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Effect of the gravity on a nonlocal thermoelastic medium with a heat source using fractional derivative

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Abstract. The purpose of this paper is to depict the effect of gravity on a nonlocal thermoelastic medium with initial stress. The Lord-Shulman and Green-Lindsay theories with fractional derivative order serve as the foundation for the formulation of the fundamental equations for the problem. To address the problem and acquire the exact expressions of physical fields, appropriate non-dimensional variables and normal mode analysis are used. MATLAB software is used for numerical calculations. The projected outcomes in the presence and absence of the gravitational field, along with a nonlocal parameter, are compared. Additional comparisons are made for various fractional derivative order values. It is evident that the variation of physical variables is significantly influenced by the fractional derivative order, nonlocal parameter and gravity field.

Keywords: fractional derivative heat transfer; gravity field; internal heat source; nonlocal parameter

1. Introduction

The classical thermoelasticity theory, first presented by Biot (1956) and based on Fourier's equation of heat conduction, has the drawback of not taking into account thermal signals that propagate at infinite speed. To resolve this contradiction, generalized theories have been developed during the past few decades that include a finite speed of heat transportation (hyperbolic heat transport equation) in elastic substances. The extended thermoelasticity theory, which has one thermal relaxation time parameter, is the first generalization theory provided by Lord and Shulman (L-S) (1967). Green and Lindsay (G-L) (1972) presented the second generalization hypothesis, which involves two thermal relaxation times. The energy equation and constitutive equations were altered by the (G-L), which is known as temperature rate-dependent thermoelasticity. The two generalized theories proposed one or two relaxation times in the thermoelastic interaction, which modified the energy formula and the Neumann-Duhamel equation to correct Fourier's heat conduction formula and define a limited speed for thermal wave propagation. Changes made to the two generalized theories caused heat propagation to be depicted as a wave phenomenon rather than a diffusion one, which is typically connected to the impact of the second sound. It is impossible to understand these two theories as a singular instance of one to the other because of their dissimilar structural designs. To solve the issue of infinite heat propagation that exists in the traditional coupled dynamical thermoelasticity theory, generalized thermoelasticity theories were developed. Various problems related to the above theories have been investigated by Abbas and Zenkour (2013), Said and Othman (2016), Youssef and Al-Lehaibi (2019), Othman *et al.* (2020), Khalil *et al.* (2021), Said (2023), Lamba (2023), and Abouelregal et al. (2021a, b, 2023).

Numerous current models of physical processes have been successfully modified by the application of fractional calculus. Abel (1881) demonstrated the first use of fractional derivatives when he solved an integral equation that arose from the formulation of the tautochrone problem using fractional calculus. Using fractional derivatives to describe viscoelastic materials, Caputo and Mainardi (1971a, b) and Caputo (1974) found good agreement with experimental results and established the relationship between fractional derivatives and the theory of linear viscoelasticity. A novel time-fractional derivative model for a micropolar thermo-viscoelastic medium with two temperatures was examined by Deswal and Kalkal (2013). Bahraini *et al*. (2021) discuss an electrically operated fractional model analysis of viscoelastic microbeams. Generalized photo-thermoelasticity with fractional derivatives was introduced by Abbas *et al*. (2018) to analyze the linked thermal, elastic, and plasma waves on an unbounded semiconductor medium. The following authors introduced generalized thermoelasticity: Abouelregal (2020), Kaur *et al.* (2021), Marin *et al*. (2021), Ellahi *et al.* (2017, 2018, 2019), Said (2022), Atta (2022), and Zenkour *et al*. (2023).

Applications in nanomechanics such as lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, fracture mechanics, surface tension fluids, etc. were studied using the nonlocal theory of elasticity. The nonlocal elasticity theory was first presented by Eringen (2002) who states that the stress at a place depends not only on the strain there but also on the strains at other sites in the domain. The nonlocal stress field is obtained by convolutioning the local strain field with a

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smoothing kernel, as described by Eringen's nonlocal elasticity theory. The properties of surface waves in a nonlocal elastic media with microstructure were examined by Kaliski *et al.* (1992). Different modes of Rayleigh waves in a nonlocal micropolar elastic half-space were discussed by Khurana and Tomar (2017). The solutions for various problems support the nonlocal thermoelasticity theory as Zenkour and Abouelregal (2014), Yu *et al.* (2015), Luo *et al.* (2021), Lata (2020), Said (2022), and Abouelregal *et al.* (2023).

The increasing interest in the theory of thermoelasticity has many applications in various fields, including geophysics, acoustic wave damping in a magnetic field, machine element design of equipment such as heat exchangers, boiler tubes, nuclear devices emitting electromagnetic radiations, development of highly sensitive and superconducting magnetometers, electrical power engineering, plasma physics, etc. The current theoretical findings could offer intriguing details and a mathematical framework for further research on the topic. In this paper, a nonlocal thermoelastic solid with a moving internal heat source is considered under the influence of the gravitational field and initial stress. The problem is discussed in the context of the Lord-Shulman (L-S), and Green-Lindsay (G-L) theories with fractional derivative heat transfer. The resulting non-dimensional equations are solved using the method of normal mode analysis. A comparison is carried out between the results for local and nonlocal thermoelasticity. Other comparisons are carried out between the results with and without gravitational field. Comparisons also are carried out between the physical variables for different values of fractional derivative order.

2. Formulation of the problem and the basic equations

The problem of a nonlocal thermoelastic solid with a moving internal heat source under the influence of gravitational field and initial stress. The dynamic displacement is given as $\underline{u} = (u, 0, w)$.

The constitutive equations as Hetnarski and Eslami (2009) and Eringen (2002)

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta\delta_{ij} - P(w_{ij}+\delta_{ij}),$$

$$w_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}), \tag{1}$$

where, $\varepsilon = a_0 e_0$ is the elastic nonlocal parameter having a dimension of length, a_0, e_0 respectively, are an internal characteristic length and a material constant (see Eringen *et al.* (1972a), (1972b) for details), σ_{ij} are the components of stress, e_{ij} are the components of strain, e_{kk} is the dilatation, λ, μ are elastic constants, α_t is the thermal expansion coefficient, $\theta = T - T_0$, where T is the temperature above the reference temperature T_0, δ_{ij} is the Kronecker's delta, P is the initial stress, and w_{ij} is the rotation vector.

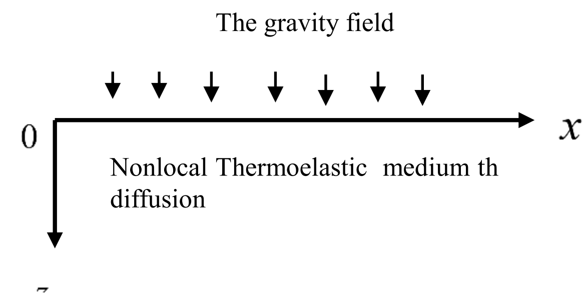

Fig. 1 The sketch of the problem

The heat conduction equation as Youssef (2016)

$$K\nabla^2\theta = \left(1+(\tau_0+\tau_2)\frac{\partial^s}{\partial t^s}\right)\rho C_E\dot{\theta} + \gamma T_0\left(1+\tau_0\frac{\partial^s}{\partial t^s}\right)\dot{e} - \left(1+\tau_0\frac{\partial^s}{\partial t^s}\right)Q, \tag{2}$$

where K is the coefficient of thermal conductivity, ρ is the mass density, C_E is the specific heat at constant strain, τ_0, τ_1, τ_2 are thermal relaxation times and Q is an internal heat source.

In the above fractional-order heat conduction Eq. (2), we have taken the following definition of fractional-order derivative.

$$\frac{\partial^s f(x,t)}{\partial t^s} = \begin{cases} f(x,t) - f(x,0), & s \to 0 \\ I^{1-s}\,\dfrac{\partial f(x,t)}{\partial t}, & 0 \prec s \prec 1 \\ \dfrac{\partial f(x,t)}{\partial t}, & s = 1 \end{cases} \tag{3}$$

Where I^s is introduced as Miller and Ross (1993) and Podlubny (1999)

$$I^s f(t) = \frac{1}{\Gamma(s)}\int_0^t \frac{1}{(t-\tau)^{1-s}} f(\tau)\,d\tau, \quad 0 \prec s \prec 1 \tag{4}$$

where $\Gamma(s)$ is the Gamma function and $f(t)$ is a Lebesgue integrable continuous function satisfies

$$\lim_{s\to 1}\frac{\mathrm{d}^s f(t)}{\mathrm{d}t^s} = f'(t) \tag{5}$$

The equation of motion

$$\rho\ddot{u}_i = \sigma_{ji,j} + F_i, \tag{6}$$

Where $F_1 = \rho g\dfrac{\partial w}{\partial x}$, $F_2 = 0$, $F_3 = -\rho g\dfrac{\partial u}{\partial x}$ are force due to the existence of the gravitational field as Said and Othman (2016).

Using Eqs. (1) in Eqs. (6), we obtain

$$\rho(1-\varepsilon^2\nabla^2)\,\ddot{u} = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda+\mu+\frac{P}{2})\frac{\partial^2 w}{\partial x\,\partial z} + (\mu-\frac{P}{2})\frac{\partial^2 u}{\partial z^2} - \gamma\,(1+\tau_1\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x} + \rho g\,(1-\varepsilon^2\nabla^2)\,\frac{\partial w}{\partial x}, \tag{7}$$

$$\rho(1-\varepsilon^2\nabla^2)\,\ddot{w} = (\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} + (\lambda+\mu+\frac{P}{2})\frac{\partial^2 u}{\partial x\,\partial z} + (\mu-\frac{P}{2})\frac{\partial^2 w}{\partial x^2} - \tag{8}$$

$$\gamma\,(1+\tau_1\frac{\partial}{\partial t})\frac{\partial\theta}{\partial z}-\rho g\,(1-\varepsilon^2\nabla^2)\,\frac{\partial u}{\partial x},$$

we introduce the non-dimensional quantities as

$$(x',z',\varepsilon',u',w')=\frac{1}{l_0}\,(x,z,\varepsilon,u,w),\quad g'=\frac{l_0}{d_0^2}\,g,\quad \sigma'_{ij}=\frac{\sigma_{ij}}{\mu},$$

$$(t',\tau_0',\tau_1',\tau_2')=\frac{d_0}{l_0}(t,\tau_0,\tau_1,\tau_2),\quad \theta'=\frac{\gamma\theta}{(\lambda+2\mu)},\quad Q'=\frac{\gamma Q}{(\lambda+2\mu)},\tag{9}$$

$$l_0=\sqrt{\frac{K}{\rho C_E\,T_0}},\qquad d_0=\sqrt{\frac{\lambda+2\mu}{\rho}}\,.$$

From Eqs. (9) in Eqs. (7), (8) and (2), we obtain

$$(1-\varepsilon^2\nabla^2)\,\ddot{u}=\frac{\partial^2 u}{\partial x^2}+A_1\frac{\partial^2 w}{\partial x\,\partial z}+A_2\frac{\partial^2 u}{\partial z^2}-(1+\tau_1\frac{\partial}{\partial t})\frac{\partial\theta}{\partial x}+ g\,(1-\varepsilon^2\nabla^2)\,\frac{\partial w}{\partial x},\tag{10}$$

$$(1-\varepsilon^2\nabla^2)\,\ddot{w}=A_2\frac{\partial^2 w}{\partial x^2}+A_1\frac{\partial^2 u}{\partial x\,\partial z}+\frac{\partial^2 w}{\partial z^2}-(1+\tau_1\frac{\partial}{\partial t})\frac{\partial\theta}{\partial z}- g\,(1-\varepsilon^2\nabla^2)\,\frac{\partial u}{\partial x},\tag{11}$$

$$\nabla^2\theta=\left(1+(\tau_0+\tau_2)\frac{\partial^s}{\partial t^s}\right)A_3\,\dot{\theta}+A_4\left(1+\tau_0\frac{\partial^s}{\partial t^s}\right)\dot{e}-A_5\left(1+\tau_0\frac{\partial^s}{\partial t^s}\right)Q.\tag{12}$$

Where $A_1=\dfrac{\lambda+\mu+\frac{P}{2}}{\rho d_0^2}$, $A_2=\dfrac{\mu-\frac{P}{2}}{\rho d_0^2}$, $A_3=\dfrac{\rho C_E\,d_0\,l_0}{K}$,

$A_4=\dfrac{\gamma^2\,T_0\,d_0\,l_0}{K\,(\lambda+2\mu)}$, $A_5=\dfrac{l_0^2}{K}$.

3. Normal mode analysis

Actually, if all the field values are sufficiently smooth on the real line for normal mode analysis of these functions to exist, then the normal mode analysis is to search for the solution in the Fourier transform domain. It has the capacity to offer novel insights into the dynamical and structural characteristics of complex systems. According to normal mode analysis, the physical field solution can be broken down into the following as Othman *et al.* (2020)

$$\left(u,w,\theta,\sigma_{ij}\right)(x,z,t)=\left(u^*,w^*,\theta^*,\sigma^*{}_{ij}\right)(z)\exp\,\left(mt+\mathrm{i}\,ax\right),\qquad Q=Q^*\exp(mt+\mathrm{i}\,ax),\qquad Q^*=Q_0V_0,\tag{13}$$

where $u^*(z)$, etc. is the amplitude of the function $u(x,z,t)$ etc., i is the imaginary unit, m is the complex time constant, a is the wavenumber in the $x-$ direction, V_0 is the velocity of a moving internal heat source and Q_0 is the magnitude of an internal heat source.

Using Eqs. (13) in Eqs. (10)-(12), we obtain

$$\left(N_1\,\mathrm{D}^2-N_2\right)u^*+\mathrm{i}a\left(-g\,\varepsilon^2\mathrm{D}^2+A_1\mathrm{D}+N_3\right)w^*-\mathrm{i}a\,N_4\,\theta^*=0,\tag{14}$$

$$\mathrm{i}a\left(g\,\varepsilon^2\mathrm{D}^2+A_1\mathrm{D}-N_3\right)u^*+\left(N_5\,\mathrm{D}^2-N_6\right)w^*-N_4\,\mathrm{D}\,\theta^*=0,\tag{15}$$

$$\mathrm{i}a\,N_7u^*+N_7\,\mathrm{D}\,w^*+\left(N_{10}-\mathrm{D}^2\right)\theta^*=N_9,\tag{16}$$

Where, $m^*=e^{-mt}\,t^{-s}\sum_{j=1}^{\infty}\dfrac{(mt)^j}{\Gamma(j+1-s)}$, $N_1=\varepsilon^2m^2+A_2$,

$N_2=a^2+m^2(1+a^2\varepsilon^2)$, $N_3=g(1+a^2\varepsilon^2)$, $N_4=1+m\tau_1$,

$N_5=\varepsilon^2m^2+1$, $N_6=A_2a^2+m^2(1+a^2\varepsilon^2)$, $N_7=A_4m(1+m^*\tau_0)$,

$N_8=A_3m(1+m^*(\tau_0+\tau_2))$, $N_9=A_5Q_0V_0\,(1+m^*\,\tau_0)$,

$N_{10}=N_8+a^2$, and $\mathrm{D}=\dfrac{\mathrm{d}}{\mathrm{d}z}$.

With the solution of Eqs. (14)- (16), we obtain

$$\left(\mathrm{D}^6-C_1\,\mathrm{D}^4+C_2\,\mathrm{D}^2-C_3\right)u^*(z)=-\frac{\mathrm{i}aN_4N_6N_9}{C_0}.\tag{17}$$

The bound solution of Eq. (17) is

$$u^*(z)=\sum_{j=1}^{3}M_j\,\exp(-k_jz)+\frac{\mathrm{i}aN_4N_6N_9}{C_0C_3}.\tag{18}$$

Similarly

$$w^*(z)=\sum_{j=1}^{3}H_{1j}\,M_j\,\exp(-k_jz)+\frac{a^2N_3N_4N_9}{C_0C_3},\tag{19}$$

$$\theta^*(z)=\sum_{j=1}^{3}H_{2j}\,M_j\,\exp(-k_jz)+\frac{a^2N_3^2N_9-N_2N_6N_9}{C_0C_3}.\tag{20}$$

$$\sigma_{zz}{}^*(z)=\sum_{j=1}^{3}H_{3j}\,M_j\,\exp(-k_jz)-C_4,\tag{21}$$

$$\sigma_{xz}{}^*(z)=\sum_{j=1}^{3}H_{4j}M_j\,\exp(-k_jz)+C_5,\tag{22}$$

$$\sigma_{zx}{}^*(z)=\sum_{j=1}^{3}H_{5j}M_j\,\exp(-k_jz)+C_6,\tag{23}$$

Where $k_j^2\,(j=1,2,3)$ are the roots of the equation: $(k^6-C_1\,k^4+C_2\,k^2-C_3=0)$.

$$C_0=g^2a^2\varepsilon^4-N_1N_5,$$

$$C_1=\frac{1}{C_0}\left\{2g\,N_3a^2\varepsilon^2+g^2a^2\varepsilon^4N_{10}+A_1^2a^2-N_1N_4N_7-N_1N_6-N_2N_5-N_1N_5N_{10}\right\},$$

$$C_2=\frac{1}{C_0}\left\{\begin{array}{l}2gN_{10}N_3a^2\varepsilon^2+2A_1a^2N_4N_7+A_1^2a^2N_{10}+N_3^2a^2-N_2N_4N_7-\\ N_2N_6-N_{10}N_1N_6-N_2N_5N_{10}-a^2N_4N_5N_7\end{array}\right\},$$

$$C_3=\frac{1}{C_0}\left\{N_{10}N_3^2a^2-N_2N_6N_{10}-a^2N_4N_6N_7\right\},$$

$$C_4=\frac{N_4N_9\left(\lambda a^2N_6+(\lambda+2\mu)(a^2N_3^2-N_2N_6)\right)}{\mu C_0C_3(1+\varepsilon^2a^2)}+\frac{P\exp\,(-mt-\mathrm{i}\,ax)}{\mu(1+\varepsilon^2a^2)},$$

$$C_5=\frac{\mathrm{i}a^3N_3N_4N_9(\mu-\frac{P}{2})}{\mu C_0C_3(1+\varepsilon^2a^2)},\quad C_6=\frac{\mathrm{i}a^3N_3N_4N_9(\mu+\frac{P}{2})}{\mu C_0C_3(1+\varepsilon^2a^2)},$$

$$H_{1n}=\frac{N_1k_n^3-g\,a^2\varepsilon^2k_n^2+(a^2A_1-N_2)k_n+N_3a^2}{\mathrm{i}a\left(g\,\varepsilon^2k_n^3+(A_1-N_5)k_n^2-N_3k_n+N_6\right)},$$

$$H_{2n}=\frac{N_1k_n^2-N_2+\mathrm{i}a\,(N_3-A_1k_n-g\,\varepsilon^2k_n^2)\,H_{1n}}{\mathrm{i}a\,N_4},$$

$$H_{3n}=\frac{\mathrm{i}a\lambda-(\lambda+2\mu)\left(k_n\,H_{1n}+N_4H_{2n}\right)}{\mu\,(1+\varepsilon^2a^2-\varepsilon^2k_n^2)},$$

$$H_{4n}=\frac{\mathrm{i}a\,(\mu-\frac{P}{2})H_{1n}-(\mu+\frac{P}{2})k_n}{\mu\,(1+\varepsilon^2a^2-\varepsilon^2k_n^2)},\quad H_{5n}=\frac{\mathrm{i}a\,(\mu+\frac{P}{2})H_{1n}-(\mu-\frac{P}{2})k_n}{\mu\,(1+\varepsilon^2a^2-\varepsilon^2k_n^2)}.$$

4. The boundary conditions

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The following boundary conditions on the plane $z=0$ are taken into consideration when determining the parameters M_n $(n=1,2,3)$ as Said (2020)

a) The solid's surface must adhere to a thermally insulated boundary.

$$\frac{\partial\theta}{\partial z}=0 \tag{24}$$

b) The mechanical boundary condition is that the surface of the solid is subjected to mechanical force.

$$\sigma_{zz}=-f_0\,G(x,t)-\frac{P}{(1+\varepsilon^2a^2)}. \tag{25}$$

c) The mechanical boundary condition is that the surface of the solid is subjected to traction-free.

$$\sigma_{xz}=0 \tag{26}$$

Where f_0 is a constant and $G(x,t)$ is an arbitrary function.

Introducing the expressions of the variables considered in Eqs. (24)- (26), we get

$$\sum_{j=1}^{3}k_jH_{2j}M_j=0,\ \sum_{j=1}^{3}H_{3j}M_j=-f_0+C_7,\ \sum_{j=1}^{3}H_{4j}M_j=-C_5. \tag{27}$$

Where $C_7=\dfrac{N_4N_9\left(\lambda a^2N_6+(\lambda+2\mu)(a^2N_3^2-N_2N_6)\right)}{\mu C_0C_3(1+\varepsilon^2a^2)}$.

Solving the above system in Eq. (27), we get

$$\begin{pmatrix}M_1\\M_2\\M_3\end{pmatrix}=\begin{pmatrix}k_1H_{21}&k_2H_{22}&k_3H_{23}\\H_{31}&H_{32}&H_{33}\\H_{41}&H_{42}&H_{43}\end{pmatrix}^{-1}\begin{pmatrix}0\\C_7-f_0\\-C_5\end{pmatrix} \tag{28}$$

Using the method of the matrix method in Eq. (28), we get the values of the variables $M_j(j=1,2,3)$.

5. Validation and applications

We take into consideration the numerical results for the physical fields as follows to demonstrate the theoretical results acquired in the preceding sections and to compare these in the context of the Green-Lindsay (G-L) theory and the Lord-Shulman (L-S) theory. For the computation, Matlab Software package is used as a tool. The physical constants are taken as as Lata (2020)

$$\lambda=3.76\text{x}10^9\ \text{N.m}^{-2},\ \mu=7.78\text{x}10^{10}\ \text{N.m}^{-2},\ \rho=8954\,\text{kg.m}^{-3},$$
$$a=0.6,\ T_0=293\text{K},\ C_E=383.3\ \text{J.kg}^{-1}.\text{K}^{-1},\ \varepsilon=0.7,$$
$$\alpha_t=1.78\text{x}10^{-3}K^{-1},\ \tau_0=0.3\text{s},\ \tau_1=0.9\ \text{s},\ \tau_2=0.7\text{s},\ f_0=0.5,$$
$$K=386\text{w.m}^{-1}.\text{K}^{-1}.\text{s}^{-1},\ m=m_0+i\xi,\ m_0=0.5,\ \xi=-0.5,$$
$$P=5\ \text{x}10^{10}\text{N.m}^{-2},\ Q_0=10\text{K},\ V_0=0.5\text{m.s}^{-1},\ x=-0.5.$$

The computations were running out for non-dimension time t=0.5. The thermodynamic temperature, stress

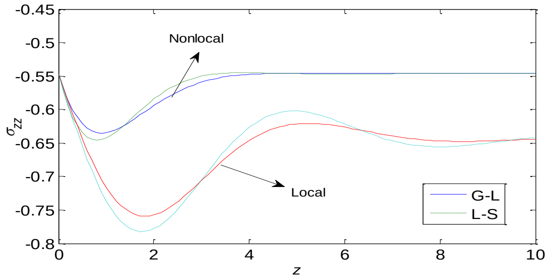

Fig. 2 Variation of stress component σ_{xx} for local and nonlocal theories

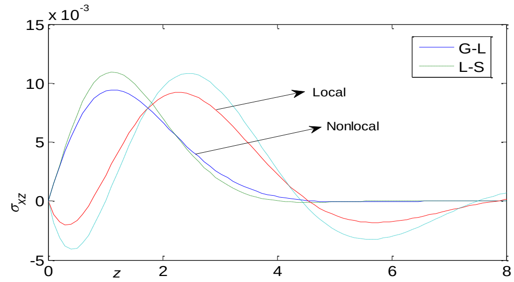

Fig. 3 Variation of stress component σ_{xz} for local and nonlocal theories

components, and displacement distributions are shown in graphs 1-9. Figs. 2-4 display the stress components σ_{zz},σ_{xz} and thermodynamic temperature distributions, respectively, for local and nonlocal thermoelastic half-space solid. Fig. 2 introduces the variation of stress component σ_{zz} with the distance z. σ_{zz} starts with negative values and satisfies the boundary conditions in Eq. (24). It minimizes reaching its minimum values, then increases, and the latter decreases. The existence of nonlocal parameter leads to increasing the values of σ_{zz} Fig. 3 displays the variation of stress component σ_{xz} begins from a zero value at $z=0$ and satisfies the boundary conditions in Eq. (26). In the existence of the nonlocal parameter, σ_{xz} maximized its maximum values in the range $0\le z\le1$, and then decreases. However, for the local thermoelastic medium, σ_{xz} minimizes its minimum values in the range $0\le z\le0.5$, then increases reaching its maximum values in the range $0.5\le z\le2.5$ with a wave behavior. The existence of the nonlocal parameter leads to an increase the values of σ_{xz} then decrease and the latter increase. Fig. 4 displays the variation of thermodynmic temperature θ with the distance z. In the existence of the nonlocal parameter, the values of θ decrease in the range $0\le z\le8$. However, for local thermoelastic medium, the values of θ attains its maximum values and then decrease with a wave behavior. The existence of the nonlocal parameter leads to an increase in the values of θ then decrease and later increase.

Figs. 5-7 display the displacement components u, w and stress component σ_{xz} variation in the absence $(g=0)$ and presence $(g=9.8)$ of the gravity field. Fig. 5 depicts the variation of horizontal displacement variation u starts with

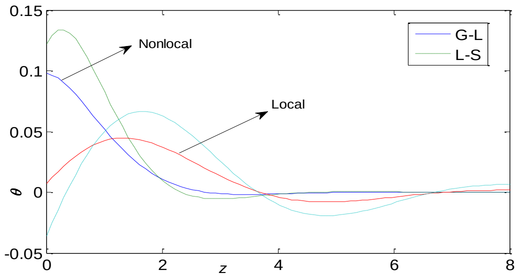

Fig. 4 Thermal temperature variation θ for local and nonlocal theories

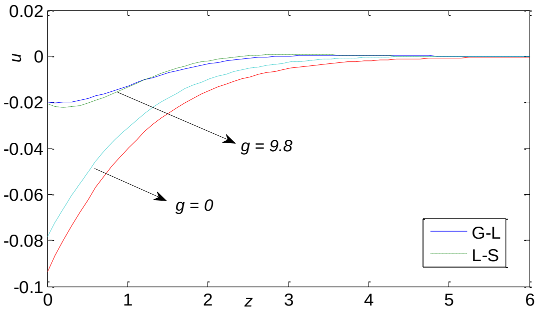

Fig. 5 Horizontal displacement variation u with and without the gravitational field

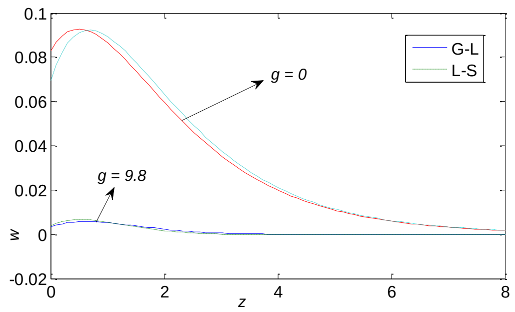

Fig. 6 Vertical displacement variation w with and without the gravitational field

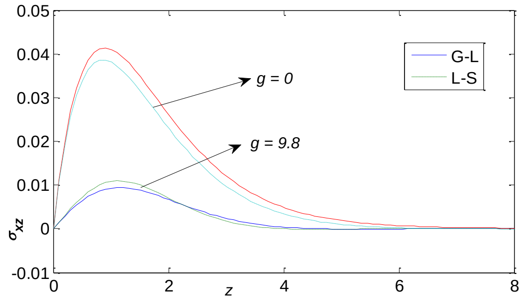

Fig. 7 Variation of stress component σ_{xz} with and without the gravitational field

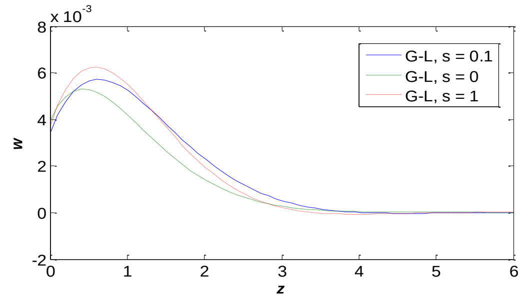

Fig. 8 Vertical displacement variation w for different values of fractional derivative order s

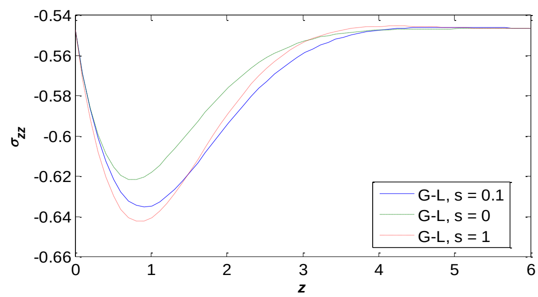

Fig. 9 Variation of stress component σ_{zz} for different values of fractional derivative order

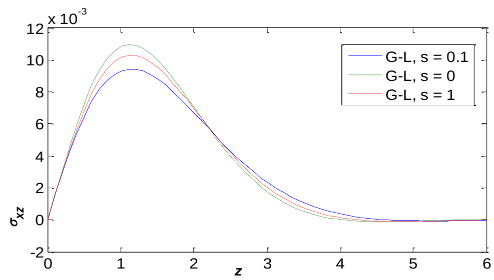

Fig. 10 Variation of stress component σ_{xz} for different values of fractional derivative order s

negative values. It increases in the range $0 \le z \le 8$. The existence of the gravitational field increase values of u. Fig. 6 depicts that the variation of vertical displacement distribution w starts with positive values. In the existence of the gravity field, values of w increase and then nearly constant. However, in the absence of the gravity field, it reaches its maximum values in the range $0 \le z \le 0.7$ and then decreases in the range $0.7 \le z \le 8$. The existance of the gravitational field decrease values of w. Fig. 7 depicts that the variation of stress component σ_{xz} begins from a zero value at $z = 0$. The values of σ_{xz} attains its maximum values in the range $0 \le z \le 1$, and then decreases. In the existence of the gravitational field, the values of σ_{xz} decrease.

Figs. 8-10 display the vertical displacement w and stress component σ_{xz}, σ_{zz} distributions for different values of fractional derivative order $s\,(s = 0, 0.1, 1)$. Fig. 8 depicts that the variation of vertical displacement distribution w maximized reaches its maximum values and then decreases.

The increasing of the value of fractional derivative order s lead to increase values of w. It is clear from Fig. 9 that the values of σ_{zz} minimized reach their minimum values then increase and again decrease. The increasing of the value of fractional derivative order s lead to decrease values of σ_{zz}. Fig. 10 depicts that the variations of stress component σ_{xz} begin from a zero value at $z = 0$. The values of σ_{xz} attain its maximum values in the range $0 \le z \le 1$, and then decrease. The increasing of the value of fractional derivative order s leads to decrease values of σ_{xz}.

6. Conclusions

We have examined the two-dimensional dynamic response of a nonlocal thermoelastic solid to a moving heat source within the frameworks of the Lord-Shulman (L-S) and Green-Lindsay (G-L) theories. We draw the following findings from the conversations that were held:

- As shown by Figs. 1-3, the nonlocal parameter significantly influences how physical variables vary.
- As seen in Figs. 4-6, the gravitational field is a significant factor in the variation of physical variables.
- It is evident that the variation of physical variables is significantly influenced by the fractional derivative order.
- Because thermal relaxation times occur, the two theories' outcomes differ from one another.
- This observation demonstrates that generalized thermoelasticity theory is, in fact, what the Green-Lindsay (G-L) and Lord-Shulman (L-S) theories are.
- The problem has been theoretically solved through the use of normal mode analysis. That applies to many different hydrodynamics problems.

This process is still applicable when an elastic solid is substituted for a nonlocal elastic one.

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