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Passive vibration control of sandwich beams with FGM faces and viscoelastic core resting on nonlinear foundation system

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Abstract. The aim of the present investigation is focused on the nonlinear forced vibration analysis of porous FGM sandwich beam with a viscoelastic core resting on nonlinear elastic foundation. The analytical formulation incorporates both normal and shear deformations in the core by utilizing the Zig-Zag theories. The harmonic balance method is integrated with a one-mode Galerkin's procedure designed for a simply supported beam. The nonlinear geometric coupling and viscoelastic effects result in a frequency amplitude equation that is nonlinear and governed by multiple complex coefficients. The damping and frequency response curves are depicted and analyzed across various geometrical and mechanical configurations of sandwich beams. The results indicate that the porosity effects and elastic coefficients of the foundation exert a significant influence on the damping and nonlinear vibration response of these beams.

Keywords: analytical modeling; damping; porosity; FGM; nonlinear elastic foundation; vibration; viscoelastic; Zig-Zag theories

1. Introduction

Structures made of functionally graded materials (FGMs) have garnered significant attention in various studies owing to their tailored properties for specific applications. Several theories have been developed to dynamically investigate structures composed of composite materials. Commonly, many of these theories are formulated within the context of classical theories, notably the Euler-Bernoulli (CBT) theory, the Timoshenko beam theory (TBT), and the first-order shear deformation theory (HSDBT). It is worth noting that these theories fail to fully satisfy the condition of displacement compatibility at the interlayer interfaces throughout the thickness, particularly in multi-layered structures. From a conceptual point of view, modern automotive and aerospace structures are subjected to harsh environmental conditions , creating the need for an economical structural arrangement capable of minimizing displacements and stresses. This has become a challenging problem. The most popular candidate for addressing this issue is the use of sandwich structures.

These are proposed as alternative solutions to replace standard metallic components, primarily those made with functionally graded material (FGM) faces and a viscoelastic core. Within this context, the viscoelastic core sandwich structures are currently employed to mitigate vibrations among the array of available technical solutions. To address this issue, various new kinematical models based on zig-zag theories have been developed to ensure inter-laminar displacement continuity. Finite element methods (FEM) and verification with artificial neural networks were employed by Yaylaci *et al.* (2023a) for analyzing the mechano-bactericidal effect of nanopatterned surfaces Based on a new-generation technology. Yaylaci *et al.* (2023b) analysed the mechano-bactericidal effects of nanopatterned surfaces on implant-derived bacteria using the FEM. Based on the first-order shear deformation, free vibration and buckling analysis of functionally graded porous beams (FGM-P) subjected to various boundary conditions are studied by Turan *et al.* (2023). Yaylacı *et al.* (2024) examined the effect of continuous and discontinuous contact problems of a functionally graded layer using the theory of elasticity and finite element method. Yaylacı *et al.* (2023c) represented a numerical research in vibration and buckling of functionally graded material (FGM) beam comprising edge crack by using finite element method (FEM) and multilayer perception (MLP). Evaluation of the contact problem of functionally graded layer resting on rigid foundation

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pressed via rigid punch by analytical and numerical (FEM and MLP) methods are evaluated By Yaylacı *et al.* (2022a). Yaylacı *et al*. (2022b) analysed the contact problem of the functionally graded layer resting on rigid foundation pressed via rigid punch . Yaylacı *et al.* (2022c) investigated the artificial neural network (ANN) to predict the dimensionless parameters for contact pressures and contact lengths under the rigid punch. Based on analytical and numerical methods the contact problem of functionally graded layers resting on a HP and pressed with a uniformly distributed load is solved by Yaylacı *et al.* (2022d). Adıyaman *et al.* (2023) presented a study on the contact problem of a layer consisting of functionally graded material (FGM) in the presence of body force. The effect of various parameters was analyzed, including material and flow properties on fluid-structure interaction in cage systems, Simulate of edge and an internal crack problem and estimation of stress intensity factor, the contact problem of a functionally graded layer containing crack. Using FEM (Ozdemir *et al.* 2023,Yayalci 2022e, Yaylacı *et al.* 2022f). Yaylacı (2016)solved the crack problem through numerical analysis. The frictionless double receding contact problem for two functionally graded (FG) layers pressed by a uniformly distributed load is presented by Öner *et al.* (2022). Moreover, several studies of FGM beams have been devoted to nonlinear vibration analysis studies that can be found in the available literature. Wattanasakulpong and Ungbhakorn (2014) investigated the linear and nonlinear vibration of FGM porous beams, using the classical beam theory (CBT) and the differential transformation method (DTM) coupled with an iterative procedure with different kinds of elastic supports at the ends. Gao, Xiao and Zhu (2019) studied nonlinear free vibration of FGM beams with various boundary conditions with many kinds of functionally graded distribution. Wang and Mao (2011) carried out the nonlinear active control analysis of FGM beam with piezoelectric sensor and actuator layers using higher order shear deformation theory. Esfahani *et al.* (2014) studied the vibration of an FGM on nonlinear hardening foundation with linear and nonlinear coefficients subjected to nonlinear thermal load in the case of small amplitude. Shen and Wang (2014) analysed the nonlinear free vibration of FGM beams resting on elastic foundations under thermal conditions. In this work, the Kelvin-Voigt model and Mori–Tanaka micromechanics models are considered by employing the higher order shear deformation beam theory. Ke, Yang and Kitipornchai (2010) presented an analytically study on the nonlinear free vibration of Euler-Bernoulli beam with various boundary conditions, Galerkin procedure and Runge-Kutta were used to solve the differential equation problems. Xie, Wang and Fu (2020) investigated the nonlinear forced vibration of FGM beams by using the third-order shear deformable theory and direct numerical integration technique. The nonlinear free vibration analysis of FGM Timoshenko beams having cracks was investigated by Kitipornchai *et al.* (2009) by means of the Ritz method and a direct iterative method to solve the nonlinear vibration frequencies problem. Based on the perturbation method and nonlocal strain gradient theory, nonlinear free vibration of Bernoulli - Euler FG nano-beam resting on elastic foundation under various boundary conditions is studied by Mahmoudpour, Hosseini-Hashemi and Faghidian (2018). Babaei and Eslami (2021) used the Neutral/Mid-plane formulations to study the large amplitude free vibration of thick FGM beam resting on nonlinear elastic foundation subjected to uniform temperature elevation using the third-order shear deformation beam theory. Large amplitude vibrations of laminated beam made of Nano-composite materials under thermal loading are studied by Yang, Huang and Shen (2020) by considering negative Poisson's Ratio. El-Borgi, Rajendran and Trabelssi (2023) investigated the impact of the nonlocal and surface effects on the free and forced vibrations of FGM beams resting on a nonlinear elastic foundation based on Timoshenko's model and by using the differential quadrature method. Fallah and Aghdam (2012) and in another work Bagheri, Kiani and Eslami (2023) analysed the nonlinear response of FGM on Winkler-Pasternak foundation. Mohamed (2024) studied the nonlinear post-buckling of movable simply supported BDFG porous plates rested on elastic foundations. Xu and She (2023) considered the problem of the influence of temperature on the resonance of doubly curved shells. Alimoradzadeh and Akbas (2023) investigated the nonlinear vibration of composite beams resting on a nonlinear foundation. The nonlinear snap-through buckling and forced vibration of curved composite beams under various boundary conditions were addressed by Gan and She (2023). Akbas (2022) analyzed the response of axially functionally graded (AFG), doubly supported beams under moving loads and rising temperatures. The effect of the various shear deformation theories on the nonlinear free vibration of FGM beam are presented by Xie *et al*. (2020) .The buckling and nonlinear vibration behaviour of Euler-Bernoulli FGM porous Micro-beam resting on linear elastic foundation are studied by Dang and Nguyen (2022) Viscoelastic damping technology has experienced rapid development over the past half-century. Gibson and Plunkett (1977) as well as Gibson and Wilson (1979) have conducted earlier research on this topic. For an extensive overview of the existing literature concerning vibration behaviour in the presence of viscoelastic materials, Nakra's articles (Nakara 1981, 1984) serve as valuable resources. Youzera *et al.* (2012) presented a simply study on nonlinear damping and forced vibration of sandwich with viscoelastic core and metal faces. In this work the balance method and Zig-zagtheories are used in the context of FGM sandwich structures with a viscoelastic core. Moita *et al.* (2018) developed a finite element model for analysis the static and dynamic behaviour properties of sandwich beams with functionally graded skins and viscoelastic core with complex frequency dependent material. Analysing the vibration and damping characteristics of sandwich plates composed of both viscoelastic core and functionally graded materials faces are studied by Yang *et al.* (2016) using a modified Fourier-Ritz solution. An analytical model has been developed to evaluate the nonlinear damping and frequency characteristics of a sandwich beam with FGM faces and viscoelastic core layers are given by Youzera *et al.* (2023). Among the composite structures, that composed with carbon or glass fibre emerged in epoxy matrix are the most employed in automotive and aeronautical structure due to their structural efficiency in normal environmental conditions. However, in extreme and harsh load condition arising from high temperature, the composite structures fail to guaranty the needed resistance. In this work, the damping analysis of porous FGM sandwich beams with viscoelastic core resting on nonlinear foundation under uniform distributed load, Zig-Zag theory considers in the core and Euler Bernoulli theory in the faces. By employing Hamilton's principle, the governing nonlinear differential equations are condensed into a single equation, following which the Harmonic Balance method coupled with the Galerkin procedure is applied to discover an analytical solution. The study includes an examination of nonlinear damping and frequency response curves for sandwich beams, delving into the impact of vibration amplitude, nonlinear foundation effects, and material geometric properties.

2. Formulation

2.1 Description of the FGM sandwich beams with viscoelastic core layer

Porous FGM sheets with a thickness denoted as 'hf' are produced through a specific combination of two phases, primarily metal and ceramic. This combination entails a gradual variation in composition and structural transition throughout the volume. This process allows for the evaluation of their intrinsic mechanical properties, denoted as '*P*,' on the faces of the FGM, following a specified law

$$P=\left(P_m-P_c\right)V_m\left(z\right)+P_c-\frac{\alpha}{2}\left(P_m+P_c\right) \quad (1)$$

The properties of ceramics and metals are designated as P_c and P_m for the upper and lower surfaces, respectively.

α represents the volume fraction that accounts for porosity concentration.

Vm (z) represents the volume fraction of either the metal or the glass material. This function is evaluated based on power law variation, as given by

$$V_m(z)=\left(\frac{1}{2}-\frac{z-z_1}{h_f}\right)^n \quad \frac{h_c}{2}\le z\le\frac{h}{2} \quad z_1=\frac{h_c+h_f}{2} \quad (2)$$

$$V_m(z)=\left(\frac{1}{2}+\frac{z-z_3}{h_f}\right)^n \quad -\frac{h}{2}\le z\le-\frac{h_c}{2} \quad z_3=-z_1 \quad (3)$$

Where n denotes the generic coefficient of the FGM material.

2.2 Viscoelastic core layer description

For efficient passive vibration control, a viscoelastic composite material with a thickness denoted as 'hc' is integrated into the core between the FGM faces. This viscoelastic material follows a constitutive law based on a linear viscoelastic model, outlined as follows

$$\sigma_{ij}\left(t\right)=\int_{-\infty}^{t}Q^{*}_{ijkl}\left(t-\tau\right)d\varepsilon_{kl}\left(\tau\right) \quad (4)$$

For the sake of simplicity, the widely recognized Kelvin-Voigt viscoelastic material model can be generalized to Hook's standard constitutive law in the frequency domain. This is represented by complex moduli consisting of real and imaginary parts, expressed as

$$Q^{*}_{ijkl}=Q^{'}_{ijkl}+iQ^{''}_{ijkl} \quad (5)$$

Q^{*}_{ijkl} and $Q^{'}_{ijkl}$ present the storage and loss moduli respectively .

Table 1 Properties of sandwich FGM beam

FGM layers properties		Viscoelastic layer	Geometrical
Aluminium	Ceramic		
E_m= 69G Pa	E_{ci}=380 GPa	E_c= 1.794 Gpa	b=5h
ν_m=0.3	ν_{ci}=0.3	ν_c=0.3	h=0.01 m
ρ_m=2766 kg/m^3	ρ_{ci}=3960 kg/^{3}	ρ_c=968.1 kg/m^3	

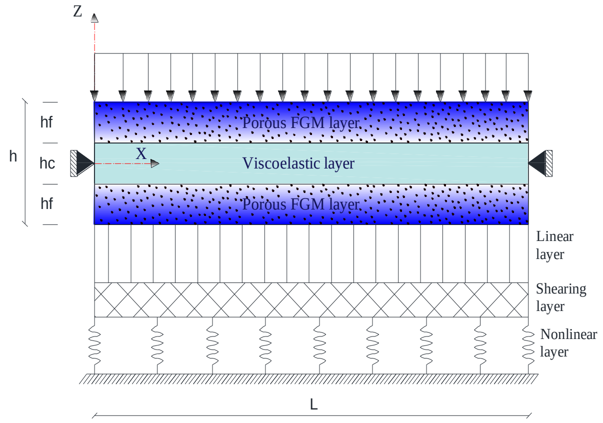

Fig. 1 Schematic figure of Sandwich beam

2.3 Kinematics

Simply supported sandwich beam with FGM faces and viscoleastic core layer, subjected to uniform distribution load q and resting on elastic foundation is indicated in Fig. 1 and referenced to the ordinarily coordinate system (x, y, z), in which the axis x is oriented along the length L, y along the width and z through the thickness direction.

The equilibrium state of the sandwich beam under consideration is formulated in a nonlinear regime, based on the following assumptions.

1. The sandwich beam deflects with the same transverse displacement.
2. The displacement continuity condition is fulfilled at the FGM-viscoelastic interlayer zones.
3. The shear deformations are neglected in the two faces but taken into account in the core layer.

4. The sandwich beam is supported on an elastic foundation described by the Pasternak model, including a nonlinear term.

The derivation of the general governing equations is based on the inter-laminar continuous shear stress Zig-Zag theories (IC-ZZT)as

$$U(x,z,t)=\begin{cases} u_0(x,t)-zw'(x,t)+\left[f(z)+\frac{h_c}{2}k_0\right]\beta(x,t) & \frac{h_c}{2}\le z\le\frac{h}{2}\\ u_0(x,t)-zw'(x,t)+\left[f(z)+k_0 z\right]\beta(x,t) & -\frac{h_c}{2}\le z\le\frac{h_c}{2}\\ u_0(x,t)-zw'(x,t)+\left[f(z)-\frac{h_c}{2}k_0\right]\beta(x,t) & -\frac{h}{2}\le z\le-\frac{h_c}{2}\end{cases} \quad (6a)$$

In the present formulation (.') denotes the derivative in x direction, *f(z)*is the shear function as proposed by Touratier model (Touratier 1991) $f(z)=\frac{h}{\pi}\sin\frac{\pi z}{h}$.

And

$$k_0=\left(\frac{G_f}{G_c}-1\right)\left.\frac{df(z)}{dz}\right|_{z=\frac{h_c}{2}} \quad (6b)$$

The nonlinear strain–displacement relations for each layer can be expressed in the following form

$$\varepsilon_x=U'+\frac{1}{2}(w')^2 \quad (7a)$$

$$\gamma_{xz}=\frac{\partial U}{\partial z}+w' \quad (7b)$$

3. Governing equations

The strain energy of sandwich beams with a viscoelastic layer can be determined

$$P_{\text{int}}(\delta U)=\int_{V_1}\left(\sigma_{xx}\delta\varepsilon_{xx}+\tau_{xz}\delta\gamma_{xz}\right)dv_t$$

$$=\int_{v_f}\left(\begin{array}{l}\sigma_{xx}\left(\begin{array}{l}\delta u_0'(x,t)-z\delta w''(x,t)+\left[f(z)+\frac{h_c}{2}k_0\right]\delta\beta'(x,t)\\+w'(x,t)\delta w'(x,t)\end{array}\right)+\\ \tau_{xz}\left(\frac{\partial\left(f(z)+\frac{h_c}{2}k\right)_0}{\partial z}\delta\beta(x,t)\right)\end{array}\right)dv_f-$$

$$\int_{v_c}\left(\begin{array}{l}\sigma_{xx}\left(\begin{array}{l}\delta u_0'(x,t)-z\delta''w(x,t)+\left[f(z)+k_0 z\right]\delta\beta'(x,t)\\+w'(x,t)\delta w'(x,t)\end{array}\right)+\\ \tau_{xz}\left(\frac{\partial\left(f(z)+k_0 z\right)}{\partial z}\delta\beta(x,t)\right)\end{array}\right)dv_c- \quad (8)$$

$$\int_{v_f}\left(\begin{array}{l}\sigma_{xx}\left(\begin{array}{l}\delta u_0'(x,t)-z\delta''w(x,t)+\left[f(z)-\frac{h_c}{2}k_0\right]\delta\beta'(x,t)+\\ w'(x,t)\delta w'(x,t)\end{array}\right)\\ +\tau_{xz}\left(\frac{\partial\left(f(z)-\frac{h_c}{2}k\right)}{\partial z}\delta\beta(x,t)\right)\end{array}\right)dv_f$$

(.") stands the second derivative according to *x* direction.

The kinetic energy of the considered sandwich beam structure is given by

$$P_{acc}=\frac{1}{2}\left(\begin{array}{l}\int_{v_1}\rho_f\left(\left(\frac{\partial U}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dv_f+\\ \int_{v_2}\rho_c\left(\left(\frac{\partial U}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dv_c+\\ \int_{v_3}\rho_f\left(\left(\frac{\partial U}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right)dv_f\end{array}\right) \quad (9)$$

Where ρ_c and ρ_f are the density for each porous FGM face sheets and the viscoelastic core. In the present model the shear deformation and the rotary inertia of kinetic energy are neglected.

The deformation energy expended by the system of an elastic foundation, formulated by the Pasternak model and incorporating a nonlinear component, is expressed as follows

$$P_{foun}=\frac{1}{2}\int_0^L\left(K_L w^2+K_s\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}K_{NL}w^4\right)dx \quad (10)$$

Where KL and KNL are the linear and nonlinear coefficient of elastic foundation respectively , and Ks stands for the coefficients of shear stiffness of the elastic foundation as suggested in Pasternak model .

The virtual work done by applied uniform distributed force is

$$P_{ext}=\int_0^L q(t)w \quad (11)$$

The inertial works are

$$\delta P_{acc}=\rho_T\int_0^L\frac{\partial^2 w(x,t)}{\partial t^2}\delta w(x,t)dx \quad (12)$$

And

$$\rho_T=b\int_{h_c/2}^{h/2}\rho_f dz+\rho_c S_c+b\int_{-h/2}^{h_c/2}\rho_f dz \quad (13)$$

The principle of Hamilton's is considered to derive the equations of motion. The Hamilton's principle is expressed as:

$$\delta\int_{t_0}^{t_1}\left(P_{in}+P_{acc}-P_{ext}+P_{foun}\right)dt=0 \quad (14)$$

In the above equations, δ indicates the first variation operator and, t_1 and $t_2$define the time interval.

According to Eqs. (8) and (13), the variational formulation of Eq. (14) can be rewritten as

$$\begin{array}{l}N_T\left(\delta u_0'(x,t)+w'(x,t)\delta' w(x,t)\right)+M_\beta\delta\beta'(x,t)\\ +M_w\delta''w(x,t)+T\delta\beta(x,t)+\rho_T\int_0^L\frac{\partial^2 w(x,t)}{\partial t^2}\delta w(x,t)\\ +k_L w(x,t)\delta w+k_{NL}w(x,t)^3\delta w-k_s w''(x,t)\delta w\\ =\int_0^L q(t)\delta w(x,t)dx\end{array} \quad (15)$$

With

$$N_T = \left(b\int_{h_c/2}^{h/2} E_f dz + E_c S_c + b\int_{-h/2}^{h_c/2} E_f dz\right)\left(u_c' + \frac{1}{2}(w')^2\right) \quad (16a)$$

$$M_w = S_1 w'' - S_2 \beta' \quad (16b)$$

$$M_\beta = -S_2 w'' + S_3 \beta' \quad (16c)$$

$$T = S_4 \beta \quad (16d)$$

In this context, N_T represents the normal force, and T signifies the shear load defined on the cross-section of the sandwich beam. M_w and M_β correspond to the bending moments associated with deflection w and section rotation β, respectively.

The stiffness coefficients indicated above are calculated as

$$C_1 = \int_{hc/2}^{h/2} E_f z^2 dz + \int_{-hc/2}^{hc/2} E_c z^2 dz + \int_{-h/2}^{hc/2} E_f z^2 dz \quad (17a)$$

$$C_2 = \int_{hc/2}^{h/2} E_f z\left(f(z) + \frac{h_c}{2}k_0\right)dz + \int_{-hc/2}^{hc/2} E_c z\left(f(z) + k_0 z\right)dz + \int_{-h/2}^{hc/2} E_f z\left(f(z) - \frac{h_c}{2}k_0\right)dz \quad (17b)$$

$$C_3 = \int_{hc/2}^{h/2} E_f\left(f(z) + \frac{h_c}{2}k_0\right)^2 dz + \int_{-hc/2}^{hc/2} E_{c2}\left(f(z) + k_0 z\right)^2 dz + \int_{-h/2}^{hc/2} E_f\left(f(z) - \frac{h_c}{2}k_0\right)^2 dz \quad (17c)$$

$$C_4 = \int_{hc/2}^{h/2} E_f\left(\frac{\partial\left(f(z) + \frac{h_c}{2}k_0\right)}{\partial z}\right)^2 dz + \int_{-hc/2}^{hc/2} E_c z\left(\frac{\partial\left(f(z) + k_0 z\right)}{\partial z}\right)^2 dz + \int_{-h/2}^{hc/2} E_f z\left(\frac{\partial\left(f(z) - \frac{h_c}{2}k_0\right)}{\partial z}\right)^2 dz \quad (17d)$$

4. Approximate solution

The deflection w and sectional rotation β of the sandwich beam are assumed to follow a harmonic pattern and are normalized with respect to the linear vibration mode.

$$w(x,t) = \frac{1}{2}\left(Ae^{i\omega t} + \overline{A}e^{-i\omega t}\right)w(x) \quad (18a)$$

$$\beta(x,t) = \frac{1}{2}\left(Ae^{i\omega t} + \overline{A}e^{-i\omega t}\right)B(x) \quad (18b)$$

The displacement solutions presented above are designed to completely meet the boundary conditions, where A represents the complex displacement amplitude, and its conjugate is denoted as Ā.

Referring to Eq. (15) and employing the differentiation by parts with respect to the internal incremental variables δβ and δw following Galerkin's method, the resulting equilibrium condition is expressed as follows

$$M'_\beta - T = 0 \quad (19)$$

$$M''_w = \rho_T \omega^2 w \quad (20)$$

Where

$$w(x) = \sin(kx) \quad (21a)$$

$$B(x) = b_0 \cos(kx) \quad (21b)$$

$$k = n\pi / L \quad (21c)$$

And

$$b_0 = \frac{C_2 k^3}{C_3 k^2 + C_4} \quad (21d)$$

In the case of the nonlinear vibration, the governing differential equations are:

$$N'_T(t) = 0 \quad (22a)$$

$$M'_\beta(x,t) - T(x,t) = 0 \quad (22b)$$

$$-N_T(t)w''^2(x,t) + M''^2_w(x,t) + k_L w(x,t) + k_{NL} w(x,t)^3 - k_s w''(x,t) + \rho_T \frac{\partial w^2(x,t)}{\partial t^2} = q(t) \quad (22c)$$

According the integral between 0 and L of Eq. (22(a)), the nonlinear stretching force is given by

$$N(t) = \frac{1}{2L}\left(b\int_{h_c/2}^{h/2} E_f dz + E_c S_c + b\int_{-h/2}^{h_c/2} E_f dz\right)\overline{N} \quad (23a)$$

$$\overline{N} = \int_0^L \left(w'(x,t)\right)^2 dx \quad (23b)$$

After some manipulations of the Eq. (22(c)), the nonlinear forced vibration problem under consideration is reduced to the following form of six (06) order nonlinear differential equation formulated as

$$D_1 w^{VI}(x,t) + D_2 w^{IV}(x,t) + D_3 w''(x,t) + D_4 \frac{\partial^2 w''(x,t)}{\partial t^2} + D_5 \frac{\partial^2 w(x,t)}{\partial t^2} + D_6 q(t) + D_7 w(x,t) + D_8 w(x,t)\left(w'(x,t)\right)^2 + D_9 w(x,t)^3 + D_{10} w''(x,t) w(x,t)^2 = 0 \quad (24)$$

Where the stiffness coefficients D_i (i=1..10) are expressed as

$$D_1 = C_2 - \frac{C_1 C_3}{C_2} \quad (25a)$$

$$D_2 = \frac{C_3}{C_2} N(t) + \frac{C_1 C_4}{C_2} + \frac{C_3}{C_2} k_s \quad (25b)$$

$$D_3 = -\frac{C_4}{C_2} N(t) - \frac{C_3}{C_2} k_l - \frac{C_4}{C_2} k_s \quad (25c)$$

$$D_4 = \frac{C_3}{C_2} \rho_T \quad (25d)$$

$$D_5 = -\frac{C_4}{C_2} \rho_T \quad (25e)$$

$$D_6 = -\frac{C_4}{C_2} \quad (25f)$$

$$D_7 = \frac{C_4}{C_2} k_l \quad (25g)$$

$$D_8 = -6\frac{C_3}{C_2} k_{NL} \quad (25h)$$

$$D_9 = \frac{C_4}{C_2} k_{NL} \quad (25i)$$

$$D_{10} = -3\frac{C_3}{C_2} k_{NL} \quad (25j)$$

The derivation of the nonlinear amplitude-frequency equation involves injecting Eqs. (18(a)) and (18(b)) into Eq. (24) and applying the harmonic balance method. This process results in a complex scalar amplitude equation as follows

$$-\omega^2 MA + \widehat{K}_L A + \widehat{K}_{NL} A^2 \overline{A} = Q \quad (26)$$

In which $\widehat{K}_L$, $\widehat{K}_{NL}$ and M represent the linear and the nonlinear and mass terms respectively of the amplitude equation expressed as

$$\begin{aligned}\widehat{K}_{NL} = & -\frac{3C_4}{16LC_2}\left(A_0^{(1)} + A_0^{(2)} + A_0^{(3)}\right)\overline{N}\int_0^L w''(x,t)\,dx + \\ & \frac{3C_3}{16LC_2}\left(A_0^{(1)} + A_0^{(2)} + A_0^{(3)}\right)\overline{N}\int_0^L w^{IV}(x,t)\,dx \\ & -\frac{9C_3 k_{NL}}{8C_2}\int_0^L w''(x,t)\,w(x,t)^2 dx - \\ & \frac{9C_3 k_{NL}}{4C_2}\int_0^L \left(w'(x,t)\right)^2 w(x,t)\,dx + \frac{3C_4 k_{NL}}{8C_2}\int_0^L w(x,t)^3 dx\end{aligned} \quad (27a)$$

$$\begin{aligned}\widehat{K}_L = & -\frac{1}{2}\frac{C_3 C_1}{C_2}\int_0^L w^{VI}(x,t)\,dx + \frac{1}{2}\frac{C_3 k_s}{C_2}\int_0^L w^{IV}(x,t)\,dx + \\ & \frac{1}{2}\frac{C_1 C_4}{C_2}\int_0^L w^{IV}(x,t)\,dx - \frac{1}{2}\frac{C_4 k_s}{C_2}\int_0^L w''(x,t)\,dx \\ & +\frac{1}{2}C_2\int_0^L w^{IV}(x,t)\,dx + \frac{1}{2}\frac{C_4 k_l}{C_2}\int_0^L w(x,t)\,dx\end{aligned} \quad (27b)$$

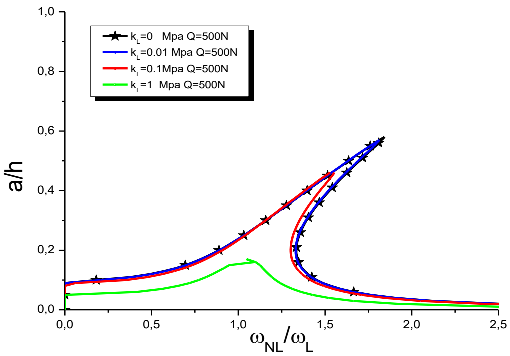

Fig. 2 Effect of kL on the frequency ratio of SS FG beam ($k_S = 0$, $k_{NL} = 0$, $n = 1$, $h = 0.5$, $L = 70\,h$, $h_f = 7h_c$)

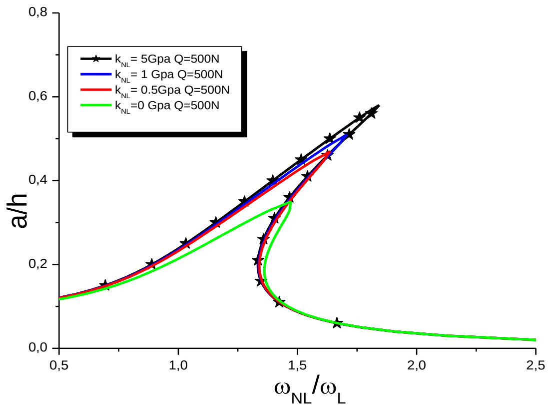

Fig. 3 Effect of k_{NL} on the frequency ratio of SS FG beam ($k_S = 0$, $k_L = 0$, $n = 1$, $h = 0.5$, $L = 70\,h$, $h_f = 7h_c$)

$$M = -\frac{1}{2}\frac{C_3 \rho_T}{C_2}\int_0^L w''(x,t)\,dx + \frac{1}{2}\frac{C_4 \rho_T}{C_2}\int_0^L w(x,t)\,dx \quad (27c)$$

5. Results and discussion

In this section, nonlinear forced vibration analysis of sandwich beams made from Functionally Graded Materials (FGM) with a viscoelastic core is carried out. The considered sandwich beams are resting on a nonlinear elastic foundation. The material and geometrical properties of the considered sandwich beam structure are given in Table 1. The graphs depicting the changes in the nonlinear to linear frequency ratio (ω_{NL}/ω_L) and the loss factor ratio (η_{NL}/η_L) of the sandwich beams are presented. The impact of the elastic foundation on the nonlinear behavior of the sandwich beams is assessed using Eq. (26) and are represented in Figs. 2 through 4, that illustrate the effects of the linear (K_L), nonlinear (K_{NL}), and shear (K_S) elastic foundation coefficients on the nonlinear frequency response (ω_{NL}/ω_L). Fig. 2 displays the impact of the linear coefficient (K_L) on the nonlinear frequency amplitude response. This

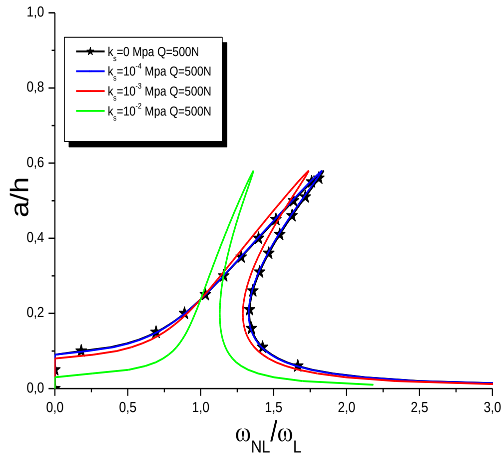

Fig. 4 Effect of ks on the frequency ratio of SS FG beam ($k_L = 0$, $k_{NL} = 0$, $n = 1$, $h = 0.5$, $L=70\ h$, $h_f=7h_c$)

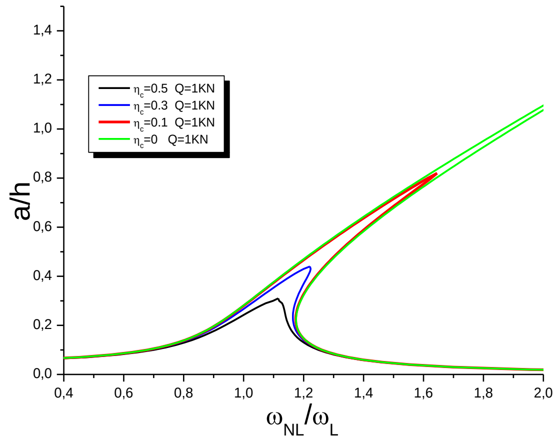

Fig. 6 Effect of loss factor η_c on the frequency ratio of SS FG beam ($k_L = 1$ Mpa, $k_S = 10^{-2}$Mpa, $k_{NL} = 5$ Gpa, $L=70\ h$, $h_f=7h_c$, $n=1$)

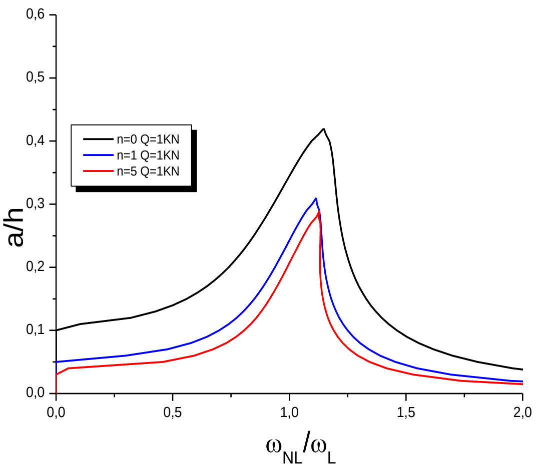

Fig. 5 Effect of volume fraction exponent (n) on the frequency ratio of SS FG beam ($k_L = 1$ Mpa, $k_S = 10^{-2}$Mpa, $k_{NL} = 5$ Gpa , $h = 0.5$, $L=70\ h$, $h_f=7h_c$)

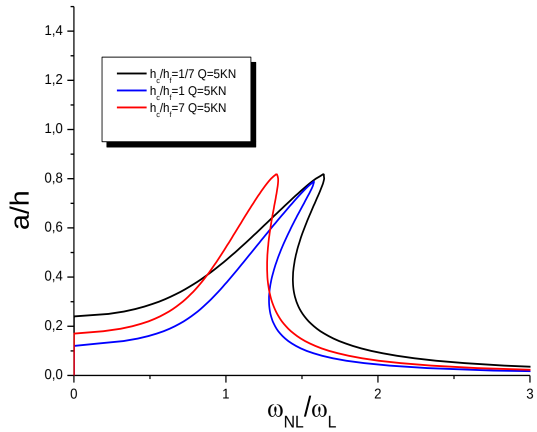

Fig. 7 Effect of h_c /h_f ratio on the frequency ratio of SS FG beam ($k_L = 1$ Mpa, $k_S = 10^{-2}$Mpa, $k_{NL} = 5$ Gpa , $L=70\ h$,$h_c=0.5$, $n=1$)

figure shows that the nonlinear behaviour is characterized by a hardening effect and depicts that the amplitude ratio decreases with the linear coefficient K_L increasing. Thus, the nonlinear effect can be managed by manipulating K_L, in order to mitigate as far as possible the nonlinear impact.

Fig. 3 indicates the effect of the nonlinear component K_{NL} of the foundation on the frequency curves. Contrary to the influence of the linear coefficient (K_L), Fig. 3 demonstrates that the frequency curves exhibit significant displacement amplification with an increase in K_{NL}.

The findings depicted in Fig. 4 highlight a clear correlation: an escalation in the shear elastic foundation coefficients (K_S) markedly influences the nonlinear frequency-amplitude response and its concurrent hardening behavior. In fact, this emphasizes that decreasing the shear elastic foundation coefficients K_S in the sandwich structure leads to a more pronounced manifestation of hardening behavior.

The impact of the volume fraction exponent, *n*' on the nonlinear frequency response of functionally graded (FG) sandwich beams is depicted in Fig. 5. The results illustrate that the frequency response of the typically functionally graded material (FGM) sandwich beam faces is lower compared to the isotropic ones, particularly when $n=0$.

Fig. 6 demonstrates the dependence of the amplitude-frequency curves on the variation of the viscoelastic core layer loss factor, ηc. As anticipated, increasing ηc results in a notable reduction in vibration amplitude.

Fig. 7 depicts the impact of the ratio of core to skin thickness (h_c/h_f) on the nonlinear frequency response, indicating that frequency alterations are dependent on the core's thickness. The substantial thickness of the core poses challenges in mitigating stress within the core through shear mechanisms. This mitigation of shear stress significantly contributes to the reduction in damping properties within the sandwich beam structure.

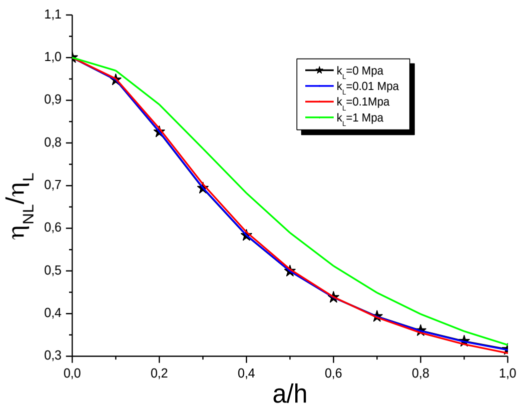

Fig. 8 Effect of k_L on the Loss factor of SS FG beam (Mpa), k_S =10^{-2}Mpa, k_{NL} = 5 Gpa, L=70 h, h_f=7h_c, n=1)

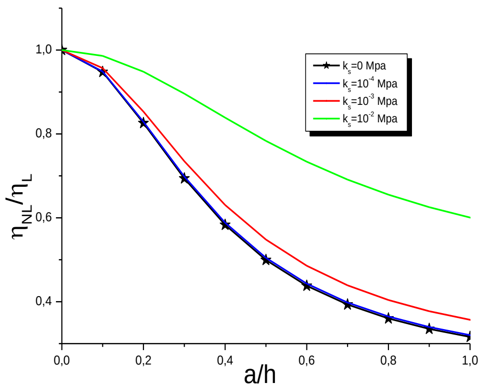

Fig. 10 Effect of k_s on the Loss factor of SS FG beam (k_L = 1 Mpa, k_S =10^{-2} Mpa, k_{NL} = 5 Gpa , L=70 h, h_f=7h_c, n=1)

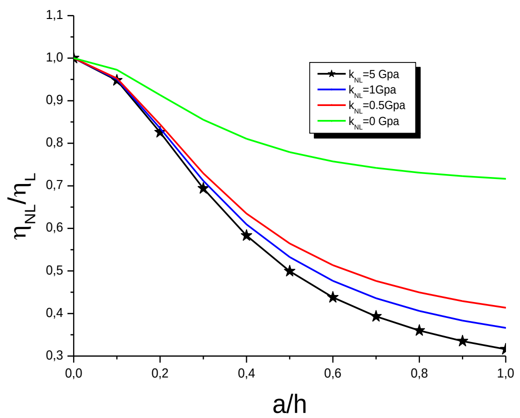

Fig. 9 Effect of k_L on the Loss factor of SS FG beam (k_L = 1 Mpa, k_S =10^{-2}Mpa, Gpa , L=70 h, h_f=7h_c, n=1)

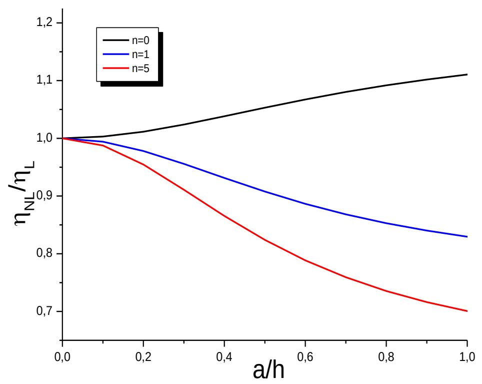

Fig. 11 Effect of volume fraction exponent (n) on the Loss factor of SS FG (k_L = 1 Mpa, k_S =10^{-2} Mpa, k_{NL} = 5 Gpa, L=70 h, h_f=7h_c)

As previously outlined, the impacts of the indicated parameters on the amplitude-frequency curves are re-evaluated to illustrate how they affect the ratio of the loss factors η_{nl}/η_l as it varies with the amplitude of vibration ratio *a/h.*

The inspection of the evolution of the loss factor ratio η_{nl}/η_l concerning the vibration amplitude in the nonlinear regime is illustrated in Fig. 8. This variation is achieved by altering the linear term of the rigidity, K_L, within the foundation system. Remarkably, it becomes evident from this figure that the ratio η_{nl}/η_l consistently diminishes alongside the magnitude of the amplitude ratio *a/h*.

This trend persists regardless of the value assigned to the foundation rigidity parameter, K_L.

Fig. 9 demonstrates the alteration of the loss factor ratio η_{nl}/η_l concerning the amplitude magnitude ratio *a/h,* for different values of the nonlinear foundation stiffness parameter K_{NL}. It is evident from this figure that the parameter K_{NL} does not significantly influence the evolution of the loss factor η_{nl}/η_l concerning *a/h*. However, a marginal decrease in the loss factor is observed for K_{NL} =0GPa compared to the other values of K_{NL}.

Fig. 10 highlights the influence of the shear stiffness parameter, K_s on the variation of the loss factor ratio versus *a/h* . The figure distinctly illustrates a significant decrease in η_{nl}/η_l as the vibration amplitude magnitude increases. However, this decrease appears to diminish as the values of K_s increase.

Fig. 11 illustrates the impact of the volume fraction exponent, *'n,'* on the evolution of the nonlinear loss factor η_{nl}/η_l during large amplitude vibrations.

This figure distinctly portrays the substantial influence of the *'n'* parameter on the damping characteristics of the FGM sandwich beam. Specifically, there is a moderate improvement observed for large displacements in a sandwich beam composed of isotropic faces *(n=0)* compared to typically faced FGM sandwich beams, which exhibit a slight degradation in the loss factor.

6. Conclusions

This study delves into the analysis of nonlinear forced vibrations occurring in an FGM sandwich beam featuring a viscoelastic core layer supported by a nonlinear foundation.

The analytical formulation considers both normal and shear deformations within the core layer, utilizing a refined higher-order theory. The governing differential equations, formulated within the context of large amplitude vibrations, are derived based on Hamilton's principle. Utilizing the harmonic balance method and Galerkin's procedure, a scalar complex nonlinear amplitude–frequency relationship is established, leading to a closed-form analytical solution for the nonlinear forced vibration problem. The parametric study primarily centres on exploring the effects of foundation stiffness parameters on both the amplitude frequency curve and the evolution of the nonlinear loss factor ratio η_{nl}/η_l concerning vibration amplitude. Additionally, this investigation encompasses the impact assessment of the geometrical and material parameters within the sandwich beam components. The obtained outcomes yield the following observations:

i: An increase in the linear stiffness parameter KL results in a notable reduction in amplitude vibration. However, this parameter does not affect the degradation of the nonlinear loss factor, which remains consistent regardless of the values assigned to K_L.

ii: The magnitude of the vibration amplitude increases with the nonlinear stiffness foundation parameter K_{NL}. The study of nonlinear vibration in the sandwich beams indicates that when K_{NL} approaches zero, a moderate degradation in the nonlinear loss factor is observed.

iii: The foundation shear stiffness K_s notably impacts the nonlinear hardening behavior of FGM sandwich beams. Increasing this parameter reduces the degradation in the loss factor associated with amplitude vibration.

iv: The influence of the volume fraction exponent, *'n,'* on the amplitude frequency curve suggests a significant decrease in vibration amplitude for higher 'n' values, accompanied by a pronounced degradation in the loss factor. It's noteworthy that sandwich beams composed of isotropic faces (*n=0*) display an effective improvement in the loss factor for large vibration amplitudes."

As expected, sandwich beams with thin core layers exhibit a more pronounced nonlinear hardening behavior compared to those with thicker core layers. Additionally, enhancing the loss factor of the core viscoelastic material significantly contributes to the improvement of damping properties, thereby enhancing overall performance.

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