

Influence of gravity and viscosity on a fiber-reinforced thermoelastic solid with voids via the three-phase-lag model

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Abstract. The goal of this work is to examine how a fiber-reinforced thermoelastic half-space with voids is affected by the gravity field and viscosity. The displacement, stress, and temperature distributions' analytical formulas are obtained from the normal mode analysis. The problem is analytically addressed using a three-phase-lag model. Use MATLAB programming to determine the physical fields with appropriate boundary conditions and carry out numerical computations. Examine the outcomes in the absence and presence of gravity, viscosity as well as reinforcement. Graphs are used to visualize and analyze computational findings. Theoretical as well as numerical results reveal that all the field variables are sensitive towards the gravity field, the viscosity, and the reinforcement.

Keywords: gravity; fiber-reinforced; normal mode analysis; three-phase-lag model; visco-thermoelastic

1. Introduction

Fiber-reinforced composites are commonly used in engineering design because they outperform structural materials in applications where lightweight components need to be extremely rigid and robust. Nowadays, the fiber-reinforced polymers are gradually replacing metallic materials in many applications, such as aerospace, marine, automotive, sporting goods, and infrastructure. Pipkin (1973) and Rogers (1975) conducted groundbreaking research on the topic. The stress boundary value problem for finite plane deformation of a fiber-reinforced material was examined by Craig and Hart (1979). Belfield *et al.* (1983) proposed the concept of continual self-reinforcement at every point of an elastic solid. Sengupta and Nath (2001) examined the issue of surface waves in an anisotropic elastic material reinforced by fibers. The reflection of plane waves on the free surface of an elastic half-space reinforced with fibers was examined by Singh and Singh (2004). Singh (2007) talked about the propagation of waves in an elastic material reinforced with fibers that is incompressible and transversely isotropic. The plane waves of a study of three distinct theories for fiber-reinforcement magneto-thermoelastic analysis were examined by Othman and Said (2013). Abd-Alla *et al.* (2015) studied surface waves in an anisotropic thermoelastic material reinforced with fibers under the influence of a gravitational field. Othman *et al.* (2015) discussed the effect of rotation and initial stress on generalized micropolar thermoelastic medium with the three-phase lag model. The impact of Coriolis acceleration and multi-phase lag on a fiber-reinforced isotropic thermoelastic medium was examined by Alharbi *et al.*

(2021). In order to investigate the behavior of fiber-reinforced viscoelastic media, Abouelregal and Alesemi (2022) developed a modified thermo-viscoelastic mathematical model based on the Moore-Gibson-Thompson equation. A dual reciprocity boundary element approach was used by Fahmy and Almehmadi (2022a) to investigate problems involving rotating functionally graded anisotropic fiber-reinforced magneto-thermoelastic composites. In the context of Lord-Shulman theory, Kalkal *et al.* (2024) addressed the two-dimensional deformations in a rotating functionally graded (non-homogeneous), fiber-reinforced, transversely isotropic thermoelastic half-space under the influence of a magnetic field. Various vital approaches and real-time applications of problems in thermodynamics and thermoelasticity have been introduced by Vlase *et al.* (2017), Marin *et al.* (2020), and Fahmy (2022b, c).

Elastic solids with voids are the most basic generalization of the classical theory of elasticity. It is crucial to remember that porous materials are used in a wide range of engineering disciplines, such as biology, material science, and the petroleum industry. This theory includes the void volume as one of the kinematic variables and focuses on elastic materials composed of a distribution of tiny pores (voids). The linear theory of elastic materials with vacancies was discussed by Cowin and Nunziato (1973). The first investigation into the theory of thermoelastic materials with voids was carried out by Nunziato and Cowin (1979). The behavior of plane waves in a linear elastic material with voids was investigated by Puri and Cowin (1985). Several theorems in the theory of viscoelastic materials with voids were covered by Ciarletta and Scalia (1991). Dhaliwal and Wang (1994) discussed the sphere of effect theorem in the linear theory of elastic materials with voids. Under initial load, Kumar and Kumar (2011) investigated the propagation of waves in transversely isotropic generalized thermoelastic half-space with voids.

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Plane waves in a generalized magneto-thermo-viscoelastic medium with voids under the influence of initial stress and laser pulse heating were examined by Othman et al. (2020). Rayleigh waves in porous orthotropic media with phase delays were covered by Biswas (2021). The displacement components of the solid and fluid phases in poro-thermo-elastic media, temperature increments, stress, and a new model for poro-thermoelastic waves under a fractional time derivative are all studied by Marin et al. (2021). A new boundary element approach model for studying the propagation of ultrasonic waves in three-temperature anisotropic visco-elastic porous media was created by Fahmy et al. (2023a). A poro-thermoelastic half-space problem with temperature-dependent properties is covered by Said (2024a), and an inclined load is investigated within the framework of a three-phase lag model while accounting for the influence of gravity and magnetic fields. The influence of viscosity on thermoelastic solids has been addressed by Othman et al. (2020) and Fahmy et al. (2021, 2023b, c, 2024).

Potential fields of use for wave propagation research in generalized thermo-elasticity materials include geophysics, seismology, and structural engineering. This kind of inquiry becomes more possible when gravity is included. Bromwich (1898) was the first to study the effects of gravity on wave propagation in solids, especially on elastic spheres. Love (1911) examined the impact of gravity and demonstrated that, at large wavelengths, the gravitational field significantly increases the velocity of Rayleigh waves. Nath and Sengupta (1999) looked into the axisymmetric problem of wave propagation under gravity in a medium consisting of a liquid layer and an underlying solid half-space. Abd-Alla and Ahmed (2003) examined the surface waves within an orthotropic elastic inhomogeneous orthotropic elastic medium, under the influence of a gravity field. Ailawalia and Narah (2009) studied how various forces acting at the interface under the effect of gravity could cause the deformation of a rotating generalized thermoelastic solid with an overlaying infinite thermo-elastic fluid. The impact of gravity and a heat source on a fractional order fiber-reinforced thermoelastic media was examined by Jain et al. (2018). A sophisticated multi-phase-lags theory for photo-thermal waves of a gravitating semiconducting half-space was presented by Zenkour (2019). A 2D problem involving a porous magneto-thermo- elastic material subjected to gravity was examined by Alharbi et al. (2021) within the framework of the three-phase lag model and a memory dependent derivative frame. The effect of gravity was examined by Othman and Eraki (2018) on generalized thermoelastic diffusion due to a laser pulse using a dual-phase-lag model. Said (2024b) discussed the influence of gravity, locality, and rotation on thermoelastic half-space via the dual-phase-lag model.

The current problem dealt with the two-dimensional (2D) problem of a fiber-reinforced visco-thermoelastic solid with voids under the impact of the gravity field within the framework of a three-phases lag (3PHL) model. Through the use of normal mode analysis, which produces analytical solutions for the relevant field variables, precise formulations for physical quantities are obtained. Visual

representations of the outcomes of numerical simulations are used to demonstrate how gravity and viscosity affect a thermo-elastic fiber-reinforced solid with voids. Comparisons are made between the gravity, viscosity, and reinforcement results.

2. Formulation of the problem

A fiber-reinforced visco-thermoelastic isotropic solid with voids in a half-space ($x \geq 0$). Plane strain in the xy -plane with the displacement vector $u = (u, v, 0)$, $u = u(x, y, t)$, $v = v(x, y, t)$.

The constitutive equations are identical to those found in Said (2020) and Belfield et al. (1983).

$$\sigma_{ij} = \lambda^* e_{kk} \delta_{ij} + 2\mu_T^* e_{ij} + \alpha^* (a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) + \beta^* a_i a_j a_k a_m e_{km} + 2(\mu_L^* - \mu_T^*) (a_i a_k e_{kj} + a_j a_k e_{ki}) + (b^* \psi - \gamma^* \theta) \delta_{ij} \tag{1}$$

The motion equations as stated by Said (2016)

$$\rho u_{i,n} = \sigma_{ji,j} + F_i, \quad F_1 = \rho g \frac{\partial v}{\partial x}, \quad F_2 = -\rho g \frac{\partial u}{\partial x}, \quad F_3 = 0 \tag{2}$$

The equations of voids as Iesan (2011)

$$\zeta^* \nabla^2 \psi - b_1 \psi - b_2 \frac{\partial \psi}{\partial t} - B^* e + (G \nabla^2 + m) \theta = \rho \chi \frac{\partial^2 \psi}{\partial t^2} \tag{3}$$

The parameters $\lambda^*, \alpha^*, \beta^*, \mu_L^*, \mu_T^*, \gamma^*, B^*, \zeta^*$ and b^* are defined as

$$\begin{aligned} \lambda^* &= \lambda(1 + \lambda_0 \frac{\partial}{\partial t}), \quad \alpha^* = \alpha(1 + \alpha_0 \frac{\partial}{\partial t}), \quad \mu_T^* = \mu_T(1 + \alpha_1 \frac{\partial}{\partial t}), \quad \mu_L^* = \mu_L(1 + \alpha_2 \frac{\partial}{\partial t}), \\ \beta^* &= \beta(1 + \beta_0 \frac{\partial}{\partial t}), \quad \gamma^* = \gamma(1 + \gamma_0 \frac{\partial}{\partial t}), \quad b^* = b(1 + \alpha_3 \frac{\partial}{\partial t}), \quad B^* = b(1 + \alpha_4 \frac{\partial}{\partial t}), \\ \zeta^* &= \zeta(1 + \zeta_0 \frac{\partial}{\partial t}) \end{aligned} \tag{4}$$

Equation for heat conduction as stated by Choudhuri (2007)

$$\begin{aligned} (K^* + \tau_v^* \frac{\partial}{\partial t} + K \tau_\theta \frac{\partial^2}{\partial t^2}) \nabla^2 \theta = \\ (1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2} \tau_q^2 \frac{\partial^2}{\partial t^2}) (\rho C_E \frac{\partial^2 \theta}{\partial t^2} + \gamma^* T_0 \frac{\partial^2 e}{\partial t^2} + m T_0 \frac{\partial \psi}{\partial t}) - \eta \nabla^2 \frac{\partial \psi}{\partial t} \end{aligned} \tag{5}$$

where $\tau_v^* = K + K^* \tau_v$

Introducing Eqs. (1) in Eqs. (2), we obtain

$$\begin{aligned} \rho \frac{\partial^2 u}{\partial t^2} = A_1 \frac{\partial^2 u}{\partial x^2} + A_2 \frac{\partial^2 v}{\partial x \partial y} + A_3 \frac{\partial^2 u}{\partial y^2} + b(1 + \alpha_3 \frac{\partial}{\partial t}) \frac{\partial \psi}{\partial t} \frac{\partial \psi}{\partial x} \\ - \gamma(1 + \gamma_0 \frac{\partial}{\partial t}) \frac{\partial \theta}{\partial x} + \rho g \frac{\partial v}{\partial x} \end{aligned} \tag{6}$$

$$\begin{aligned} \rho \frac{\partial^2 v}{\partial t^2} = A_3 \frac{\partial^2 v}{\partial x^2} + A_2 \frac{\partial^2 u}{\partial x \partial y} + A_4 \frac{\partial^2 v}{\partial y^2} + b(1 + \alpha_3 \frac{\partial}{\partial t}) \frac{\partial \psi}{\partial t} \frac{\partial \psi}{\partial y} \\ - \gamma(1 + \gamma_0 \frac{\partial}{\partial t}) \frac{\partial \theta}{\partial y} - \rho g \frac{\partial u}{\partial x} \end{aligned} \tag{7}$$

where

$$A_1 = \lambda(1 + \lambda_0 \frac{\partial}{\partial t}) + 2\alpha(1 + \alpha_0 \frac{\partial}{\partial t}) - 2\mu_r(1 + \alpha_1 \frac{\partial}{\partial t}) + 4\mu_L(1 + \alpha_2 \frac{\partial}{\partial t}) + \beta(1 + \beta_0 \frac{\partial}{\partial t}),$$

$$A_2 = \lambda(1 + \lambda_0 \frac{\partial}{\partial t}) + \alpha(1 + \alpha_0 \frac{\partial}{\partial t}) + \mu_L(1 + \alpha_2 \frac{\partial}{\partial t}), \quad A_3 = \mu_L(1 + \alpha_2 \frac{\partial}{\partial t}),$$

$$A_5 = A_2 - A_3, \quad A_4 = \lambda(1 + \lambda_0 \frac{\partial}{\partial t}) + 2\mu_r(1 + \alpha_2 \frac{\partial}{\partial t})$$

Take into account the subsequent dimensionless variables

$$(x', y') = \frac{w_1}{c_1} (x, y), \quad (u', v') = \frac{\rho c_1 w_1}{\gamma T_0} (u, v), \quad \theta' = \frac{\theta}{T_0}, \quad \psi' = \frac{\rho c_1^2}{\gamma T_0} \psi,$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\gamma T_0}, \quad g' = \frac{g}{c_1 w_1},$$

$$(t', \tau'_q, \tau'_v, \tau'_\theta, \lambda'_0, \alpha'_0, \alpha'_1, \alpha'_2, \alpha'_3, \alpha'_4, \beta'_0, \xi'_0) = \dots,$$

$$w_1(t, \tau_q, \tau_v, \tau_\theta, \lambda_0, \alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_0, \gamma_0, \xi_0)$$

$$w_1 = \frac{\rho C_E c_1^2}{K^*}, \quad c_1 = \sqrt{\frac{\lambda + 2\mu_T}{\rho}}$$

Introducing the non-dimensional Eqs. (8) in Eqs. (3), (5), (6), and (7), we obtain

$$\frac{\partial^2 u}{\partial t^2} = h_1 \frac{\partial^2 u}{\partial x^2} + h_2 \frac{\partial^2 v}{\partial x \partial y} + h_3 \frac{\partial^2 u}{\partial y^2} + h_6 \frac{\partial \psi}{\partial x} - (1 + \gamma_0 \frac{\partial}{\partial t}) \frac{\partial \theta}{\partial x} + g \frac{\partial v}{\partial x} \quad (9)$$

$$\frac{\partial^2 v}{\partial t^2} = h_3 \frac{\partial^2 v}{\partial x^2} + h_2 \frac{\partial^2 u}{\partial x \partial y} + h_4 \frac{\partial^2 v}{\partial y^2} + h_6 \frac{\partial \psi}{\partial y} - (1 + \gamma_0 \frac{\partial}{\partial t}) \frac{\partial \theta}{\partial y} - g \frac{\partial u}{\partial x} \quad (10)$$

$$(1 + \xi_0 \frac{\partial}{\partial t}) \nabla^2 \psi - d_7 \psi - d_8 \frac{\partial \psi}{\partial t} - d_9 (1 + \alpha_4 \frac{\partial}{\partial t}) e + (d_{10} \nabla^2 + d_{11}) \theta = d_{12} \frac{\partial^2 \psi}{\partial t^2} \quad (11)$$

$$(d_1 + d_2 \frac{\partial}{\partial t} + d_3 \frac{\partial^2}{\partial t^2}) \nabla^2 \theta = (1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2} \tau_q^2 \frac{\partial^2}{\partial t^2}) [\frac{\partial^2 \theta}{\partial t^2} + d_4 (1 + \gamma_0 \frac{\partial}{\partial t}) \frac{\partial^2 e}{\partial t^2} + (d_5 - d_6 \nabla^2) \frac{\partial \psi}{\partial t}] \quad (12)$$

where

$$(h_1, h_2, h_3, h_4, h_5, h_6) = [\frac{A_1}{\rho c_1^2}, \frac{A_2}{\rho c_1^2}, \frac{A_3}{\rho c_1^2}, \frac{A_4}{\rho c_1^2}, \frac{A_5}{\rho c_1^2}, \frac{b}{\rho c_1^2} (1 + \alpha_3 \frac{\partial}{\partial t})],$$

$$d_1 = \frac{K^*}{\rho C_E c_1^2}, \quad d_2 = \frac{w_1 K}{\rho C_E c_1^2} + d_1 \tau_v, \quad d_3 = \frac{K w_1 \tau_\theta}{\rho C_E c_1^2}, \quad d_4 = \frac{\gamma^2 T_0}{\rho^2 C_E c_1^2},$$

$$d_5 = \frac{m \gamma T_0}{\rho^2 C_E c_1^2 w_1}, \quad d_6 = \frac{\eta \gamma w_1}{\rho^2 C_E c_1^4}, \quad d_7 = \frac{b_1 c_1^2}{\xi w_1^2}, \quad d_8 = \frac{b_2 c_1^2}{\xi w_1}, \quad d_9 = \frac{b c_1^2}{\xi w_1^2},$$

$$d_{10} = \frac{G c_1^2 \rho}{\xi \gamma}, \quad d_{11} = \frac{m c_1^4 \rho}{\xi \gamma w_1^2}, \quad d_{12} = \frac{\chi c_1^2 \rho}{\xi}$$

3. Normal mode analysis

The normal mode method has the advantage of finding the exact solutions without any assumed constraints on the field variables. In this approach, the solution of the physical

variables is decomposed in terms of normal modes and one gets exact solution without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. The problem of a fiber-reinforced visco-thermoelastic solid with voids can be resolved as follows using normal mode analysis

$$(u, v, \psi, \theta, \sigma_{ij})(x, y, t) = (\bar{u}, \bar{v}, \bar{\psi}, \bar{\sigma}_{ij})(x) \exp(i r y - s t) \quad (13)$$

where $\bar{u}(x)$, etc. is the amplitude of the function $u(x, y, t)$ etc.

Using Eq. (13) in Eqs. (9)-(12), we obtain

$$(\bar{h}_1 D^2 - B_1) \bar{u} + B_2 D \bar{v} + \bar{h}_6 D \bar{\psi} - B_3 D \bar{\theta} = 0 \quad (14)$$

$$B_4 D \bar{u} + (\bar{h}_3 D^2 - B_5) \bar{v} + i r \bar{h}_6 \bar{\psi} - i r B_3 \bar{\theta} = 0 \quad (15)$$

$$B_6 D \bar{u} + i r B_6 \bar{v} + (d_6 s D^2 - B_7) \bar{\psi} + (B_8 D^2 + B_9) \bar{\theta} = 0 \quad (16)$$

$$B_{10} D \bar{u} + i r B_{10} \bar{v} + (B_{12} - B_{11} D^2) \bar{\psi} + (B_{13} - B_{10} D^2) \bar{\theta} = 0 \quad (17)$$

where

$$(\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4, \bar{h}_5, \bar{h}_6) = [\frac{\bar{A}_1}{\rho c_1^2}, \frac{\bar{A}_2}{\rho c_1^2}, \frac{\bar{A}_3}{\rho c_1^2}, \frac{\bar{A}_4}{\rho c_1^2}, \frac{\bar{A}_5}{\rho c_1^2}, \frac{\bar{b}}{\rho c_1^2} (1 - s \alpha_3)],$$

$$\bar{A}_1 = \lambda(1 - \lambda_0 s) + 2\alpha(1 - \alpha_0 s) - 2\mu_r(1 - \alpha_1 s) + 4\mu_L(1 - \alpha_2 s) + \beta(1 - \beta_0 s),$$

$$\bar{A}_2 = \lambda(1 - \lambda_0 s) + \alpha(1 - \alpha_0 s) + \mu_L(1 - \alpha_2 s), \quad \bar{A}_3 = \mu_L(1 - \alpha_2 s), \quad \bar{A}_5 = \bar{A}_2 - \bar{A}_3,$$

$$\bar{A}_4 = \lambda(1 - \lambda_0 s) + 2\mu_r(1 - \alpha_2 s), \quad B_1 = r^2 \bar{h}_3 + s^2, \quad B_2 = i r \bar{h}_2 + g,$$

$$B_3 = 1 - \gamma_0 s, \quad B_4 = i r \bar{h}_2 - g, \quad B_5 = r^2 \bar{h}_4 + s^2,$$

$$B_6 = d_4 s^2 (1 - s \tau_q + \frac{1}{2} \tau_q^2 s^2) (1 - \gamma_0 s), \quad B_7 = d_5 s (1 - s \tau_q + \frac{1}{2} \tau_q^2 s^2) + d_6 s r^2,$$

$$B_8 = d_2 s - d_1 - d_3 s^2,$$

$$B_9 = d_1 r^2 - d_2 s r^2 + d_3 s^2 r^2 + s^2 (1 - s \tau_q + \frac{1}{2} \tau_q^2 s^2),$$

$$B_{10} = d_9 (1 - \alpha_4 s), \quad B_{11} = 1 - \xi_0 s, \quad B_{12} = B_{11} r^2 + d_7 + d_{12} s^2 - d_8 s,$$

$$B_{13} = d_{10} r^2 - d_{11}.$$

Omitting the function $\bar{v}(x), \bar{\theta}(x), \bar{\psi}(x)$ from Eqs. (14)-(17), we get

$$(D^8 - E_1 D^6 + E_2 D^4 - E_3 D^2 + E_4) \bar{u}(x) = 0 \quad (18)$$

The answer to Equation (18) can be expressed as

$$\bar{u}(x) = \sum_{j=1}^4 N_j \exp(-k_j x) \quad (19)$$

$$\bar{v}(x) = \sum_{j=1}^4 N_j R_{1j} \exp(-k_j x) \quad (20)$$

$$\bar{\theta}(x) = \sum_{j=1}^4 N_j R_{2j} \exp(-k_j x) \quad (21)$$

$$\bar{\psi}(x) = \sum_{j=1}^4 N_j R_{3j} \exp(-k_j x) \tag{22}$$

where $k_j^2 (j=1,2,3,4)$ are the roots of the characteristic equation: $(k^8 - E_1 k^6 + E_2 k^4 - E_3 k^2 + E_4 = 0)$

Where, $E_1 = (L_1 + L_2 - L_3) / L_0$,

$$E_2 = (L_4 + L_5 + L_6 + L_7 - L_8 - L_9 - L_{10} - L_{11}) / L_0,$$

$$E_3 = (L_{12} - L_{13} + L_{14} + L_{15} - L_{16} + L_{17}) / L_0, \quad E_4 = (L_{19} - L_{18}) / L_0,$$

$$L_0 = \bar{h}_1 \bar{h}_3 (B_8 B_{11} - s d_6 B_{10}),$$

$$L_1 = B_2 B_4 B_8 B_{11} + \bar{h}_3 B_1 B_8 B_{11} - \bar{h}_3 B_3 B_6 B_{11} + \bar{h}_1 B_5 B_8 B_{11} - \bar{h}_1 \bar{h}_3 B_7 B_{10},$$

$$L_2 = \bar{h}_1 \bar{h}_3 B_8 B_{12} - \bar{h}_1 \bar{h}_3 B_9 B_{11} - \bar{h}_3 \bar{h}_6 B_6 B_{10} - \bar{h}_3 \bar{h}_6 B_8 B_{10} - d_6 s B_2 B_4 B_{10},$$

$$L_3 = \bar{h}_3 d_6 s B_1 B_{10} + \bar{h}_3 d_6 s B_3 B_{10} + \bar{h}_1 d_6 s B_5 B_{10} + \bar{h}_1 \bar{h}_3 d_6 s B_{13},$$

$$L_4 = B_1 B_5 B_8 B_{11} - B_2 B_4 B_7 B_{10} - B_3 B_5 B_6 B_{11} + B_2 B_4 B_8 B_{12} - B_2 B_4 B_9 B_{11},$$

$$L_5 = \bar{h}_3 B_1 B_8 B_{12} - \bar{h}_3 B_1 B_9 B_{11} - \bar{h}_3 B_1 B_7 B_{10} - \bar{h}_3 B_3 B_7 B_{10} - \bar{h}_1 B_5 B_7 B_{10},$$

$$L_6 = \bar{h}_1 B_5 B_8 B_{12} - \bar{h}_3 B_3 B_6 B_{12} - \bar{h}_1 B_5 B_9 B_{11} - \bar{h}_6 B_5 B_6 B_{10} - \bar{h}_6 B_5 B_8 B_{10},$$

$$L_7 = \bar{h}_3 \bar{h}_6 B_9 B_{10} - \bar{h}_1 \bar{h}_3 B_7 B_{13} - \bar{h}_1 \bar{h}_3 B_9 B_{12} - \bar{h}_3 \bar{h}_6 B_6 B_{13} - d_6 s B_1 B_5 B_{10},$$

$$L_8 = d_6 s B_3 B_5 B_{10} + d_6 s B_2 B_4 B_{13} + \bar{h}_3 d_6 s B_1 B_{13} + \bar{h}_1 d_6 s B_5 B_{13} + i r \bar{h}_6 B_2 B_6 B_{10},$$

$$L_9 = i r \bar{h}_6 B_2 B_8 B_{10} + i r \bar{h}_6 B_4 B_6 B_{10} + i r \bar{h}_6 B_4 B_8 B_{10} + r^2 \bar{h}_1 B_3 B_6 B_{11} + i r B_2 B_3 B_6 B_{11},$$

$$L_{10} = i r B_3 B_4 B_6 B_{11} + r^2 \bar{h}_1 \bar{h}_6 B_6 B_{10} + r^2 \bar{h}_1 \bar{h}_6 B_8 B_{10} + r^2 \bar{h}_1 d_6 s B_3 B_{10} + i r d_6 s B_2 B_3 B_{10},$$

$$L_{11} = i r d_6 s B_3 B_4 B_{10},$$

$$L_{12} = B_1 B_5 B_8 B_{12} - B_1 B_5 B_7 B_{10} - B_3 B_5 B_7 B_{10} - B_1 B_5 B_9 B_{11} - B_2 B_4 B_7 B_{13},$$

$$L_{13} = B_3 B_5 B_6 B_{12} + B_2 B_4 B_9 B_{12} + \bar{h}_3 B_1 B_7 B_{13} + \bar{h}_3 B_1 B_9 B_{12} + \bar{h}_1 B_5 B_7 B_{13},$$

$$L_{14} = \bar{h}_6 B_5 B_9 B_{10} - d_6 s B_1 B_3 B_{13} - \bar{h}_6 B_5 B_6 B_{13} - i r \bar{h}_6 B_2 B_6 B_{13} + i r \bar{h}_6 B_2 B_9 B_{10},$$

$$L_{15} = i r \bar{h}_6 B_4 B_9 B_{10} - i r \bar{h}_6 B_4 B_6 B_{13} - r^2 B_1 B_3 B_6 B_{11} - r^2 \bar{h}_1 B_3 B_7 B_{10} - r^2 \bar{h}_1 B_3 B_6 B_{12},$$

$$L_{16} = r^2 \bar{h}_6 B_1 B_6 B_{10} + r^2 \bar{h}_6 B_1 B_8 B_{10} + i r B_2 B_3 B_7 B_{10} + i r B_2 B_3 B_6 B_{12} + i r B_3 B_2 B_7 B_{10},$$

$$L_{17} = r^2 \bar{h}_1 \bar{h}_6 B_3 B_{12} - i r B_3 B_4 B_6 B_{12} + r^2 \bar{h}_1 \bar{h}_6 B_6 B_{13} - r^2 d_6 s B_1 B_3 B_{10},$$

$$L_{18} = B_1 B_5 B_9 B_{12} + B_1 B_5 B_7 B_{13} + r^2 B_1 B_3 B_7 B_{10} + r^2 B_1 B_3 B_6 B_{12} + r^2 \bar{h}_6 B_1 B_6 B_{13},$$

$$L_{19} = r^2 \bar{h}_6 B_1 B_9 B_{10}, \quad R_{1j} = \frac{i r (\bar{h}_1 k_j^2 - B_1) - B_4 k_j^2}{i r B_2 k_j + B_5 k_j - \bar{h}_3 k_j^3},$$

$$R_{2j} = \frac{\bar{h}_6 k_j^2 B_6 - (d_6 s k_j^2 - B_7)(\bar{h}_1 k_j^2 - B_1) - (i r \bar{h}_6 B_6 k_j - B_2 k_j (d_6 s k_j^2 - B_7)) R_{1j}}{B_3 k_j (d_6 s k_j^2 - B_7) + \bar{h}_6 B_8 k_j^3 + \bar{h}_6 B_9 k_j}.$$

Using Eqs. (8), (13) and (19)- (22) in Eqs. (1), we obtain

$$\bar{\sigma}_{xx}(x) = \sum_{j=1}^4 N_j R_{4j} \exp(-k_j x) \tag{23}$$

$$\bar{\sigma}_{xy}(x) = \sum_{j=1}^4 N_j R_{5j} \exp(-k_j x) \tag{24}$$

Where

$$R_{4j} = -\bar{h}_1 k_j + i r \bar{h}_5 R_{1j} + \bar{h}_6 R_{3j} - (1 - \gamma_0 s) R_{2j}$$

$$R_{5j} = \bar{h}_{31} (i r - k_j R_{1j})$$

4. Boundary conditions

To get the constants $N_i (i=1,2,3)$, we take the initial and regularity conditions used for solving the current problem at $x=0$ are as Othman *et al.* (2019):

a) The thermal boundary condition for the medium exposed to

$$\theta = M_1 \tag{25}$$

b) Condition on the volume fraction field

$$\psi = M_2 \tag{26}$$

c) The mechanical boundary condition for the medium is subjected to

$$\sigma_{xx} = -f^* \exp(i r y - s t), \quad \sigma_{xy} = 0 \tag{27}$$

where, f^*, M_1, M_2 are constants.

Eqs. (21)-(24) are inserted into Eqs. (25)-(27) to get

$$\sum_{j=1}^4 N_j R_{2j} = M_1, \quad \sum_{j=1}^4 N_j R_{3j} = M_2, \tag{28}$$

$$\sum_{j=1}^4 N_j R_{4j} = -f^*, \quad \sum_{j=1}^4 N_j R_{5j} = 0$$

Eq. (28) is solved as follows using the matrix's assistance

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} = \begin{pmatrix} R_{21} & R_{22} & R_{23} & R_{24} \\ N_1 R_{31} & N_2 R_{32} & N_3 R_{33} & N_4 R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \\ R_{51} & R_{52} & R_{53} & R_{54} \end{pmatrix}^{-1} \begin{pmatrix} M_1 \\ M_2 \\ -f^* \\ 0 \end{pmatrix} \tag{29}$$

5. Numerical results and discussion

We now provide numerical results for a few physical constants to demonstrate the theoretical findings from the preceding section, by Othman and Said (2013)

$$\lambda = 3.76 \times 10^8 \text{ N.m}^{-2}, \quad \mu_T = 2.86 \times 10^9 \text{ N.m}^{-2}, \quad \rho = 7800 \text{ kg.m}^{-3},$$

$$b = 1.13849 \times 10^9 \text{ N.m}^{-2}, \quad T_0 = 293 \text{ K}, \quad C_E = 383.1 \text{ J.kg}^{-1}.\text{K}^{-1},$$

$$\alpha_r = 2.78 \times 10^{-4} \text{ K}^{-1}, \quad \tau_\theta = 0.9 \text{ s}, \quad \tau_\theta = 0.7 \text{ s}, \quad \tau_v = 0.6 \text{ s}, \quad f^* = 0.5,$$

$$K = 386 \text{ w.m}^{-1}.\text{K}^{-1}.\text{s}^{-1}, \quad s = m_0 + i\omega, \quad \beta = 2 \times 10^9 \text{ N.m}^{-2}, \quad m_0 = -4,$$

$$M_1 = 1.9, \quad M_2 = 0.5, \quad K^* = 386 \text{ w.m}^{-1}.\text{K}^{-1}.\text{s}^{-1}, \quad \lambda_0 = 1.97, \quad \alpha_0 = 1.98,$$

$$\alpha = -1.78 \times 10^9 \text{ N.m}^{-2}, \quad \mu_L = 7.86 \times 10^9 \text{ N.m}^{-2}, \quad \alpha_2 = 1.95, \quad \alpha_3 = 1.92,$$

$$\alpha_4 = 1.86, \quad \beta_0 = 1.96, \quad \gamma_0 = 1.97, \quad \zeta_0 = 1.02, \quad \chi = 1.75 \times 10^{-15} \text{ m}^2,$$

$$\eta = 1 \times 10^{-6} \text{ N.m}^{-2}, \quad b_1 = 1.475 \times 10^{10} \text{ N.m}^{-2}, \quad b_2 = 1.78 \times 10^{-12} \text{ N.m}^{-2} \text{ s}^{-1},$$

$$m = 2 \times 10^5 \text{ N.m}^{-2} \text{ K}^{-1}, \quad r = 0.78, \quad \alpha_1 = 1.93, \quad \omega = 0.5, \quad G = 1 \times 10^{-6} \text{ N.m}^{-2} \text{ K}^{-1},$$

$$\mu = 3.86 \times 10^9 \text{ N.m}^{-2}, \quad \zeta = 1.688 \times 10^8 \text{ N.m}^{-2}.$$

For the computation, the Matlab Software package is used as a tool. At the time value $t = 0.5 \text{ s}$ thermal temperature change θ , the volume fraction field ψ , horizontal displacement component u as well as stress

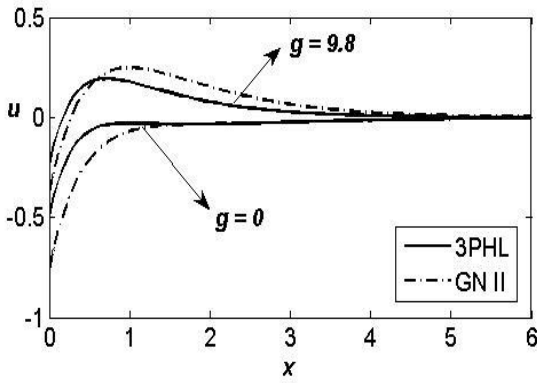


Fig. 1 Variation of the horizontal displacement u in the absence and presence of gravity

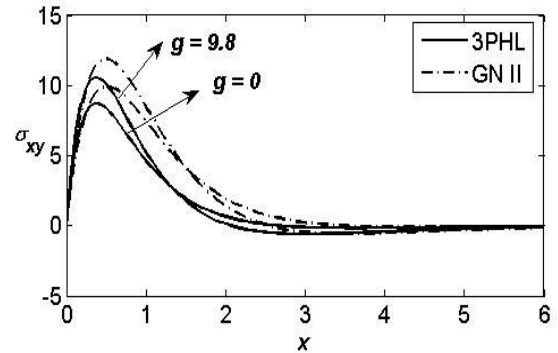


Fig. 4 Variation of the stress component σ_{xy} in the absence and presence of gravity

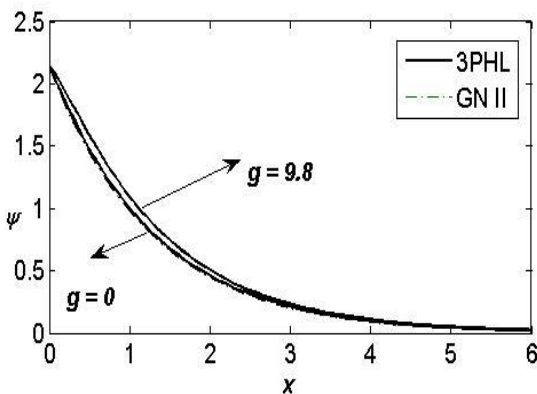


Fig. 2 Variation of the volume fraction field ψ in the absence and presence of gravity

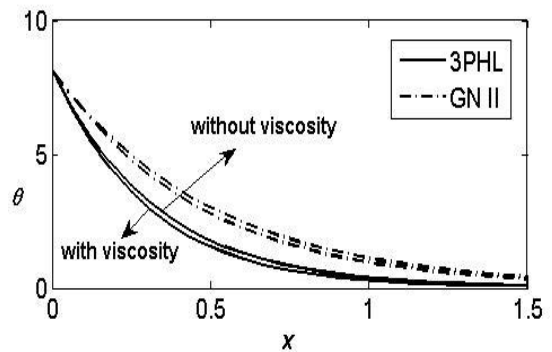


Fig. 5 Variation of the thermal temperature θ in the absence and presence of viscosity

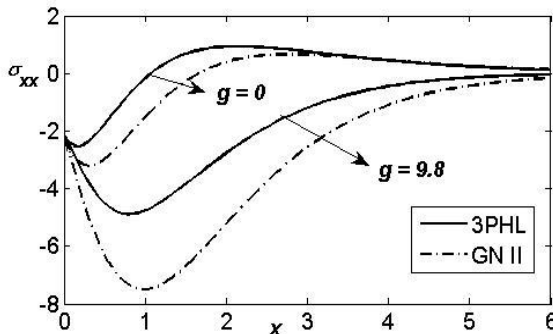


Fig. 3 Variation of the stress component σ_{xx} in the absence and presence of gravity

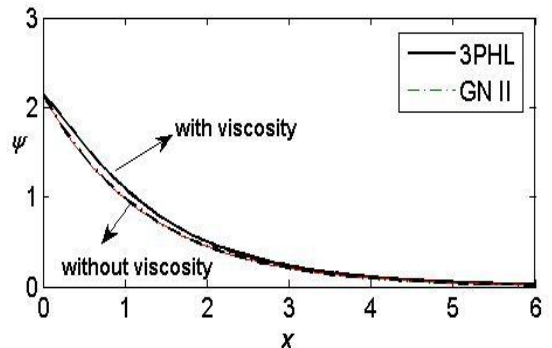


Fig. 6 Variation of the volume fraction field ψ in the absence and presence of viscosity

components σ_{xx} and σ_{xy} in plane $y = -0.9$ for the considered problem are calculated based on the (3PHL) model. The results are given in Figs. 1-14, all results are dimensionless.

In Figs. 1 to 4, the influence of the gravity field on ψ , u as well as σ_{xx} and σ_{xy} are analyzed. Fig. 1 shows that u starts with negative values. The values of u increase to a maximum value, then decrease. The presence of the gravity field causes an increase of u . Fig. 2 depicts that ψ starts with positive values. The values of ψ decrease in the range

$0 \leq x \leq 6$. The presence of the gravity field causes an increase of ψ . Fig. 3 shows that σ_{xx} starts from negative values and obeys the boundary conditions. σ_{xx} decreases, then increases and again decreases. The presence of the gravity field causes a decrease of σ_{xx} . Fig. 4 shows that σ_{xy} starts from a zero value and obeys the boundary conditions. σ_{xy} increases to a maximum value and then decreases. The presence of the gravity field causes an increase of σ_{xy} , and then a decrease.

Figs. 5-7 show plots of thermal temperature change θ , the volume fraction field ψ , as well as stress components

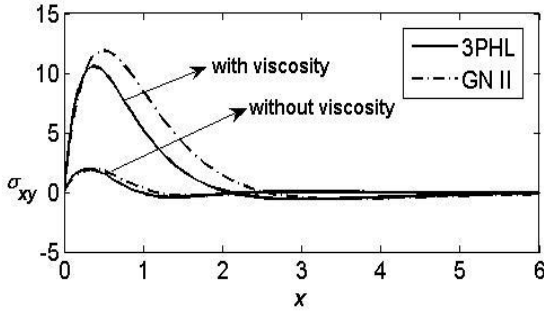


Fig. 7 Variation of the stress component σ_{xy} in the absence and presence of viscosity

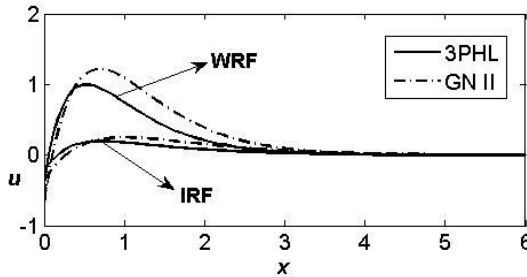


Fig. 8 Variation of the horizontal displacement u in the absence and presence of reinforcement

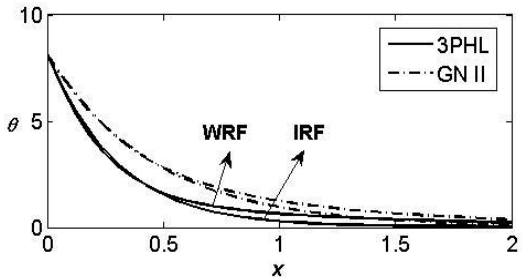


Fig. 9 Variation of the thermal temperature θ in the absence and presence of reinforcement

σ_{xy} in the absence and presence of viscosity. Fig. 5 depicts that θ starts with positive values. The values of θ decrease in the range $0 \leq x \leq 6$. The presence of viscosity causes a decrease of θ . Figure 6 depicts that ψ starts with positive values. The values of ψ decrease in the range $0 \leq x \leq 6$. The presence of viscosity causes an increase of ψ . Fig. 7 shows that σ_{xy} starts from a zero value and obeys the boundary conditions. σ_{xy} increases to a maximum value and then decreases. The presence of viscosity causes an increase of σ_{xy} and then a decrease.

Figs. 8-11 show plots of horizontal displacement component u , thermal temperature change θ , the volume fraction field ψ , as well as stress components σ_{xy} in the absence (WRF) and presence of reinforcement (IRF). Fig. 8 shows that the presence of the reinforcement causes a decrease of u . Fig. 9 depicts that the presence of the reinforcement causes a decrease of θ . Fig. 10 depicts that

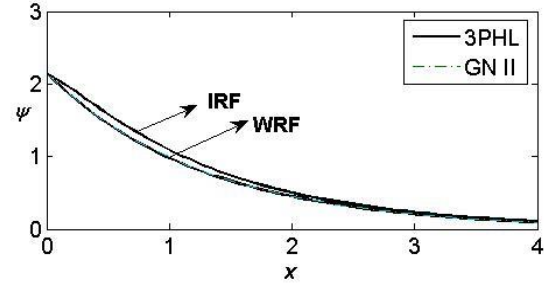


Fig. 10 Variation of the volume fraction field ψ in the absence and presence of reinforcement

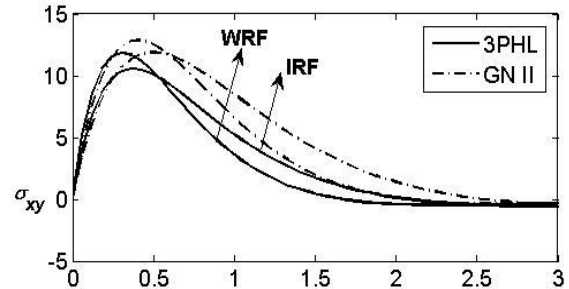


Fig. 11 Variation of the stress component σ_{xy} in the absence and presence of reinforcement

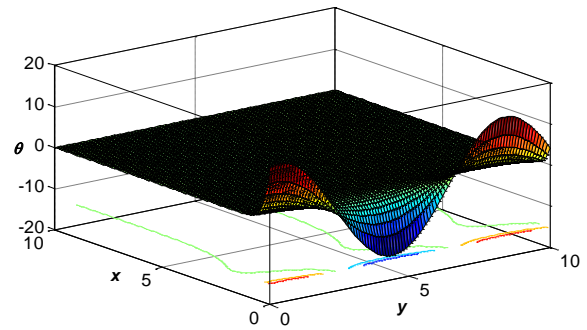


Fig. 12 Distribution of the thermal temperature θ in the context of 3PHL

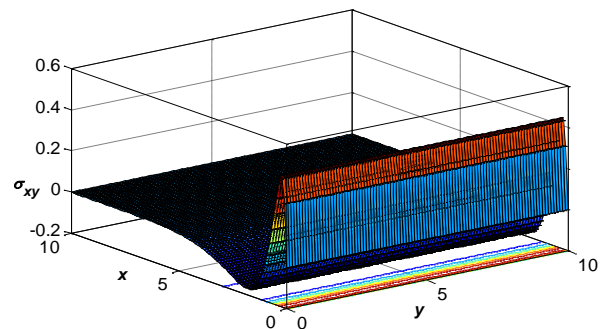


Fig. 13 Distribution of stress component σ_{xy} in the context of 3PHL

the presence of the reinforcement causes an increase of ψ . Fig. 11 displays that the presence of the reinforcement causes a decrease of σ_{xy} then an increase.

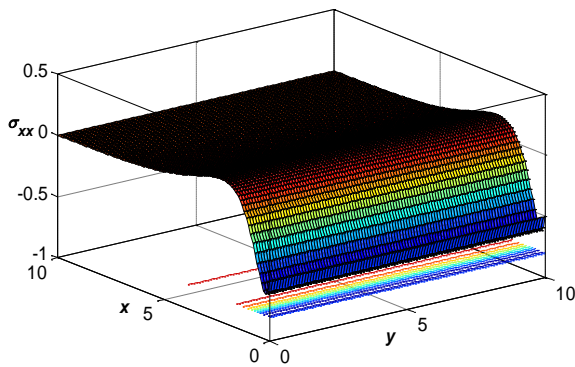


Fig. 14 Distribution of stress component σ_{xx} in the context of 3PHL

Figs. 12-14 show the 3D surface curves of thermal temperature change θ as well as stress components σ_{xx} and σ_{xy} in the context of the 3PHL model. These figures are important for studying the dependence of these quantities on the vertical component of displacement. The obtained curves strongly depend on the vertical displacement, and all physical quantities are moving in wave propagation.

6. Conclusions

The 2D problem of a fiber-reinforced thermoelastic material with voids under the effect of gravity field and viscosity was successfully solved using harmonic wave analysis. We possess:

- The gravity field clearly has a significant impact on the physical fields.
- Because all the physical quantities under consideration have fluctuating (growing or decreasing) amplitudes, viscosity plays a significant influence in all of them.
- The reinforcement clearly has a significant impact on the physical fields.
- Both the horizontal and vertical distances have a significant impact on the physical fields.
- The normal mode analysis is applicable to a wide range of problems in thermodynamics and thermoelasticity. It can be employed for boundary-layer problems which are described by linearized Navier-Stokes equations in electro-hydrodynamics.
- There are significant differences in the field quantities under the G-N II theory and the 3PHL model due to the phase-lag of the temperature gradient and the phase-lag of heat flux.

The three-phase-lag model is useful in the problems of heat transfer, heat conduction, nuclear boiling, exothermic catalytic reactions, and phonon-electron interactions, phonon-scattering. So, the 3PHL model is the most adequate theory to describe the present problem. Viscoelastic materials with complex structural composition and time and temperature-dependent properties, such as elastomers or rubber, are widely utilized in engineering uses such as automobile belts and tires, seals, and biomedical devices.

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List of symbols

σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
T	absolute temperature
T_0	temperature of medium in its natural state assumed to be such that $ (T - T_0)/T_0 < 1$, $\theta = T - T_0$.
τ_θ	phase-lag of the temperature gradient
τ_q	phase-lag of heat flux
τ_v	phase-lag of the thermal displacement gradient, $\tau_v^* = K + K^* \tau_v$.
a	fiber-direction, $a \equiv (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. Here taken as $a \equiv (1, 0, 0)$.
λ, μ	Lame's constant
$\alpha, \beta, \mu_L, \mu_T$	reinforcement parameters
$\lambda_0, \alpha_0, \alpha_1, \alpha_2, \alpha_3,$ $\alpha_4, \beta_0, \gamma_0, \xi_0$	viscoelastic variables
$b, b_1, b_2, \chi, m, G,$ ξ, η	the material constants due to presence of voids
ψ	the volume fraction field
δ_{ij}	Kronecker's delta
K	coefficient of thermal conductivity
K^*	additional material
ρ	density
C_E	specific heat at constant strain
i	imaginary unit, $i = \sqrt{-1}$
r	wave number
s	complex constant
α_i	linear thermal expansion coefficient