

Influence of inclined load, locality, and gravity on a thermoelastic medium with temperature-dependent properties using Lord-Shulman model

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Abstract. The impact of inclined load on a nonlocal thermoelastic solid is discussed in this work. Based on the Lord-Shulman model, the fundamental equations for a nonlocal thermoelastic half-space medium are developed. Given as a linear function of the reference temperature is the modulus of elasticity. To address the problem and acquire the exact expressions of physical fields, appropriate non-dimensional variables and normal mode analysis are used. The anticipated outcomes for various gravity field values, the nonlocal parameter, the empirical material constant, and the inclined load are compared. Physical variables are affected by nonlocal thermoelasticity, the empirical material constants, and the inclined load.

Keywords: gravity field; inclined load; nonlocal parameter; temperature-dependent properties

1. Introduction

The classical thermoelasticity theory, which was first presented by Biot (1965) and is based on Fourier's equation of heat conduction, it is flawed in that it does not allow for the admission of thermal signals that propagate at infinite speed. To resolve this contradiction, generalized theories have been developed during the past few decades that include a finite speed of heat transportation (hyperbolic heat transport equation) in elastic substances. Lord and Shulman (L-S) (1967) provided the first generalization theory, referred to as the extended thermoelasticity theory, which has a single thermal relaxation time parameter. The Lord and Shulman energy equation includes the strain's first and second-time derivatives. Rather than using the law of Fourier analysis, they looked at a novel law of heat conduction. Based on their theory, the linear correlation between temperature and heat flux includes temperature rate and thermal rates. According to this theory, the temperature propagation speed is finite due to the hyperbolic heat equation. Actually, as is well known, the term "generalized" usually refers to thermodynamic theories based on hyperbolic (wave-type) heat equation, so that a finite propagation speed for thermal signals is admitted. This theory was extended by Dhaliwal and Sherief (1980) to include the anisotropic case. The uniqueness of the solution to this theory was proved under different conditions by Ignaczak (1979, 1982), by Dhaliwal and Sherief (1980), and by Sherief (1987). The model of the equations of generalized thermoelasticity based on Lord-Shulman theory in an isotropic elastic medium under the dependence of the modulus of elasticity on the reference temperature is established by Othman (2002). The

generalized thermoelasticity has drawn extensive attention due to its applications in diverse fields such as earthquake engineering, nuclear reactor design, and high-energy particle accelerators. The L-S theory has lately been established in numerous studies covering a wide range of topics, such as Othman (2002), Baksi *et al.* (2005), Othman and Said (2012), Ailawalia and Singla (2015), Youssef and El-Bary (2021), Marin *et al.* (2014, 2020, 2021), Saeed (2022), Said and Othman (2024).

Kumar *et al.* (2005, 2010) investigated different problems in the micropolar elastic medium due to inclined load. Sharma *et al.* (2015) investigated the two-dimensional deformation in homogeneous, transversely isotropic thermoelastic solids with two temperatures in the context of the Green-Naghdi theory of type II as a result of an inclined load. Zenkour *et al.* (2016) discussed the thermoelastic interaction due to inclined load on a homogeneous isotropic half-space in the context of the two-temperature generalized theory of thermoelasticity with dual-phase-lags. Othman *et al.* (2017) discussed the effect of inclined load on a micropolar thermoelastic medium possessing cubic symmetry with energy dissipation. In studies by Lata and Singh (2019), Alharbi (2021), Barak and Dhankhar (2022), Marin *et al.* (2022, 2024), Das *et al.* (2024), and Purkait and Kanoria (2023), you can find more significant papers on the subject.

Noda (1986) conducted a thorough investigation into thermal stress in a material having temperature-dependent characteristics. Temperature affects a material's elasticity modulus and thermal conductivity, among other characteristics. The characteristics of materials remain consistent when there is little movement in the temperature from the initial load. Ezzat *et al.* (2004) examined how the modulus of elasticity in generalized thermoelasticity with thermal relaxation depends on the reference temperature. The equation of generalized thermo-piezoelectricity in an isotropic elastic media with temperature-dependent

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mechanical properties was established by Aouadi (2006). The problem of generalized thermoelastic media with temperature-dependent properties for various theories under the influence of gravity field was first presented by Othman *et al.* (2013). In studies by Said and Othman (2016), Vlase *et al.* (2017), Abouelregal (2018), Sheoran *et al.* (2021), Kaur *et al.* (2022), and Said (2023), you can find more significant papers on the subject.

This work examines the impact of the gravity field, location, inclined load, and temperature-dependent features on a thermoelastic half-space medium. The Lord-Shulman (L-S) model with one relaxation time is used to discuss the issue. Normal mode analysis is utilized to solve the ensuing non-dimensional equations. The anticipated outcomes for various gravity field values, the nonlocal parameter, empirical material constants, and the inclined load are compared.

2. Formulation of the problem

The problem of a nonlocal thermoelastic half-space medium under effects of the gravity field and initial stress. We are interested in the Cartesian coordinates x and z with the dynamic displacement vector is given as $\underline{u} = (u, 0, w)$, $v = 0$, $\frac{\partial}{\partial y} = 0$.

The constitutive equations as Hetnarski and Eslami (2009) and Eringen (2002)

$$(1 - \varepsilon^2 \nabla^2) \sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma \theta \delta_{ij}. \quad (1)$$

The heat conduction equation as Lord and Shulman (1967)

$$K \nabla^2 \theta = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \rho C_E \dot{\theta} + \gamma T_0 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \dot{\varepsilon}. \quad (2)$$

We assume that as Said (2023)

$$\begin{aligned} \mu &= \mu_1 (1 - \theta_0 T_0), & \lambda &= \lambda_1 (1 - \theta_0 T_0), \\ \gamma &= \gamma_1 (1 - \theta_0 T_0). \end{aligned} \quad (3)$$

In the case of the temperature-independent modulus of elasticity $\theta_0 = 0$.

The equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + F_i, \quad (4)$$

where $F_1 = \rho g \frac{\partial w}{\partial x}$, $F_2 = 0$, $F_3 = -\rho g \frac{\partial u}{\partial z}$ are force due to the presence of the gravity field.

Introducing Eqs. (1) in Eqs. (4), we get

$$\rho(1 - \varepsilon^2 \nabla^2) \ddot{u} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 w}{\partial x \partial z} + \mu \frac{\partial^2 u}{\partial z^2} -$$

$$\gamma \frac{\partial \theta}{\partial x} + \rho g (1 - \varepsilon^2 \nabla^2) \frac{\partial w}{\partial x},$$

$$\rho(1 - \varepsilon^2 \nabla^2) \ddot{w} = (\lambda + 2\mu) \frac{\partial^2 w}{\partial z^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial x \partial z} + \mu \frac{\partial^2 w}{\partial x^2} -$$

$$\gamma \frac{\partial \theta}{\partial z} - \rho g (1 - \varepsilon^2 \nabla^2) \frac{\partial u}{\partial x}.$$

For convenience, we introduce the non-dimensional variables as:

$$(x', z', \varepsilon', u', w') = \frac{1}{l_0} (x, z, \varepsilon, u, w), \quad (t', \tau'_0, \tau'_1, \tau'_2) = \frac{d_0}{l_0} (t, \tau_0, \tau_1, \tau_2),$$

$$\theta' = \frac{\gamma \theta}{(\lambda + 2\mu)}, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\mu}, \quad g' = \frac{l_0}{d_0} g, \quad (7)$$

$$l_0 = \sqrt{\frac{K}{\rho C_E T_0}}, \quad d_0 = \sqrt{\frac{\lambda + 2\mu}{\rho}}.$$

Introducing Eqs. (7) in Eqs. (5), (6), and (2), we get

$$(1 - \varepsilon^2 \nabla'^2) \ddot{u} = \frac{\partial^2 u}{\partial x'^2} + A_1 \frac{\partial^2 w}{\partial x' \partial z'} + A_2 \frac{\partial^2 u}{\partial z'^2} - \frac{\partial \theta}{\partial x'} + g (1 - \varepsilon^2 \nabla'^2) \frac{\partial w}{\partial x'}, \quad (8)$$

$$(1 - \varepsilon^2 \nabla'^2) \ddot{w} = A_2 \frac{\partial^2 w}{\partial x'^2} + A_1 \frac{\partial^2 u}{\partial x' \partial z'} + \frac{\partial^2 w}{\partial z'^2} - \frac{\partial \theta}{\partial z'} - g (1 - \varepsilon^2 \nabla'^2) \frac{\partial u}{\partial x'}, \quad (9)$$

$$\nabla'^2 \theta = \left(1 + \tau_0 \frac{\partial}{\partial t'}\right) A_3 \dot{\theta} + A_4 \left(1 + \tau_0 \frac{\partial}{\partial t'}\right) \dot{\varepsilon}, \quad (10)$$

where

$$A_1 = \frac{\lambda + \mu}{\rho d_0^2}, \quad A_2 = \frac{\mu}{\rho d_0^2}, \quad A_3 = \frac{\rho C_E d_0 l_0}{K},$$

$$A_4 = \frac{\gamma^2 T_0 d_0 l_0}{K (\lambda + 2\mu)}, \quad A_5 = \frac{l_0^2}{K}.$$

3. Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal modes, as Othman *et al.* (2013)

$$(u, w, \theta, \sigma_{ij})(x, z, t) = (u^*, w^*, \theta^*, \sigma^*_{ij})(z) \exp (mt + iax) \quad (11)$$

where $u^*(z)$, etc. is the amplitude of the function $u(x, z, t)$ etc.

Introducing Eqs. (11) in Eqs. (8)-(10), we obtain

$$(N_1 D^2 - N_2) u^* + ia(-g \varepsilon^2 D^2 + A_1 D + N_3) w^* - ia N_4 \theta^* = 0, \quad (12)$$

$$ia(g \varepsilon^2 D^2 + A_1 D - N_3) u^* + (N_5 D^2 - N_6) w^* - N_4 D \theta^* = 0, \quad (13)$$

$$ia N_7 u^* + N_7 D w^* + (N_9 - D^2) \theta^* = 0, \quad (14)$$

where

$$N_1 = \varepsilon^2 m^2 + A_2, \quad N_2 = a^2 + m^2(1 + a^2 \varepsilon^2), \quad N_3 = g(1 + a^2 \varepsilon^2),$$

$$N_4 = 1, \quad N_5 = \varepsilon^2 m^2 + 1, \quad N_6 = A_2 a^2 + m^2(1 + a^2 \varepsilon^2),$$

$$N_7 = A_4 m(1 + m \tau_0), \quad N_8 = A_3 m(1 + m \tau_0), \quad N_9 = N_8 + a^2, \quad D = \frac{d}{dz}.$$

Obviate $w^*(z)$ and $\theta^*(z)$ between Eqs. (12)-(14), we get:

$$(D^6 - C_1 D^4 + C_2 D^2 - C_3) u^*(z) = 0. \quad (15)$$

The bound solution of Eq. (15), as $z \rightarrow \infty$, is

$$u^*(z) = \sum_{j=1}^3 M_j \exp(-k_j z). \quad (16)$$

Similarly

$$w^*(z) = \sum_{j=1}^3 H_{1j} M_j \exp(-k_j z), \quad (17)$$

$$\Phi^*(z) = \sum_{j=1}^3 H_{2j} M_j \exp(-k_j z). \quad (18)$$

Using the above equations, we get

$$\theta^*(z) = \sum_{j=1}^3 H_{3j} M_j \exp(-k_j z), \quad (19)$$

$$\sigma_{zz}^*(z) = \sum_{j=1}^3 H_{4j} M_j \exp(-k_j z), \quad (20)$$

$$\sigma_{xz}^*(z) = \sum_{j=1}^3 H_{5j} M_j \exp(-k_j z), \quad (21)$$

where k_j^2 ($j=1,2,3$) are the roots of the characteristic equation: $(k^6 - C_1 k^4 + C_2 k^2 - C_3 = 0)$.

$$C_0 = g^2 a^2 \varepsilon^4 - N_1 N_5,$$

$$C_1 = \frac{1}{C_0} \{ 2g N_3 a^2 \varepsilon^2 + g^2 a^2 \varepsilon^4 N_{10} + A_1^2 a^2 - N_1 N_4 N_7 - N_1 N_6 - N_2 N_5 - N_1 N_5 N_{10} \},$$

$$C_2 = \frac{1}{C_0} \left\{ 2g N_9 N_3 a^2 \varepsilon^2 + 2A_1 a^2 N_4 N_7 + A_1^2 a^2 N_9 + N_3^2 a^2 - N_2 N_4 N_7 - \right. \\ \left. N_2 N_6 - N_9 N_1 N_6 - N_2 N_5 N_{10} - a^2 N_4 N_5 N_7 \right\},$$

$$C_3 = \frac{1}{C_0} \{ N_9 N_3^2 a^2 - N_2 N_6 N_9 - a^2 N_4 N_6 N_7 \},$$

$$H_{1n} = \frac{N_1 k_n^3 - g a^2 \varepsilon^2 k_n^2 + (a^2 A_1 - N_2) k_n + N_3 a^2}{i a (g \varepsilon^2 k_n^3 + (A_1 - N_5) k_n^2 - N_3 k_n + N_6)},$$

$$H_{2n} = \frac{N_1 k_n^2 - N_2 + i a (N_3 - A_1 k_n - g \varepsilon^2 k_n^2) H_{1n}}{i a N_4},$$

$$H_{3n} = \frac{i a \lambda - (\lambda + 2\mu) (k_n H_{1n} + N_4 H_{2n})}{\mu (1 + \varepsilon^2 a^2 - \varepsilon^2 k_n^2)},$$

$$H_{4n} = \frac{i a \mu H_{1n} - \mu k_n}{\mu (1 + \varepsilon^2 a^2 - \varepsilon^2 k_n^2)}.$$

4. Boundary conditions

In order to determine the parameter M_n ($n=1,2,3$), we take the initial and regular conditions used to solve the present problem at $z = 0$, are as Said (2024)

$$\frac{\partial \theta}{\partial z} = 0, \quad \sigma_{zz} = -f_0 \cos(\varphi) \exp(mt + i ax), \quad (22)$$

$$\sigma_{xz} = -f_0 \sin(\varphi) \exp(mt + i ax),$$

where f_0 is a constant and φ is an arbitrary angle of the inclined load.

Substituting the expressions of the variables considered in Eq. (22), we can obtain the following equations

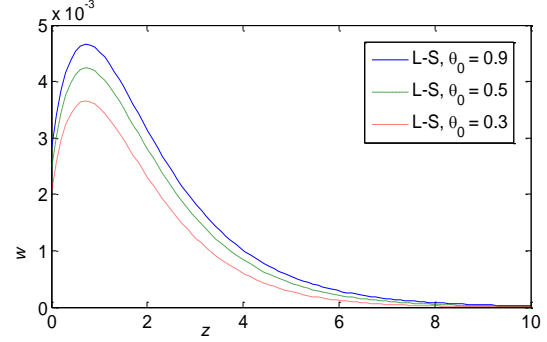


Fig. 1 Vertical displacement distribution w for different values of an empirical material constant

$$\sum_{j=1}^3 k_j H_{2j} M_j = 0, \quad \sum_{j=1}^3 H_{3j} M_j = -f_0 \cos(\varphi), \quad (23)$$

$$\sum_{j=1}^3 H_{4j} M_j = -f_0 \sin(\varphi).$$

Solving the above system in Eq. (23), we get

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} k_1 H_{21} & k_2 H_{22} & k_3 H_{23} \\ H_{31} & H_{32} & H_{33} \\ H_{41} & H_{42} & H_{43} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -f_0 \cos(\varphi) \\ -f_0 \sin(\varphi) \end{pmatrix}. \quad (24)$$

Using the inverse of the matrix method of Eq. (23), we obtain the values of the coefficients M_j ($j = 1, 2, 3$).

5. Numerical results and discussion

To compare the results in the context of the Lord-Shulman (L-S) model, we consider the numerical results for the physical constants as, Said and Othman (2024)

$$\lambda_1 = 3.76 \times 10^{10} \text{ N.m}^{-2}, \quad \mu_1 = 7.78 \times 10^{10} \text{ N.m}^{-2}, \quad \rho = 8954 \text{ kg.m}^{-3},$$

$$a = 0.6, \quad T_0 = 293 \text{ K}, \quad C_E = 3833.3 \text{ J.kg}^{-1} \cdot \text{K}^{-1},$$

$$\alpha_t = 1.78 \times 10^{-4} \text{ K}^{-1}, \quad \tau_0 = 0.3 \text{ s}, \quad f_0 = 0.7, \quad K = 386 \text{ w.m}^{-1} \cdot \text{K}^{-1} \cdot \text{s}^{-1},$$

$$m = m_0 + i\xi, \quad m_0 = -0.8, \quad \xi = 0.65, \quad x = -0.5.$$

For the calculation, the Matlab Software package is utilized as an instrument. The computations were running out of non-dimensional time $t = 0.6$. The thermodynamic temperature, stress components, and displacement distributions are shown in graphs 1-11. Figs. 1-3 display the vertical displacement w , the stress components σ_{zz} and thermodynamic temperature distributions θ for different values of an empirical material constant θ_0 . Fig. 1 depicts that the variation of vertical displacement distribution w starts with positive values. w starts to increase to reach its maximum values in the range $0 \leq z \leq 1$, and then decreases in the range $1 \leq z \leq 10$. Fig. 2 displays the variation of thermodynamic temperature θ starts with positive values. Values of θ decrease in the range $0 \leq z \leq 10$. The increase of the value of an empirical material constant θ_0 causes the increasing values of w, θ . Fig. 3 introduces that the variation

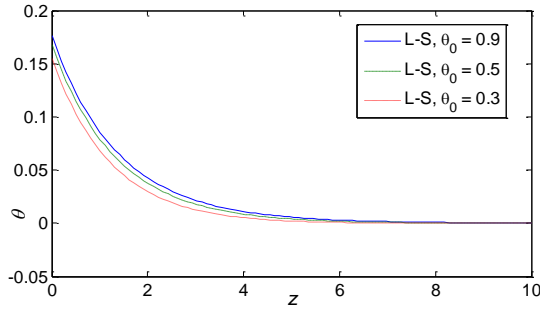


Fig. 2 Thermal temperature distribution θ for different values of an empirical material constant

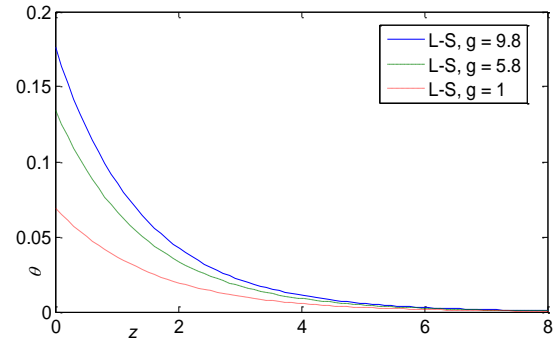


Fig. 5 Thermal temperature distribution θ for different values of the gravity field

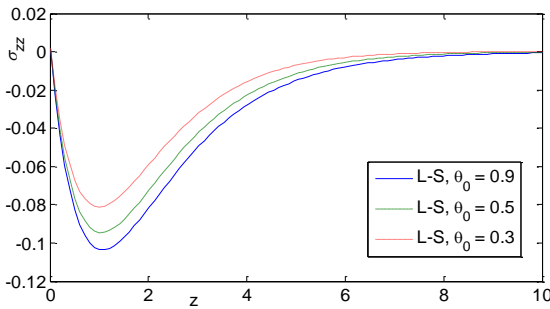


Fig. 3 Distribution of stress component σ_{zz} for different values of an empirical material constant

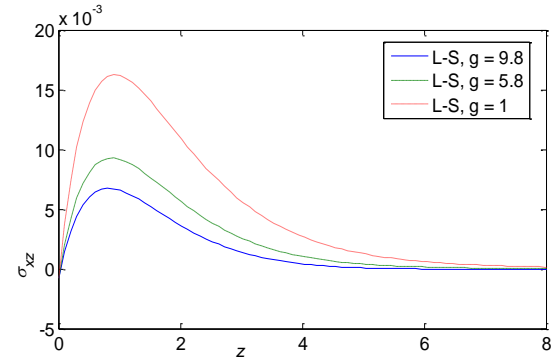


Fig. 6 Distribution of stress component σ_{xz} for different values of the gravity field

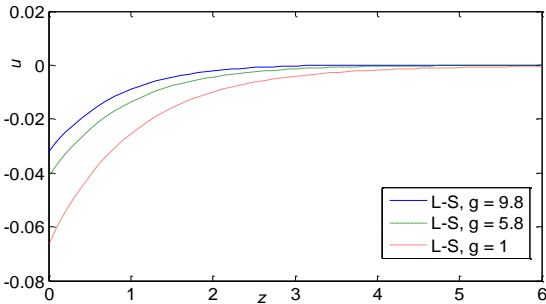


Fig. 4 Horizontal displacement distribution u for different values of the gravity field

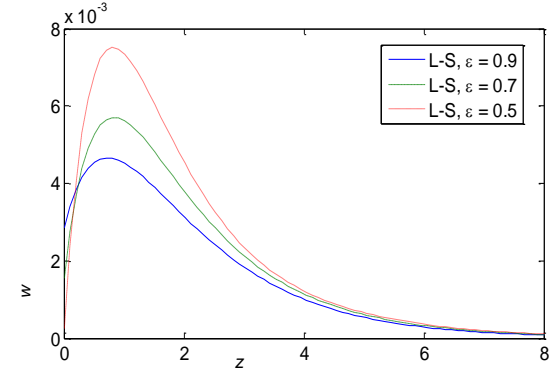


Fig. 7 Vertical displacement distribution w for different values of a nonlocal parameter

of stress component σ_{zz} begins with negative values and satisfies the boundary conditions in Eq. (21). It attains reaching its minimum values in the range $0 \leq z \leq 1$, then increases in the range $1 \leq z \leq 10$. The increase of the value of an empirical material constant θ_0 causes the decreasing values of σ_{zz} .

Figs. 4-6 display the displacement component u , the stress components σ_{xz} and thermodynamic temperature distributions θ for different values of the gravity field g . Fig. 4 depicts the variation of horizontal displacement distribution u starts with negative values. It increases in the range $0 \leq z \leq 6$. Fig. 5 displays the variation of thermodynamic temperature θ starts with positive values.

Values of θ decrease in the range $0 \leq z \leq 8$. The increase of the value of the gravity field causes the

increasing values of u, θ . Fig. 6 depicts that the variation of stress component σ_{xz} starts with negative values and satisfies the boundary conditions in Eq. (21). σ_{xz} starts increasing to reach its maximum values in the range $0 \leq z \leq 1$, and then decreases in the range $1 \leq z \leq 8$. The increase of the value in the gravity field cause decreasing values of σ_{xz} .

Figs. 7-9 display the vertical displacement w , thermodynamic temperature distributions θ , and stress component σ_{zz} distributions for different values of a nonlocal parameter ϵ . Fig. 7 depicts that the variation of vertical displacement distribution w starts to increase to

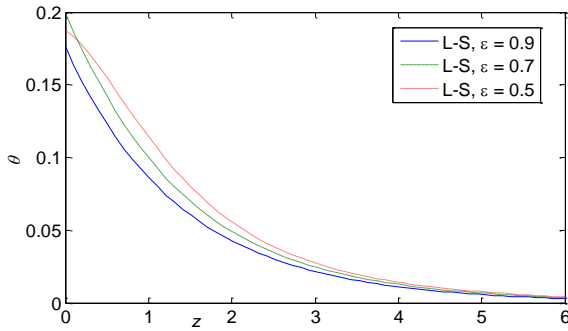


Fig. 8 Thermal temperature distribution θ for different values of a nonlocal parameter

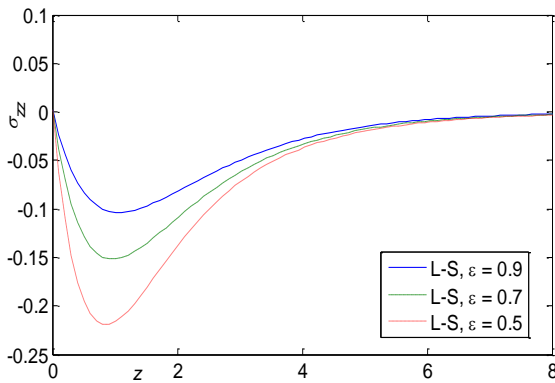


Fig. 9 Distribution of stress component σ_{zz} for different values of a nonlocal parameter

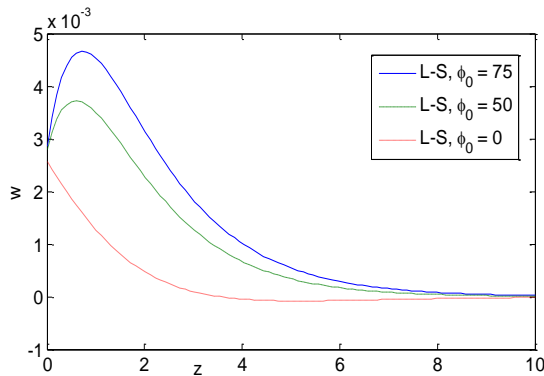


Fig. 10 Vertical displacement distribution w for different values of an inclined load

reach its maximum values and then decreases. Fig. 8 displays that the variation of thermodynamic temperature θ starting with positive values. Values of θ decrease in the range $0 \leq z \leq 6$. The increasing of the value of a nonlocal parameter ϵ causes decreasing values of w, θ . It clears from Fig. 9 that the values of σ_{zz} start with decreasing, reach their minimum values and then increase. The increase in the value of a nonlocal parameter ϵ causes increasing values of σ_{zz} .

Figs. 10 and 11 display the vertical displacement w , and thermodynamic temperature distributions θ for different values of an inclined load ϕ . Fig. 10 depicts that

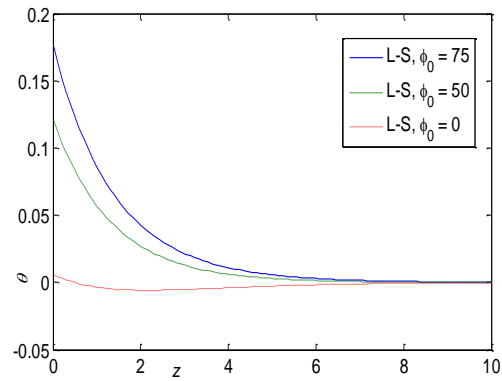


Fig. 11 Thermal temperature distribution θ for different values of an inclined load

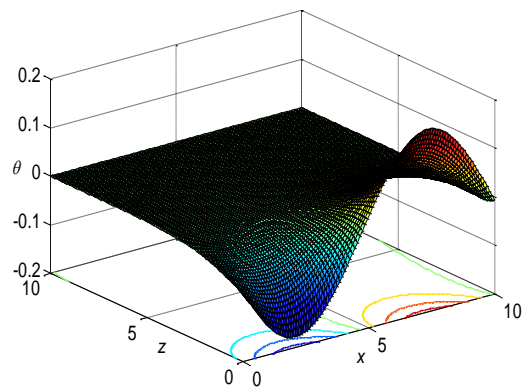


Fig. 12 Represent 3D Thermal temperature distribution θ in L-S model

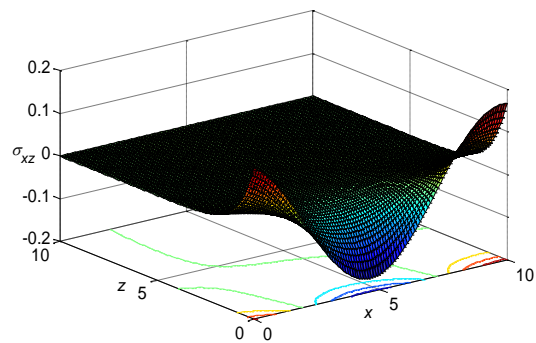


Fig. 13 represent 3D Distribution of stress component σ_{xz} in L-S model

the variation of vertical displacement distribution w starts to increase, and then decreases. Fig. 11 displays the variation of thermodynamic temperature θ starts with positive values. Values of θ decrease in the range $0 \leq z \leq 6$. The increase in the value of an inclined load ϕ causes increasing values of w, θ . Figs. 12-14 display 3D distributions of the thermodynamic temperature distributions θ , and stress component σ_{xz}, σ_{zz} . These figures are very important to study the dependence of these physical quantities on the vertical component of distance.

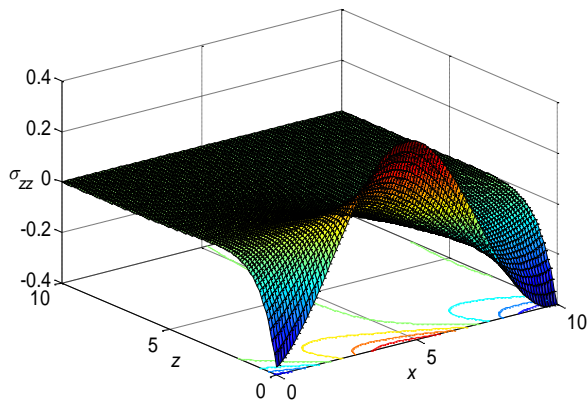


Fig. 14 represent 3D Distribution of stress component σ_{zz} in L-S model

6. Conclusions

Within the framework of Lord-Shulman (L-S) theory, we have studied the two-dimensional dynamic response of a nonlocal thermoelastic half-space solid. We draw the following findings from the conversations that were held:

- The distributions of all physical fields are significantly influenced by the nonlocal parameter.
- The distributions of all physical fields are significantly influenced by the gravity field.
- The distributions of all physical fields are significantly influenced by the empirical material constant.
- The distributions of all physical fields are significantly influenced by the inclined load.
- This observation supports the notion that the Lord-Shulman (L-S) theory is, in fact, a generalized theory of thermoelasticity.

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CC

List of Symbols

σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
$\varepsilon = a_0 e_0$	elastic nonlocal parameter
T	absolute temperature
T_0	temperature of medium in its natural state assumed to be such that $ (T-T_0)/T_0 < 1$, $\theta_0 = T - T_0$
τ_0	is thermal relaxation time
$\mu_1, \lambda_1, \gamma_1$	constants of material
λ, μ	Lame's constant
δ_{ij}	Kronecker's delta
K	coefficient of thermal conductivity
ρ	density
C_E	specific heat at constant strain
θ_0	is an empirical material constant
i	imaginary unit, $i = \sqrt{-1}$
a	wave number
m	complex constant
α_t	linear thermal expansion coefficient, $\gamma = (3\lambda + 2\mu)\alpha_t$,