. When the value of $f(x)$ is 1, the M-P Method is occasionally referred the Spencer Method (Spencer, 1967).

By substituting Q_i in Eq. (1) into Eq. (4) or (5) and Eq. (6), two equations with unknowns F and λ are obtained to find the safety factor.

Eqs. (2) and (4) can be converted into Eq. (8), which is found for each F and λ pair. Then Eq. (8) is solved by the proposed algorithm described in Section 4.

$$|\sum M| + |\sum (Q_i \cos \theta_i)| = |\sum (Q_i \cos \theta_i Y_i + Q_i \sin \theta_i X_i)| + |\sum (Q_i \cos \theta_i)| = 0 \quad (8)$$

In Eq. (8) for i th slice; θ_i is the angle of Q_i with horizontal axis, X_i and Y_i represent the coordinates of slice base's midpoint.

4. Optimization algorithm and objective function

4.1 Jaya optimization algorithm

Jaya optimization algorithm was developed by researcher Rao (2016). This artificial intelligence algorithm

was created by minimizing the use of population size and maximum number of iterations used as control parameters. The word Jaya means victory in Sanskrit. The aim of the algorithm is for individuals in the population to approach the most successful individual and move away from the worst individual. Therefore, coding Jaya and adapting it to the problem is very simple. In addition, Jaya works faster and reaches the result in a shorter time due to its simple structure. Thanks to these features, Jaya provides superiority among population-based algorithms.

The flow chart of the algorithm, which initially creates a random population, is given in Fig. 3. In the population, the best and worst individuals are determined using the problem's fitness function. Individuals are renewed with the interaction made with the idea of approaching the best individual and moving away from the worst individual. For each renewed individual, it is decided that it will remain in the population in its old or new form in the next iteration, considering the fitness function value of its state before renewal. This thought process continues with iterations until all individuals are updated and the stopping criteria for the problem are met.

The updated equation according to the individuals (solutions) is given in Eq. (9).

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i} (X_{j,best,i} - |X_{j,k,i}|) - r_{2,j,i} (X_{j,worst,i} - |X_{j,k,i}|) \quad (9)$$

In Eq. (9); $X_{j,k,i}$ represents the j . design variable for the k . individual in the i . iteration, $X_{j,best,i}$ represents the j . design variable of the best individual and $X_{j,worst,i}$ represents the j . design variable of the worst individual. The value $X'_{j,k,i}$ is the modified version of $X_{j,k,i}$ according to this algorithm and $r_{1,j,i}$ and $r_{2,j,i}$ are two random values generated in the range (0-1) for the j . design variable during the i . iteration (Rao and Waghmare 2017).

If the newly proposed individual's fitness function (error) value is less than the existing individual's fitness function value, the new individual replaces the default existing individual. Otherwise, the algorithm keeps the existing individual. Comprehensive information about Jaya algorithm is available in the literature (Rao 2016, Rao and Saroj 2017, Rao and Waghmare 2017).

There are many areas of application where optimization algorithms are integrated into civil engineering. (Kaveh 2021, Yakak *et al.* 2024). These algorithms are also used in specific geotechnical problems (Öztürk *et al.* 2020, Aydın *et al.* 2024). This study applies Jaya algorithm, which has shown many practical and successful results in civil engineering. Its focus is on determining the minimum safety factor related to the slope's critical non-circular failure surface.

4.2 Objective function

The population undergoes a series of cycles to determine F and λ which minimize the objective function value. The objective function can be shown as Eq. (10).

$$\text{Objective Function} = |\sum (Q_i \cos \theta_i Y_i + Q_i \sin \theta_i X_i)| + |\sum (Q_i \cos \theta_i)| \quad (10)$$

Where, Q_i is defined in Eq. (1), X_i and Y_i are defined in

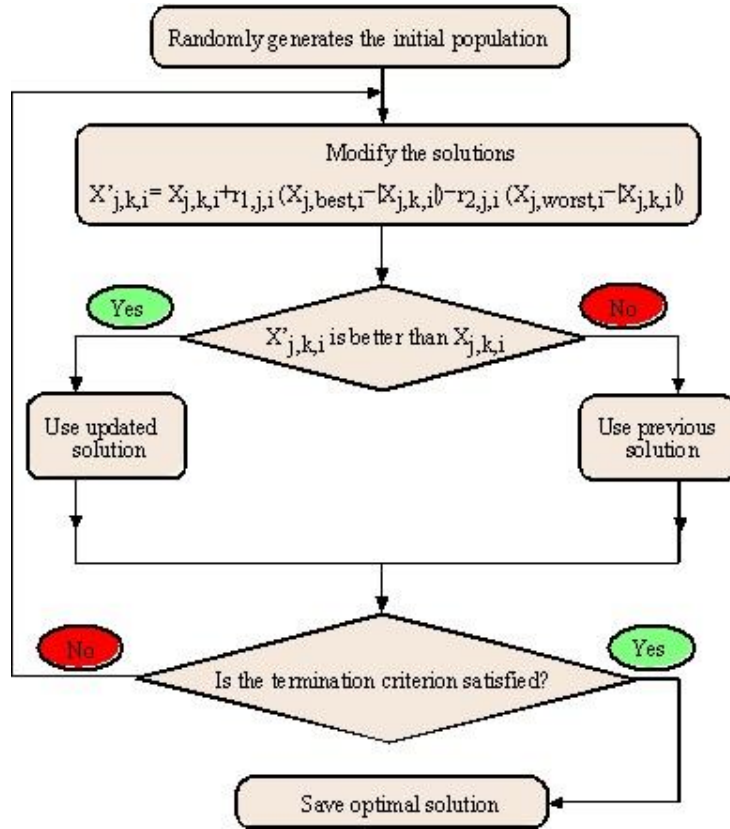
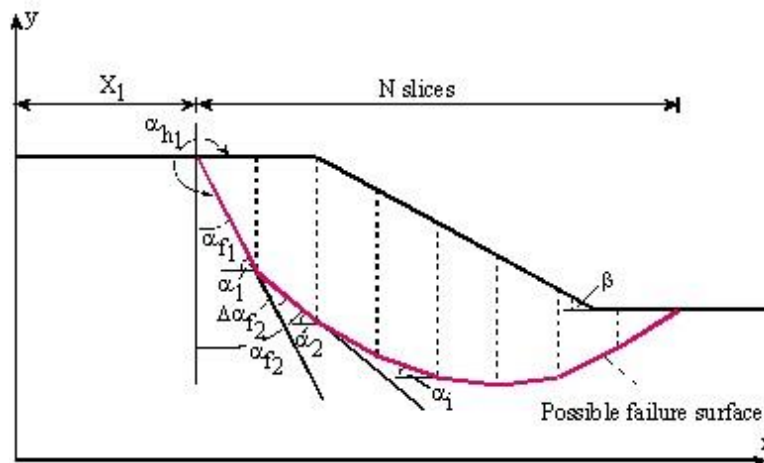


Fig. 3 Flow diagram for Jaya optimization method



$$\begin{aligned}
 \alpha_{f_1} &= \alpha_{f_1} & \alpha_{h_1} &= 270 + \alpha_{f_1} & \alpha_1 &= 360 - \alpha_{h_1} \\
 \alpha_{f_2} &= \alpha_{f_1} + \Delta\alpha_{f_2} & \alpha_{h_2} &= 270 + \alpha_{f_2} & \alpha_2 &= 360 - \alpha_{h_2} \\
 \alpha_{f_i} &= \alpha_{f_{i-1}} + \Delta\alpha_{f_i} & \alpha_{h_i} &= 270 + \alpha_{f_i} & \alpha_i &= 360 - \alpha_{h_i}
 \end{aligned}$$

Fig. 4 A possible failure surface's view

Fig. 1 and the θ_i is defined in Fig. 2.

To convert the problem into an optimization problem, all parameters to be sought in the population to which the problem will be applied must first be created. In this research depended on the M-P Method, the mass of sliding soil is divided into N slices. Then, a failure line is depicted

as a piecewise-linear curve. The parameters used to characterize various curves for various failure surfaces are based on the research by Yamagami and Ueta (1988), Zolfaghari *et al.* (2005), as shown in Fig. 4. Here, the failure surface's initial x coordinate is X_1 and is generated randomly. α_{h_1} represents the failure surface's angle and

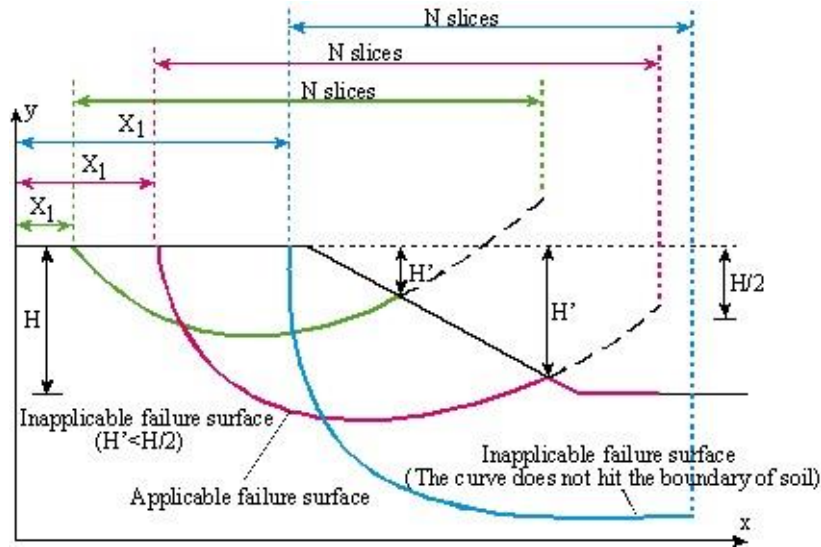


Fig. 5 Applicable and inapplicable failure surfaces

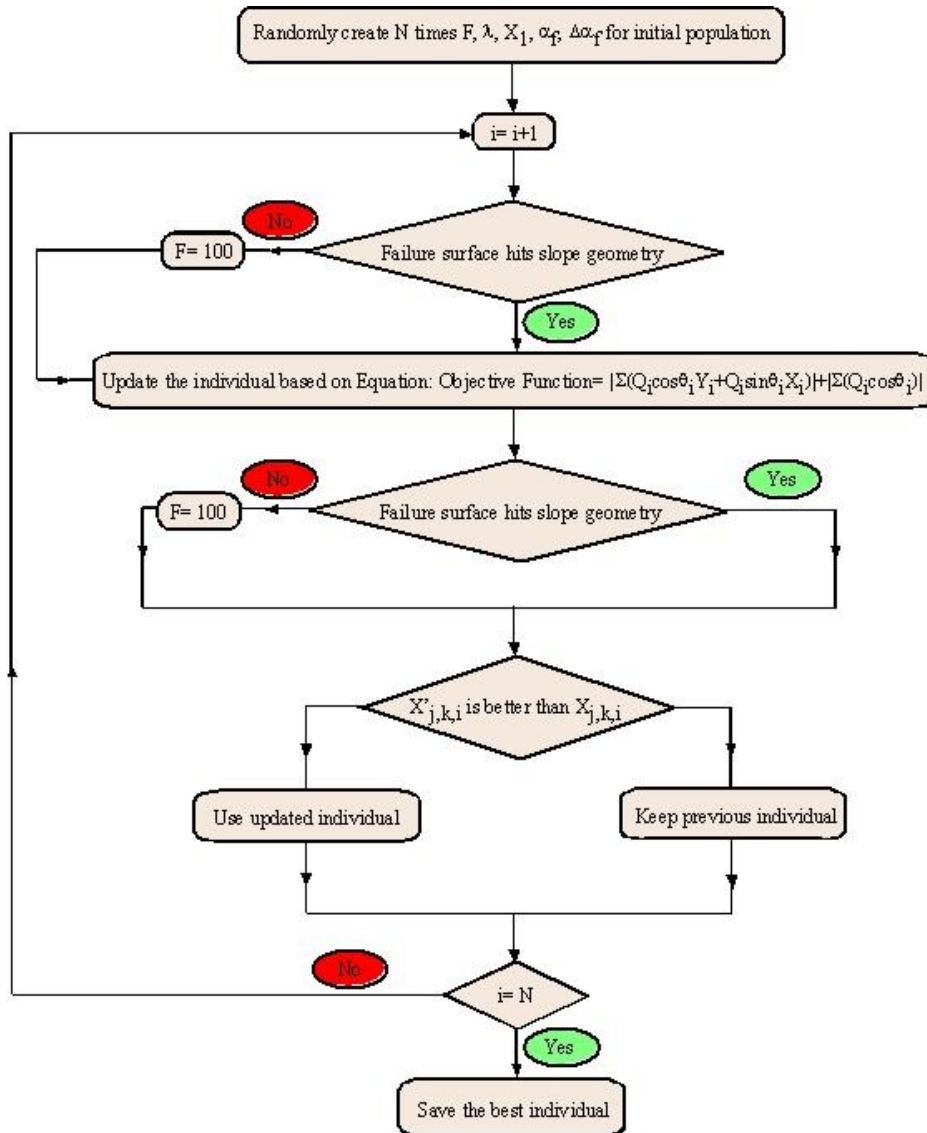


Fig. 6 Flow diagram of Jaya optimization method for this study

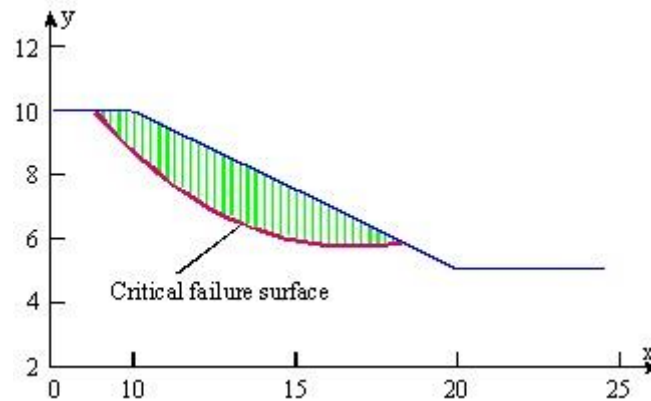


Fig. 7 Example-1's slope geometry and critical failure surface

generated randomly, too. α_{f1} 's initial value is randomly selected within the range of $45-\phi/2$ and $45+\phi/2$ that is Rankine failure angle. $\Delta\alpha_{f1}$ represents the angular difference between the failure surface lines of two consecutive slices and is randomly selected. It is presumed that the soil mass moves to the right in the problems addressed in this study.

The failure surface generally follows the curve starting from the initial point as seen in Fig. 5. However, the failure surface instead follows boundary of the soil when this curve hits the boundary of the soil. If this curve does not hit the boundary of the soil at all, it is deemed inapplicable. Additionally, if the vertical distance from the intersection point to the slope's crest is shorter than the slope height's half, the curve is considered as a local failure and is consequently deemed inapplicable.

To determine the critical failure surface, the variables' combination in Q_i that gives the minimum safety factor must be identified. It is evident that failure surfaces with smaller safety factors will result in the objective function's lower values. Therefore, the problem of determining the failure surface that provides the minimum safety factor can be defined as a minimization problem of Q_i . In this study, the safety factor's great value is assigned to every infeasible curve. In the light of all this information, in Matlab, a computer program has been created using Jaya model to define the critical failure surface and the related minimum safety factor by minimizing the objective function. The flow diagram of Jaya optimization method for this study is presented in Fig. 6.

5. Results and discussion

To research the validity and efficiency of Jaya algorithm used in this study, two examples were solved. Both selected examples include problems related to homogeneous soils. Vertical slices with the same slice width were taken in both examples. For the accuracy of the acquired results, the algorithm was run 100 times randomly for both examples. For obtaining the best solution, the best solution was searched in these 100 runs. This means that the problem with the minimum safety factor in 100 runs will be accepted

as the best solution. Consequently, the best solutions obtained were compared with reported results in the past.

5.1 Example-1

The Example-1 regarded as a homogeneous soil slope was extracted from the study of Yamagami and Ueta (1988), where it was examined using different optimization methods. In their original study, Yamagami and Ueta (1988) used the M-P Method in combination with some traditional optimization methods based on the Davidon-Fletcher-Powell (DFP) and Broyden-Fletcher-Goldfarb-Shanno (BFGS) Methods which are derivative methods, Simplex and Powell Methods which are non-derivative methods (Fletcher 1987, Nocedal and Wright 2006). In their study, they took into account that the function $f(x)$ is considered to be 1 and this refers to the Spencer Method. After Yamagami and Ueta (1988), many researchers used the same example and obtained results that were parallel to the results obtained from Yamagami and Ueta (1988)'s study. Greco (1996) used the M-P Method ($f(x)=1$) to estimate the safety factor, in combination with the Pattern Search and Monte-Carlo Methods. Malkawi *et al.* (2001) developed a program where the limit equilibrium-based methods (ordinary method of slice, Janbu, Bishop, M-P and Spencer) were incorporated with the suggested method depended on the Monte Carlo techniques' principle. Solati and Habibagahi (2006) used Generalized Janbu Method in combination with Genetic Algorithms (GA). Cheng *et al.* (2007b) utilized the Spencer Method in combination with Particle Swarm Optimization (PSO) and Modified Particle Swarm Optimization (MPSO). Cheng *et al.* (2008) applied the M-P Method ($f(x)=1$) in combination with Modified Harmony Search Optimization (MHS). Jianping *et al.* (2008) utilized the Spencer Method ($f(x)=1$) in combination with Genetic Algorithms (GA; line, spline). Kahatadeniya *et al.* (2009) applied the M-P Method ($f(x)=1$) in combination with Ant Colony Optimization (ACO). Li *et al.* (2010) used the M-P Method ($f(x)=1$) in combination with real-coded Genetic Algorithm (real-coded GA). Kalatehjari *et al.* (2012) applied the Simplified Bishop Method in combination with Particle Swarm Optimization (PSO). Kang *et al.* (2013)

Table 1 The soils' physical-mechanical properties and the slopes' coordinates for Example-1 and 2

| Problem | Cohesion, c (kN/m ²) | Internal friction angle, ϕ (°) | Unit weight, γ (kN/m ³) | The coordinates of the slope's crest (m) | | The coordinates of the slope's toe (m) | |
|-----------|---------------------------------------|--|---|---|----|---|------|
| | | | | x | y | x | y |
| Example-1 | 9.8 | 10 | 17.64 | 10 | 10 | 20 | 5 |
| Example-2 | 15 | 20 | 19 | 15 | 50 | 32 | 41.5 |

Table 2 The maximum, minimum and increment values of variables for Example-1 and 2

| Example | X_1 | | | α_{f1} | | | $\Delta\alpha_{f1}$ | | |
|---------|-------|-----|-----------|---------------|-----|-----------|---------------------|-----|-----------|
| | Min | Max | Increment | Min | Max | Increment | Min | Max | Increment |
| 1 | 0 | 10 | 0.001 | 40 | 50 | 0.1 | 0 | 1 | 0.04 |
| 2 | 0 | 15 | 0.001 | 30 | 50 | 0.1 | 0 | 1 | 0.04 |

Table 3 Summary of the results from prior studies and the current study for Example-1

| Study | Limit equilibrium methods | Optimization methods | Safety factor, F |
|-----------------------------------|---------------------------|----------------------|--------------------|
| Yamagami and Ueta (1988) | M-P ($f(x)=1$) | BFGS | 1.338 |
| | | DFP | 1.338 |
| | | Powell | 1.338 |
| | | Simplex | 1.339-1.348 |
| Greco (1996) | M-P ($f(x)=1$) | Pattern search | 1.326-1.330 |
| | | Monte Carlo | 1.327-1.333 |
| Malkawi <i>et al.</i> (2001) | Combined method of LEMs | Monte Carlo | 1.238 |
| Solati and Habibagahi (2006) | Generalized Janbu | GA | 1.380 |
| | | PSO | 1.3261 |
| Cheng <i>et al.</i> (2007b) | Spencer | MPSO | 1.3259 |
| Cheng <i>et al.</i> (2008) | M-P ($f(x)=1$) | MHS | 1.3224 |
| | | GA (line) | 1.324 |
| Jianping <i>et al.</i> (2008) | Spencer | GA (spline) | 1.321 |
| | | ACO | 1.311 |
| Kahatadeniya <i>et al.</i> (2009) | M-P ($f(x)=1$) | ACO | 1.311 |
| Li <i>et al.</i> (2010) | M-P ($f(x)=1$) | Real-coded GA | 1.327 |
| Kalatehjari <i>et al.</i> (2012) | Simplified Bishop | PSO | 1.3128 |
| Kang <i>et al.</i> (2013) | Spencer | ABC | 1.3206 |
| Kashani <i>et al.</i> (2016) | M-P ($f(x)=1$) | ICA | 1.3206 |
| | | Grid search | 1.268 |
| Singh <i>et al.</i> (2018) | Fellenius | GA | 1.237 |
| | | GSA | 1.226 |
| | | Jaya | 1.303 |
| Current Study | M-P ($f(x)=1$) | Jaya | 1.303 |

utilized the Spencer Method in combination with Artificial Bee Colony (ABC). Kashani *et al.* (2016) applied the M-P Method ($f(x)=1$) in combination with Imperialistic Competitive Algorithm (ICA). Singh *et al.* (2018) used Fellenius Method in combination with Gravitational Search Algorithm (GSA), Grid Search and Genetic Algorithm (GA).

For Example-1, the run is utilized as 100, the population number is utilized as 300 and the iteration number is utilized as 200. The number of vertical slices of the sliding

soil was selected as 200 and the width of each slice was selected as 0.25m. α_{f1} 's initial value was randomly selected within the range of $45-\phi/2$ and $45+\phi/2$ that is Rankine failure angle.

The slope geometry of Example-1 and the critical failure surface of this geometry are given in Fig. 7.

The soils' physical-mechanical properties and the slopes' coordinates used in the proposed algorithm for both examples are presented in Table 1. The maximum and minimum values of the variables X_1 , α_{f1} and $\Delta\alpha_{f1}$ and the

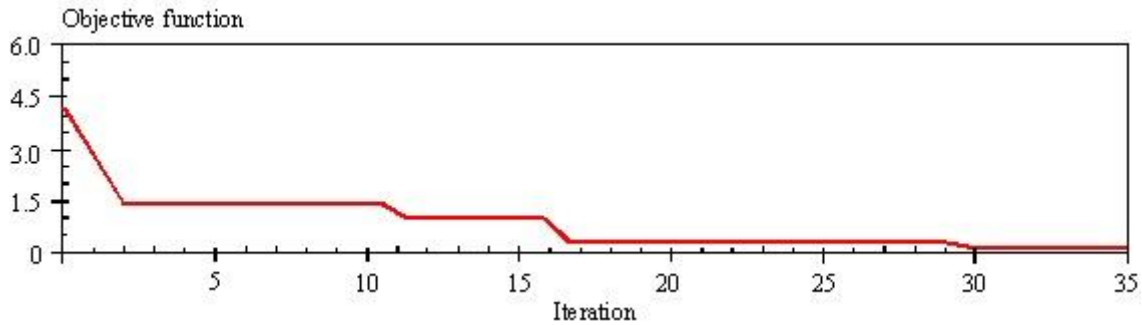


Fig. 8 Example-1's convergence graph

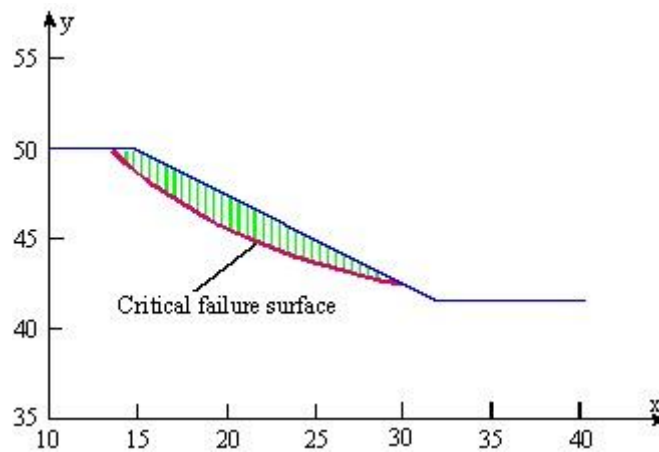


Fig. 9 Example-2's slope geometry and critical failure surface

increments of these variables for both examples are depicted in Table 2.

Table 3 presents as a summary of the minimum safety factors derived from both the majority of prior studies and the current research. The result obtained in this study is found to be consistent with the results reported in the literature. As shown, the table lists different minimum safety factors derived from various limit equilibrium methods and various optimization methods. To better evaluate the performance and efficiency of an optimization method, it would be more appropriate to compare values obtained using the same limit equilibrium method. As depicted in table, the minimum safety factor got in this research is more favorable than the minimum safety factor reported in previous studies using the M-P Method.

As seen in Fig. 8, the graph started to converge after the first 30 iterations.

5.2 Example-2

The Example-2 regarded as a homogeneous soil slope was extracted from Zolfaghari *et al.* (2005)'s study. In their original paper, Zolfaghari *et al.* (2005) utilized a Simple Genetic Algorithm (SGA) combined with the Simplex Method (SM) based on the methods of Bishop and M-P. After Zolfaghari *et al.* (2005), some researchers used the same example and obtained results that were consistent with those from Zolfaghari *et al.* (2005)'s study. Cheng *et al.* (2007a) utilized the Spencer Method to estimate the safety

factor, in combination with the Ant Colony Algorithm (ACO), Simulated Annealing Algorithm (SA), Particle Swarm Optimization Algorithm (PSO), Genetic Algorithm (GA), Tabu Search Algorithm, Simple Harmony Search Algorithm (SHM) and Modified Harmony Search Algorithm (MHM). Kalatehjari *et al.* (2012) applied the Bishop Method to estimate the safety factor, in combination with the Particle Swarm Optimization (PSO). Himanshu *et al.* (2021) used the M-P Method to estimate the safety factor, in combination with the Grey Wolf Optimization (GWO). Alok *et al.* (2024) applied the Janbu, Bishop and M-P Methods to calculate the safety factor, in combination with the VBA (Visual Basic for Applications) program on excel platform.

As in the previous example, in Example-2, the run is utilized as 100, the population number is utilized as 300 and the iteration number is utilized as 200, too. The number of vertical slices of the sliding soil was selected as 200 and the width of each slice was selected as 0.25 m, too.

The slope geometry of Example-2 and this geometry's critical failure surface are given in Fig. 9.

The maximum and minimum values of the variables X_1 , α_{j1} and $\Delta\alpha_{j1}$ and the increments of these variables are taken as the same as in the first example. These values are depicted in Table 2.

The minimum safety factors produced from the majority of prior studies as well as the current research are summarized in Table 4. The findings of this study are consistent with those found in the literature. As seen in the

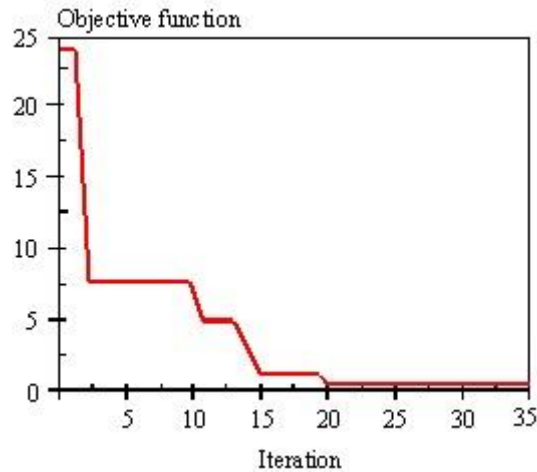


Fig. 10 Example-2's convergence graph

Table 4 Summary of the results from prior studies and the current study for Example-2

| Study | Limit equilibrium methods | Optimization methods | Safety factor, F |
|----------------------------------|---------------------------|--------------------------|--------------------|
| Zolfaghari <i>et al.</i> (2005) | Bishop (Circular) | | 1.74 |
| | M-P (Non-Circular) | SGA combined with the SM | 1.75 |
| | M-P (Circular) | | 1.76 |
| Cheng <i>et al.</i> (2007a) | Spencer | SA | 1.7267 |
| | | GA | 1.7297 |
| | | PSO | 1.7282 |
| | | SHM | 1.7264 |
| | | MHM | 1.7279 |
| | | Tabu Search | 1.7415 |
| | ACO | 1.7647 | |
| Kalatehjari <i>et al.</i> (2012) | Bishop | PSO | 1.7197 |
| Himanshu <i>et al.</i> (2021) | M-P | GWO | 1.7295 |
| Alok <i>et al.</i> (2024) | Bishop | | 1.742 |
| | Janbu | VBA | 1.604 |
| | M-P | | 1.732 |
| Current Study | M-P ($f(x)=1$) | Jaya | 1.715 |

table, the minimum safety factor estimated in this research is more favorable than those reported in previous studies using the M-P Method.

As seen in Fig. 10, the graph started to converge after the first 20 iterations.

6. Conclusions

In the past, researchers believed that the slope of the base of any slice in the nonlinear failure surface was not influenced by the slope of the adjacent slices. Nevertheless, in this study, it has been observed that connecting the slopes of neighboring slices leads to a significant improvement in computational efficiency. The focus of this paper is to analyze a slope's safety factor by applying the method of M-P, which relies on the limit equilibrium method. To

achieve of this goal, Jaya optimization method was utilized to estimate the critical failure surface and solved the objective function for a selection of benchmark slope stability problems. The Jaya optimization algorithm succeed to locate a critical failure surface which satisfies its kinematics and geometric constraints. The method eliminated invalid failure surfaces, so that a valid failure surface was calculated within its boundary limits. From results, it has been seen that the M-P Method can be applied solved using Jaya optimization method to calculate the safety factor for different slope geometries and this algorithm can be utilized to locate the non-circular failure surface with the lower safety factor than the other algorithms in the existing literature. The proposed Jaya optimization method is straightforward in design and easy to implement. Additionally, a few examples are presented in this study to demonstrate the accuracy of the method. The

effectiveness of the suggested method in this study relies primarily on the precision of the limit equilibrium method and the reliability of the optimization methods. Therefore, two distinct slope problems with homogenous geometric profile were examined using Jaya algorithm, and the findings were documented. Based on the examples examined, it can be concluded that with a run number of 100, a population number of 300, and 200 iterations, the proposed method is capable of identifying the critical non-circular surface and determining the interslice forces. The findings show that Jaya algorithm is effective and valid in addressing slope stability issues. With this, this study's results, when compared with existing researches in the past, demonstrated that the limit equilibrium methods used to create trial failure surfaces have a direct impact on the safety factor assessments, as a result, various outcomes may be produced for the same slope problem. It is not also surprised to achieve a considerably higher safety factor value when using limit equilibrium methods (Generalized (Rigorous) Janbu method of slices, Spencer and M-P etc.) that take into account both the force and moment equilibrium equations, compared to methods (Ordinary Fellenius, Simplified Janbu and Simplified Bishop etc.) that only consider either the force or the moment equilibrium equations. But although the values obtained from the methods that take into account both the force and moment equilibrium equations are higher than the other group, they give more realistic results. So, it is not unexpected that the proposed method yields a higher safety factor, as it takes into account both the force and moment equilibrium equations, in contrast to methods which only consider either the force or the moment equilibrium equation. As a result, Jaya optimization method has been considered to outperform over other implemented optimization algorithms such as Monte Carlo, Pattern Search, Ant Colony Optimization and Particle Swarm Optimization in terms of fast, stable convergence and local minima avoidance, a more realistic value trend for the safety factor.

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