

Exploring the limitations of applying ground reaction curve in hard rock

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Abstract. Excavation in underground construction causes changes in the stress of the ground which causes motion in the surrounding regions particularly the tunnel's ceiling and walls. It is imperative to perform a stability analysis for the mineral and tunneling industries during the excavation process, and the modern methods of tunneling rest on the ground reaction curve (GRC). While GRC is a good tool to visualize these displacements, there exist gaps between the analytical and numerical outcomes as analytical solutions do not extend beyond isotropic circumstances for tunnels that are deep. As a result, there is an increased demand for an equation which focuses on the numerical methods. This paper seeks to address this pressing question by looking into GSI=75 where $K=0.5, 1, 1.5$ and 2 . GRC was used alongside numerical and analytical methods to see the ratio of maximum displacement on point. The purpose of the evaluation was to determine the boundaries of the analytical technique and its effectiveness; conditions that aren't suitable for use of the analytical method were also evaluated. FLAC^{2D} was used for the numerical methods and Duncan-Fama's method was employed for the analytical approaches. The gap in tunnel wall displacement estimates between the analytical and numerical methods was acceptable under isotropic stress conditions. However, large discrepancies were revealed between the methods under anisotropic stress, and wall displacement results of the tunnel weren't overly influenced by the excavation depth. In particular, numerical and analytical displacement estimates for the tunnel crown diverged in shallow models. This was because previously, wall displacements were less affected than crown displacements by excavation depths. Finally, equations were presented for the tunnel ceiling and walls in elastic shapes, which compared favorably with numerical data as opposed to the method of analysis. The presented equations are said to be new especially due to their treatment of issues associated with non-isotropic stress analytical solutions but only for tunnels which are situated at shallow depths and that covers only the tunnel centroid. The equations render better performance in terms of alignment with numerical outcomes that is achieved over a wide range of stress ratios and depths as compared to the current methods in use; this increases the effectiveness of segments in real tunneling conditions.

Keywords: displacement; ground reaction curve; numerical method; underground space

1. Introduction

The changes caused by initial underground excavation occur at the site of the rock mass, and with these changes, ground movements in the areas surrounding the excavation begin (Tabaroei and Chenari 2024, Chen and Lee 2020, Zhou Yang 2021). There are mainly two types of deformation that can occur at the ground surface when encountering shallow tunnels: on one hand, ground subsidence (downward movement), and on the other hand, ground heave (upward movement) (Wang *et al.* 2023, Ahn *et al.* 2022). Brown *et al.* (1983) introduced the concept of the GRC method, emphasizing their role in determining displacement magnitude within tunnels.

The Convergence-Confinement Method (CCM), also known as the GRC method, is commonly employed in tunnel maintenance system design. This method simplifies the three-dimensional interaction between support and rock into a two-dimensional model (Rahimpour *et al.* 2022).

Further investigation involves finite element methods to plot GRC method for both shallow and deep tunnels. It becomes evident that ground response curves significantly depend on tunnel depth, underscoring the need for accurate excavation depth determination during curve plotting. Comparing closed-form analytical solutions with numerical methods for elastic-plastic analysis of circular tunnels, research reveals errors in analytical approaches for brittle elastoplastic materials under hydrostatic stresses, particularly when using Hook's and Brown's criteria. Closed form methods are inadequate for such materials, as demonstrated by modeling accuracy (Sharan 2003).

The tunnel shape is an important factor in modeling, while minor discrepancies exist in results related to tunnel geometry ratios, these differences remain practically acceptable. Additionally, presenting an improper standard Longitudinal Displacement Profile (LDP) method for non-uniform stress is incorrect (Vlachopoulos and Diederichs 2014, Rooh *et al.* 2018). Sharan (2005) conducted studies on displacement analysis around circular tunnels using Hook's and Brown's criteria in elastoplastic and brittle elastic environments under hydrostatic stresses. This led to

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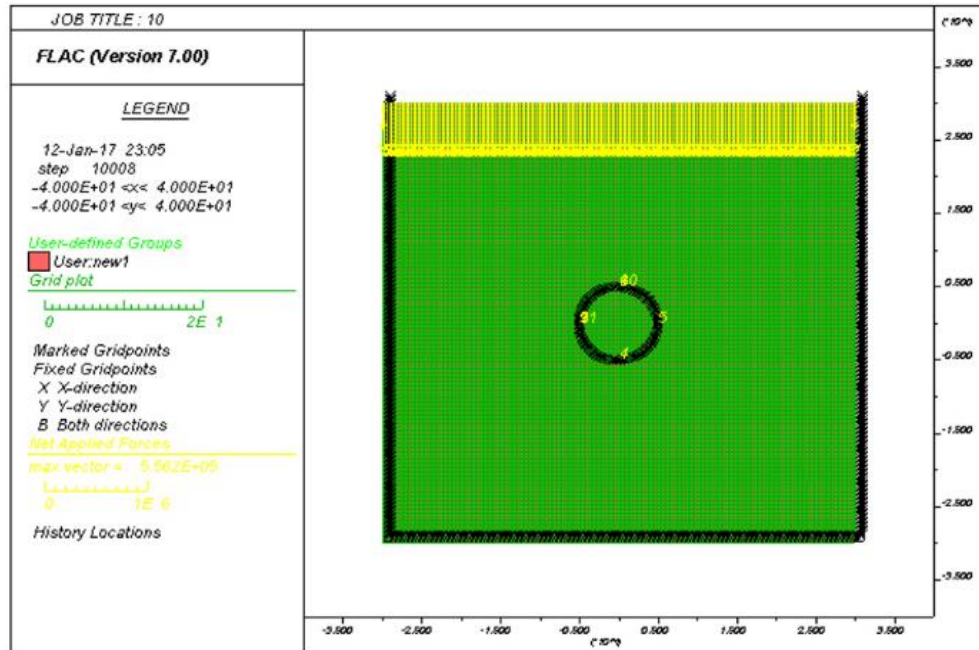


Fig. 1 Dimensions, model, meshing and section geometry

the development of a new method for determining displacements around circular tunnels in elastoplastic and brittle elastic environments using considering Hook's and Brown's criteria.

The assumptions underlying analytical methods include: 1) The tunnel cross-section is circular, 2) The surrounding rock mass is assumed homogeneous, 3) The in-situ stress field is considered isotropic, 4) Plane strain conditions (long tunnel) are taken into account, and 5) Gravity force is neglected. In most cases, these assumptions are typically violated in actual tunneling conditions (Brown *et al.* 1983).

Continuing, GRC are examined at depths of 10, 20, 35, 100, 200, and 400 meters, considering horizontal-to-vertical in-situ stress ratios of $K=0.5$ and 1 and 1.5 and 2 on tunnel walls and roof in hard rock. Displacement values are compared using analytical and numerical methods through comparative graphs. After assessing differences in displacement at various depths and in-situ stress ratios, the limitations of the GRC method are identified. Subsequently, conditions where the analytical method is not applicable and where its discrepancy with the numerical method is significant are determined. An empirical equation is proposed to mitigate these limitations, allowing practical use at different depths and in-situ stress ratios. Finally, displacement graphs for the numerical and empirical methods are plotted, and the limitations of the empirical approach are examined.

2. Methodology

This article utilizes both analytical and numerical methods. In the analytical approach, the Duncan-Fama method is employed after reviewing various techniques. The Duncan-Fama method is used since it has been proven

to be reliable in simulating tunnel stability under Mohr-Coulomb criterion in hard rock environments. This method does an excellent job of modeling geomechanical interactions which allows one to accurately predict tunnel displacement analytically. This method, based on the linear criterion of Mohr-Coulomb failure, was introduced in 1993. The required parameters for plotting the GRC method include the modulus of elasticity, Poisson's ratio, internal friction angle, and rock mass compressive strength (Mahetaji *et al.* 2023, Ranjbarnia *et al.* 2020). The data of interest in the elastic region are examined according to Eq. (1).

$$u_r = -\frac{1+\nu}{E}(p_o - p_i)R \quad (1)$$

One of the important issues in choosing a numerical method is the conditions of discontinuities and the rock mass environment. Given that continuity is one of the assumptions of the problem, the FLAC software, which is one of the most practical geotechnical software available for numerical modeling in a continuous environment, is suitable for conducting this research. Since the third dimension is not considered in this research due to the assumption of the plane strain condition of the tunnel, the two-dimensional FLAC software is one of the most suitable software for numerical analysis. The FLAC software is a program based on the finite difference numerical method that operates based on Lagrangian analysis.

The modeling was conducted at depths of 10, 20, 35, 100, 200, and 400 meters in hard rock with $GSI=75$ and with in-situ stress ratios of 0.5, 1, 1.5, and 2. Considering the mentioned depths, the GRC falls within the elastic regions. The GRC was plotted analytically and on the roof and left wall of the tunnel using numerical methods.

Assumptions:

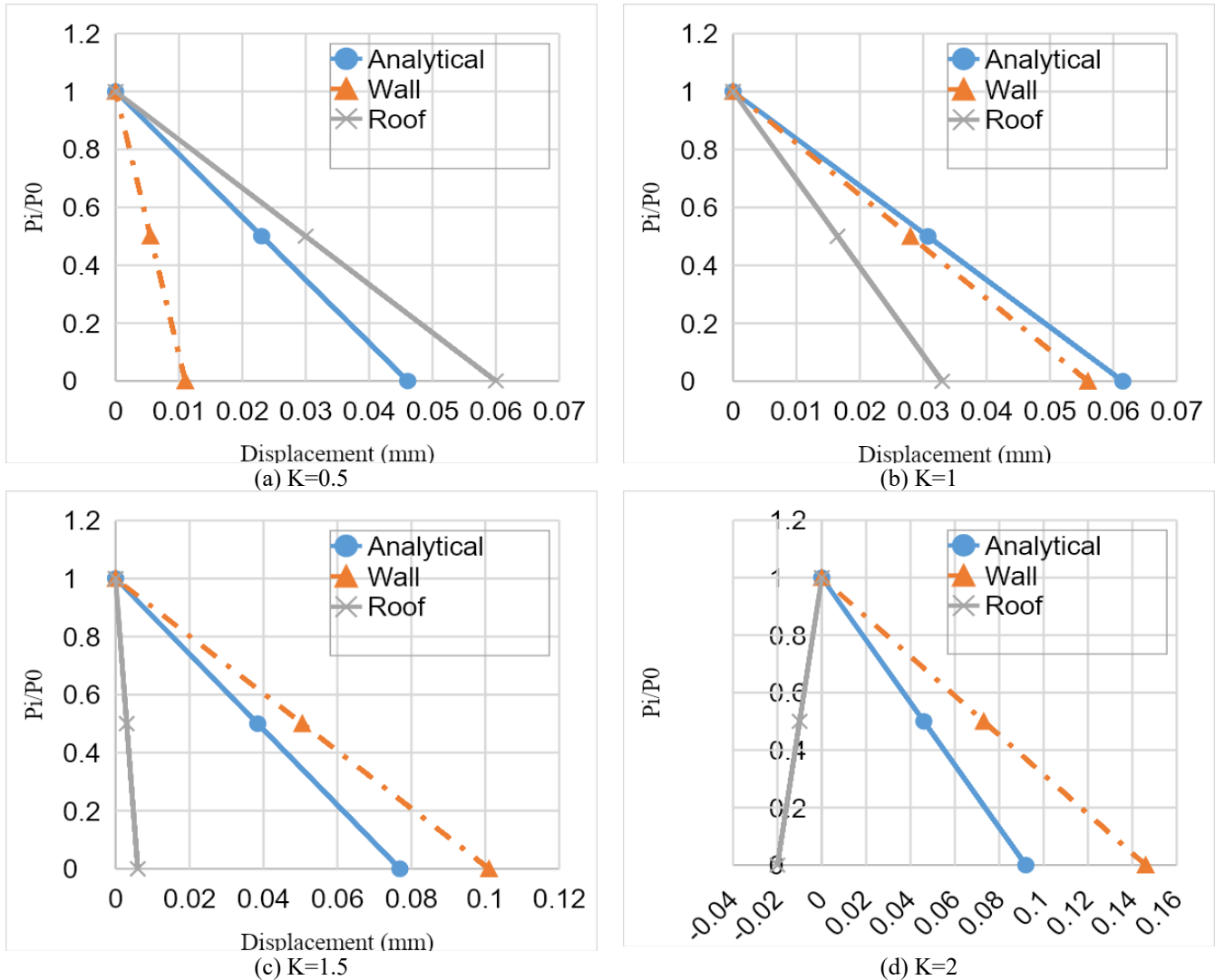


Fig. 2 GRC in hard rock environment, depth 10 m and $k=0.5$ and 1 and 1.5 and 2

- The models are two-dimensional and under plane strain conditions.
- The behavior of the materials in the environment is considered to be in hard rock with resistant properties.
- The Mohr-Coulomb failure criterion is used.
- The tunnel cross-section is assumed to be circular and excavated in a single stage.
- The load applied to the tunnel is due to the weight of the ground and the excavation in a rocky environment.
- There are no discontinuities or joints in the materials surrounding the tunnel.

Given the 10 m diameter of the tunnel, the optimal model dimensions were considered to be 60×60 meters with a total of 40,000 meshes. Also, considering the assumptions, a circular model was used; Fig. 1 shows the dimensions of the model and the tunnel before excavation. However, since the tunnel was also modeled at depths of 10 and 20 meters, the dimensions of the model in the y-direction are not necessarily 60 m and can be less.

Table 1 Properties of rock mass in GSI= 75

	Index	Unit	Value
Compressive strength of rock mass	σ_{cm}	MPa	47.317
Modulus of elasticity of rock mass	E_m	MPa	26939
special crime	ρ	Kg/m^3	2700
Poisson's ratio	ν	-	0.25
Angle of internal friction	ϕ	Degree	42.45
cohesion	C	MPa	10.421
Tensile strength of rock mass	σ_t	MPa	-1.072

3. Environments used and their parameters

This analysis was conducted in hard rock (GSI=75). Hypothetical parameters for the intact rock were established, input into RocData software, and based on Table 1, the required parameters for the rock mass were obtained from the software. The environment is situated in hard rock, and the material of the environment is considered to be sandstone with the hypothetical properties of intact rock as $GSI=75$, $\sigma_{ci}=120$ MPa, $m_i=17$, $D=0$, and $E_i=33,000$

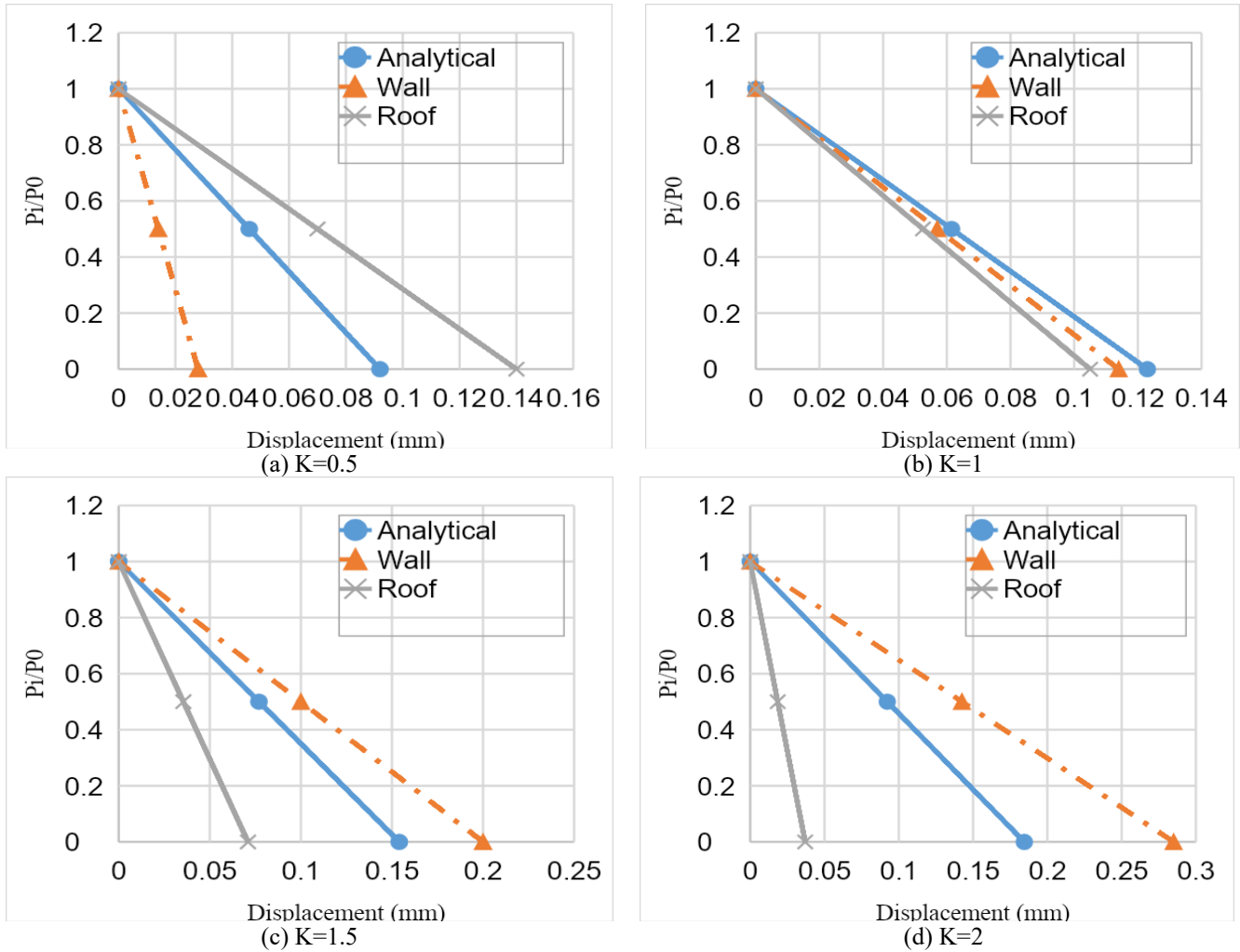


Fig. 3 GRC in hard rock environment, depth 20 m and k=0.5 and 1 and 1.5 and 2

MPa. Using RocData software, the characteristics of the rock mass were obtained as shown in Table 1.

4. Exploring of GRCs using numerical and analytical methods

After solving the desired parameters analytically and numerically, 24 graphs were plotted, each containing three curves: one using the analytical method and two using the numerical method, one at the roof and the other at the wall. Subsequently, the GRC in a hard rock environment was plotted at depths of 10, 20, 35, 100, 200, and 400 meters, with horizontal to vertical in-situ stress ratios of 0.5, 1, 1.5, and 2.

Fig. 2 shows the GRC at a depth of 10m. At K=0.5, it is observed that the maximum displacement obtained for the tunnel wall is significantly less than that for the roof and the analytical method, due to the greater vertical stress compared to horizontal stress. In other cases, the wall displacement is greater than the roof displacement. At K=1.5 and 2, the reason is the greater horizontal stress compared to vertical stress, and at K=1, the lower roof displacement is due to lower overburden pressure compared

to lateral pressure. At k=1, the displacement obtained for the tunnel wall in both the analytical and numerical methods is almost identical. However, for the tunnel crown, the shallow depth and surface excavation cause the numerical displacement to deviate from the analytical value, indicating less roof displacement at shallow depths. At in-situ stress ratios greater than 1, due to increased lateral stress effects, the GRC for the tunnel crown shifts to the left, such that at k=2, this displacement becomes negative and uplift is observed.

Fig. 3 shows the GRC at a depth of 20 m. At K = 0.5 with increasing depth, the displacement in the analytical method and the roof separate from each other. In isotropic stress, with increasing depth and the depth of overburden approaching the model dimensions, the displacement in the roof approaches the displacement in the wall and the analytical method, but the roof displacement is still less than the analytical and wall displacement. However, given the small difference, the results are acceptable. With increasing depth at in-situ stress ratios above 1, the roof displacement increases and uplift is not observed, which is due to the increase in overburden pressure. Also, the roof displacement approaches the displacement of the analytical method and the wall.

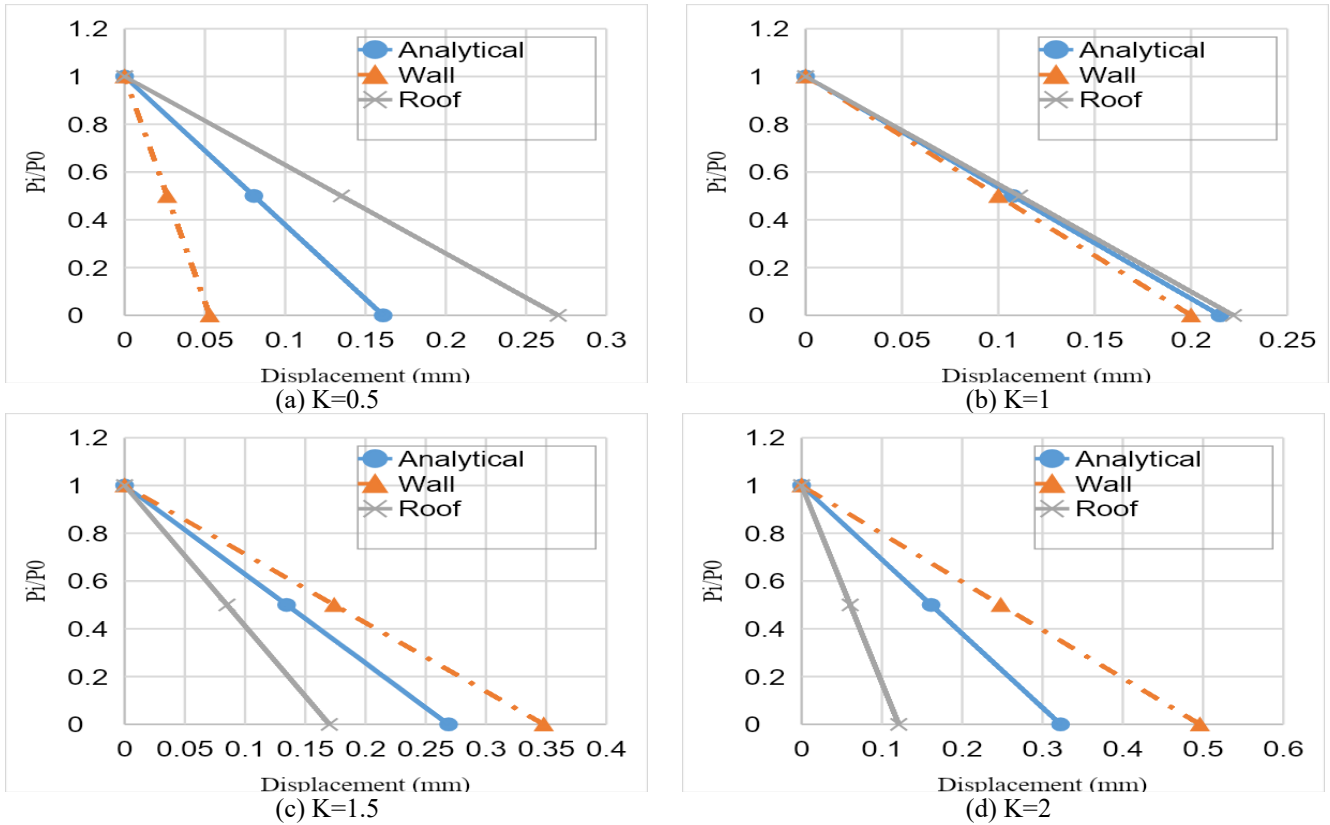


Fig. 4 GRC in hard rock environment, depth 35 m and $k=0.5$ and 1 and 1.5 and 2

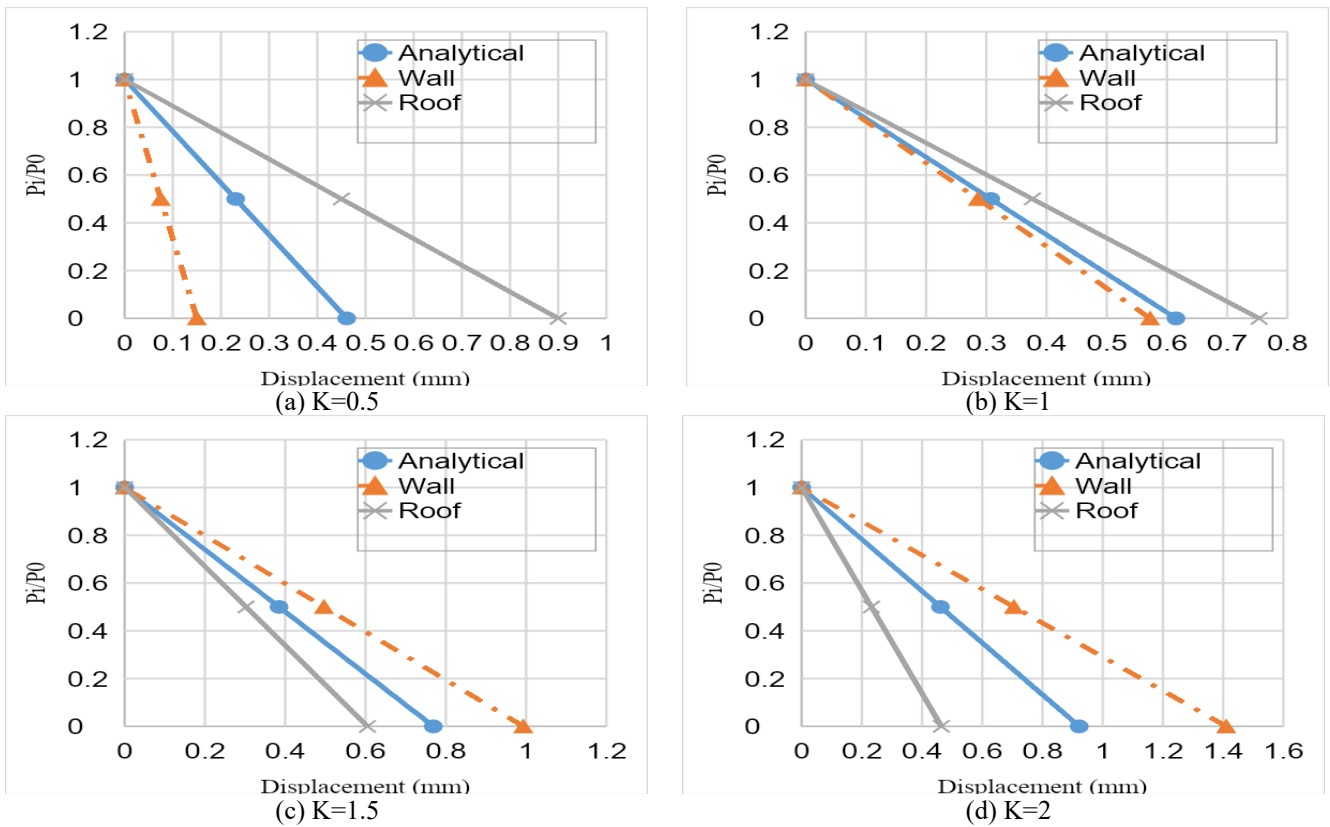


Fig. 5 GRC in hard rock environment, depth 100 m and $k=0.5$ and 1 and 1.5 and 2

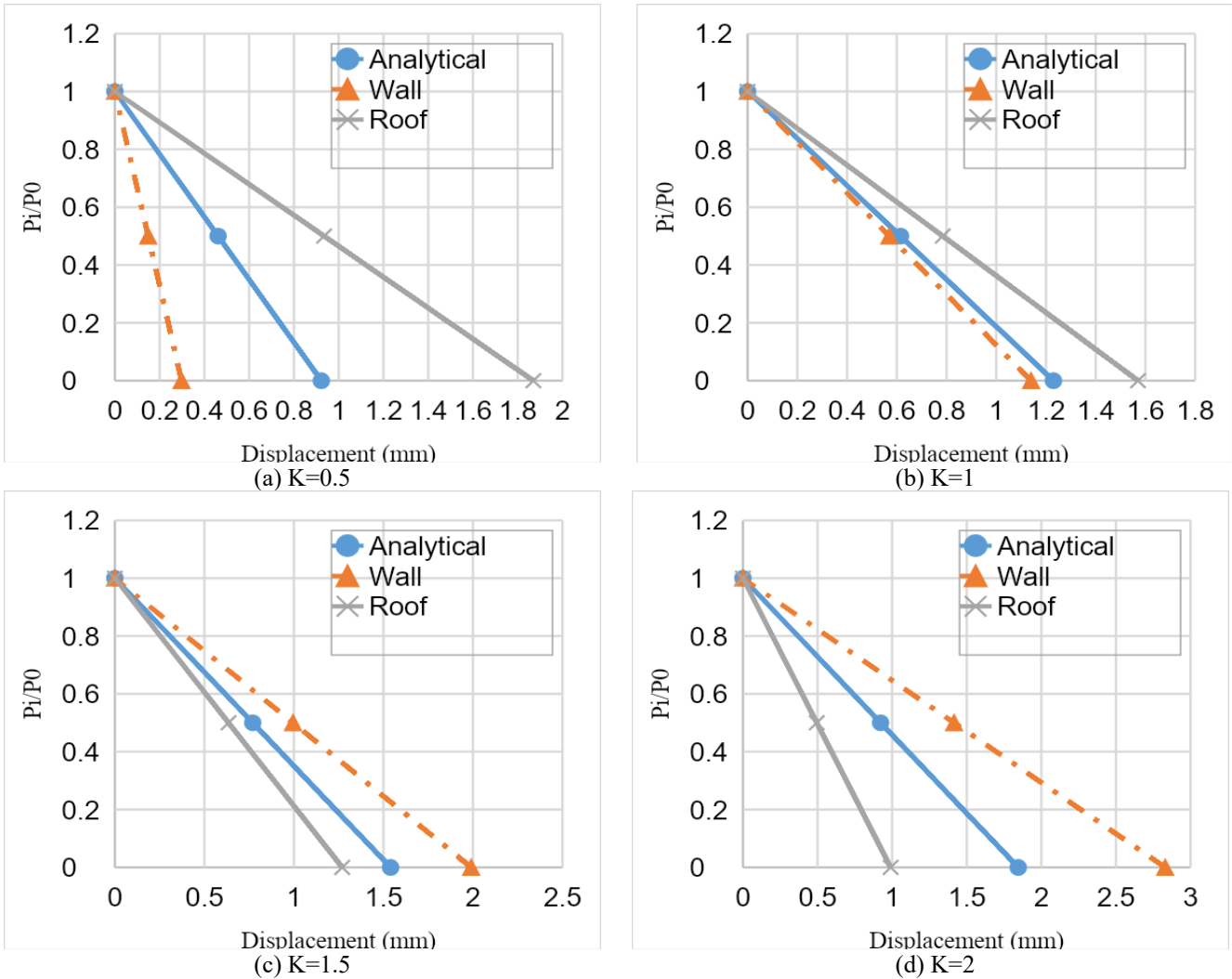


Fig. 6 GRC in hard rock environment, depth 200 m and $k=0.5$ and 1 and 1.5 and 2

Fig. 4 shows the GRC at a depth of 35 m. At $K=0.5$ due to lower horizontal stress, the tunnel roof displacement is greater compared to the tunnel wall and the analytical method. At isotropic stress, because the tunnel depth is greater than the mesh dimensions, as shown in the figure, the roof displacement is greater than the wall displacement. In the hydrostatic stress ratio ($k=1$), considering the mesh dimensions, at lesser depths, the wall displacement is greater, and at greater depths, the roof displacement is greater. At $K=1.5$ and 2, considering the increased depth, the GRC at the roof becomes closer to the GRC at the wall and the analytical method at lesser depths. However, due to higher in-situ horizontal stress, the displacement at the wall is greater, resulting in a significant and unacceptable discrepancy.

Fig. 5 shows the GRC at a depth of 100 m. With increasing depth at $K=0.5$ it is evident that the displacement ratio between the sidewall and the analytical method does not differ significantly compared to shallower depths. However, increasing depth results in an increase in the displacement ratio on the roof compared to the analytical method. As depicted in the figure, at $K=1$, with increasing depth, the displacement ratio on the roof relative to the

sidewall and the analytical method has increased. At $K=1.5$ and 2, displacements converge to a similar extent, but do not differ significantly from shallower depths.

Fig. 6 shows the GRC at a depth of 200 m. At $K=0.5$ significant differences between analytical and numerical methods are observed at shallower depths. At $K=1$ there is little difference in displacement between the analytical method and the wall, but there is some displacement in the roof due to greater tunnel arching relative to the model dimensions applied as pressure in the software. As this value increases, the ratio of roof displacement to wall displacement and analytical method increases. With increasing stress ratio, displacements differ more, and since the pressure is higher along the wall, wall displacement will be greater.

Fig. 7 shows the GRC at a depth of 400 m. At $K=0.5$ significant discrepancies between numerical and analytical methods are observed in deeper hard rock, particularly in the roof and wall, which is unacceptable, possibly due to increased vertical in-situ stress and tunnel depth. At $K=1$ at greater depths, the GRC is similar between the wall and analytical method, but there are differences with the roof curve due to depth, which increase further with depth. At

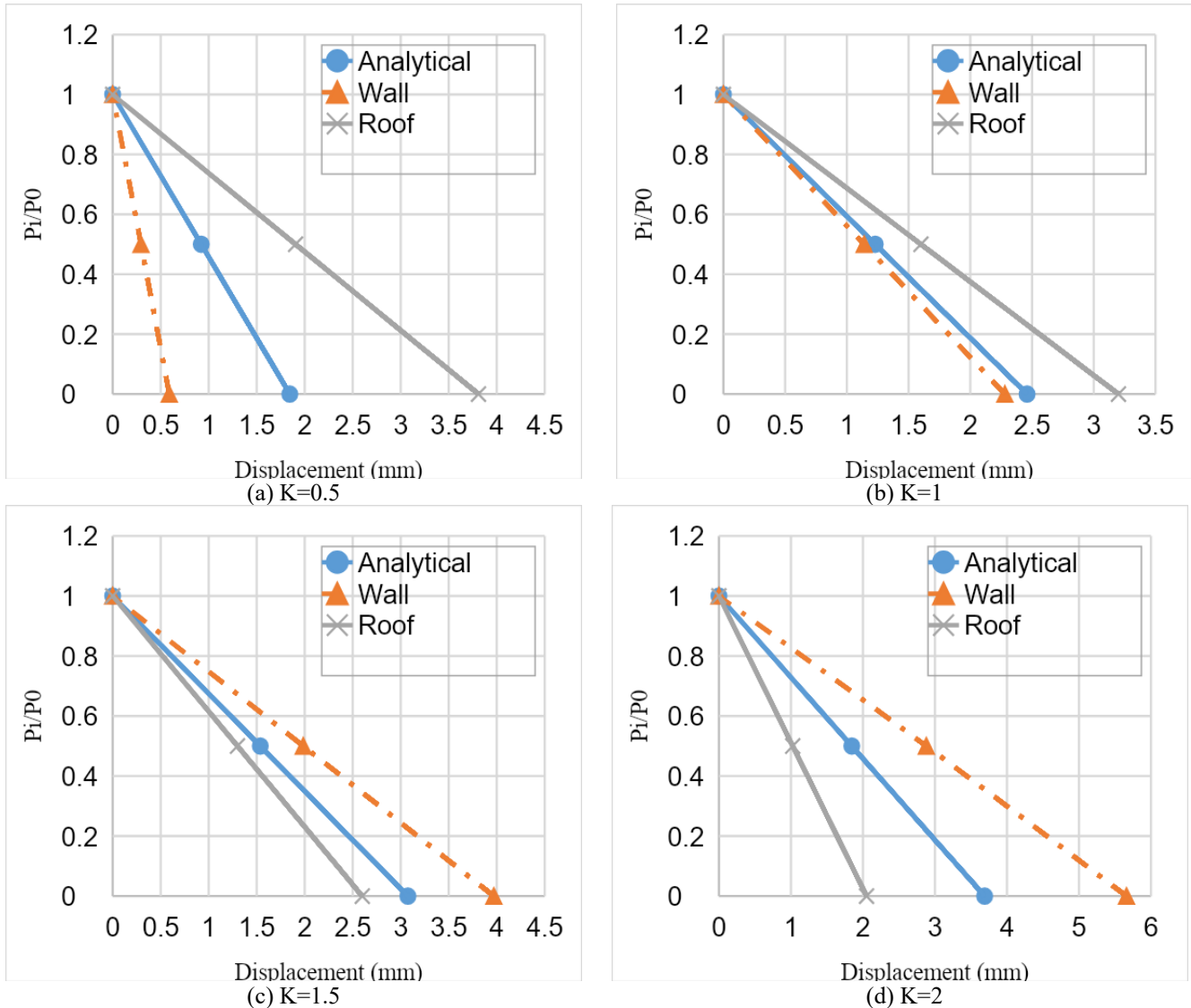


Fig. 7 GRC in hard rock environment, depth 400 m and $k=0.5$ and 1 and 1.5 and 2

greater depths and $K > 1$ in hard rock, the greater the in-situ stress ratio, the greater the disparities in displacements will be.

Given that the analysis is conducted in hard rock, all GRCs are placed within the elastic region, and for this reason, the curves are straight lines and due to the elasticity of the displacements, there is no curvature in the curves.

5. Exploring of analytical method limitations for GRCs

The primary objective of this study is to examine the limitations of the GRC drawing in the analytical method for tunnels. To achieve this, a comparison between analytical and numerical solutions is conducted by introducing the parameter S (the maximum displacement ratio of numerical to analytical methods) and considering the effects of the initial in-situ stress ratio (K) and tunnel depth.

5.1 Exploring of Limitations of Tunnel Wall GRC

Based on the conducted modeling and obtained maximum displacements, graphs were plotted to compare the S ratio (the maximum displacement ratio of numerical to analytical methods) and the in-situ stress ratio for tunnel walls at various depths. Figs. 8 and 9 illustrate these graphs plotted against S and excavation depth for different in-situ stress ratios.

Fig. 8 depicts the discrepancy between results from the numerical and analytical solution methods for tunnel walls in hard rock, plotted against the in-situ stress ratio K . In hard rock, the graphs are nearly aligned at all depths, indicating close agreement between numerical and analytical solutions under homogeneous stress conditions, with an S ratio of 0.93. However, with variations in initial in-situ stress ratios, the analytical solution does not yield satisfactory results. It is evident that as the comparative results between numerical and analytical solutions converge, the S ratio approaches 1. When the S value deviates further from 1, the results of the numerical and analytical solutions diverge, rendering the analytical

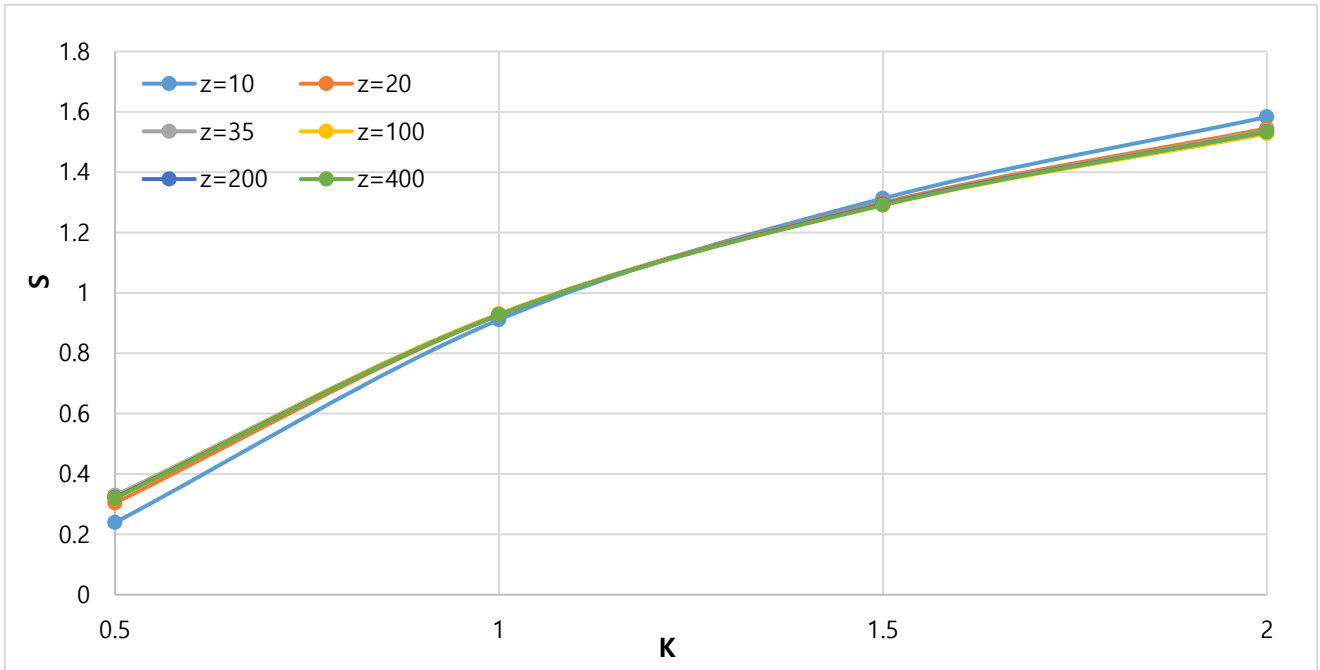


Fig. 8 Displacement ratio of numerical method to analytical method (S) in the tunnel wall at different depths

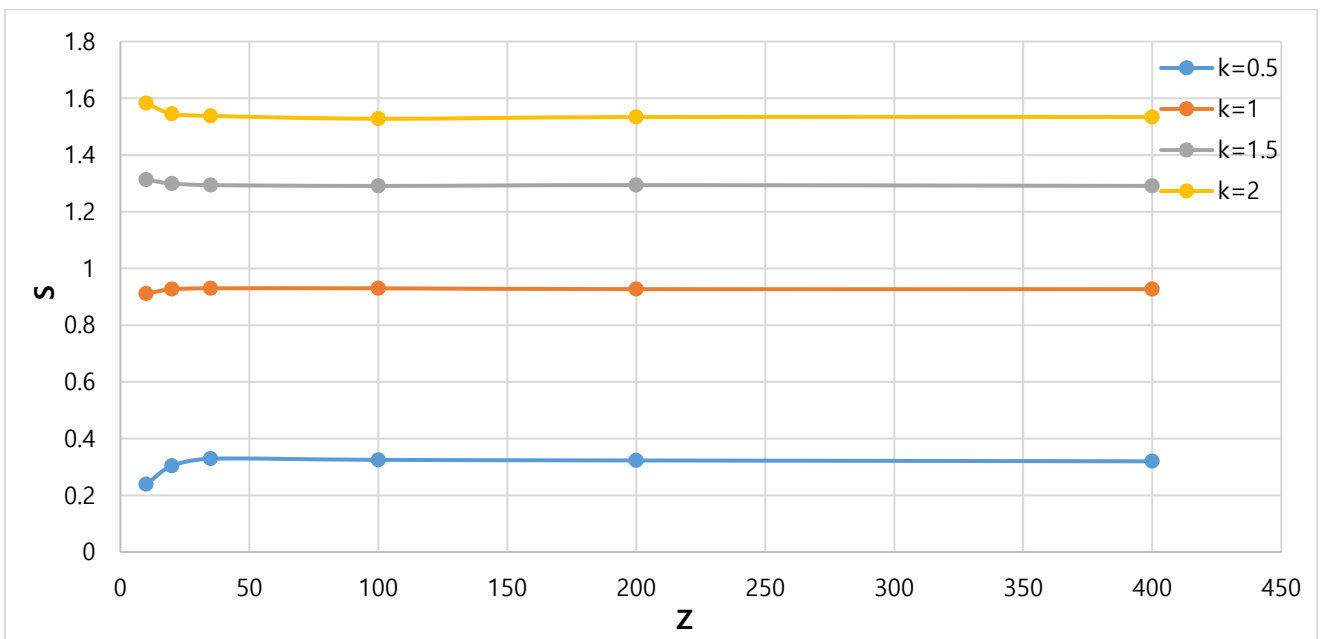


Fig. 9 Displacement ratio of numerical method to analytical method (S) in the tunnel wall at different K

solution unacceptable. Additionally, as shown in Fig. 8, for all depths, analytical displacement is greater than numerical for $K < 1$, while numerical displacement is greater than analytical for $K > 1$. Significant differences between numerical methods and analytical solutions at $K = 0.5$ and $K = 2$ have resulted in S ratios of approximately 0.30 and 1.55, respectively. In other words, analytical solution results will not be sufficiently credible at these stress ratios.

In Fig. 9, it is observed that in hard rock, with increasing excavation depth under elastic conditions, there are minimal variations between the numerical and analytical methods. This indicates that the influence of K on final displacements

is more significant and crucial compared to excavation depth. Additionally, for the isotropic stress field ($K = 1$), both numerical and analytical solutions for the tunnel wall are consistent with each other, independent of depth.

5.2 Exploring the limitations of tunnel roof GRC

To examine the constraints of the crown tunnel, similar diagrams to those drawn for the tunnel wall have been created to analyze the results of the crown tunnel. Graphs based on the ratio of S and K for the crown tunnel at different depths, as well as a graph based on the ratio of S and tunnel depth for different K, have been plotted.

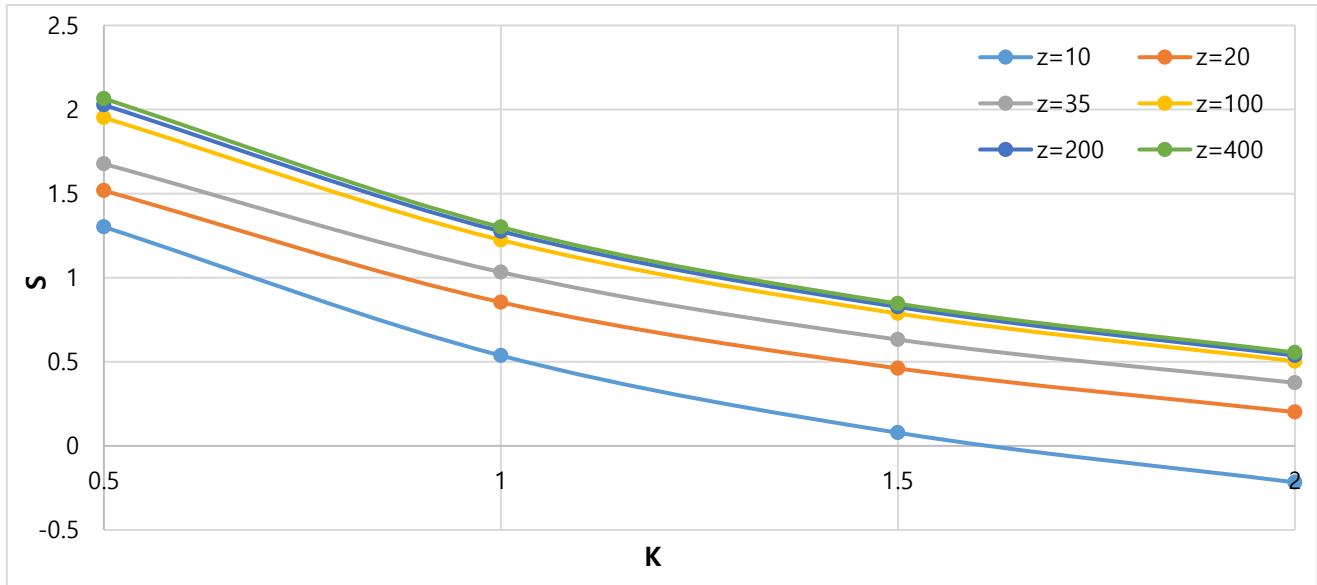


Fig. 10 Displacement ratio of numerical method to analytical method (S) in the tunnel roof, at different depths

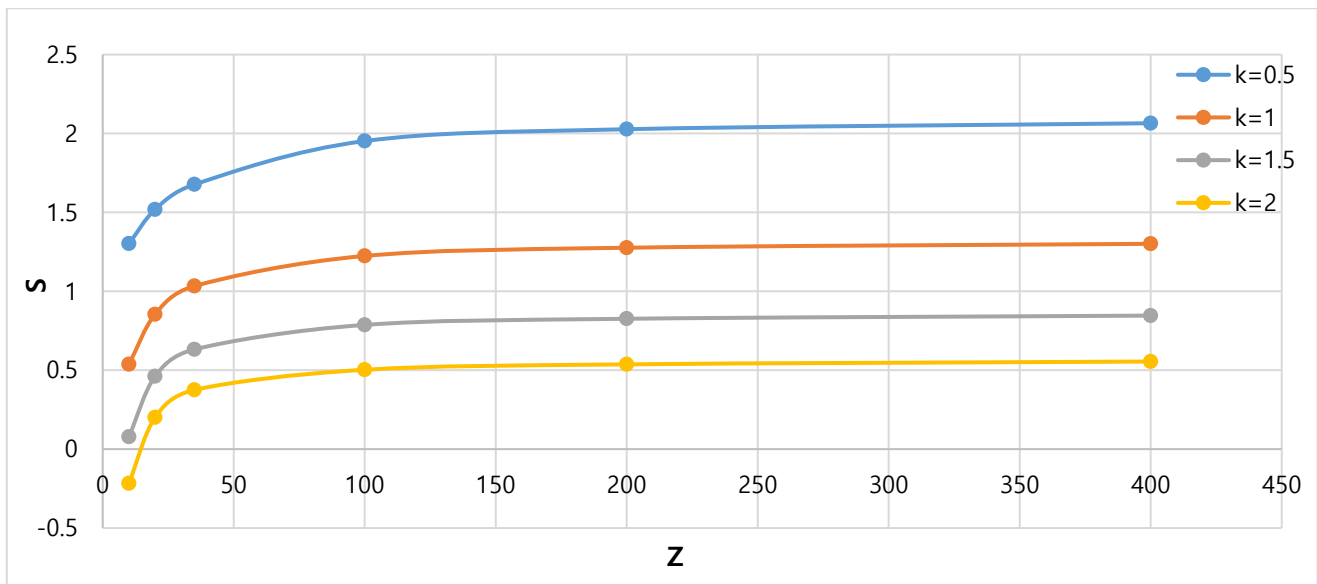


Fig. 11 Displacement ratio of numerical method to analytical method (S) in the tunnel roof at different K

Fig. 10 illustrates that the displacement of the crown under isotropic conditions, particularly at shallow depths, does not conform well to that of the tunnel wall. This discrepancy is because drilling depth has a greater effect on crown displacement compared to wall displacement. Under in-situ stress conditions, when the load is less than 30 meters, which equals the model dimensions, the numerical-to-analytical solution ratio is less than 1. Conversely, when the load exceeds 30 meters, the numerical-to-analytical solution ratio exceeds 1. Additionally, it should be noted that at $K=2$ and a depth of 10 meters, due to the shallow tunnel depth, crown displacements become negative, meaning that the crown tunnel moves upward instead of downward. This issue arises because the confining pressure in the horizontal direction has been sufficiently large. Based on the above considerations and the provided figure, it is concluded that the analytical method does not present

acceptable displacements compared to the roof displacements.

Fig. 11 shows that in the elastic region, drilling depth has a greater effect on crown displacements compared to tunnel sidewalls, especially with much greater displacements at shallower depths. Under isotropic stress conditions in the elastic region, analytical and numerical solutions diverge more at greater depths. The best match for crown displacement and the analytical method in the elastic region at greater depths is at $K=1.3$. Additionally, with increasing K , the value of S significantly decreases

6. Presentation of an empirical equation

Given the discrepancy between the analytical and numerical methods, particularly in non-isotropic in-situ

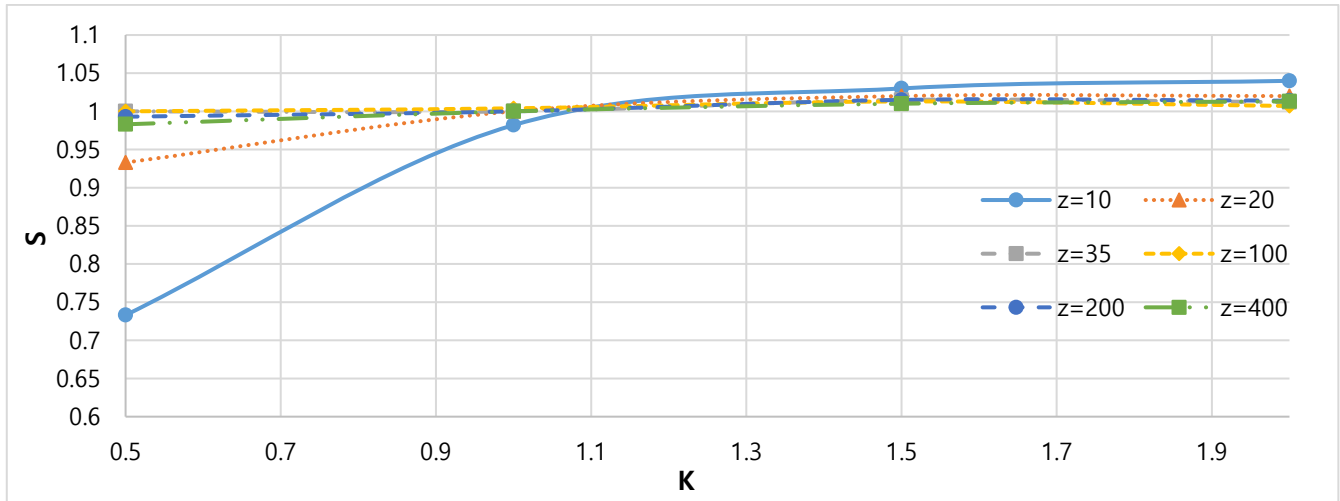


Fig. 12 Displacement ratio of numerical method to experimental equation (S) in the tunnel wall at different depths

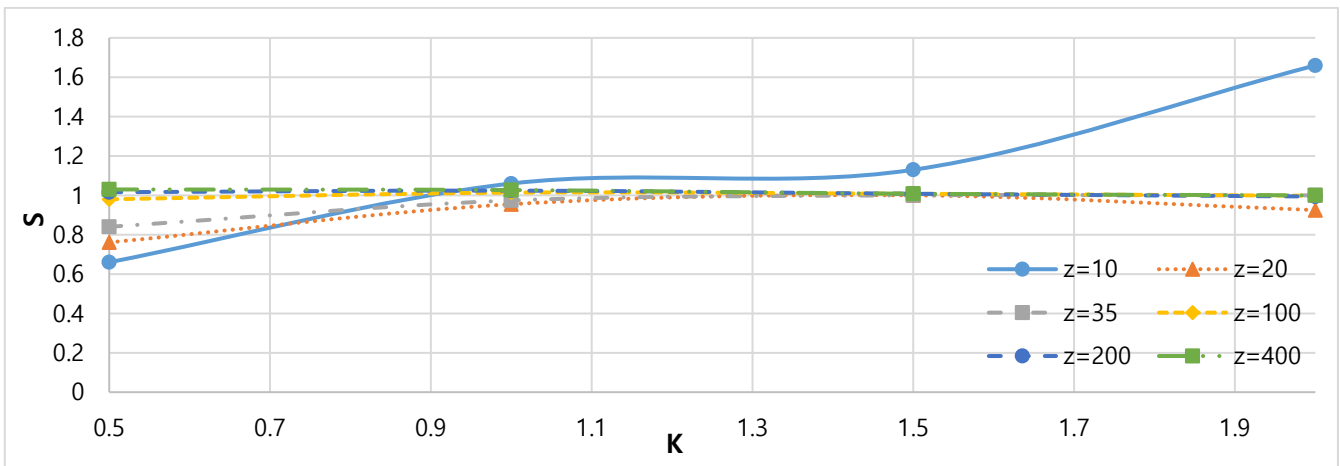


Fig. 13 Displacement ratio of numerical method to experimental equation (S) in the tunnel roof at different K

stress ratios, and the significant displacement differences at the crown, we aimed to develop a new empirical equation. This equation is intended to reduce the displacement difference between the analytical and numerical methods while enhancing the accuracy of the analytical method. Moreover, it can be applied across various K values. These equations were derived using Excel software, particularly in cases requiring dual regression. Eqs. (2) and (3) represent displacement in the tunnel sidewall and tunnel crown under elastic conditions, respectively.

$$u_{xe} = \frac{Z}{E} (2.24 - 0/716) \times 10^8 \quad (2)$$

$$u_{ye} = \frac{Z - (k^2 - 1/5k - \frac{6/1}{k} + 12/7)}{E - 0/07K} \times 10^9 \quad (3)$$

In Eqs. (2) and (3), a tunnel radius of 5 and a Poisson's ratio of 0.25 have been assumed. To formulate a general equation, the effects of tunnel radius and Poisson's ratio must also be considered.

As seen in Fig. 12, according to Eq. (2), which represents displacement in the tunnel sidewall under elastic conditions, the ratio of numerical to empirical results

approaches 1 across all in-situ stress ratios. This indicates significantly higher accuracy of the empirical equation compared to the equation presented in the analytical method.

Fig. 13 illustrates the ratio of numerical to empirical results on the tunnel roof under elastic conditions. Considering the influence of K on parameter Z, the discrepancy between the numerical and empirical methods has slightly widened. However, the empirical equation presented here demonstrates significantly greater accuracy compared to the equation provided by the analytical method. Unlike the analytical approach, it clearly shows an increase in its findings.

These findings suggest that it is possible to construct a more accurate equation in the elastic region, capable of being used for various in-situ stress ratios.

7. Conclusions

The GRC method, analyzed using both analytical and numerical approaches, was examined on the tunnel's wall and roof. Considering different depths and anisotropic stress

conditions, elastic displacement occurred, and the results obtained are detailed below.

- With an increasing in-situ stress ratio, displacement on the tunnel walls increased in all cases and at different depths. However, varied conditions were observed on the tunnel roof due to lateral stress increases, resulting in some cases of uplift. Displacement on the roof decreased with an increase in the in-situ stress ratio.
- At depths less than the dimensions of the mesh, wall displacement increased more significantly compared to the roof. However, at depths exceeding 30 meters, roof displacement surpassed that of the wall, resulting in a steeper slope in roof displacement with increasing depth.
- Under elastic conditions in the tunnel wall, numerical and analytical modeling solutions were nearly congruent under isotropic stress and hydrostatic conditions at all depths. The value of the S ratio under isotropic stress was approximately 0.93. The approximate values of the S ratio for in-situ stress ratios of 0.5, 1.5, and 2 were respectively 0.30, 1.3, and 1.55 at all depths. Checking the results for the tunnel wall showed that the numerical-to-analytical ratio diagrams for in-situ stress ratios were somewhat congruent, indicating that the depth of the rock had little effect on the numerical-to-analytical ratio in the elastic range of the tunnel wall, and only the stress ratio (k) was influential. As the in-situ stress ratio deviates from the isotropic value in the tunnel wall, the numerical and analytical solutions also diverge, and an accurate and acceptable analytical solution is not obtained.
- Under isotropic stress conditions, displacement in the crown of a shallow tunnel (less than 30 meters) did not conform well to the tunnel wall with an $S=1$. This discrepancy is because drilling depth has a greater effect on crown displacement compared to wall displacement, especially at shallower depths. When the span dimensions of the model, which are 30 meters from the tunnel center, decrease, the numerical-to-analytical ratio (S) is less than 1. Conversely, when the span dimension exceeds 30 meters, S becomes greater than 1. For large depths under isotropic stress conditions, analytical and numerical solutions diverge. Furthermore, with an increase in K, the S ratio decreases significantly.
- At $K=2$ and a depth of 10 meters, due to the shallow tunnel depth, crown displacements become negative, indicating upward movement of the tunnel crown instead of downward, caused by sufficiently large confining pressure in the horizontal direction.
- Considering the limitations mentioned in the analytical method, efforts have been made to present more accurate equations capable of closer calculations in various in-situ stress ratios at roof and wall points. Equations were developed for the tunnel wall and roof in elastic environments based on numerical method data.
- For the tunnel wall, the ratio of numerical to empirical results was close to 1 at all K values, indicating

significantly greater accuracy of the empirical equation compared to the analytical method.

- For the tunnel roof, the ratio of numerical to empirical results diverged slightly. However, even here, the presented empirical equation shows much greater accuracy compared to the analytical method, and contrary to the analytical method, uplift is evident.
 - The formulated equations can be utilized in tunnels forecasting to determine movements more precisely and under different stress conditions, thereby assisting in support structures design and evaluation of risk. They, for example, are especially useful in anisotropic stress with shallow tunnels where great movements are not captured well by normal methods. This enhances the matching of the design parameters and the observed ground behavior.
 - In summary, it is possible to create more precise equations applicable to the elastic region, considering different in-situ stress conditions in both tunnel walls and roof. Further investigation should account for variations in rock hardness, tunnel radius, and Poisson's ratio.
 - Although the equations yield favorable results in elastic situations and shallow waters, their reliability may be reduced in rock compositions that are greatly fragmented or in settings that possess intricate discontinuities which were not addressed in this research.
 - Suggested directions include applying the proposed empirical equations to different rock types, integrating artificial intelligence techniques for more adaptive modeling, and validating the models with field data from ongoing tunneling projects.

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