

A new prediction model of force evolution behavior of a conical pick by indentation tests

Xiang Wang^{1,3a}, Ming S. Gao^{1b}, Okan Su^{*2} and Dan Huang^{3c}

¹School of Mechanical Engineering, Chongqing University of Technology, Chongqing 400054, China

²Department of Mining and Mineral Extraction, Zonguldak Bülent Ecevit University, Zonguldak 67600, Türkiye

³Institute of Mining Engineering, Beijing General Research Inst. of Min. and Metall., Beijing 102628, China

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Abstract. In this study, a prediction model for the cutting force evolution in brittle rocks was developed. This model is based on indentation tests using a conical pick at a cutting depth of 9 mm. The behavior of the indentation mechanism was analyzed in three phases by using Evans' cutting mode. The peak values in the force history identified these phases. The variation in the local strength of the rock caused a large offset in the model prediction of chipping. Regression analyses showed that there is a strong power relationship between the upper bound of the cutting force along with chipping and depth of cut. The slope of the three crushing phases has been found to increase sequentially ($\alpha_1 < \alpha_2 < \alpha_3$). In addition, a positive correlation existed between the Schmidt hardness and brittleness index that affects the lower and upper bounds of chipping. Consequently, the results clearly demonstrate that the new model can reasonably predict the evolution of the cutting force based on experimental data. These results would be beneficial for engineers to design and select the optimum excavation machine to reduce mechanical vibration and enhance cutting efficiency.

Keywords: conical pick; Evans' cutting mode; indentation test; rock cutting

1. Introduction

In modern tunneling techniques, mechanical excavation, which particularly developed later than 2000s, has been playing an important role for driving mining galleries and tunnels. The roadheaders, raise boring machines and TBM technologies have quite advanced recently as parts of mechanical excavation. Such technological machines provide multiple advantages including productivity, reliability, and safety both in mining and civil engineering.

Roadheaders have been driven in hard strength rocks since machine designers developed heavy-duty roadheaders which have the weight ranging from 70 to 110 tons. Even extra heavy roadheaders over 110 tons in weight with a cutting power of 400 kW are offered for some conditions as well.

The cutting tools used on the cutterhead are essential components in the process of cutting. Depending on the type of rock and strength, there are radial and conical types of cutters employed on the roadheaders. In order to get the best machine performance in a project, a proper pick and

appropriate machine selection should be performed. Due to a longer lifetime and strong rock cutting ability of conical (point attack) picks on the roadheaders, they have been generally selected on the coal measures and in stable rock conditions of low to medium hardness. However, in harder and abrasive rocks, the use of conical picks can be limited because of increases in tool cost although extra heavy-duty roadheaders are available to overcome these limitations in the market. The excavation capability of roadheaders can considerably change a continuous intact rock to a highly fractured rock mass although the existence of rock joints and other discontinuities govern the excavation behavior of the machine.

A medium-duty roadheader (EBZ160) has been driven to develop roadways in granite formation at the Sanshandao gold mine (Huang *et al.* 2022). Although a number of scholars focused on the cuttability behavior of granite for geothermal drilling by applying different techniques such as plasma treatment, high-power (24 kW) microwave radiation and so on, the application of conical picks is not common on granite for cutting purposes (Li *et al.* 2018, Kazi *et al.* 2021). Disc cutters were the only ones which could cut granites in the laboratory conditions by using linear cutting machines (LCM) (Xu *et al.* 2021). In this sense, it is quite difficult to measure the cutting force of conical picks and examine the chipping modes of granite. For this reason, pick indentation test as an alternative method instead of linear cutting tests could be taken into account. Both of the methods have some similarities. As can be seen from Fig. 1, which was obtained from sandstone cutting at the LCM, the forces acting on the pick fluctuate through the cutting

*Corresponding author, Professor

E-mail: okansu@beun.edu.tr

^aAssociate Professor

E-mail: wangxiang@cqut.edu.cn

^bM.Sc student

E-mail: miso_gao@163.com

^cAssociate Professor

E-mail: huangdan@bgrimm.com

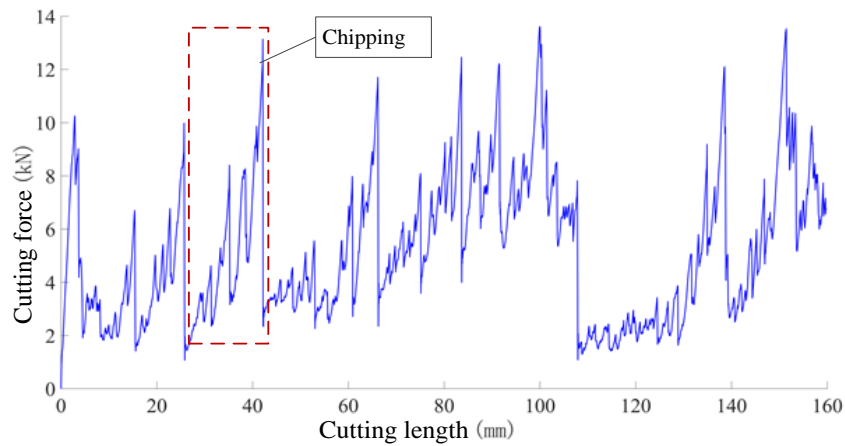


Fig. 1 Brittle failure and chipping process of rock during a conical pick cut

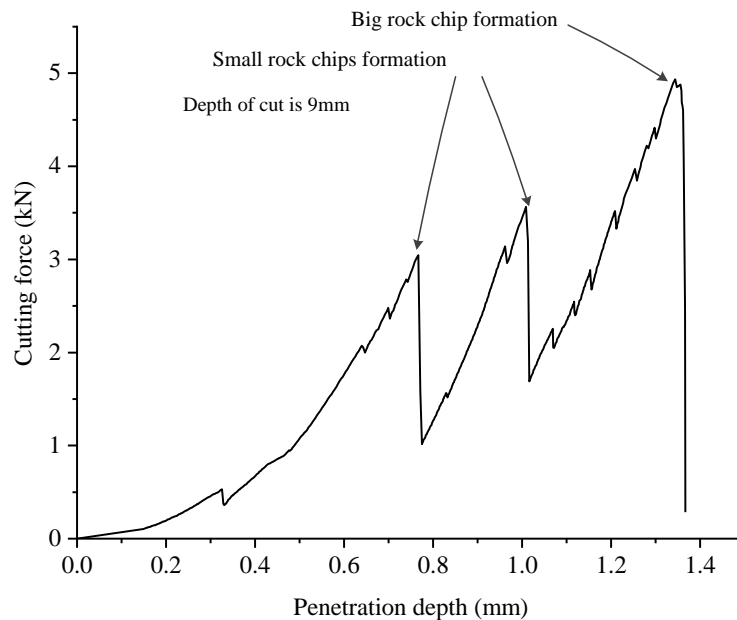


Fig. 2 Cutting force-indentation curves for conical picks

process. An increase of the force over several kilonewtons up to a peak identifies that a crack is initiated. The abrupt drop of the force after the peak is associated with unstable propagation of the crack which leads ultimately to the formation of a chip. Deep and shallow cut mechanisms explain the frequency and magnitude of chipping modes. At a shallow depth of cut, the cutting mode can be defined as ductile and the shape of the chips is formed in small or powder grains. However, at larger cutting depths, brittle failure, where macroscopic cracks are initiated from the tool tip and propagated ahead of the cutter, occurs.

The cutting tests performed in the LCM reveal the peak and mean values of the cutting forces. The chipping mode, in other words, the fluctuations of the cutting force depicted in Fig. 1, is the main vibration source of the roadheader. The successive increase and abrupt release of stress ahead of the cutter can generate, in the case of very hard and abrasive rocks, vibrations of the machine. This is more important when a machine is driven in granite formation.

Wang and Su (2019) indicated that the ratio of FC_p/FC_m is the superior parameter to determine the vibration characteristic of a machine. The upper values of the FC_p/FC_m ratio vary from 2.09-2.16 where a safe and confident excavation would be expected on sandstones. An increase in this ratio means that the vibration on the machine would increase and it could be very destructive in terms of pick and roadheader life. Therefore, it is necessary to pay attention to the fluctuation range of the forces in the process of rock cutting. Li *et al.* (2024) has numerically modelled the test in order to estimate the penetration coefficient.

Besides, Fig. 2 shows the force-indentation depth curve. The depth of pick indentation at the peak, where the first chip is formed, is similar to the one given in Fig. 1. At the beginning of the cutting test, a steeper increase is observed since the pick has high compressive stresses in the elastic state. However, indentation test forms a compressive failure zone surrounding a small portion of the high compressive

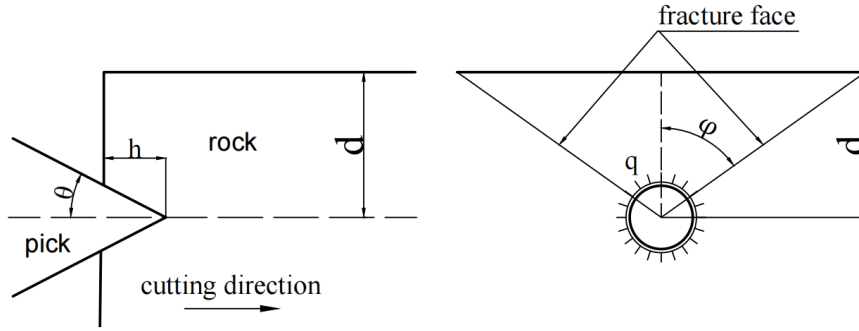


Fig. 3 Evans' (1984) theoretical cutting mode

elastic area under the pick. Increasing indentation gradually reduces material strength and expands the compressive zone. This results in a soft increase to the peak values in indentation tests. Thus, it can be reported that both of the tests have similar characteristics and chipping modes.

In this study, a detailed analysis of the evolution of the cutting force when breaking brittle granites is studied and some models are further developed based on rock indentation tests used for predicting the evolution of the cutting force. Zhang *et al.* (2023) reported statistical characteristic of the indentation indices by conducting macro-indentation tests on 47 granite specimens and plotting their force–penetration curves.

2. Mechanism of cutting forces

2.1 Theory of rock cutting and the prediction methods of forces

There are some parameters to predict the machine performance in the laboratory. Tool forces, wear rate of pick, specific energy or advance rate of the machine are some of those indicating the cuttability operation of a roadheader (Kwak and Ko 2022). Cutting force acting on the pick plays a vital role within these parameters since it allows us to evaluate whether the selected rock can be cut or not. A considerable amount of literature has been published relating to the prediction of cutting force so far. Analytical models, which were based on theoretical assumptions depending on geometry of the bit and mechanical characteristics of the rocks are also available. The first serious discussion had emerged on metal cutting. Later, Evans (1962) made attempts on coal ploughing machines. The models were then improved and modified by different researchers (Nishimatsu 1972, Evans 1984, Roxborough and Liu 1995, Göktań 1997). Evans (1984) proposed a theoretical cutting model and a failure criterion based on the tensile failure and it is widely accepted. As shown in Fig. 3, he proposed that compressive stresses are initiated in the radial direction while tensile stresses are generated in the circumferential direction. Tensile cracks are produced at the intersection of the picks and the rock until the tensile stress is equal to the tensile strength of the rock. If the conditions are favorable, the cracks extend to the free surface of the rock to form a "V" shaped chip. The

formula for the peak cutting force of the conical picks is provided in Eq. (1).

$$FC_p = \frac{16\pi\sigma_t^2 d^2}{\sigma_c \cos^2 \theta} \quad (1)$$

where FC_p is the peak cutting force, d is the depth of cut, and θ is the half tip angle of the pick, σ_t and σ_c are the tensile strength and unconfined compressive strength of the rock, respectively.

Roxborough and Liu (1995) and Göktań (1997) stated that the friction between the pick and the rock has a significant effect on the peak cutting force, which was neglected in Evans' model. Thus, they included them in the modified Eqs. (2) and (3).

$$FC_p = \frac{4\pi\sigma_t d^2 \sin^2(\theta+\psi)}{\cos(\theta+\psi)} \quad (2)$$

$$FC_p = \frac{16\pi\sigma_c d^2 \sigma_t^2}{\left[2\sigma_t + \sigma_c \cos\theta \left(\frac{1 + \tan\psi}{\tan\psi}\right)\right]^2} \quad (3)$$

where FC_p is the peak cutting force, d is the depth of cut, and θ is the half tip angle of the pick, ψ is the friction angle between the pick and the rock, σ_t and σ_c are the tensile strength and unconfined compressive strength of the rock, respectively.

The cutting force values calculated by the theoretical models often differ greatly from the actual values. This is because the theoretical models ignore many factors, such as attack angle, the friction between the pick and rock, etc. It is found that there is a relatively substantial difference between the calculated and measured values of the cutting force. Goktan and Gunes (2005) attributed this deviation to model assumptions where the symmetrical cutting conditions have only been considered. Therefore, they proposed a semi-empirical model for estimating the cutting force considering the effect of the attack angle as Eq. (4).

$$FC_p = \frac{12\pi\sigma_t d^2 \sin^2[(90-\gamma)/2+\theta+\psi]}{\cos((90-\gamma)/2+\theta+\psi)} \quad (4)$$

where d is the depth of cut, σ_t is the tensile strength, ψ is the friction angle between the pick and the rock γ is the attack angle, θ is the half tip angle of the pick.

Moreover, the most practical and valuable method, as well as the analytical methods, is to measure the forces during tool-rock interaction in linear cutting sets (LCM).

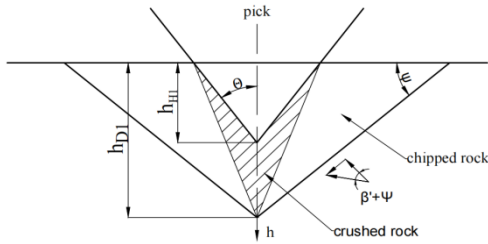


Fig. 4 Chipping process of indentation test (Pang *et al.* 1989)

These sets not only provide effective insights but also offer a convenient approach to analyze the process. A number of empirical models were developed from these sets (Copur *et al.* 2003, Balci and Bilgin 2007, Balci 2009, Entacher *et al.* 2013, Bilgin *et al.* 2014, Pan *et al.* 2018, Yasar 2018, Cheng *et al.* 2019, Entacher and Rostami 2019, Hamzaban *et al.* 2022, Kim *et al.* 2022, She *et al.* 2024). Wang *et al.* (2017) proposed a general cutting force model considering various influential factors. This model was then improved through the use of principal component regression methods. Yasar (2020) proposed that the cutting force acting on a conical pick is closely related to the brittleness parameter presented as the ratio of rock compressive strength and tensile strength.

Furthermore, taking advantage of numerical modeling codes with excellent functionalities and high performance, well-calibrated models have also served successfully the rock cutting forces in the last couple of decades (Rojek *et al.* 2011, Su and Akcin 2011, Moon and Oh 2012, Labra *et al.* 2017, Jiang *et al.* 2021, Stopka 2021). In these studies, various material constitutive models were implemented, different types of rocks were modeled, and the status of the chip formation and fragmentation processes were analyzed in detail. Accordingly, as long as the boundary conditions and rock calibration are set up successfully, such engineering modelling tools were found to be very beneficial.

2.2 Rock indentation mechanism of conical picks

Estimation of the forces for cutting a certain type of rock with a conical pick is an essential aspect of roadheader performance prediction as stated above. Basic studies on pick indentation into rock have been of interest to many investigators since they cover some important events of conical picks in the failure sequence. They describe the details of crushing, chip formation, and quantitative evaluation of the stress and displacement field by taking into account the contact area between the conical pick and rock surface. Some models were developed from different aspects by Bao *et al.* (2011) and Su and Wang (2019). They improved the models of cutting force based on fracture mechanics of Evans' cutting mode. Yin *et al.* (2014) studied the rock fragmentation processes with different confining stress levels in the indentation tests. Crack initiation and propagation processes during the indentation test were gained by examining the AE activity and force-indentation depth curve. Zhang *et al.* (2021) developed an in-situ

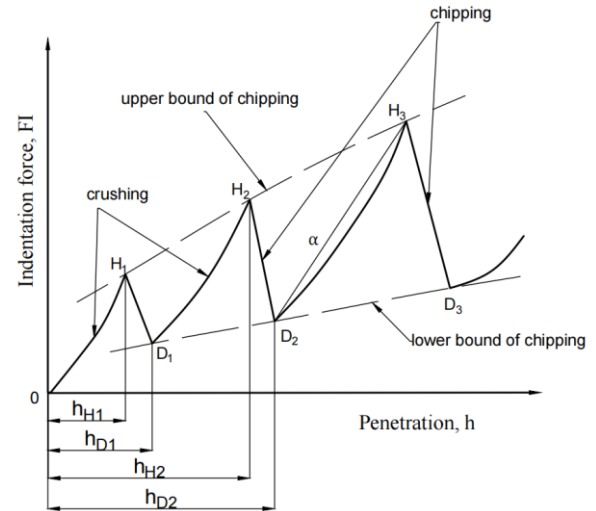


Fig. 5 Evolutionary curve of indentation force (Pang *et al.* 1989)

indentation testing system for performance prediction of TBM. A crushing phenomenon was revealed by Paul and Sikarskie (1965), Miller and Sikarskie (1968) and Pang *et al.* (1989) that the rock is fragmented and compacted during rock indentation tests when the loading exceeds the compressive strength of the rock. Subsequently, as the strain energy of the rock around the pick reaches its critical state, the separation of small rock chips is observed in a comparatively short period, a process referred to chipping, as shown in Fig. 4.

For the entire test, the evolution of the indentation force (indentation force-indentation curve) exhibits nonlinearity and discontinuity. Namely, the forces in a state of constant alternating cycles of crushing and chipping are illustrated in Fig. 5 where the curves $0H_1, D_1H_2, D_2H_3...$ show crushing stages and the curves $H_1D_1, H_2D_2, H_3D_3...$ designate chipping process. Meanwhile, the upper and lower bounds of chipping represent the ending and beginning of each phase, respectively.

As a result, the prediction of the evolution of the indentation force is reasonably divided into four parts: crushing process, upper bound of chipping, chipping process, and lower bound of chipping.

(1) Crushing process

Initially, the prediction of crushing trends in both wedge and conical pick is assumed to be linearly increasing (Bao *et al.* 2011). In contrast, Pang *et al.* (1989) determined that the indentation force is squarely related to indentation when using a conical pick. In addition, an indentation indices (α) was employed to characterize the growth trend of the indentation force during each crushing (Fig. 5), where α is defined as the ratio of the indentation force (F') to the indentation depth (h') at which chip forms as given in Eq. (5) (Morris 1969, Kahraman *et al.* 2000, Liu *et al.* 2016, Xie *et al.* 2021).

$$\alpha = F'/h' \quad (5)$$

(2) Upper bound of chipping

Paul and Sikarskie (1965) proposed a predictive model for the upper limit of chipping in wedge picks based on the Cullen criterion. Then, another model for conical picks given in Eq. (6) was proposed by Miller and Sikarskie (1968) using the same method.

$$FI = Kh^2 \quad (6)$$

$$K = \frac{\pi c \sin(\pi/4 - \phi/2 + \tan^{-1}\mu_f) \tan^2\theta \cos\phi}{\tan^2(\pi/4 - \phi/2) \sin\Psi \tan\Psi \cos(\beta' + \Psi + \phi)} \quad (7)$$

where FI is the indentation force, K is a coefficient reflecting the rock resistance to the intrusion of tools depending on rock hardness and tool shape and size, h is the indentation, c is the cohesive strength of the material, ϕ is the angle of internal friction of the rock, μ_f is the coefficient of friction between the crushed and chipped rock, θ is the half conical angle of the pick, Ψ is the angle between fracture plane and the initial rock surface.

Afterwards, Pang *et al.* (1989) realized that Eq. (6) overestimates the indentation force and they revised it based on the size effect (Eq. (8)).

$$FI = (Kh^2)(Ch^{-2m}) = \bar{K}h^{2(1-m)} \quad (8)$$

here $\bar{K} = KC$ is a constant, m is a constant for a given material and bit geometry.

(3) Chipping process

Since the chipping process of hard rock is classified as brittle fracture, it is found that the indentation force in this stage is an abruptly decreasing straight line (Bao *et al.* 2011, Wang and Su 2019). When the indentation force reaches a sufficient level, one or more large chips are formed by lateral cracks propagating from beneath the tip of the pick to the surface. Copur *et al.* (2003) also stated experimentally that the growth rate of force during crushing and the decreasing rate of force during chipping are both associated with the brittleness index of the rock.

(4) Lower bound of chipping

Paul and Sikarskie (1965) and Miller and Sikarskie (1968) proposed that the indentation force reduces to zero after each chipping in the constant displacement rate test. However, Pang *et al.* (1989) claim that only the shear force generated during this process decreases to zero and the indentation force retains a positive value depending on the geometric parameters of the conical pick and the rock properties.

2.3 Rock cutting mechanism of conical picks

Although conical pick has been used for cutting operations in the field, it was used in indentation tests due to limited cutting conditions of hard rock in the laboratory. Thus, the results of the indentation tests would also be explained in terms of the results from the cutting tests. Fig. 6 demonstrates the evolution of the cutting force acting on

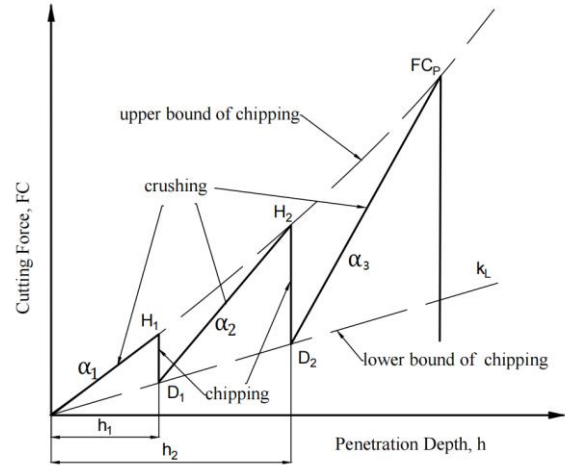


Fig. 6 Prediction model of the evolution of the cutting force

the conical pick while penetrating it into the brittle rock. The behavior of the braking mechanism evolves from small chips to larger chips in three phases as the tip penetrates deeper. These phases reveal the transition from small to large chips which are particularly characterized by the peak values in the force history. The evolution curves of the indentation force and cutting force display remarkably similar behavior prior to the formation of the largest chip. As can be seen in Fig. 6, the upper and lower bounds of chipping in the indentation force prediction model are adopted to describe the ending and beginning of chipping in the cutting force prediction model.

However, there is a maximum value of the cutting force (FCp) because of the boundary effect in the rock cutting test, which eventually contributes to the fracture and falling of the largest rock chip. It is very well known that plastic rocks are generally crushed and penetrated by an indenter, giving linear curves, whereas brittle rocks are cracked and chips are formed, giving non-linear and discontinuous curves (Kahraman *et al.* 2000). The test results showed that most of the cutting force evolution curves conducted in this study are syllogism (three-stage), that is, there are two extreme values (H_1) and (H_2) and two minimal values (D_1) and (D_2) as shown in Fig. 6. There are two simplifications applied to make a better prediction of the cutting force evolution curve. On the one hand, the time occurring in the chipping process is negligible compared to the crushing process for brittle rocks, so the indentation of H_i and D_i are presumed to be equal in the prediction model using the same approaches of Paul and Sikarskie (1965) and Miller and Sikarskie (1968). On the other hand, approximating crushing as a straight line with a slope of α proposed by Morris (1969) and α_1 , α_2 and α_3 are defined as the penetration rate in each crushing process.

3. Indentation tests

Although indentation tests have prospect applications in rock engineering and rock mechanics, they are also used to estimate the rock cuttability and drillability under rock

Table 1 Physical and mechanical properties of the rocks

| Rock name | Density (g/cm ³) | σ_c (MPa) | σ_t (MPa) | SH | B ₁ | B ₂ | B ₃ (MPa ²) |
|-----------|------------------------------|------------------|------------------|-------|----------------|----------------|------------------------------------|
| Granite-1 | 2.62 | 237.18 | 12.17 | 58.57 | 19.49 | 0.90 | 1443.24 |
| Granite-2 | 2.76 | 189.56 | 10.40 | 59.13 | 18.23 | 0.90 | 985.71 |
| Granite-3 | 2.65 | 140.23 | 5.40 | 56.67 | 25.97 | 0.93 | 378.62 |
| Granite-4 | 2.59 | 136.17 | 5.69 | 57.53 | 23.93 | 0.92 | 387.40 |
| Granite-5 | 2.59 | 76.20 | 5.19 | 54.27 | 14.68 | 0.87 | 197.74 |
| Granite-6 | 2.64 | 148.47 | 5.62 | 56.77 | 26.42 | 0.93 | 417.20 |
| Granite-7 | 2.69 | 64.94 | 2.60 | 51.53 | 11.60 | 0.84 | 181.83 |
| Sandstone | 2.44 | 27.22 | 2.74 | 45.37 | 9.93 | 0.82 | 37.29 |

excavation technology (Fattahi and Bayat 2024). For this reason, we measured the indentation (cutter) forces in our study. Before performing the tests in the laboratory, a series of physical and mechanical tests on granites were performed.

3.1 Physical and mechanical properties of the rocks

The rock samples subjected to physical, mechanical and indentation tests were collected from some mining sites in Chongqing and Sichuan province in China, including seven different granites and one sandstone. These physical-mechanical tests were conducted in compliance with the guidelines established by ISRM (Ulusay 2016).

- (1) Density: The rock specimens were prepared in a standard shape. The specimens were first dried in an air-drying oven for 12 hours to eliminate water content. Then, the volume and mass were measured with calipers and sensitive electronic scales, respectively. The average value was determined after performing the test several times. The dry density of the rock was obtained by the ratio of mass to volume as given in Table 1.
- (2) Uniaxial compressive strength test: The collected samples were cored for uniaxial compressive tests. The cylindrical samples with 25 mm radius and 100 mm height were prepared. During the tests, the stress rate was applied within the limits of 0.5-1 MPa/s. The test results are summarized in Table 1.
- (3) Brazilian tensile strength: In order to test the Brazilian tensile strength of granites, the samples were drilled into standard cylinders with diameters and heights of 50 mm and 30 mm, respectively. The tests were carried out at a uniform rate of 0.1 mm/min.
- (4) Schmidt hammer tests: This test method is the most common method for determining the hardness values of rock materials. Depending on the impact energy applied by the hammer, the rebound value of the tip on the rock surface was determined by the rock rebounder. The peak rebound values from 10 continuous impacts at the surface and averaging the peaks of the three sets of tests conducted at three separate points (Poole and Farmer 1980). The average hardness values of the studied samples ranged from 45 to 59 as listed in Table 1.
- (5) Brittleness index: Different measures of rock brittleness are available in the literature based on experimental tests

as well as some empirical approaches. It presents an opinion about the breakage characteristics under different loading conditions and it is relevant to the mechanical excavation of rocks (Copur *et al.* 2003). According to the characteristics of rock brittleness, industry scholars regard the ratio of compressive strength to tensile strength of rocks as an index of rock brittleness as shown in Eq. (9) (Hucka and Das 1974).

$$B_1 = \sigma_c / \sigma_t \quad (9)$$

Andreev (1995) proposed to use Eq. (10) to determine the brittleness index of the rock.

$$B_2 = (\sigma_c - \sigma_t) / (\sigma_c + \sigma_t) \quad (10)$$

Altindag (2002) recommended that the brittleness index of the rock can be estimated using Eq. (11). He defined the index as the multiplication of compressive strength and tensile strength of rocks and divided the product by 2.

$$B_3 = \sigma_c \sigma_t / 2 \quad (11)$$

Based on mechanical test results, the brittleness index values obtained from Eqs. (9)-(11) are presented in Table 1.

3.2 Test equipment

The test equipment employed for indentation tests is an electro-hydraulic servo testing machine which is essentially used for the mechanical property tests of rocks (Fig. 7). To reduce the rigid body motion of the specimen and the potential risk of sudden fractures, the left and right sides of specimens are confined in a heavy steel box during the tests.

A commonly used conical pick is mounted as the loading indenter. The tip angles of conical picks are generally chosen within a range about 60°-100° in the industry. In order to measure the indentation forces in our tests, a sharp pick with a tip angle of 80° and a tip diameter of 22 mm was selected. The alloy tip of the pick is made of YG13 grade tungsten carbide. The picks are sharpened before each test with a diameter lower than 0.5 mm since blunt picks would increase indentation force and specific energy. During the tests, a constant velocity rate was applied to control the vertical displacement travel. The variation of indentation (vertical) force and indentation depth (h) of conical pick is automatically recorded and

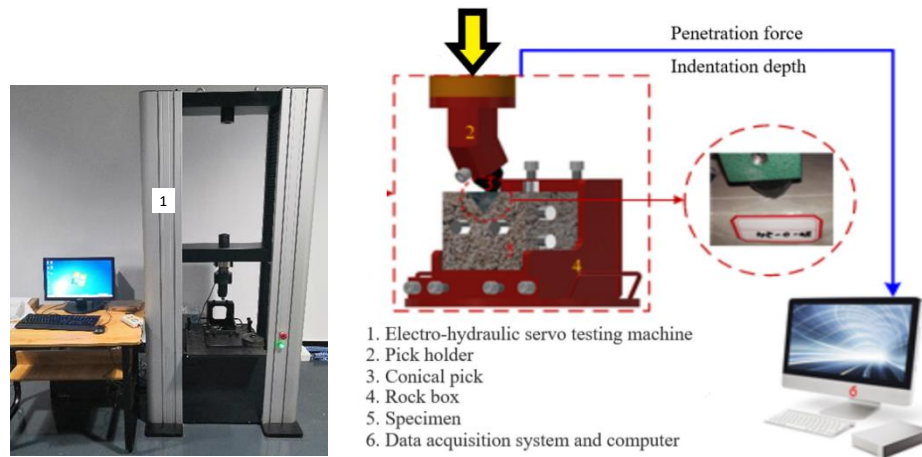


Fig. 7 Experimental test setup

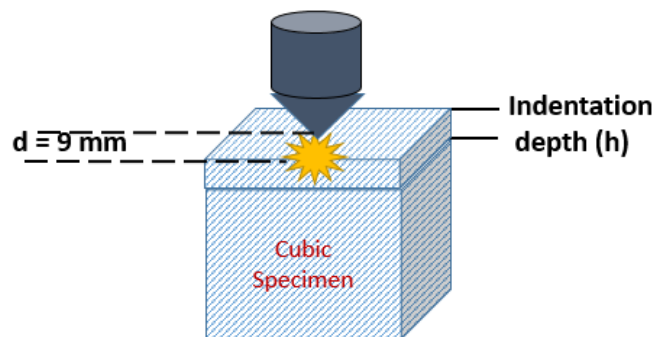


Fig. 8 A general view of the specimen and conical pick

transferred to the data acquisition system by using a 100 kN load cell where the electronic signals are processed in a microprocessor based display as force.

3.3 Testing conditions

Indentation tests were conducted on 7 granites and 1 sandstone brittle specimen in rectangular shape with dimensions of 150 x 150 x 150 mm. All carefully selected rock samples were free of visible cracks and discontinuity planes within the structures. The specimen surfaces were polished carefully to obtain flat surfaces, reduce the toughness, and minimize the effect of surface morphology because variations in the mineral composition, grain size, and the presence of micro-cracks or inclusions can lead these inherent differences in the force-depth relationships.

In addition, variations in how the sample is mounted or held during the test can influence the results because the surface area is not very large. Consistent and well-defined boundary conditions are important for reproducibility of the tests. While some variation in the force-depth curve is to be expected due to these factors, careful control of testing conditions and procedures can help to reduce this variability and improve the reproducibility of the results.

At the beginning of the tests, conical pick is positioned perpendicular to the surface in accordance with Evans' criterion (Fig. 8). Thus, both attack and clearance angles are

set to 0° . The initial position of the pick is placed close to the edge of the specimen depending on the cutting depth and it starts penetrating with a dent in the rock. Since previous studies by Copur *et al.* (2003) and Huang *et al.* (2022) have shown that an increase of specific energy during indentation tests is caused at the cutting depths (d) smaller than 9 mm, we selected a critical depth of 9 mm in our tests. Thus, the braking mechanism of specimens is assumed to be brittle. On the other hand, there is no evidence proposing that the indentation speed has a considerable effect on the forces and specific energy when it is relatively low (Nishimatsu 1972, Bilgin *et al.* 2006, Copur *et al.* 2017). Therefore, we applied 1 mm/min of speed for all tests in order to provide the equilibrium state of the specimen. The resulting horizontal indentation force signals were collected and filtered by a data acquisition system.

4. Analysis of test results from rock properties

4.1 Prediction of the peak cutting force

Table 2 summarizes the measured data obtained from indentation tests including both the mean and peak forces during the experiments obtained from granite samples and a sandstone. In addition, the theoretical models of Evans (1984), Roxborough

Table 2 Comparison of experimental and theoretical cutting forces

| Rock name | Experimental cutting force (kN) | | Theoretical cutting force (kN) | | |
|-----------|---------------------------------|--------|--------------------------------|---------|---------|
| | Mean | Peak | Eq. (1) | Eq. (2) | Eq. (3) |
| Granite-1 | 1.8422 | 5.8687 | 4.3462 | 4.9762 | 4.2743 |
| Granite-2 | 1.9038 | 6.1166 | 3.9713 | 4.2525 | 3.8829 |
| Granite-3 | 1.6052 | 4.7500 | 1.4473 | 2.2080 | 1.4537 |
| Granite-4 | 1.1840 | 4.3222 | 1.6548 | 2.3266 | 1.6532 |
| Granite-5 | 1.1812 | 3.5368 | 2.4603 | 2.1221 | 2.3543 |
| Granite-6 | 1.4058 | 5.0348 | 1.4806 | 2.2980 | 1.4888 |
| Granite-7 | 1.3382 | 4.5003 | 3.3610 | 2.2898 | 3.1239 |
| Sandstone | 0.9166 | 2.7367 | 1.9197 | 1.1204 | 1.7439 |

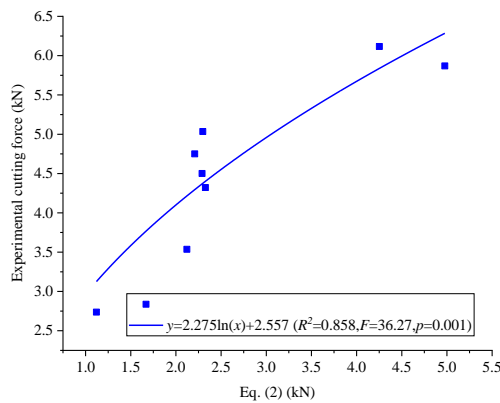
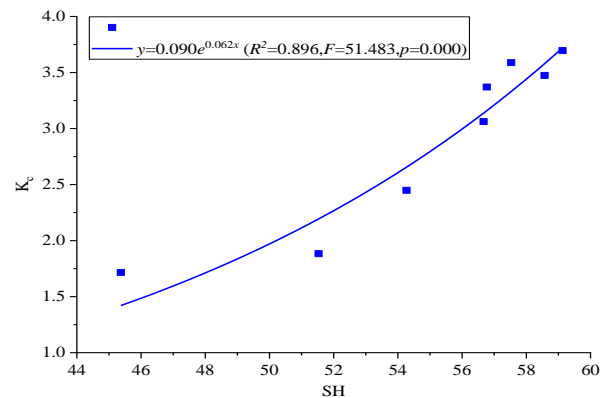


Fig. 9 Relationship between experimental and Goktan's theoretical cutting force

Fig. 10 Relationship between Schmidt hardness and K_C value

and Liu (1995) and Gökten (1997) are presented as peak cutting forces. The theoretical models cannot always represent the actual cutting conditions of rock mass because variations on some rock properties such as the joints, uniaxial compressive strength, or rock quality designation (RQD) may lead to inaccurate results. It can be seen from Table 2 that the peak cutting force is about three times the mean cutting force, which is consistent with the theories of Rånman (1985) and Goktan and Gunes (2005) as well.

Most of the proposed theoretical models use different cutting parameters and mechanical properties of rock acquired from laboratories. Some of the mechanical tests are usually time-consuming. Thus, an extremely large dataset is trained to develop the model. In this context, it is examined that the theoretically calculated values are significantly different from the experimental values. It is worth noting that a strong logarithmic correlation ($R^2 = 0.858$) was found between the Gökten (1997) Eq. (2) and the experimental tests as shown in Fig. 9. Therefore, a semi-empirical model is obtained by modifying Goktan's theoretical model as given in Eq. (12). This good agreement highlights the comparability between the experimental indentation force and the theoretical cutting force. Since Goktan's equation is based on the friction between pick and rock, it can be proposed that friction is also a dominant parameter in indentation tests.

$$FC_p = 2.275 \ln(FC_c) + 2.557 \quad (12)$$

where FC_p is Goktan's (1997) theoretical value of the peak cutting force.

4.2 Prediction of the upper bound of chipping

The force-indentation history indicates the phases of crushing and chipping which are repeated sequentially until the force is reduced to zero. During the chipping action, the target force decreases with increasing indentation. The peak values in the force history describe the upper bound of chipping which was analyzed as a result of the experimental tests. The results for each specimen are summarized in Table 3 where the predictor equations for the critical values of cutting force H_1 , H_2 and peak cutting force (FC_p) are given. Statistical analysis reveals a strong correlation between these values and indentation depth, and Eq. (13) is significant at a confidence level of 99% ($p=0.000$) as a result of statistical analyses.

$$FC_U = K_c h^{2(1-m_c)} \quad (13)$$

where FC_U is the cutting force of upper bound of chipping, K_c and m_c are both constants.

This equation agrees well with Eq. (8) given by Pang *et al.* (1989) for the estimation of the upper bound of chipping and the K_c and m_c values are also listed in Table 3. where R^2 is the proportion of the variance in the dependent variable, t-value is a ratio that compares the difference

Table 3 Regression equations to predict the extreme and peak cutting force

| Rock name | Regression equation | R^2 | F | p | K_C | m_c |
|-----------|-----------------------|-------|--------|-------|-------|-------|
| Granite-1 | $FC_U=3.474h^{0.880}$ | 0.916 | 141.41 | 0.000 | 3.474 | 0.560 |
| Granite-2 | $FC_U=3.695h^{0.915}$ | 0.882 | 89.52 | 0.000 | 3.695 | 0.543 |
| Granite-3 | $FC_U=3.062h^{1.045}$ | 0.806 | 62.41 | 0.000 | 3.062 | 0.478 |
| Granite-4 | $FC_U=3.588h^{1.020}$ | 0.976 | 454.30 | 0.000 | 3.588 | 0.490 |
| Granite-5 | $FC_U=2.449h^{1.031}$ | 0.882 | 89.94 | 0.000 | 2.449 | 0.484 |
| Granite-6 | $FC_U=3.370h^{1.049}$ | 0.947 | 180.40 | 0.000 | 3.370 | 0.476 |
| Granite-7 | $FC_U=1.883h^{1.043}$ | 0.932 | 81.72 | 0.000 | 1.883 | 0.478 |
| Sandstone | $FC_U=1.716h^{1.054}$ | 0.879 | 65.17 | 0.000 | 1.716 | 0.473 |

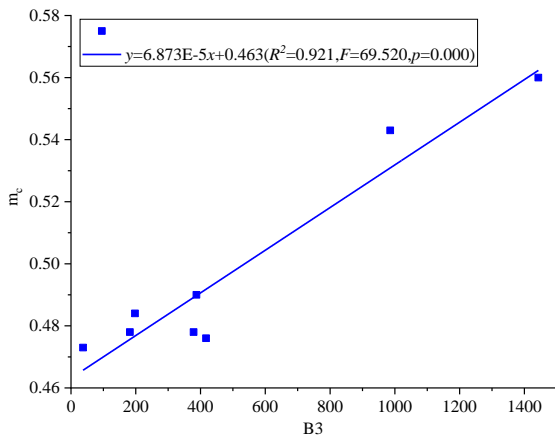


Fig. 11 Relationship between brittleness index B3 and m_c value

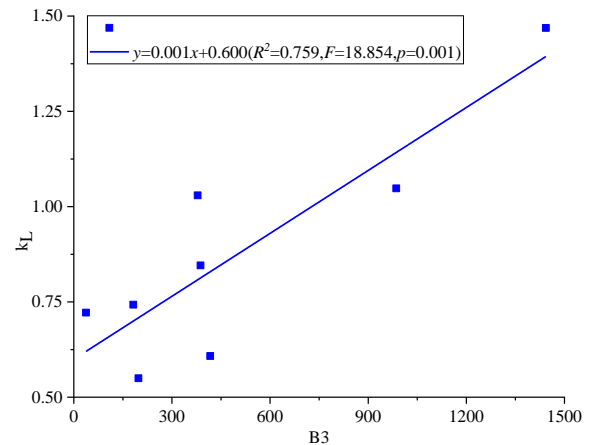


Fig. 12 Relationship between brittleness index B3 and k_L

between the sample mean and the population mean relative to the variability in the sample data, p value is the probability and other notations are as given above.

From Eqs. (14) and (15), it is found that there is an extremely strong exponential relationship between the value of K_C and the rebound strength of the rock, and the m_c value increases linearly with the increase of the brittleness index B3, as shown in Figs. (10) and (11), respectively.

$$K_C = 0.09e^{0.062SH} \tag{14}$$

$$m_c = 6.873E^{-5}B_3 + 0.463 \tag{15}$$

Finally, a new equation emerges from the substitution of Eqs. (14) and (15) into Eq. (13). The predictive model presented in Eq. (16) for the upper bound of chipping is based on Schmidt hardness and brittleness index.

$$FC_U = 0.09e^{0.062SH} h^{6.873e^{-5}B_3+0.463} \tag{16}$$

4.3 Prediction of the lower bound of chipping

At the initial stage of rock cutting, intense crushing of the rock occurs under the tool, and the cracks lead to the

formation of a chip. However, in indentation test, a pure crushing failure mode with fewer cracks occurs. Hence, the lower bound of chipping in the predictive model for the evolution of the cutting force consists of a few straight lines across the 0 and D_1 points with slope k_L using Eq. (17). According to the results of the peak cutting force, it can be reported that the coefficients of the model are influenced by brittleness index.

$$FC_L = k_L h \tag{17}$$

where FC_L is the cutting force of lower bound of chipping.

In this regard, as depicted in Fig. 12, it is evident that brittleness index B3 is highly correlated with the slope where a strong linear relationship ($R^2 = 0.759$) exists as shown in Eq. (18).

$$k_L = 0.001B_3 + 0.6 \tag{18}$$

Thus, the prediction equation for the lower bound of chipping can be reached as follows

$$FC_L = (0.001B_3 + 0.6)h \tag{19}$$

where FC_L is the cutting force of lower bound of chipping.

A recent study by Shaterpour-Mamaghani et al. (2023) also has taken into account the effect of brittleness index to predict the field performance of raise boring machines.

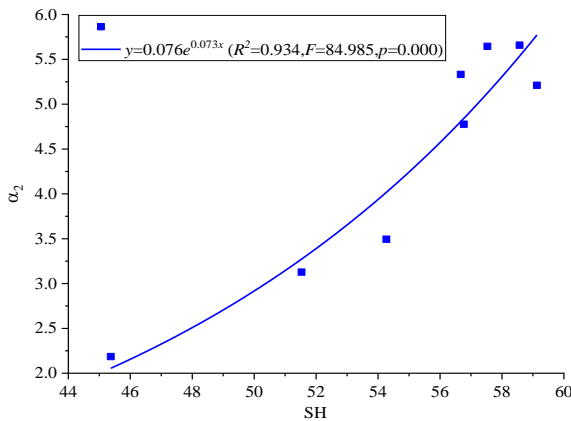


Fig. 13 The relationship between rebound strength and α_2

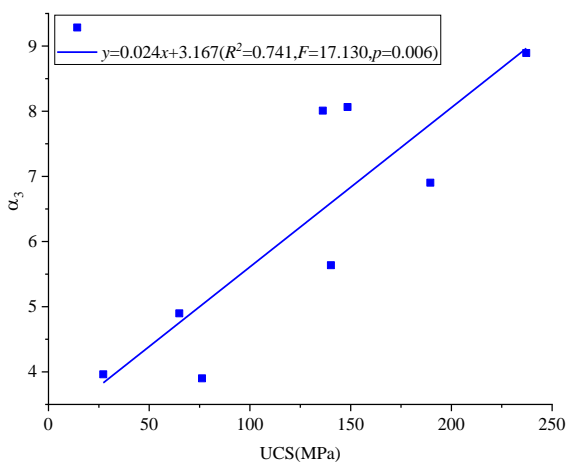


Fig. 14 The relationship between the uniaxial compressive strength of the rock and α_3

They have suggested the indentation test as a promising method as well as rock cutting tests.

4.4 Prediction of the evolution curve of the cutting force

Since indentation is a basic process related to rock cutting, it is widely used to study rock basic deformation, failure mechanism, and fragmentation of rocks. Kahraman *et al.* (2000) built some models from indentation tests based on rock properties by employing statistical analysis techniques and Weibull parameter. Accordingly, they found that Schmidt hardness has a non-linear relationship with α while compressive strength has a linear relationship. As can be seen in Figs. 13 and 14, α_2 increases exponentially with increasing Schmidt hardness while α_3 increases linearly as rock compressive strength increases.

It is important to highlight that the penetration of H1 and D1, shown in Fig. 6 as h_1 , is equal. Similarly, the penetration (h_2) of both H2 and D2 is identical. It is worth noting that the lower bound model of chipping can be obtained by using Eq. (7) as a straight line. The FC_p is calculated using Eq. (12). Hence, the maximum depth of penetration, which results in the largest chip formation, can

be achieved through the use of FC_p and Eq. (13). By combining FC_p , α_3 , and the lower bound model for chipping from Eq. (17), we can effectively determine the D2 value and penetration (h_2) as depicted in Fig. 6. We can derive H2 point by substituting h_2 into Eq. (13). Then, the point D1 and the penetration depth h_1 can accurately be determined by combining α_2 and K from Eq. (7). By inserting h_1 into Eq. (13), the exact value of the point H1 can be verified. The final evolution curve of rock cutting force is obtained by connecting the points of H1, D1, H2, D2 and FC_p in a straight line.

5. Experimental results and discussion on the prediction model

The experimental test results and prediction models of the rock cutting force are shown in Fig. 15. It can be seen that the upper bound of chipping determined by Eq. (13) is effective in predicting the extreme and peak values of the cutting force, especially in Fig. 15(a), 15(b), and 15(e). Meanwhile, the limitations of the upper bound of chipping equation proposed by Pang *et al.* (1989), in which the parameters are complicated to measure and have to be re-determined in each test, are successfully addressed by regression analysis using rock properties such as rebound strength and brittleness index B3.

The rock cutting tests yielded promising results, revealing a noteworthy force remaining after each chip. This is due to the fact that not all rock fragments are completely expelled from the area surrounding the conical pick, supporting Pang *et al.* (1989) findings. Furthermore, Deketh *et al.* (1998) clarified that the more brittle the rock is the stronger the fluctuation in the chipping process, which reasonably explains the existence of a better correlation between the rock brittleness index B3 and the slope of the lower bound of chipping k_L in the prediction model. It is important to highlight that this study does not provide a specific mathematical formula for the lower bound of chipping after removing all D1 and D2 points from the test data. The 0, D1 and D2 points in each set of data are approximately on a straight line, as seen in Figs. 15(a)-15(h). Therefore, a straight line with slope k_L crossing the points 0 and D1 is utilized in this prediction model to describe the lower bound of the chipping. The model presents a unique formulation of the lower bound of chipping, which is based on brittleness index B3. Unlike previous studies, this approach yields highly accurate predictions that closely align with experimental results.

The prediction model for the evolution of rock cutting force utilizes the method proposed by Paul and Sikarskie (1965) along with the approach presented by Miller and Sikarskie (1968). It approximates the termination indentation of chipping to be equivalent to the starting indentation of the subsequent crushing phase. The reason is the extremely brief duration of the chipping completion time. The experimental data also proved that this method can effectively characterize the trend of chipping. However, it is noticed that the prediction of the indentation occurring in each chipping shows some deviation in Figs. 15(a), 15(d), 15(f), and 15(g). The main reason for this phenomenon is that the rock is a nonlinear material with inhomogeneity and anisotropy.

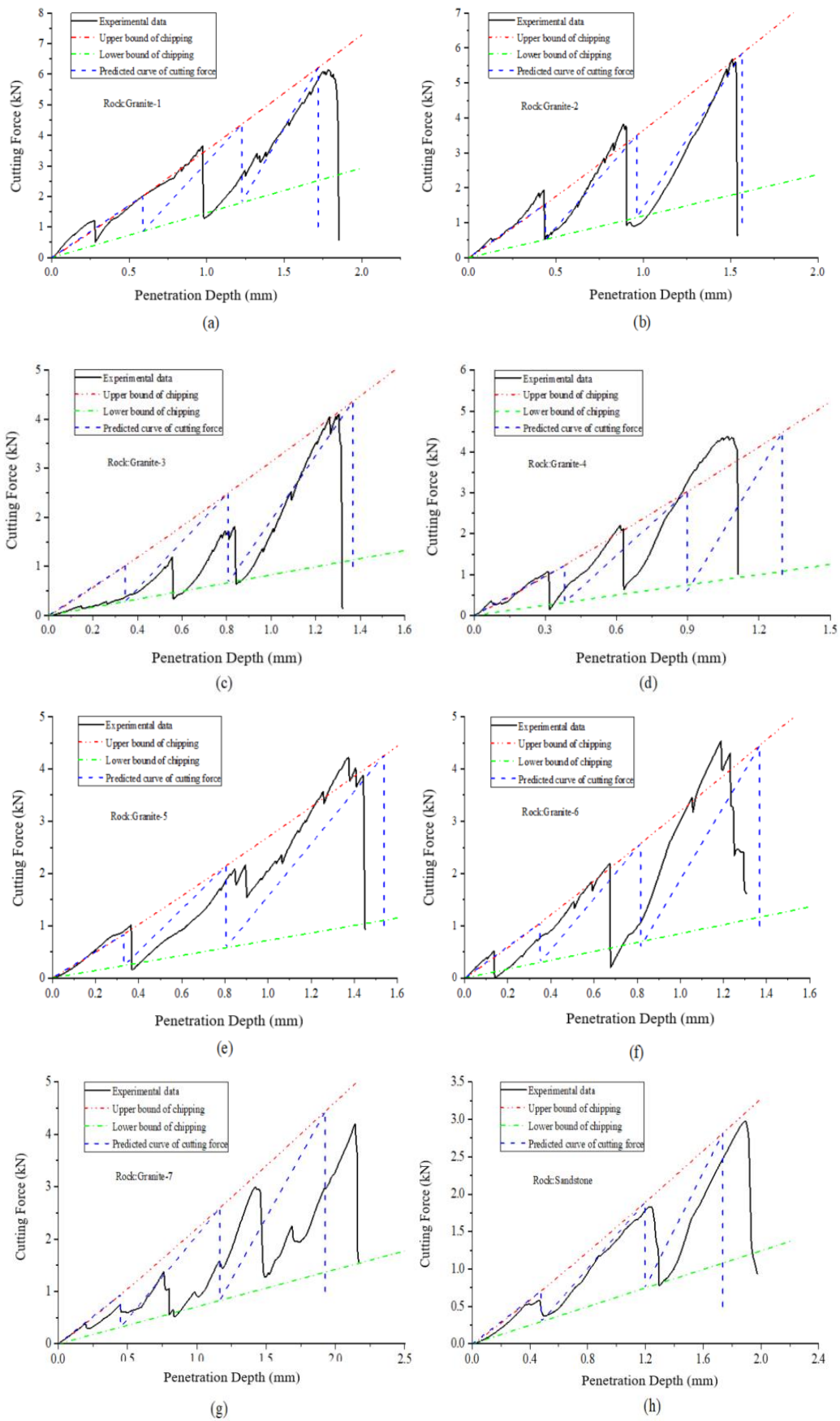


Fig. 15 Comparison of experimental results and the prediction model

It is well known that chipping is generated by the critical state of local stress in the rock around the conical pick and it would cause a large disturbance to the prediction of chipping when the local strength of the rock fluctuates seriously.

When we compare the experimental data with the prediction model, we observe that the conical pick with a tip angle of 80° , when vertically penetrating rocks at a cutting depth of 9 mm, exhibits a three-stage evolution curve for the cutting force. This curve also indicates the occurrence of three distinct crushing processes, of which the crushing duration of the first process is significantly smaller than the last two processes, and it is very close to the upper bound of chipping as shown in Figs. 15(b), 15(d), 15(e), 15(f), and 15(h). Meanwhile, the cutting force-indentation depth curve has almost a linear shape in the initial loading stage, which is the linear elastic deformation stage of each rock, and it reveals that there is little damage on the rock. It is also noted that the slopes α_2 and α_3 are both higher than the slope of the upper bound of chipping for the corresponding period and the statistical analysis shows that the slope α tends to increase gradually, i.e., $\alpha_1 < \alpha_2 < \alpha_3$.

In summary, the evolution curve of cutting force can be reasonably predicted based on the measured physical and mechanical properties of the rock. The results can be used for the fluctuation study of rock cutting to design the tunneling machine and reduce mechanical vibration. Otherwise, higher cutting forces would cause increased machine vibration which can also lead to costly breakdowns and consequent loss of production. In addition, it is important to mention that the predicted evolution curve of cutting force is valid when considering a specific cutting condition. This refers to the implementation of conical pick with a tip angle of 80° , penetrating into the rock vertically with a 9 mm of cutting depth which is widely encountered in actual production. It can be concluded that the evolutionary characteristics of the cutting force are influenced by the shape of the pick and the variation of the cutting parameters. Therefore, the present research should be further developed in the future.

6. Conclusions

Based on the summary on the indentation tests by using a conical pick, force evaluation behavior of granites was analyzed at a cutting depth of 9 mm. In this context, a new prediction model has been developed to predict the changes in cutting force by examining the properties of rocks. This model was developed by conducting a series of tests on eight specimens using a conical pick. Through a comparison of the test data and the prediction model, several significant conclusions were drawn as follows:

- We found that there is a strong power relationship between the upper bound of the cutting force along with chipping and depth of cut. As the depth of cut increases, the upper bound of the cutting force also increases. A small change in the depth of cut can lead to a significant change in the force depending on the value of the exponent n . In

other words, larger fragments that are produced during cutting process influences chipping because higher force cause more intense deformation and potential breakage of the rock. Nevertheless, no noteworthy mathematical relationship was observed for the lower bound of chipping because the minimum size chips is influenced by a tool geometry, cutting speed, and other process parameters, making it more complex to mathematically model.

- In addition, the slope of the three crushing included in the evolution curve of the cutting force tends to increase sequentially ($\alpha_1 < \alpha_2 < \alpha_3$). In the first chip crushing, force increases at a certain rate to a level. The second crushing event increase higher than the first level. The third one increases even more rapidly compared to second level due to reaching to a deeper depth of cut.

- Furthermore, the variation in the local strength of the rock causes a large offset in the model prediction of chipping because the rock's strength is not uniform, the model might predict that a certain force will cause chipping at a specific point, but in reality, the chipping occurs elsewhere or under different conditions. Thus, the varying strength does not allow to accurately predict the behavior of the rock, posing challenges in industries relying on precise rock-breaking predictions.

According to the distribution characteristics and the correlation accuracy of regression analyses, a strong power relationship was found between the upper bound of the cutting force and depth of cut. In addition, it was stated that chipping is also affected by cutting depth while predicting the cutting force. Schmidt hardness and brittleness index tests are suggested to plot the lower and upper bounds of chipping. As a result, the developed model provides references for the design of cutting picks as well as improving the cutting efficiency of excavation machines.

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