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Limit equilibrium and swarm intelligence solutions in analyzing shallow footing's bearing capacity located on two-layered cohesionless soils

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Abstract. The research findings of two nonlinear machine learning and soft computing models- the Cuckoo optimization algorithm (COA) and the Teaching-learning-based optimization (TLBO) in combination with artificial neural network (ANN)- are presented in this article. Detailed finite element modeling (FEM) of a shallow footing on two layers of cohesionless soil provided the data sets. The models are trained and tested using the FEM outputs. Additionally, various statistical indices are used to compare and evaluate the predicted and calculated models, and the most precise model is then introduced. The most precise model is recommended to estimate the solution after the model assessment process. When the anticipated findings are compared to the FEM data, there is an excellent agreement, which indicates that the TLBO-MLP solutions in this research are reliable (R^2=0.9816 for training and 0.99366 for testing). Additionally, the optimized COA-MLP network with a swarm size of 500 was observed to have R^2 and RMSE values of (0.9613 and 0.11459) and (0.98017 and 0.09717) for both the normalized training and testing datasets, respectively. Moreover, a straightforward formula for the soft computing model is provided, and an excellent consensus is attained, indicating a high level of dependability for the suggested model.

Keywords: artificial neural network; bearing capacity; cohesionless soil; limit equilibrium; shallow foundation

1. Introduction

The bearing capacity of shallow footings on two-layered cohesionless soils is a crucial aspect of geotechnical engineering, particularly in scenarios where soil layers have different properties. This situation commonly arises in practice, especially in areas with varying soil stratifications, and understanding it is vital for safe and economical foundation design (Shariati *et al*. 2019). Typically, the soil layer is directly under the footing, which could be weaker or stronger than the lower layer. The second layer beneath the upper layer influences the overall bearing capacity depending on its relative strength and thickness. Their relative densities and strengths heavily influence the interaction between the layers. A stronger upper layer over a weaker lower layer might provide higher initial resistance. Still, it could fail if the weaker layer cannot support the loads transferred through the upper layer. If the upper layer is thin, the footing may penetrate the lower layer, making the bearing capacity largely dependent on the properties of the lower layer. The interface between the two layers can significantly influence bearing capacity. If the upper layer is weaker, sliding at the interface could become a critical failure mechanism (Rao *et al*. 2015).

Many scholars study the Failure Mechanisms of two-layered Cohesionless Soils. The punching shear failure occurs when the footing penetrates the upper layer and directly affects the lower layer. A weaker lower layer could reduce bearing capacity (Boufarh *et al*. 2019, Lyu *et al.* 2022).

Many theoretical, numerical or even empirical methods have been used to calculate the bearing capacity for foundations on multilayer soils. For instance, in Terzaghi's bearing capacity theory (i.e., traditionally applied to homogenous soils)(Terzaghi 1943), adaptations can be made for two-layered systems, considering both layers' strength parameters (angle of internal friction). In addition, Finite element methods (FEM) or finite volume methods (Sohrabi *et al.* 2024) are recommended and feasible solutions for solving very complex engineering problems. For instance, FEM is often used to model the interaction between layers more accurately and to simulate potential failure mechanisms (e.g., also called numerical simulation). Thirdly, the evaluation of the bearing capacity through empirical and semi-empirical methods is based on various available formulas and charts. They offer practical solutions based on experimental and field data for two-layered soils.

The limit equilibrium technique (LET) is a widely used

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method in geotechnical engineering to analyze the stability of slopes, retaining walls, and the bearing capacity of soils, including multilayered soils (Dong-ping 2021). In multilayered soils, the method is applied to evaluate the ultimate bearing capacity of shallow foundations, accounting for the complex interaction between different soil layers (Ghazavi and Eghbali 2008). LET involves dividing the soil mass into slices or elements and analyzing the equilibrium of forces and moments acting on each slice. The method seeks to find the point at which the soil mass is on the verge of failure, hence determining the safety factor and ultimate bearing capacity. The technique assumes that the soil mass is critical, where any additional load will lead to failure. This approach allows for assessing various failure mechanisms and can be adapted for different soil conditions, including multilayered systems. In multilayered soils, a shallow foundation's bearing capacity depends on each soil layer's strength and deformation properties (Kuo *et al*. 2009). Weaker or stronger layers beneath the footing influence the overall stability and load-bearing behavior. LET is used to evaluate the bearing capacity by considering the contribution of each layer to the overall stability. The technique accounts for the difference in shear strength parameters (such as cohesion and angle of internal friction) between layers (Ghazavi and Eghbali 2008).

The LET in multilayered soils involves calculating the bearing capacity factors for each layer and combining them to determine the overall bearing capacity. The analysis considers the continuity of the failure surface across the layers and the differential settlement that may occur due to the varying stiffness of the layers. In this sense, various empirical and semi-empirical methods are also used to complement LET, providing a more comprehensive understanding of the soil's behavior under load. Normally, two scenarios for failure Mechanisms are possible in layered soils. First, called general shear failure, it occurs when the upper layer is strong, and the load is distributed through the entire soil mass. In such a case, the failure surface may pass through both layers, requiring the analysis to consider the interaction between the layers. General shear failure is more likely when the upper layer is strong and thick enough to prevent penetration into the lower layer. Many scholars note that the failure surface may extend through both layers (Padmini *et al.* 2008, Lyu *et al*. 2022).

The second failure type is called punching shear failure, as the weaker layer lies beneath a stronger one, and the footing may punch through the upper layer, with the bearing capacity being controlled by the weaker layer and noting that engineers must carefully characterize the soil profile through site investigations and laboratory tests to apply LET accurately. The method is sensitive to the accuracy of input parameters, such as soil strength and layer thickness. Safety factors are typically applied to account for uncertainties in soil properties and loading conditions (Kuo *et al.* 2009).

Artificial Neural Networks (ANNs) have gained prominence in geotechnical engineering, particularly in complex tasks like predicting the bearing capacity of foundations on layered soils (Moayedi and Armaghani 2018, Moayedi and Hayati 2018). ANNs offer a data-driven approach to model the nonlinear relationships between various soil parameters and bearing capacity, complementing traditional methods like the LET. Indeed, soil behavior, particularly in layered configurations, is highly nonlinear and influenced by layer thickness, soil strength, and load distribution factors (Moayedi *et al*. 2019). ANNs excel at capturing these complex, nonlinear relationships through training on relevant data sets. Indeed, ANNs can model the interactions between multiple soil layers more effectively than purely analytical methods, providing a more nuanced understanding of how different soil layers contribute to overall bearing capacity (Moayedi and Hayati 2018, Moayedi and Rezaei 2021). Unlike traditional methods that rely on predefined equations and parameters, ANN-based methods learn directly from data.

This is particularly useful in scenarios where soil properties are variable or empirical data is available from past projects. ANNs can be trained on data from site investigations, laboratory tests, and previous bearing capacity calculations, allowing for predicting bearing capacity in new scenarios based on learned patterns. For example, an ANN can be trained to predict the ultimate bearing capacity by learning from data that includes the outcomes of LET analyses. This hybrid approach allows engineers to leverage the strengths of both methods (Moayedi and Armaghani 2018).

Nature-inspired optimization algorithms have seen significant developments and applications in geotechnical engineering, particularly in estimating the bearing capacity of shallow footings. These algorithms, which mimic natural processes such as evolution, swarm behavior, and the social interactions of species, offer powerful tools for solving complex optimization problems in engineering (Onyelowe *et al.* 2023).

As in the Swarm intelligence-based algorithms, particle swarm optimization (PSO), inspired by the social behavior of birds, has been increasingly applied to optimize the parameters of empirical models used to estimate the bearing capacity of shallow footings. Recent developments include hybrid models that combine PSO with other techniques like genetic algorithms (GA) to enhance convergence speed and accuracy (Moayedi *et al.* 2020). Researchers have used PSO to optimize the parameters of Terzaghi's and Meyerhof's bearing capacity equations, achieving more reliable predictions in varied soil conditions. Ant Colony Optimization (ACO), modeled after the foraging behavior of ants, has been applied to optimize the footing design by minimizing cost while ensuring sufficient bearing capacity (Ukritchon and Keawsawasvong 2016). This approach helps find the optimal footing size and shape combination under complex soil profiles. ACO has been hybridized with other algorithms like GA to improve its robustness in handling multi-objective optimization problems related to geotechnical design.

Similarly, in Evolutionary Algorithms, GA, inspired by natural selection, remains one of the most popular algorithms for optimizing the estimation of bearing capacity. Recent studies have focused on combining GA with ANN models, where GA optimizes the input features or hyperparameters of the ANN to enhance prediction accuracy. The adaptability of GA has led to its use in conjunction with fuzzy logic systems, providing a way to incorporate uncertainty in soil properties into the optimization process.

In addition, differential evolution (DE), a variant of GA that emphasizes mutation and recombination, has also been utilized to calibrate empirical bearing capacity formulas to reflect site-specific conditions better. Its recent applications include optimizing the design of shallow foundations under complex loading conditions. In some cases, DE has outperformed traditional GA, particularly in scenarios involving continuous and multi-dimensional search spaces.

Regarding metaheuristic approaches, Harmony Search (HS), inspired by musicians' improvisation process (e.g., metaheuristic optimization algorithm inspired by the process of musical improvisation where musicians seek to find a pleasing harmony.), has been applied in recent studies to optimize bearing capacity estimations by fine-tuning model parameters. The flexibility of HS in exploring a wide solution space without getting trapped in local minima has been a key advantage (Cheng *et al*. 2012).

Hybrid HS models have been developed to combine their strengths with other algorithms like Simulated Annealing (SA) for improved convergence in complex geotechnical problems. The Firefly Algorithm (FA), based on the flashing behavior of fireflies, has been used to solve multi-objective optimization problems in geotechnical engineering (Ghorbani *et al*. 2017). Recent advancements include the integration of FA with fuzzy systems to handle the inherent uncertainty in soil properties and loading conditions. FA's ability to explore a broad solution space while focusing on the brightest solutions has effectively optimized the bearing capacity of shallow footings on layered soils.

Few investigations have been conducted on the shallow foundation's bearing capacity on two-layered cohesionless soil using neural network-based modeling. To fine-tune ANN's inherent biases and weights, hybrid optimization algorithms may benefit from the potent population-based stochastic technique provided by the Cuckoo optimization algorithm (COA) and Teaching-learning-based optimization (TLBO). The shallow foundation's bearing capacity on two-layered cohesionless soil has never been addressed using the hybrid COA-MLP and TLBO-MLP models introduced in this research. To predict the eventual bearing capacity of a shallow foundation (q_{ult}) on two layers of cohesionless soil, nearly no research has been conducted using a hybrid COA-MLP and TLBO-MLP solution based on learning systems. In this research, finite element model simulations were offered and employed in neural network training systems to determine the best performance to approximate the behaviors of cohesionless soil (with two layers of soil). Correspondingly, numerous models were assessed using a trial-and-error process on their effective parameters to determine the most suitable ANN framework. Eventually, the best structure for each model, or the design simplifying the equation solution, was introduced. The q_{ult} in a two-layer cohesionless soil has been predicted in this article using the optimal forms of the COA-MLP and TLBO-MLP models.

2. Established database

As stated earlier, The Finite Element Method (FEM) and the Limit Equilibrium Method (LEM) are powerful tools in geotechnical engineering used for analyzing the bearing capacity of foundations, particularly when dealing with layered soils. FEM is primarily used for detailed stress-strain analysis, while LEM is traditionally employed to evaluate the stability of slopes and foundations. Combining these methods allows for a comprehensive analysis that leverages the strengths of both approaches.

Steps to Integrate FEM and LEM in bearing capacity analysis include (i) identification of soil layers, where it begins by characterizing the different soil layers, including their thickness, cohesion, internal friction angle, and unit weight. Input the material properties for each soil layer into the FEM software. These properties include Young's modulus, Poisson's ratio, and relevant nonlinear soil behavior data.

To set up the finite element model, we initially created a model of the foundation and the surrounding soil in FEM software, such as Abaqus, PLAXIS, or ANSYS. Here, we can define the geometry of the shallow footing and the layering of the soil beneath it. As for the current study, we prefer PLAXIS version 8.2 due to its simplicity. Generate a finite element mesh over the soil domain. Ensure the mesh is fine enough near the foundation to capture the stress distribution accurately but coarser at greater distances from the foundation to save computational resources. Next, we apply appropriate boundary conditions. Typically, the model's base is fixed (no displacement), and the sides are constrained to prevent horizontal movement while allowing vertical displacement. Then, we set appropriate boundary conditions. Typically, the model's base is fixed (no displacement), and the sides are constrained to prevent horizontal movement while allowing vertical displacement. In most cases, we run the FEM simulation to obtain the stress-strain distribution within the soil. This helps understand the soil's behavior under the applied loads, particularly the development of plastic zones, which are critical in identifying potential failure mechanisms.

While FEM provides detailed stress and deformation information, LEM is used to check the stability of the foundation against failure. Using the stress distribution from FEM, conduct a limit equilibrium analysis to evaluate the factor of safety (FoS) against bearing capacity failure. In addition, we can identify potential failure mechanisms, such as shear failure, and calculate the corresponding bearing capacity using LEM equations (e.g., Terzaghi's or Meyerhof's bearing capacity formulas) adjusted for layered soils. Based on the results from LEM, refine the FEM model if necessary. For example, if the LEM suggests that failure might occur at a lower load than predicted by FEM, revisit the soil parameters or mesh density in the FEM model. Therefore we perform iterative analyses, adjusting parameters and model configurations until both FEM and LEM predictions converge toward a consistent bearing capacity value.

The input layers used in the current study were footing width (B), upper layer thickness (h1), upper layer internal

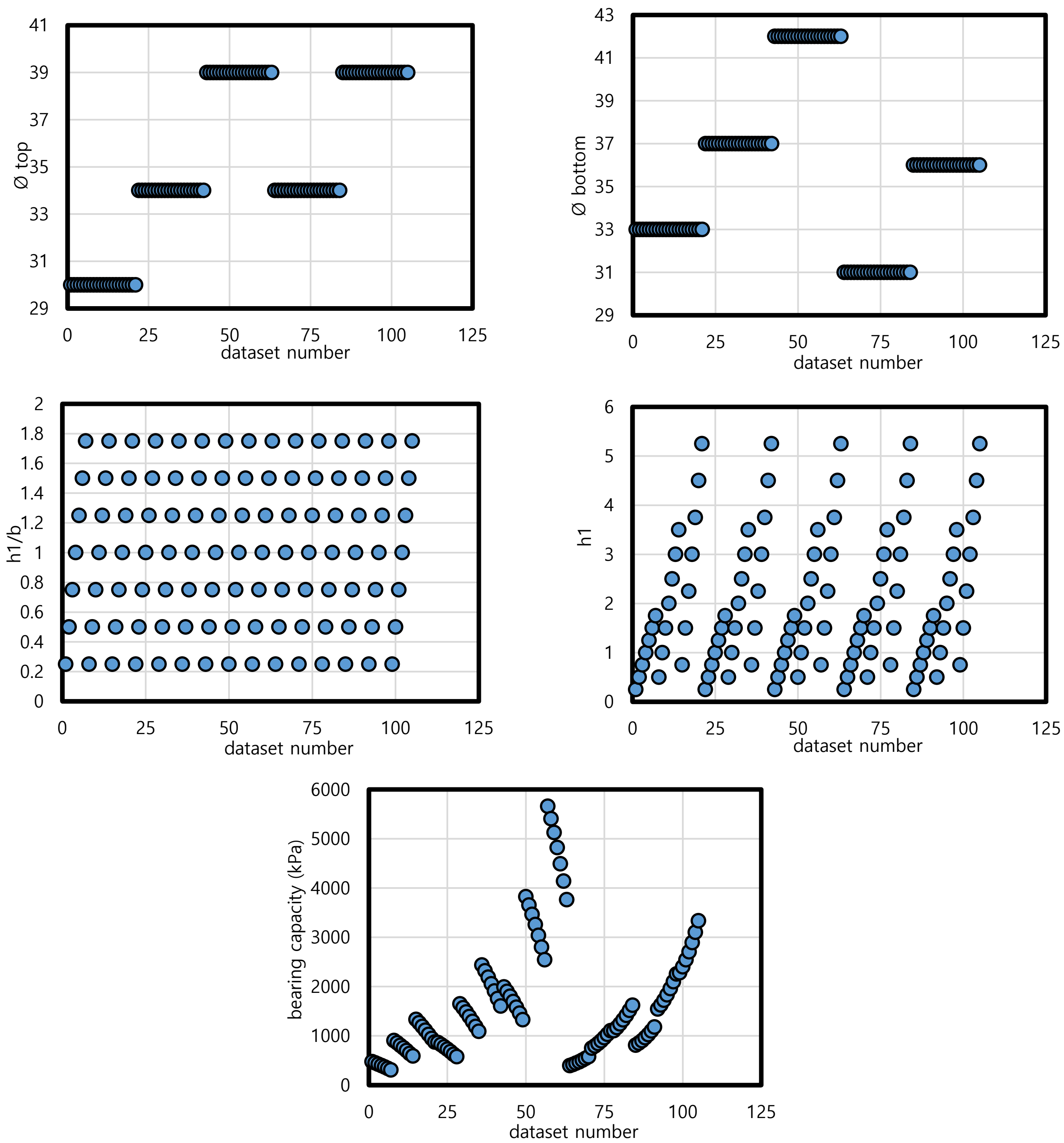


Fig. 1The graphical distribution of the input and output layers

friction angle in ($\emptyset_{top}$) and lower layer internal friction angle ($\emptyset_{bottom}$), where q_{ult} represents the applied stress network output. The results of FEM modeling resulted in over 98 different cases.

3. Methodology

3.1 Limit equilibrium approach

Calculating a shallow foundation's bearing capacity is challenging for practical engineers. Indeed, the limit equilibrium analysis for the bearing capacity of shallow footings on multilayered sandy soils is a comprehensive approach that integrates soil properties, foundation geometry, equilibrium conditions, and correction factors to assess the stability and capacity of the foundation. Accurately determining these elements is crucial for ensuring the footing design is safe and effective under the given soil conditions. In this regard, several key elements and factors must be considered. These elements ensure a comprehensive understanding of the soil behavior and the potential failure mechanisms. These elements are listed in Table 1.

Table 1 The key elements in calculating the shallow bearing capacity using limit equilibrium analysis

element	description
Soil Stratification and Properties	**Layer Thickness:** The thickness of each soil layer plays a crucial role in bearing capacity analysis, as the depth and properties of the layers influence the stress distribution and potential failure planes. **Shear Strength Parameters:** This typically involves each layer's internal friction angle (φ) for sandy soils. The friction angle determines the soil's resistance to shearing along potential failure surfaces. **Unit Weight:** The density or unit weight (γ) of each layer is essential for calculating the overburden pressure, which affects the bearing capacity of the footing.
Foundation Geometry	**Footing Dimensions:** The width (B), length (L), and depth (D) of the footing must be considered, as they directly influence the stress distribution in the soil and the overall bearing capacity. **Shape Factors:** Correction factors are applied to account for the shape of the footing (e.g., square, circular, or strip footings) in the bearing capacity calculation.
Failure Mechanism Assumptions	**Slip Surface Assumptions**: LEA typically assumes a specific failure mechanism, such as a circular or wedge-shaped failure surface. For multilayered soils, the analysis might need to adapt to complex failure mechanisms where the slip surface crosses different soil layers. **Inter-Layer Interactions:** The interaction between different layers, especially with contrasting properties (e.g., dense sand over loose sand), can significantly affect the failure mechanism.
Equilibrium Conditions	**Force Equilibrium:** The analysis ensures that the forces acting on the potential failure surface are in equilibrium, considering both the driving forces (due to applied loads) and the resisting forces (due to soil shear strength). **Moment Equilibrium:** Moment equilibrium is also checked to ensure no rotational failure, which is especially important in scenarios where the footing is subject to eccentric loading.
Bearing Capacity Factors	**Terzaghi's or Meyerhof's Factors:** These classic bearing capacity factors (Nq, Nc, Nγ) are modified to account for the presence of multiple soil layers. Adjustments may be necessary depending on each layer's relative stiffness and strength. **Correction Factors:** Various correction factors are applied for depth (D), shape, and load inclination, tailored to the specifics of multilayered soils.
Load Conditions	**Vertical and Horizontal Loads:** The analysis typically considers vertical loads, but horizontal loads and moments (if present) influence the bearing capacity. **Overburden Pressure:** The effective stress approach is used, considering the overburden pressure from the soil layers above the footing.
Factor of Safety (FoS)	**Determination of FoS:** The limit equilibrium method calculates a safety factor by comparing the available shear strength to the required shear strength for equilibrium. This ensures that the footing is safe under the applied load conditions.

Figs. 2 and 3 depict a solid base and a soft base mechanism, respectively. The foundation is supposed to be located on cohesionless soil for Nγ, and the surcharge is ignored. With a unit weight of γ, the soil beneath the footing is homogeneous but anisotropic. The preliminary plane

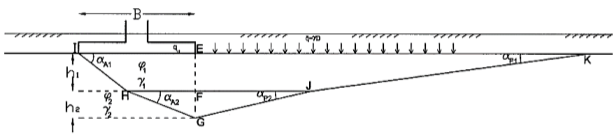


Fig. 2 Failure mechanism of shallow foundation and wedges assumed in present model; after (Ghazavi and Eghbali 2008)

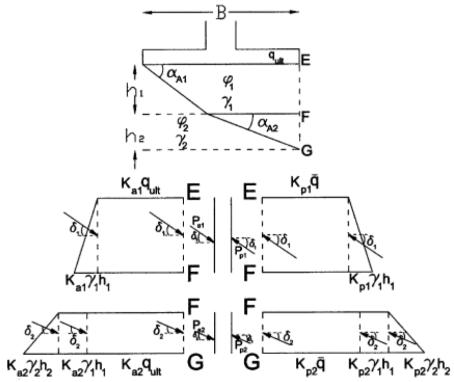


Fig. 3 Failure mechanism of shallow foundation on a soft base; after (Ghazavi and Eghbali 2008)

strain condition introduced by Terzaghi (Terzaghi 1943) assumes that the footing's out-of-plane dimension (length) is large enough to consider this condition. With the Mohr-Coulomb yield criterion, the soil is expected to demonstrate rigid-perfectly plastic conduct (Fatahi *et al.* 2014).

Understanding the potential failure mechanisms is essential for designing and assessing solid foundations, ensuring they can safely support the applied loads without experiencing excessive settlement or catastrophic failure. Each mechanism depends on the soil conditions, foundation type, and load characteristics, making site-specific analysis crucial. For instance, the failure mechanism of a solid footing on layered soil involves complex interactions between the footing and the various soil layers beneath it (Fig. 2). These interactions are influenced by the properties of each layer, such as their strength, stiffness, and thickness (Ghazavi and Eghbali 2008). Understanding these failure mechanisms is crucial for designing safe and effective foundations. General shear failure occurs in dense or stiff soils where the soil beneath and around the footing experiences a significant displacement, forming a well-defined shear zone. The failure surface typically extends from the edge of the footing to the surface some distance away. This type of failure occurs when the footing load causes large deformations in the soil, forming distinct failure surfaces that extend from the edge of the footing to the ground surface.

On the other hand, punching shear failure occurs when the footing pushes through a weaker soil layer beneath it, similar to how a punch goes through a piece of material. This results in a vertical failure surface. This is common when a stiff upper layer overlays a weak lower layer. The footing load concentrates through the upper layer, causing the lower layer to fail in a punching mode. The stiffer the upper layer, the more likely it is to confine the failure mechanism to a punching shear through the lower, weaker layer.

In terms of bearing capacity failure in stratified soils, the bearing capacity may not be a simple average of the capacities of each layer but is rather governed by the weakest layer and the interaction between layers (Fig. 3). The failure mechanism can involve a combination of shear and settlement failures, depending on the relative thicknesses and strengths of the layers. The strength contrast between layers significantly influences the type and extent of failure. A strong layer overlying a weak layer often leads to punching shear failure, while a weak layer on top may lead to more distributed shear failure. The thickness of each layer affects the development of failure surfaces. Thin, weak layers may lead to localized failure, while thicker, weak layers may cause more extensive settlement. Larger footings distribute the load over a wider area, potentially affecting the failure mechanism by reducing the stress concentration on the weaker layers. How the load is applied (e.g., uniformly vs. point loads) can also influence the failure mechanism, with concentrated loads more likely to cause localized failures.

As stated, understanding these failure mechanisms is critical for the design of foundations on layered soils. Engineers must account for the specific properties of each soil layer, their interactions, and how they influence the footing's overall stability and settlement behavior. As stated earlier, numerical methods like finite element analysis (FEA) or limit equilibrium methods are often used to model these complex interactions and predict the most likely failure mechanism under given loading conditions. Equation 1 is obtained from a comprehensive study on using limit equilibrium to calculate the bearing capacity of shallow footing (with no surcharge) rested on two-layer cohesionless sandy soil conditions. This equation is well discussed by Ghazavi and Eghbali (2008).

$$q_{ult(\ddot{q}=0)}=\frac{0.5\gamma_1 cos\delta_1\left(K_{p1}-K_{a1}\right)+cos\delta_2(0.5\gamma_2X^2+\gamma_1X)(K_{p2}-K_{a2})h_1}{(K_{a1}cos\delta_1+K_{a2}Xcos\delta_2)} \quad (1)$$

3.2 Artificial Neural Network (ANN)

The development of distributed network models known as artificial neural networks (ANNs) is deeply inspired by the behavior and intricate functioning of the human brain. These networks are designed to mimic the way biological neurons in the brain process information. Note that neurons are the fundamental units in the human brain that transmit and process information. Each neuron receives inputs through dendrites, processes this information in the cell body, and transmits outputs through axons to other neurons. In ANNs, the artificial neurons (often called nodes or units) function similarly. Each node receives input signals, processes these signals using a weighted sum, applies an activation function, and passes the output to other nodes in the network. Noteworthy, the ANNs learn by adjusting weights during training. Techniques like backpropagation are used to update weights based on the error of the network's predictions, allowing the network to learn from data. They are structured in input, hidden, and output layers.

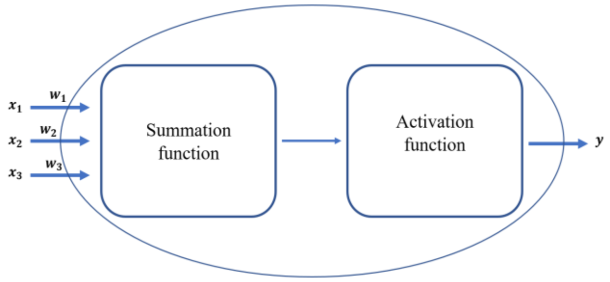


Fig. 4 The simple artificial neuron

Data is processed through these layers, with each layer extracting different levels of features or abstractions.

The ANNs have been increasingly applied in geotechnical engineering, particularly in predicting the bearing capacity of soils. Traditional methods for estimating bearing capacity often rely on empirical formulas, field tests, and analytical methods. However, these methods may be limited by site-specific conditions, soil heterogeneity, and the complexity of layered soils. In this sense, ANNs offer a powerful alternative for modeling these complex interactions. They consist of multiple layers of interconnected, parallel-operating processing elements (neurons). Fig. 4 depicts a simple neuron. A neuron's state can be described in terms of its input (x), output (y), summation function, and activation function (Fig. 4). They are inherently data-driven, meaning they can learn from large datasets of soil properties and load conditions. This is particularly useful in bearing capacity prediction, where soil behavior is highly nonlinear and influenced by various factors such as soil type, density, moisture content, and layering. Researchers collect data from field experiments, laboratory tests, and simulations to train ANN models. These models can then accurately predict the bearing capacity for new soil conditions. The accuracy and reliability of ANN models can be enhanced by integrating them with optimization algorithms. Techniques like Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE) have been used to optimize the weights and biases in ANNs, leading to improved prediction performance. This combination allows for better tuning of the model parameters, which is crucial in capturing the nonlinear behavior of soil under load.

One of the most widely used ANN architectures is the multilayered perceptron (MLP). The neurons and layers in an MLP are linked to one another through weights, and it is very effective at approximating functions in high-dimensional spaces (Du *et al*. 2002). Adjusting the weights (w) between the layers and neurons significantly impacts how well an MLP network performs. Obtaining the connection weight values minimizing the error function between the real network outputs and correlating target values constitutes the training mechanism of an MLP network. This is accomplished by training the neural network by applying a learning algorithm. Numerous learning algorithms, including back propagation (BP) and Levenberg-Marquardt (LM), can be used to train ANNs. Several variations of gradient descent are employed by these algorithms, designed to train ANNs. This is

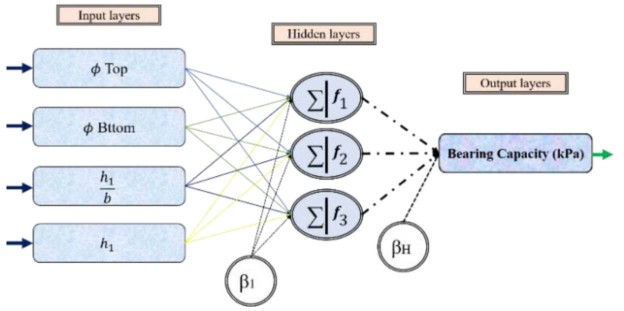


Fig. 5 Neural model for ultimate bearing capacity prediction of shallow foundations

accomplished by merely considering the error function's derivative regarding the network weights and afterward altering those weights in a gradient-related orientation. Another well-known advantage of the LM algorithm over the BP algorithm is that it offers faster and more efficient convergence.

The ability of the ANN to learn from experience and instances and generalize them is one of its distinguishing features. Training and recalling are typically the two phases of neural methods. A sizable training pattern set is typically required for ANN training. Nevertheless, it can be employed directly to replace complex system dynamics after training (Picton 1994). An MLP model, as demonstrated in Fig. 5, was suggested in this research to forecast eventual bearing capacities.

The base of the hybrid model is a standard MLP, a type of feedforward artificial neural network. It consists of multiple layers of neurons, including input, hidden, and output layers. Each neuron is connected to every neuron in the next layer, and these connections have associated weights that are adjusted during training.

3.3 Hybrid MLP model development

A Hybrid MLP model is an advanced neural network architecture that combines the strengths of traditional MLP models with other computational techniques or models to improve performance in various tasks, such as prediction, classification, or optimization. This approach is particularly useful in complex problem domains where a single model may not capture all the nuances or interactions within the data. Hybrid models often incorporate optimization techniques such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), or Differential Evolution (DE) to optimize the MLP's architecture (e.g., number of layers, number of neurons) or to fine-tune the weights and biases. This integration helps in avoiding local minima and improves the overall model performance.

As with any MLP model, data preprocessing is crucial. Hybrid models often use advanced data preprocessing techniques, including normalization, to ensure that the input features are on a similar scale. This step is essential for speeding up the convergence of the training process and improving model accuracy. Training a hybrid MLP model involves not only backpropagation (the traditional method for training MLPs) but also the chosen optimization

Table 2 The performance evaluation criteria

Evaluation criteria	Definition
Coefficient of determination (R^2)	$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(Y_{i_{predicted}} - Y_{i_{observed}}\right)^2}{\sum_{i=1}^{N}\left(Y_{i_{observed}} - \bar{Y}_{observed}\right)^2}$
Root-mean-square error (RMSE)	$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(Y_{i_{observed}} - Y_{i_{predicted}}\right)\right]^2}$
Mean average error (MAE)	$MAE = \frac{1}{N}\sum_{I=1}^{N}\left\lvert Y_{i_{observed}} - Y_{i_{predicted}}\right\rvert$

technique. For instance, if a GA is used, it may evolve the initial set of weights or optimize the MLP's hyperparameters before final fine-tuning using backpropagation.

In geotechnical engineering, a hybrid MLP model could combine the MLP's learning capabilities with physical modeling techniques or optimization algorithms to predict the bearing capacity of layered soils more accurately. Datasets of training and testing data were employed for this study's comparisons to those in the research conducted by Padmini et al. (Padmini *et al*. 2008). Accordingly, 70% and 30% of the data sets were used for training and validation. To determine the eventual bearing capacity of shallow foundations while input values were entered, the suggested method employed training the MLP model. The LM algorithm trained the neural model (Boulefrakh *et al.* 2019). Different input sets and corresponding measured values ($q_{ult_{me}}$) were used when training the MLP by the LM algorithm. The LM learning algorithm was employed to adjust the weights of the neural model based on differences between the target output ($q_{ult_{me}}$) and the MLP$q_{ult_{ANN}}$ real output. Based on the root-mean-square (RMS) error between $q_{ult_{me}}$ and $q_{ult_{ANN}}$ for the entire training set, or obtaining the highest permissible number of repetitions, the network's calculation precision was considered acceptable after each set presentation.

Table 2 lists the criteria employed in this study to evaluate the model. In the table, n represents the number of specimens, and the mean values of the measured and computed eventual bearing capacities are $q_{ult_{me}}$ and $q_{ult_{ANN}}$, respectively.

3.3.1 Cuckoo Optimization Algorithm (COA)

The Cuckoo Optimization Algorithm (COA) is a nature-inspired optimization algorithm based on the brood parasitism of some cuckoo species. Developed by Xin-She Yang and Suash Deb in 2009, the algorithm simulates how some cuckoo birds lay their eggs in the nests of other host birds. The algorithm incorporates Lévy flights to enhance the exploration of the solution space. The COA is inspired by the parasitic behavior of cuckoo birds that lay their eggs in the nests of other birds. If a host bird discovers the foreign eggs, it may either throw them away or abandon the nest and start building a new one elsewhere. This behavior is mimicked in the algorithm's approach to finding optimal solutions. A significant aspect of COA is using Lévy flights to model the random walk of cuckoos when searching for a new nest. Lévy flights are characterized by occasional long jumps, which help the algorithm explore the search space more efficiently and avoid getting trapped in local optima. In the context of COA, the "nests" represent potential solutions to the optimization problem. Each nest (solution) is evaluated based on a fitness function, and the better solutions are more likely to be retained and improved in subsequent iterations. The discovery of cuckoo eggs by the host bird is represented by a probability parameter p_a, which controls the fraction of the worst nests that are abandoned and replaced with new random solutions. This helps maintain diversity in the population and facilitates exploration of the solution space. The algorithm starts with an initial population of nests (solutions). In each iteration, new solutions are generated either by Lévy flights or by replacing some of the worst solutions with new random solutions. The algorithm iteratively improves the population until it converges to an optimal or near-optimal solution.

Steps in the Cuckoo Optimization Algorithm include (i) randomly generating an initial population of nests (solutions). (ii) Evaluate each nest's fitness based on the optimization problem's objective function. (iii) For each cuckoo, generate a new solution using Lévy flight. Evaluate the new solution and replace the host bird's nest with the cuckoo's solution if it is better. (iv) With a probability p_a, replace a fraction of the worst nests with new random solutions to introduce diversity and explore new areas of the solution space. (v) Repeat the above steps until a stopping criterion is met, such as a maximum number of iterations or a convergence threshold.

A candidate habitat matrix of size N_{pop} x N_{var} is constructed to begin an optimization algorithm. A random number of eggs is anticipated for each preliminary cuckoo ecosystem. Between 2 and 9 eggs are laid by each cuckoo in the environment; these ranges are employed in various iterations as lower or upper thresholds for each cuckoo. The so-called "Egg Laying Radius (ELR)" is the greatest distance at which actual cuckoos will lay their eggs from their lodging. It is proportional to the overall frequency of eggs, the frequency of existing cuckoo eggs, and the varying Varhi and Varlow boundaries. Therefore, Eq. (2) is the definition of ELR

$$ELR = \alpha \times \frac{Number\ of\ Cukoo\ eggs}{Ttal\ number\ of\ eggs} \times (Var_{hi} - Var_{low}) \quad (2)$$

Where α represents an integer suggested for applying the ELR greatest value (Rajabioun 2011).

3.3.2 Teaching-learning-based optimization (TLBO)

TLBO was recently established by Rao *et al.* (2012). The teaching-learning process inspires it in a classroom, where the interaction between a teacher and students helps students to improve their knowledge. This algorithm simulates this process to optimize a given objective function. The algorithm begins with a population of potential solutions called learners. Each learner represents a candidate solution to the optimization problem. In the teacher phase, the best solution in the current population is considered the "teacher." The role of the teacher is to improve the overall knowledge level of the class (i.e., move the population toward the global optimum). The teacher modifies the learners' solutions by shifting their positions toward the teacher's position, aiming to enhance their performance. In the learner phase, learners interact with each other to improve their solutions. A learner compares himself with another randomly selected learner, and depending on the outcome of this comparison, the learner either moves toward the better-performing learner or remains unchanged. This phase helps explore the search space more thoroughly and helps escape local optima. TLBO does not require algorithm-specific control parameters like crossover or mutation rates in Genetic Algorithms, making it simpler to implement and tune. The teaching and learning process continues iteratively, with the population updating each iteration. The algorithm stops when a predefined stopping criterion, such as a maximum number of iterations or a satisfactory solution quality, is met.

The initial portion is the teacher phase, in which the teacher instructs the students. In this stage, the teacher raises the class's overall performance following its capacity. The teacher is regarded as an ideal learner. The algorithm attempts to shift all learners' mean positions in favor of the teacher. Let $\mathrm{M_i}$ represent the average performance of our students:

$$DifferenceMean_i = r_i(M_{new} - T_F M_i) \quad (3)$$

Where

- r_i represents a uniform random number in the range of [0,1]
- M_{new} represents the finding acquired by the teacher

T_F represents the teaching factor determining how the mean value is going to change; it can be either 1 or 2 randomly presented by

$$T_F = round[1 + rand(0.1)(2 - 1)] \quad (4)$$

The algorithm uses the equation mentioned above to determine the value of T F, which the user does not determine.

Learner solution in the i^{th} iteration from the earlier $(i-1)^{th}$ iteration is changed, as follows

$$X_i = X_{i-1} + DifferenceMean_i \quad (5)$$

The second stage of the algorithm is the learner phase. During this stage, students attempt to interact with one another to enhance their performance. An improvement in the output is the consequence of a learner randomly interacting with another. The solution of the entire community is adapted toward more informed learners, as the one who has more awareness than the other aids the one with less awareness. Concerning the learner phase, the following equations were recommended by Rao et al. (2012).

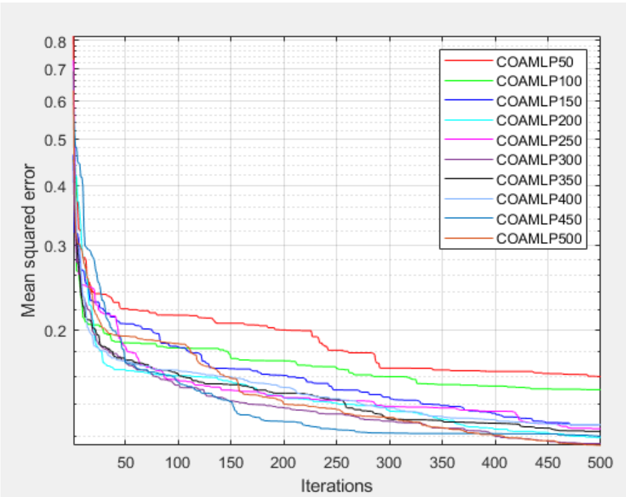
$$If\ f(X_j) < f(X_k)\ then\ X_i = X_{i-1} + r_i(X_j - X_k) \quad (6)$$

$$If\ f(X_j) > f(X_k)\ then\ X_i = X_{i-1} + r_i(X_k - X_j) \quad (7)$$

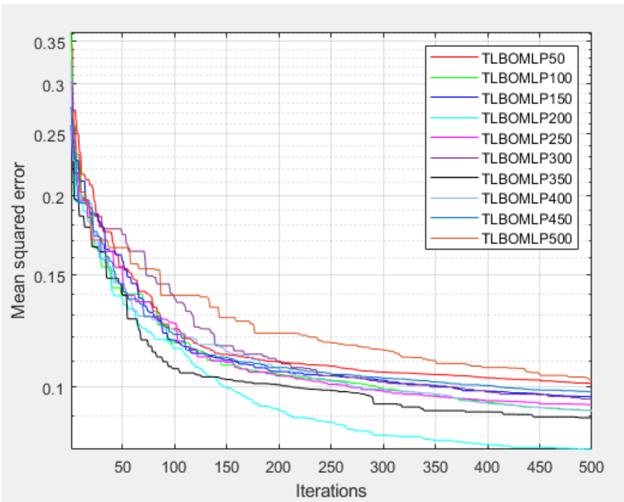
Only X_i values-producing functions that outperform the preceding iteration are deemed permissible.

Table 3 The network results for different COAMLP structures with one hidden layer

Population size	Training dataset		Testing dataset		**Scoring**				Total Score	Rank
	RMSE	R^2	RMSE	R^2	Training		Testing			
50	0.1592	0.9239	0.15719	0.94722	1	1	1	1	4	10
100	0.14902	0.9337	0.1212	0.96897	2	2	3	3	10	9
150	0.12629	0.9528	0.11753	0.97085	3	3	6	6	18	7
200	0.1186	0.9585	0.11067	0.97419	8	8	7	7	30	3
250	0.12408	0.9545	0.11929	0.96995	5	5	5	5	20	6
300	0.11564	0.9606	0.10668	0.97604	9	9	9	9	36	2
350	0.12244	0.9557	0.11017	0.97443	6	6	8	8	28	4
400	0.12575	0.9533	0.12396	0.96751	4	4	2	2	12	8
450	0.11961	0.9578	0.12109	0.96902	7	7	4	4	22	5
500	0.11459	0.9613	0.09717	0.98017	10	10	10	10	40	1



(a) COAMLP



(b) TLBOMLP

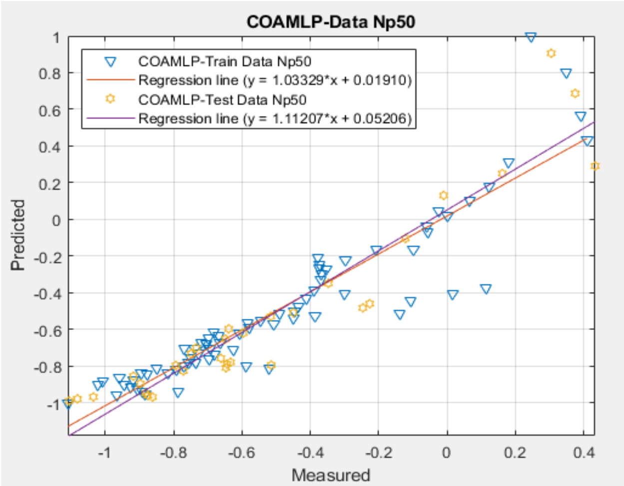
Fig. 6 The best-fit model for the (a) COAMLP and (b) TLBOMLP

4. Results and discussion

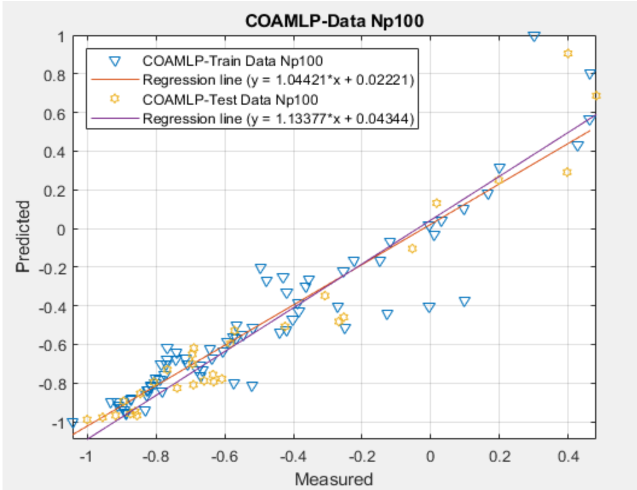
As mentioned earlier, the ultimate bearing capacity of a shallow foundation on two-layered, cohesionless soils is predicted using two distinct nonlinear machine learning models. The COA and TLBO models were used in conjunction with ANN. The data set is split into training and testing classes to develop and train the models.

The present study's primary goal was to approximate the eventual pullout bearing capacity of a shallow foundation (F_{ult}) in a two-layer cohesionless soil employing two artificial intelligence methods, such as COA-MLP and TLBO-MLP. To calculate the F_{ult} capacity, it was necessary to identify and use the main characteristics. As with several other hybrid predictive models, the initial stage separates the database into training and testing datasets, each with layers of input(s) and output(s). Consequently, the results of model optimization for each recommended model were presented as the initial step. Also, following all optimization procedures, a process known as "un-normalization" allowed the normalized values to revert to their initial values (i.e., with a novel normalization process, but regarding their original highest and lowest values). To identify the most effective approach, the acquired findings for both the training and testing data were evaluated using the performance mentioned above indices, including the root mean square error (RMSE), coefficient of determination (R^2), and mean average error (MAE). As a result, this section discusses assessing the findings acquired for estimating shallow foundations' bearing capacity, employing the COA-MLP and TLBO-MLP techniques. Additionally, a ranking system of the total ranking method (TRM), focusing on the findings of statistical indices, was used to assess the predicted network performance.

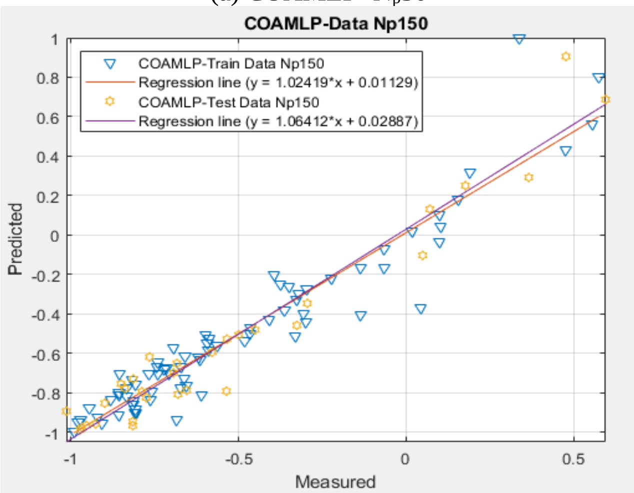
Reducing network error while improving the prediction capacity of the principal optimization algorithms are the primary goals of employing sensitivity analysis in the initial algorithm, like numerous other hybrid techniques.



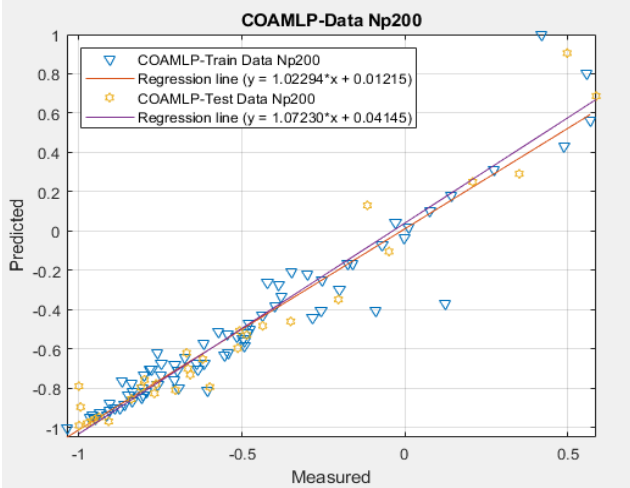
(a) COAMLP- N_p50



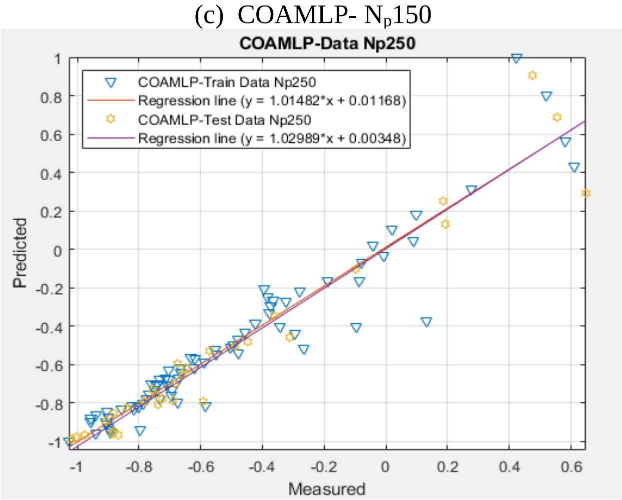
(b) COAMLP- N_p100



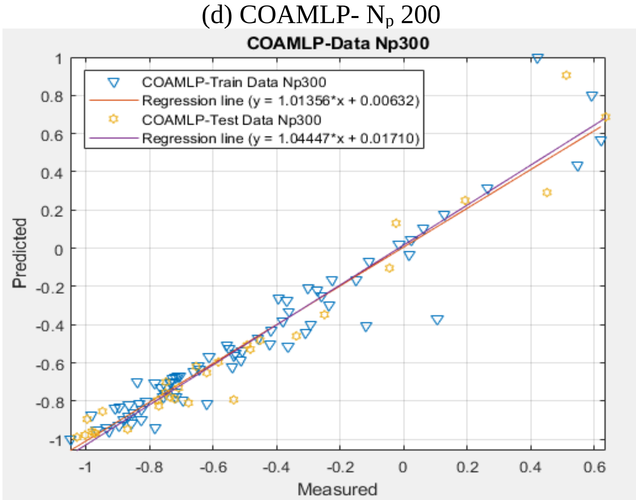
(c) COAMLP- N_p150



(d) COAMLP- N_p 200



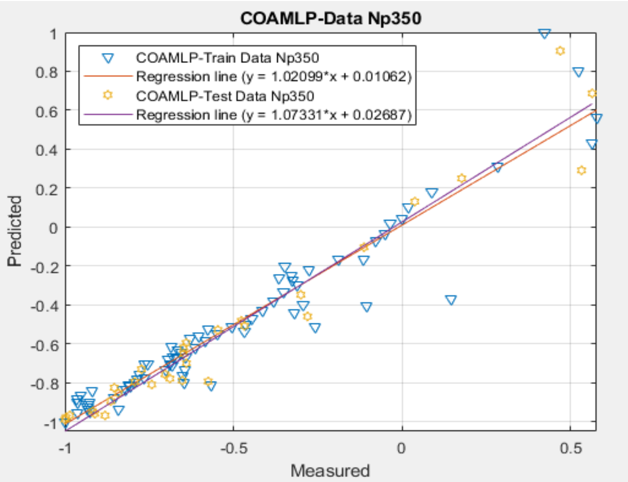
(e) COAMLP- N_p250



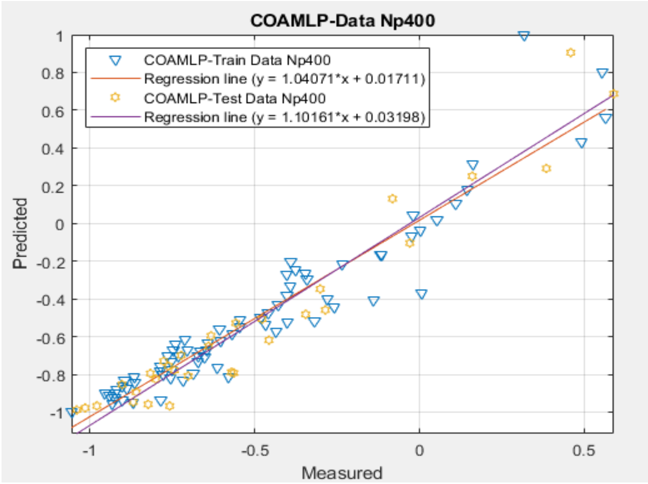
(f) COAMLP- N_p300

Fig. 7 Continued-

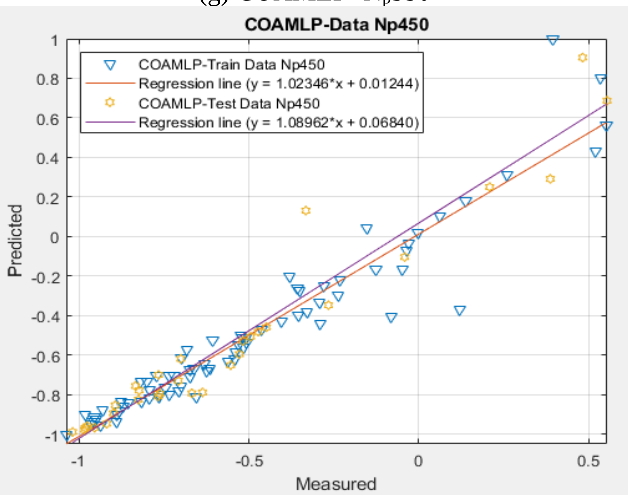
The number of swarm sizes, the most important parameter for COA-MLP and TLBO-MLP, is the subject of an additional parametric investigation. Tables 3 and 4 summarize the findings on the effectiveness of the swarm size for population sizes ranging from 50 to 500. Swarm size is also used to evaluate the mean square error, which decreases as the number of models' iterations increases (Fig. 6). Figs. 7 and 8 demonstrate the performance findings for both the training and testing datasets regarding the best-selected frameworks of COA-MLP (i.e., swarm size equal to 500) and TLBO-MLP (i.e., swarm size equal to 200).



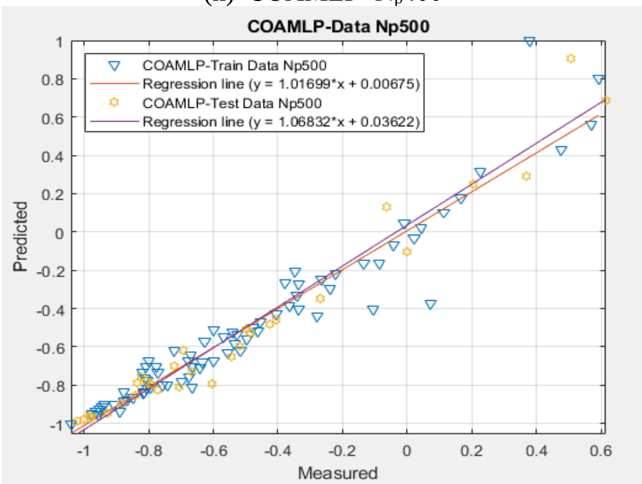
(g) COAMLP- N_p350



(h) COAMLP- N_p400



(i) COAMLP- N_p450



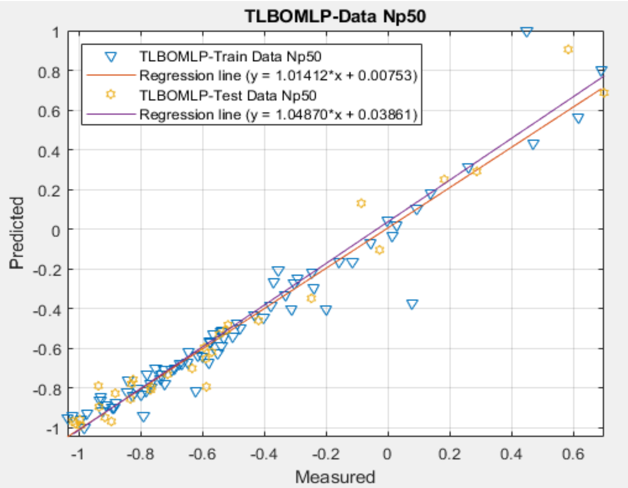
(j) COAMLP- N_p500

Fig. 7 The results of accuracy for both training and testing datasets for different structures of COAMLP

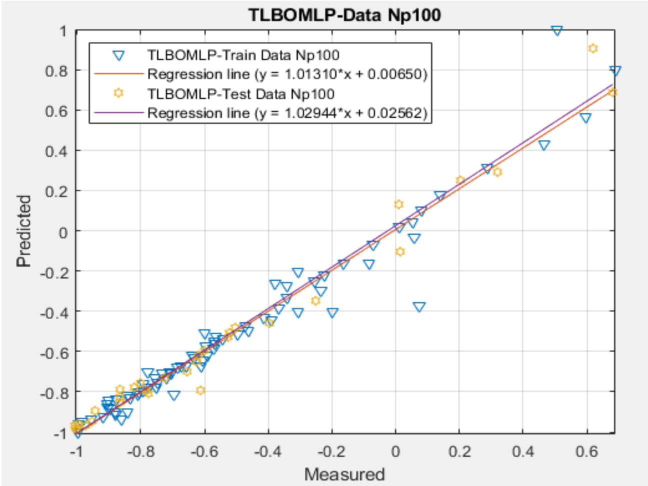
Table 4 The network results for different TLBOMLP structures with two hidden layers

Population size	Training dataset		Testing dataset		**Scoring**				Total Score	Rank
	RMSE	R^2	RMSE	R^2	Training		Testing			
50	0.1592	0.9700	0.09108	0.9826	1	2	2	2	7	10
100	0.14902	0.9754	0.07475	0.98831	2	7	8	8	25	2
150	0.12629	0.9730	0.06996	0.98977	3	4	9	9	25	2
200	0.1186	0.9816	0.05511	0.99366	8	10	10	10	38	1
250	0.12408	0.9744	0.0822	0.98585	5	6	7	7	25	2
300	0.11564	0.9733	0.09098	0.98264	9	5	3	3	20	8
350	0.12244	0.9767	0.08894	0.98341	6	9	4	4	23	5
400	0.12575	0.9755	0.086	0.9845	4	8	5	5	22	7
450	0.11961	0.9719	0.09259	0.98201	7	3	1	1	12	9
500	0.11459	0.9691	0.08475	0.98495	10	1	6	6	23	5

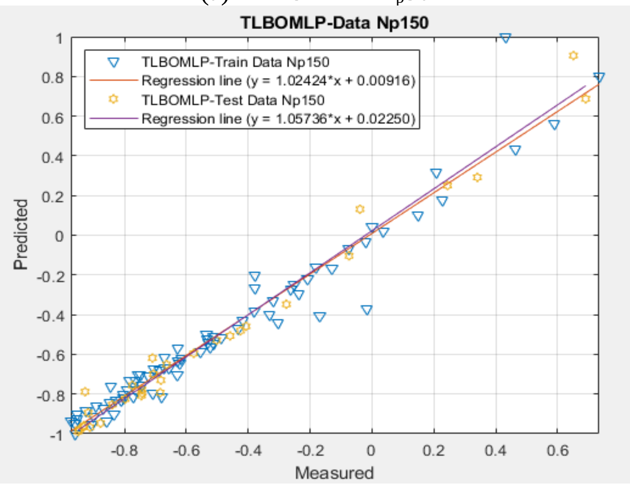
A graphical assessment of the patterns of the bearing capacity that were originally anticipated and measured can be seen in Figs. 6 and 7 (for both the training and testing datasets). As can be observed, the bearing capacity patterns can be accurately estimated using both of the recommended models, with the COA-MLP's measured R^2



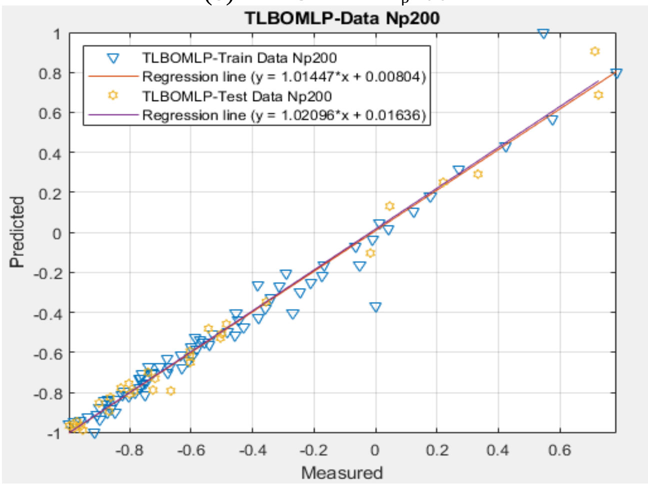
(a) TLBOMLP- N_p50



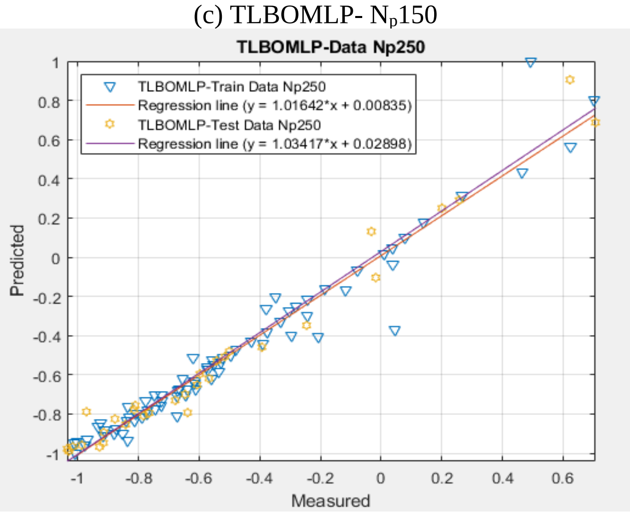
(b) TLBOMLP- N_p100



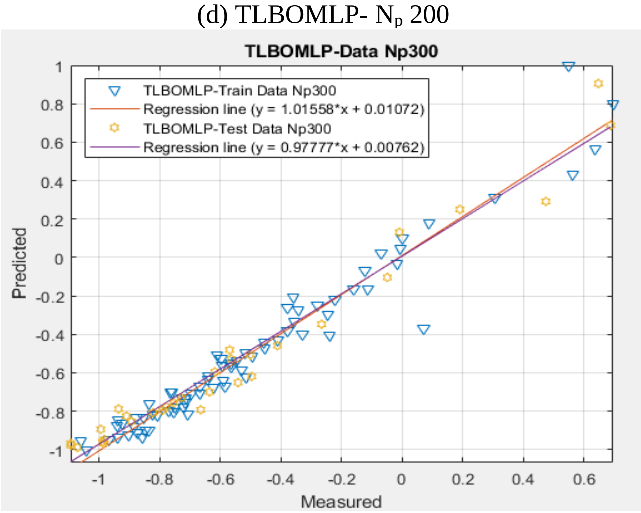
(c) TLBOMLP- N_p150



(d) TLBOMLP- N_p 200



(e) TLBOMLP- N_p250



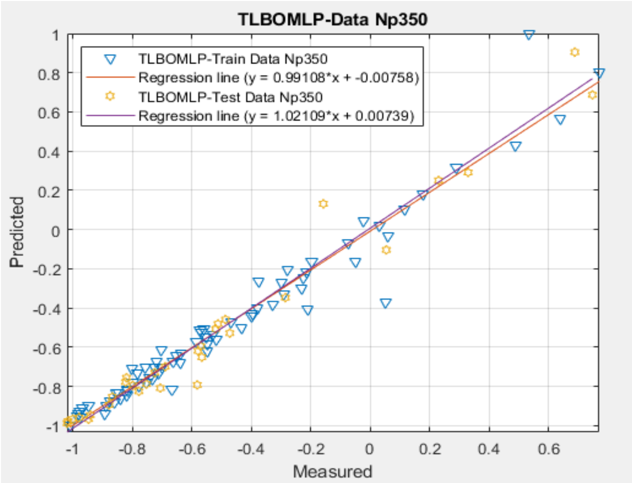
(f) TLBOMLP- N_p300

Fig. 8 Continued-

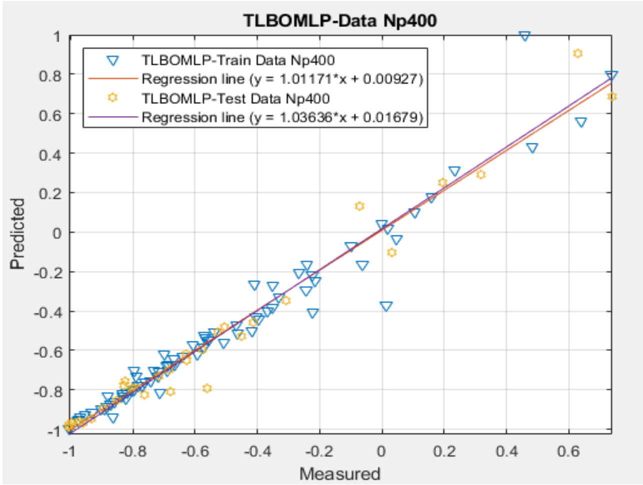
(training=0.9816, testing=0.99366) and the TLBO-MLP's measured R^2 (training=0.9613, testing= 0.98017).

The suggested structure of TLBO-MLP yields the best-fit model when the two recommended techniques are considered, followed by a nonlinear swarm optimization procedure.

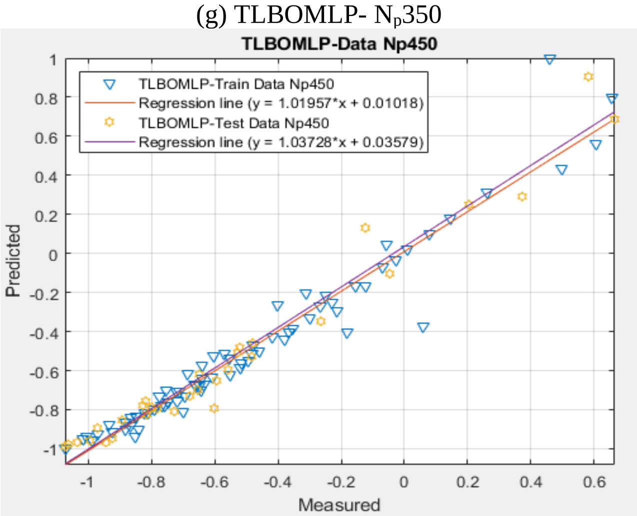
The ten populations above sizes are employed to train and test the recommended models. The previously utilized statistical indices, such as R^2, RMSE, and MAE, are calculated for each of the ten population sizes. Tables 3 and 4 show the values of the acquired indices for each model at ten population sizes and each model's acquired ranking value. Both methods had a similar and satisfactory R^2 between calculated and computed eventual bearing



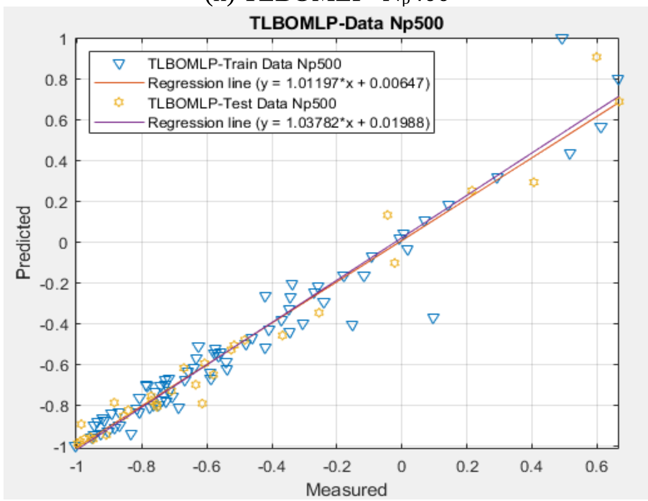
(g) TLBOMLP- N_p350



(h) TLBOMLP- N_p400

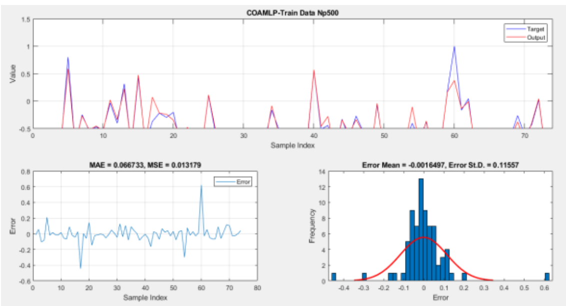


(i) TLBOMLP- N_p450

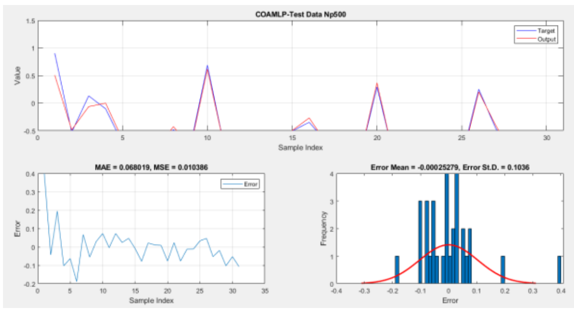


(j) TLBOMLP- N_p500

Fig. 8 The results of accuracy for both training and testing datasets for different structures of TLBOMLP



(a) COAMLP 500-training



(b) COAMLP 500-testing

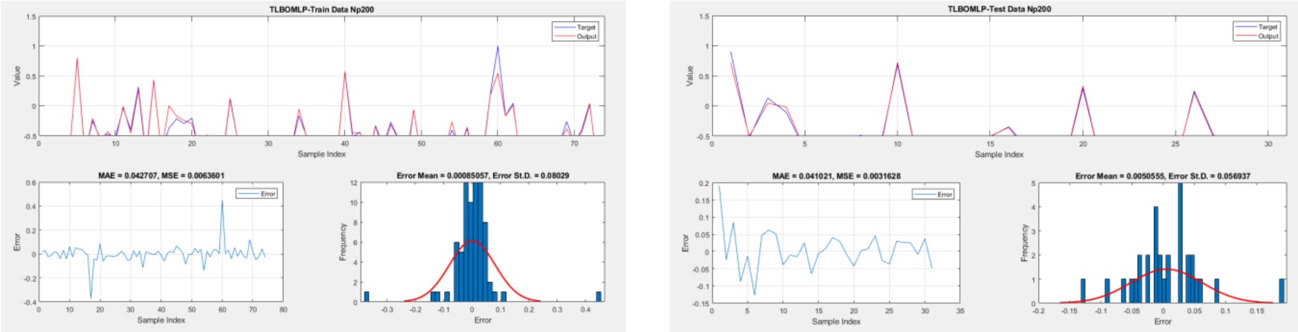
Fig. 9 The error and frequency of MAE for the best fit COAMLP proposed model

capacity. Calibration results for the suggested neural models were identically low on both the RMSE and the R^2.

Nevertheless, during validation, the suggested model had the minimum RMSE and MSE values. The RMSE values in the TLBO-MLP model were 0.1186 and 0.05511, lower than those of the COA-MLP model. The TLBO-MLP model has greater precision than the COA-MLP model, consistent with earlier findings.

Figs. 9 and 10 show the histograms of the training and testing errors for COA-MLP and TLBO-MLP, respectively.

It is evident from these figures that the findings acquired employing the TLBO-MLP model closely match the laboratory data. Furthermore, compared to the COA-MLP model, an improved comparison between the laboratory data and the recommended TLBO-MLP model for training and testing data is accomplished. The training findings indicated that these techniques adequately analyzed the relationship between the studied parameter and other factors. These two models had similar RMSEs (0.11459 and



(a) TLBOMLP 200-training (b) TLBOMLP 200-testing

Fig. 10 The error and frequency of MAE for the best fit SFOMLP proposed model

0.1186). Comparable levels of precision were also demonstrated during training of the ANN by the MAEs of 0.066733 and 0.042707 for the COA-MLP and TLBO-MLP models, respectively. The COA-MLP and TLBO-MLP techniques can accurately estimate the considered parameter, respectively, as demonstrated in the testing phase by the RMSEs of 0.09717 and 0.05511 and the MAEs of 0.068019 and 0.041021.

5. Conclusions

Based on a conjunction of the COA, TLBO, and MLP, the behavior of maximum allowable stresses on shallow footings resting on two-layer cohesionless soils is investigated and evaluated. The ideal COA-MLP and TLBO-MLP outputs compare the determined values to the anticipated ones. The findings indicate that the recommended COA-MLP and TLBO-MLP models can precisely predict the output. Nevertheless, it is discovered that the recommended TLBO-MLP is more precise than the COA-MLP model due to its simpler framework. Furthermore, concerning both the normalized training and testing datasets in the optimum COA-MLP network with swarm size equivalent to 500, R^2 and RMSE values of (0.9613 and0.11459) and (0.98017 and0.09717) have been discovered, which were marginally smaller than the TLBO-MLP model. In a population size equal to 200, the COA-MLP model with the second-highest accuracy was discovered. For both the normalized training and testing datasets, R^2 and RMSE values were equal to (0.9816 and 0.1186) and (0.99366 and 0.05511), respectively.

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