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On the thermal buckling response of FG Beams using a logarithmic HSDT and Ritz method

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Abstract. This paper presents a logarithmic shear deformation theory to study the thermal buckling response of power-law FG one-dimensional structures in thermal conditions with different boundary conditions. It is assumed that the functionally graded material and thermal properties are supposed to vary smoothly according to a contentious function across the vertical direction of the beams. A P-FG type function is employed to describe the volume fraction of material and thermal properties of the graded (1D) beam. The Ritz model is employed to solve the thermal buckling problems in immovable boundary conditions. The outcomes of the stability analysis of FG beams with temperature-dependent and independent properties are presented. The effects of the thermal loading are considered with three forms of rising: nonlinear, linear and uniform. Numerical results are obtained employing the present logarithmic theory and are verified by comparisons with the other models to check the accuracy of the developed theory. A parametric study was conducted to investigate the effects of various parameters on the critical thermal stability of P-FG beams. These parameters included support type, temperature fields, material distributions, side-to-thickness ratios, and temperature dependency.

Keywords: logarithmic shear deformation theory; P-FG beams; Ritz model; thermal stability

1. Introduction

A novel class of composite materials, designated as FGMs (Functionally graded materials), has attracted considerable attention in the scientific community in recent years (Koizumi 1993, Koizumi 1997, Delfosse 1998). The FGM concept was first proposed in 1984 by Japanese scientists in Sendai, Japan, as an ultra-high temperature resistant material. An inhomogeneous ceramic-metal composite material in which the properties change gradually and smoothly in a continuous manner from point to point is considered to be a typical continuously varying FGM. One of the advantages of FGMs is the elimination of the interfacial problem found in conventional composite materials, which results in a smooth stress variation (Li *et al.* 2008). In recent years, FGM has experienced considerable global growth following the development of significant research for general use as structural elements. This research has been conducted by numerous researchers to study the static, dynamic and stability behaviour of structures (Yaylacı 2016, Yaylacı *et al*. 2021, Yaylacı *et al.* 2022a, Öner *et al*. 2022, Yaylaci 2022, Cuong-Le *et al.* 2020, 2022, Akbas 2022, Chinnapandi *et al.* 2022, Alimoradzadeh and Akbas 2022, Du *et al.* 2022, Huang *et al.* 2022, Liu *et al.* 2022, Polat and Kaya 2022, Zhang and Ko 2022, Hagos *et al*. 2022, Zhu *et al*. 2022, Adıyaman *et al.* 2023, Özdemir and Yaylaci 2023, Turan *et al*. 2023, Yaylacı *et al.* 2023, Mohamed *et al.* 2024, Karamanli *et al*. 2024a, b). Functionally graded materials (FGM) are believed to be capable of withstanding extreme thermal environments. The exposure of this range of materials (FGM) to thermal conditions may lead to the thermal buckling phenomen of the material (FGM). A structural design point of view is required to correctly and accurately understand the thermal stability response of this advanced material (FGM) under thermal conditions.

Many scientific works have focused on stability problems (buckling) of one-dimensional FGM structures (beams) under axial mechanical loading (Yang and Chen 2008, Nguyen *et al.* 2013, Li and Batra 2013, Li *et al.* 2015, Aydogdu 2008, Shahba *et al.* 2011, Shahba and Rajasekaran 2012, Rychlewska 2014, Torki and Reddy 2016, Shvartsman and Majak 2016, Huang *et al*. 2016, Nguyen *et*

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al. 2016a, Thai *et al.* 2018). Studies related to thermal buckling problems of FGM beam have received excellent attention in the literature. Sun *et al*. (2016a) investigated the thermomechanical buckling and post-buckling of P-FG one dimensional structures (beams) resting on a nonlinear two-parameters elastic foundation. The numerical analysis is performed using Timoshenko's model and the shooting method. Also, the temperature variation across the thickness is obtained numerically by resolving the 1D with a heat conduction equation. Zhang *et al*. (2019) investigated the thermal stability of a ceramic-metal FG elastoplastic beam under temperature rise (transversely non-uniform). Elastoplastic material properties are simulated via a linear hybrid strain-hardening model. Based on classical theory (Euler-Bernouli) and shear deformation theories (Timoshenko's and HSDTs), She *et al*. (2017) examined the critical buckling temperature of buckling and post-buckling of clamped temperature dependent FG-beam under uniform temperature raising. The two-step perturbation method was used to determine the critical loads of the buckling and post-buckling paths. Also, Babaei *et al*. (2020) studied the post-buckling response due to the thermal environment of inhomogeneous position/temperature dependent clamped and clamped-rolling FG-beam is investigated with the help of the Third Order Theory (TSDT). The microvoids are assumed to vary uniformly varying in the thickness direction. An initial deflection is considered. Based on the classical theory (Euler-Bernoulli), Majumdar and Das (2018a) studied the nonlinear thermal stability of the temperature-dependent clamped FG beam under two types (nonlinear and linear) of thermal loading. Using both Timoshenko's model and FE approach, the thermal stability and dynamic behavior of the temperature-dependent FG beam are investigated by Hosseini *et al*. (2018a), and the temperature rise is assumed to be constant (uniform) through the thickness direction. The results indicate the performance of the FG beam is influenced by the temperature effect. The thermal stability of the symmetric and asymmetric FG beam with elastic support under uniformly distributed temperature rising is examined by Hosseini *et al*. (2018a) using Timoshenko's analytical model.

In the majority of published investigations of the thermal stability of FG beams, the differential stability equations have been determined by coupling the transverse and axial deformation. Subsequently, the critical thermal buckling temperature has been derived using several improved shear deformation theories which simplify and solve the system of equations into a set of algebraic eigenvalue equations. In order to accurately predict the mechanical response of the beams, various beam theories have been proposed. These beam theories can be divided into three main groups, namely Euler Beams theory, Timoshenko beams theory and HSDT. It is important to note that the Euler-Bernoulli beam theory is only valid for long (slender) beams. In contrast, for medium short beams, it overestimates the fundamental frequency and critical load of buckling and underestimates the central deflection due to the omission of transverse shear deformation. Timoshenko's theory overcame this problem. To obtain a more accurate prediction of the beam response and to avoid the need to introduce the shear correction factor in Timoshenko's model, several HSDT formulations have been proposed (Reddy 1984, Pai 1995, Panda and Singh 2013, Ahmed *et al.* 2019, Madenci 2019, Vinyas 2020, Hadji 2020, Madenci and Özütok 2020, Touratier 1991, Soldatos 1992, Karama *et al.* 2003, Akavci 2016), in which the transverse shear stress is distributed parabolically through the thickness, ensuring zero at the free surfaces and based on a nonlinear distribution of the in-plane displacements in the thickness direction. Recently, a simple and improved theory of high-order shear deformation (HSDT) has been presented (Rachedi *et al.* 2020, Merzoug *et al*. 2020) for various mechanical responses of FG plates and beams with a reduced number of variables (unknowns) by introducing the undetermined integral variables in the formulations. This model also provides the parabolic transverse stresses without the need for a correction factor. As a result, the shear stress in the top and bottom faces is zero. This theory gives results as accurate as the other theories. The improved HSDT model has been used to analyze the behavior of FG plates (2D) and beams (1D) and this theory has shown significant characteristics compared to the classical beam theory (CBT) and the first order model FSDT for thick and highly flexible FG beams.

This paper aims to improve the logarithmic integral higher order shear deformation theory (L-HSDT) in order to investigate the critical thermal buckling temperature of FG beams with different boundary conditions. The proposed theories include the undetermined integral components and satisfy the equilibrium conditions at the upper and lower surfaces of the plate without the use of shear correction factors. The displacement field of the proposed theory is chosen as a function logarithmic of the nonlinear variation of displacements in the plane through the thickness (Mohammadi 2022, Mohammadi *et al*. 2022, Mohammadi 2021, Mohammadi 2023 a, b, Mohammadi *et al.* 2024). A variational approach using Ritz method (Kiani 2017, Kiani and Mirzaei 2018) is applied to study the thermal stability of P-FG beams. The Ritz method is a methods that has demonstrated considerable efficacy in addressing solid mechanics problems. The temperature dependence is considered in the formulation to calculate the material properties of the structure. The critical buckling temperature is calculated for a uniform (UTR), linear (LTR) and nonlinear (NTR) temperature rise. In order to validate the accuracy of the proposed model, some examples are studied to obtain the results for thermal buckling of different boundary conditions (clamped-clamped, simple-simple and simple-clamped) of FG beams and compared with those of other researchers. A good agreement is obtained. The effects of volume fraction inhomogeneity index, material distribution types, slenderness ratios, temperature-dependent properties and temperature fields on the critical thermal buckling is investigated.

2. Analytical formulation

2.1 Temperature dependent FG beams modeling

The functionally graded beam is considered, which is obtained from ceramics and metals with rectangular cross-section of b × h and the span L, as illustrated in Fig. 1.

The P-FG function (power-law) is used to obtain the ceramic volume fraction as (Avcar 2019)

Table 1 Silicon Nitride, Stainless Steel, Ni, Zirconia, and Alumina coefficients (Temperature-dependent coefficients of Young's modulus E, Thermal expansion coefficient ϑ, Poisson's ratio ν) (Shen and Wang 2014, Reddy and Chin 1998)

Materials	Proprieties	P_0	P_{-1}	P_1	P_2	P_3	P at 300 K
Stainless Steel SUS304	$E(Pa)$	201.04e+9	0	3.079e-4	-6.534e-7	0	207.79e+9
	$\vartheta(1/K)$	12.330e-6	0	8.086e-4	0	0	15.321e-6
	ν	0.3262	0	-2.002e-4	3.797e-7	0	0.318
	$K\ (W/mK)$	15.379	0	-1.264e-3	2.092e-6	-7.223e-10	12.143
Silicon Nitride Si_3N_4	$E(Pa)$	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11	322.27e+9
	$\vartheta(1/K)$	5.8723e-6	0	-095e-4	0	0	7.475e-6
	ν	0.24	0	0	0	0	0.240
	$K\ (W/mK)$	13.723	0	-1.032e-3	5.466e-7	-7.876e-11	10.120
Zirconia ZrO_2	$E(Pa)$	244.27e+9	0	-1.371e-3	1.214e-6	-3.681e-10	168.06e+9
	$\vartheta(1/K)$	12.766e-6	0	-1.491e-3	1.006z-5	-6.778e-11	18.591e-6
	ν	0.2882	0	1.133e-4	0	0	0.298
	$K\ (W/mK)$	1.7	0	1.276e-4	6.648e-8	0	1.775
Alumina Al_2O_3	$E(Pa)$	349.55e+9	0	-3.853e-4	4.027e-7	-1.673e-10	320.24e+9
	$\vartheta(1/K)$	6.8269e-6	0	1.838e-4	0	0	7.203e-6
	ν	0.26	0	0	0	0	0.260
	$K\ (W/mK)$	-14.087	-1123.6	-6.227e-3	0	0	64.989
Ni	$E(Pa)$	223.95e+9	0	-2.794e-4	-3.998e-9	0	127.96e+9
	$\vartheta(1/K)$	9.9209e-6	0	8.705e-4	0	0	12.512e-6
	ν	0.3100	0	0	0	0	0.310

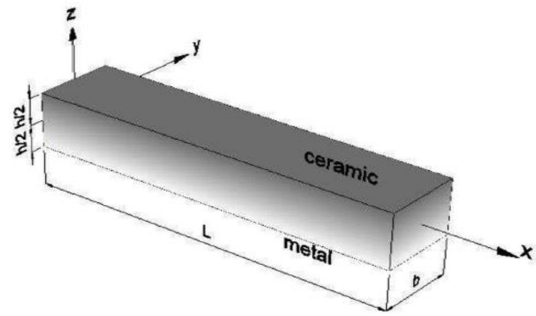

Fig. 1 Dimension of the FG beams with section b × h and length L

$$V=\left(\frac{z}{h}+\frac{1}{2}\right)^{p} \tag{1}$$

where the material parameter ($0\leq p\leq\infty$).

The thermo-elastic effective material properties are computed based on the model of Touloukian, 1967. The equation (nonlinear) of the effective properties is given as a function of the temperature as

$$P(T)=P_0\left(P_{-1}T^{-1}+1+P_1T+P_2T^2+P_3T^3\right) \tag{2}$$

where $P(T)$ denotes the properties of the materials as a function of the temperature (thermal expansion coefficient, Young's modulus and Poisson ratio). P_i are the coefficient of temperature dependent abstracted in Table 1 (Shen and Wang 2014, Reddy and Chin 1998) with (i=0, -1, 1 and 2) for materials components.

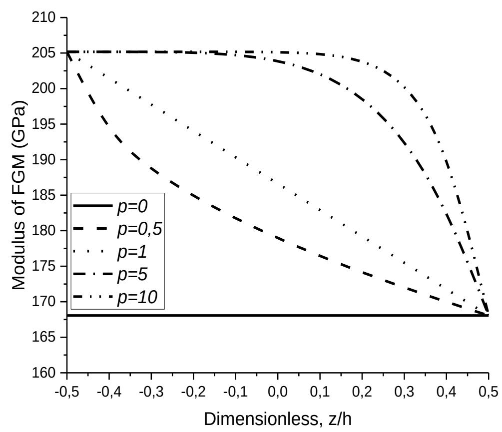

Fig. 2 Elasticity- modulus variation in the thickness direction of graded Nickel / Zirconia (at 300 K)

Fig. 2 illustrates the Young modulus variation in the thickness direction versus the volume fraction index of the Nickel/Zirconia FG-beam at the temperature 300 K. The variation of properties $P(z,T)$ of the structure computed based on the temperature-dependence (Eq. (2)) and the power rule (Eq. (1)) can be expressed as

$$P(z,T)=\left[P_c(T)-P_m(T)\right]V_c(T)+P_m(T) \tag{3}$$

The poison's ratio is evaluated by considering the average of metal/ceramic values at the temperature $T_0=300K$.

2.2 Temperature distribution

In the current investigation, the temperature varies through the thickness according to three conditions: linear, nonlinear, and uniform rise.

2.2.1 Temperature Rise Uniform-type (UTR)

In this type, the temperature rises uniformly, which is increased with ΔT from$T_0 = 300K$ to $T(z)$according to the function $(z) = T_0 + \Delta T$.

2.2.2 Temperature Rise Linear-type (LTR)

For FGM beams, the temperature change is not uniform. The temperature is supposed linearly changed in the direction of the thickness as the following function

$$T(z) = T_m + \Delta T\left(\frac{1}{2} + \frac{z}{h}\right) \tag{4}$$

where the metal/ceramic face temperatures are T_m. and T_c, respectively.

2.2.3 Temperature Rise Non-linear-type (NLTR)

For the temperature rise type-nonlinear, the temperature field $T(z)$ is obtained based on the 1D equation of steady state heat conduction given by (Houde *et al.* 2010, Praveen and Reddy 1998, Bodaghi and Saidi 2011).

$$\frac{d}{dz}\left(K(z)\frac{dT}{dz}\right) = 0 \tag{5a}$$

with the following conditions

$$T\left(\frac{h}{2}\right) = T_c, \qquad T\left(-\frac{h}{2}\right) = T_m \tag{5b}$$

The temperature field of Eq. (5) may be determined by employing polynomial-series as

$$T(z) = T_m + \frac{T_c - T_m}{C}\sum_{i=0}^{N}\frac{(-1)^i\left(\frac{K_{cm}}{K_m}\right)^i\left(\frac{z}{h}+\frac{1}{2}\right)^{ip+1}}{ip+1} \tag{5c}$$

where $C = \sum_{i=0}^{N}\frac{(-1)^i\left(\frac{K_{cm}}{K_m}\right)^i}{ip+1}$ and $K_{cm} = K_c - K_m$. With ,$T(z)$ is obtained with a number of terms N in the polynomial series.

2.3 Kinematics and constitutive equations

The novel kinematic is developed in such a way that the conditions of the nullity of transverse shear stresses will be automatically ensured, and the current model (displacement field) is

$$u(x,z) = u_0(x) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int\theta(x)dx$$
$$w(x,z) = w_0(x) \tag{6}$$

The coefficient k_1 depends on the geometry (Mantari and Granados 2015a, b) and the function $f(z)$ which represents the warping. This last can take various forms, such as polynomial (Reddy 1984, Xiang *et al.* 2011, Darijani and Shahdadi 2015, Nguyen *et al.* 2015a), trigonometric and their inverse (Touratier 1991, Mantari *et al.* 2012b), exponential (Soldatos 1992), hyperbolic (Karama *et al.* 2003, Aydogdu 2009), and can be a combination of various functions (Mantari *et al.* 2012a) in which ensures the zero transverse stresses at the upper/ lower surface of the structure. Also, for the first time, the warping function takes a logarithmic form, as proposed by (Zhu *et al.* 2019). In the current investigation, this logarithmic warping function is used with

$$f(z) = \frac{1}{2}\ln\left(\frac{h-z}{h+z}\right) + \frac{4z}{3h} \tag{7}$$

Where the terms (u, w) presented in Eq. (6) are the components of displacement of a general point (x, z) in the FG beam, (u_0, w_0, θ) are unknown displacements of the mid-plane of the 1D-structure and h is the beam thickness. Using the Eq. (6), the linear strains ε_{ij} are given as

$$\varepsilon_{xx} = \varepsilon_x^0 + zk_x^b + f(z)k_x^s \tag{8}$$

$$\gamma_{xz} = g(z)\gamma_{xz}^0 \tag{9}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_0}{\partial x^2}, \quad k_x^s = k_1\theta, \tag{10}$$

$$\gamma_{xz}^0 = k_1\int\theta\, dx \tag{11}$$

The terms of integrals defined in the Eq. (6) can be obtained as

$$\int\theta(x)\,dx = A'\frac{\partial\theta(x)}{\partial x} \tag{12}$$

where the coefficient A' is determined according to the analytical solution used, in this case via trigonometric-series solution for different boundary conditions. Therefore, A' and k_1 are expressed in Table 2 as follows.
where α is defined in section 3.

The constitutive stress-strain relations of an FG beam are expressed as

$$\sigma_{xx} = Q_{11}\left(\varepsilon_x^0 + zk_x^b + f(z)k_x^s\right) \tag{13}$$

$$\tau_{xz} = Q_{55}\,g(z)\gamma_{xz}^0 \tag{14}$$

where

$$Q_{11} = \frac{E(z,T)}{1-\upsilon^2(z,T)}, \quad Q_{55} = \frac{E(z,T)}{2[1+\upsilon(z,T)]} \tag{15}$$

Table 2 Value of A' and k_1 for different boundary conditions

Boundary conditions	A'	k_1
S-S	$-\frac{1}{\alpha^2}$	α^2
C-S	$-\frac{1}{3\alpha^2}$	$3\alpha^2$
C-C	$-\frac{1}{4\alpha^2}$	$4\alpha^2$

where (σ_x, τ_{yx})are the stresses and (ε_x , γ_{yx}) are the strain components.

2.4 Variational formulation

The strain energy corresponds to the current model and by using Eqs. (8) to (9) and (10) to (11), we can obtain the following expression

$$U = \frac{1}{2}\int_V \left(N_x \varepsilon_x^0 + M_x k_x^b + S_x k_x^s + Q_{xz}\gamma_{xz}^0\right)dx, \tag{16}$$

where

$$\left(N_x, M_x, S_x\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_{xx} dz, \tag{17}$$

$$Q_{xz} = \int_{-h/2}^{h/2} g(z)\tau_{xz} dz, \tag{18}$$

Replacing Eqs. (17) to (18) into Eqs. (16), one can obtain the Eq. (16) in the following form.

$$U = \frac{1}{2}\int_0^L \begin{bmatrix} A(\varepsilon_x^0)^2 + 2B\varepsilon_x^0 k_x^b + 2C\varepsilon_x^0 k_x^s + D(k_x^b)^2 \\ +2Fk_x^b k_x^s + H(k_x^s)^2 + A^s(\gamma_{xz}^0)^2 \end{bmatrix} dz \tag{19}$$

where the terms $\left(A, B, D, C, F, H, A^s\right)$ are the stiffness components of beams expressed by

$$(A,B,D,C,F,H) = \int_{-h/2}^{h/2} \left(1, z, z^2, f(z), zf(z), [f(z)]^2\right) Q_{11}\, dz; \quad A^s = \int_{-h/2}^{h/2} [g(z)]^2 Q_{55}\, dz \tag{20}$$

The Eq. (19) of strain energy can be re-writing in the function of the rotation and displacement by replacing Eqs. (10) and (11) with Eq. (19). Finally, we have

$$U = \frac{1}{2}\int_0^L \begin{bmatrix} A(u_{0,x})^2 - 2Bu_{0,x}w_{0,xx} + 2k_1 C u_{0,x}\theta + D(w_{0,xx})^2 \\ -2k_1 F w_{0,xx}\theta + k_1^2 H\theta^2 + k_1^2 A'^2 A^s(\theta_{,x})^2 \end{bmatrix} dx \tag{21}$$

In this investigation and for a beam embedded in high thermal condition for a long time, the uniform initial temperature T_0=300 K without stress rise to the final one's. Hence, the following thermal stresses occurs as

$$S_{xx}^T = s^T = -(1+n)Q_{11}J(z)\Delta T, \quad t_{xy}^T = t^T = 0 \tag{22}$$

in which $\vartheta(z)$ is the thermal expansion coefficient.

The energy U_T due to thermal stress is given as (Kim 2005)

$$U_T = \frac{1}{2}\int_0^L \left[A^T\left(\left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial w_0}{\partial x}\right)^2\right) - 2B^T\left(\frac{\partial u_0}{\partial x}\right)\left(\frac{\partial^2 w_0}{\partial x^2}\right) + D^T\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + 2C^T\left(\frac{\partial u_0}{\partial x}\right)k_1\theta - 2F^T\left(\frac{\partial^2 w_0}{\partial x^2}\right)k_1\theta + H^T(k_1\theta)^2 \right]dx \tag{23}$$

with

$$\left(A^T, B^T, D^T, C^T, F^T, H^T\right) = b\int_{-h/2}^{h/2} \left(1, z, z^2, f(z), zf(z), [f(z)]^2\right)\sigma_{xx}^T dz \tag{24}$$

The total potential energy of FG beams stability analysis under thermal load is expressed by

$$\Pi = U + V \tag{25}$$

3. Solution method

The analytical solution is obtained based on the Ritz method (Reddy, 1997). The following expression presents the terms of displacement

$$u_0(x,t) = \sum_{j=1}^m a_j \psi_j(x); \quad w_0(x,t) = \sum_{j=1}^m c_j \phi_j(x); \quad \theta(x,t) = \sum_{j=1}^m d_j \xi_j(x); \tag{26}$$

Where the shape functions $\psi_j(x)$, $\varphi_j(x)$ and $\xi_j(x)$ are listed in Table 3 noting that $\alpha = j\pi/L$. And the terms (a_j, c_j, d_j) are coefficients.

The trigonometric shape functions proposed in Table 3 satisfy the simply, clamped and simply-clamped BC summarized in Table 4.In the case where the functions $\psi_j(x)$, $\varphi_j(x)$ and $\xi_j(x)$ which do not satisfy the S-S, C-C and C-S BC, the Lagrangian multipliers method (Mantari and Canales 2016, Nguyen *et al*. 2015b, Nguyen *et al*. 2016b) can be employed.

Based on Lagrange's equations and by replacing the Reiley-Ritz solution with total potential energy expression, we obtain the governing equations as

$$\frac{\partial \Pi}{\partial q_j} - \frac{d}{dt}\frac{\partial \Pi}{\partial \dot{q}_j} = 0 \tag{27}$$

Table 3 Shape functions for each BC

Edges conditions	$\psi_j(x)$	$\varphi_j(x)$	$\zeta_j(x)$
S-S	$\cos\alpha x$	$\sin\alpha x$	$\sin\alpha x$
C-S	$\sin\alpha x(\cos\alpha x-1)$	$\sin\alpha x(\cos\alpha x-1)$	$\cos\alpha x$
C-C	$\sin 2\alpha x$	$\sin^2\alpha x$	$\cos 2\alpha x$

Table 4 Edges conditions of the beams

Boundary conditions	x=0	x=L
S-S	$w_0 = 0$	$w_0 = 0$
C-S	$u_0 = 0, w_0 = 0,$ $q_x = 0, w_{0,x} = 0$	$w_0 = 0$
C-C	$u_0 = 0, w_0 = 0,$ $q_x = 0, w_{0,x} = 0$	$u_0 = 0, w_0 = 0,$ $q_x = 0, w_{0,x} = 0$

With the terms q_j denote the (a_j, c_j, d_j) values.

The analytical solution of the thermal stability analysis can be expressed as

$$([K] + \lambda[K^T])\{\Delta\} = \{0\} \tag{28}$$

where the matrix occurs in Eq. (28) is the stiffness *[K]* one and the coefficient *[K^T]* of the temperature change. The parameter λ of the thermal buckling corresponds to the critical temperature ΔT_{cr}. Δ *is displacement vector. The parameter is obtained by setting the* $\det([K] + \lambda[K^T]) = 0$.

In this investigation, the temperature-dependency (TD and TID) is considered. An iterative procedure is required for TD solution. This procedure can be explained by some steps as follows:

(i) Firstly, obtaining the critical temperature ΔT_{cr} correspond to TID from Eq. (28) at $T = T_0$

(ii) Employing the obtained temperature $T = T_0 + \Delta T_{cr}$ to recalculate the matrix coefficient of Eq. (28) and compute the new critical temperature.

(iii) The same step (II) is repeated until reaching the error tolerance (δ) as

$$\delta = \left| \frac{\Delta T_{cr}^{i+1} - \Delta T_{cr}^{i}}{\Delta T_{cr}^{i}} \right| \leq 0.05\%$$

4. Results and discussion

This part contains the validation and the parametric studies sections. The first one presents the accuracy and efficiency of the present integral solutions for the buckling response of the clamped-clamped, simply-simply, and clamped-simply temperature-dependent FG beam.

A homogenous ceramic beam *(L/h*=10) isconsidered to evaluate the reliability and convergence of the developed analytical solution. The dimensionless thermal buckling load is defined as, $\lambda = \Delta T_{cr} L^2 \vartheta_m / h^2$, where Δ is the temperature change from ambient temperature and ϑ_m is the metal coefficient of the thermal expansion constituent at "T_0"as a function of the number m (series number) for various models considered of the boundary conditions (BC) are tabulated in Table 5. It is remarked that for each BC (C-C, C-S, and S-S) the results converge quickly. Thus, the non-dimensional thermal buckling λ of beams computed via the current solution is compared with those found in the ref. (Inc. 2016) calculated via the commercial software ANSYS based on FE method.

Table 5 The non-dimensional thermal buckling λ of C-C, S-S, and C-S temperature independent beams with slenderness ratio *L/h*=10

Number of series (m)	S-S	C-S	C-C
2	0.5397	1.0772	2.0091
4	0.5397	1.0772	2.0091
6	0.5397	1.0772	2.0091
8	0.5397	1.0772	2.0091
Ref. (Inc 2016)	0.540	1.070	2.004

Table 6 shows the results of the non-dimensional thermal buckling temperature λ for clamped-clamped ZrO_2/Ni FG beams subjected to the UD temperature change with L/h=50. Both TID and TD solutions are shown and compared with those by other investigators. The thermal coefficient of expansion of the material at the bottom surface is taken at T = 300 K. The results of the present L-HSDT theory are compared to the analytical solution and finite element formulation presented by Chen *et al*. 2020, Anandrao *et al.* 2013, respectively. From the comparison, It is clear that an excellent concordance is achieved between the current solution and the literature. It is observed that a substantial difference is observed in critical temperatures obtained using the TID and TD. The thermal buckling results obtained from temperature-independent solution are significantly over-predicted in comparison with those derived from temperature-dependent solution. It is clear that with the increase of *p* , the critical temperatures for clamped-clamped ZrO2/Ni beams decrease.

As verification and examination of the numerical accuracy of the present L-HSDT results of the dimensionless thermal buckling were obtained for homogeneous isotropic beams (p = 0) via the developed model are verified with those obtained by (Anandrao *et al*. 2013) and commercial finite element software ANSYS (Inc. 2016) as shown in Table 7 for various ratios of slenderness "L/h." The agreeability between the present L-HSDT results and the published models is clearly noted. The highest critical temperatures, irrespective of the L/h ratio, are observed for the C-C beam, followed by the C-S beam and the S-S beam. It can be observed that the critical temperatures for these boundary conditions are higher for higher L/h ratios.

Table 8 presents the results and comparisons of the different ceramics/ SUS304 simply supported FG-beam with slenderness ratio L=20h versus the material index p. The temperature independent and dependent beams are subjected to UTR type of temperature rise. The tabulated results of the dimensionless buckling temperature show that the higher results values of thermal buckling λ are always provided from solutions of the TID beam. The increase of the values of p (material index) leads to a reduction in the results values of the thermal buckling temperature.

Except for the ZrO_2/SUS304 power law FG beam, this relation is inversed (the results of the buckling temperature increase with the power law index). Comparing the buckling temperature of Al_2O_3/SUS304, Si_3N_4/SUS304 and ZrO_2/SUS304 beams, values for Al_2O_3/SUS304 is lower.

Table 6 Non-dimensional thermal buckling λ for clamped-clamped ZrO_2/Ni power law FG beams with L/h =50 under uniform temperature rise

	Theory	$p = 0$	0.5	1	10
TID	Present	2.205119218	2.428867869	2.597076675	3.214536993
	Anandrao *et al.* (2013)	2.2056	2.4996	2.6452	3.0870
	Chen *et al.* (2020)	2.2141	2.5093	2.6553	3.0986
TD	Present	1.829558325	1.99637745	2.136515575	3.008649821
	Anandrao *et al.* (2013)	1.8299	2.1056	2.2578	2.8084
	Chen *et al.* (2020)	1.8358	2.1126	2.2653	2.8180

Table 7 Non-dimensional thermal buckling λ of homogeneous beams under UTR at condition TID

BC	C-C			S-S			C-S		
L/h	100	50	10	100	50	10	100	50	10
Present	2.2119	2.2051	2.0091	0.5534	0.5530	0.5397	1.1329	1.1311	1.0772
Anandrao *et al.* (2013)	2.2120	2.2060	2.0090	0.5530	0.5530	0.5400	1.1320	1.1300	1.0710
ANSYS (Inc. 2016)	2.2120	2.2050	2.0040	0.5530	0.5530	0.5400	1.1320	1.1300	1.0700

Table 8 Dimensionless thermal buckling λ of S–S beams subjected to UTR with L/h=20

Material	Theory	TID				TD			
		p=0	*1*	*5*	*10*	*p=0*	*1*	*5*	*10*
Si_3N_4/SUS304	Present	1.3507	0.8667	0.7246	0.6898	1.1866	0.7812	0.6795	0.6502
	Wattanasakulpong *et al.* (2011)	1.3480	0.8760	0.7500	0.7120	1.1850	0.8050	0.6970	0.6640
Al_2O_3/SUS304	Present	1.3793	0.8714	0.7254	0.6906	1.3290	0.8080	0.6854	0.6529
	Wattanasakulpong *et al.* (2011)	1.3760	0.8800	0.7470	0.7080	1.3260	0.8270	0.6980	0.6620
ZrO_2/SUS304	Present	0.5206	0.5661	0.5877	0.5984	0.4171	0.4800	0.5306	0.5504
	Wattanasakulpong *et al.* (2011)	0.5180	0.5650	0.5890	0.6000	0.4160	0.4810	0.5300	0.5490

Table 9 The values of the $\Delta T_{cr}(K)$ of the clamped Si_3N_4/SUS304 FG beams with $L/h = 25$ under UTR

	Theory	$p = 0$	0.5	1	2	5	10	∞
TID	Present	692.8489	510.0635	458.7973	423.5428	394.3212	376.1062	338.0357
	Chen *et al.* (2019)	692.8500	510.0000	458.7800	423.6200	394.4900	376.2200	338.0200
	Esfahani *et al.* (2013)	692.7000	509.8900	458.6800	423.5300	394.3900	376.1400	337.9400
	Sun *et al.* (2016)	693.0500	510.1400	458.9100	423.7500	394.6100	376.3400	338.1200
	Hosseini *et al.* (2018b)	698.9500	–	461.8200	–	–	376.7000	–
TD	Present	508.2858	399.6238	367.4054	345.1588	326.3320	313.5572	285.1305
	Chen *et al.* (2019)	508.2600	399.5700	367.3900	345.2200	326.4600	313.6500	285.1200
	She *et al.* (2017)	508.1700	399.5000	367.3200	345.1500	326.4000	313.5800	285.0600

The thermal buckling temperature of the clamped Si_3N_4/SUS304 FG beam is presented in Table 9. The results are computed for a beam under UTR temperature rise and a slenderness ratio L/h = 25 for various values of index p. From the performed comparisons, it is clear that an excellent agreement is obtained between the current model and Ref. (Chen *et al*. 2019). It is also confirmed that the present TID solutions and temperature $\Delta T_{cr}(K)$ agree well with those given via the Refs (Chen *et al.* 2019, Esfahani *et al.* 2013, Sun *et al*. 2016b, Hosseini *et al*. 2018b). This table also shows that the TD solution gives significantly lower values than the TID solution, highlighting the importance of temperature dependence in FG beams.

It is important to note that the critical buckling temperature decreases as the power law index (p) increases. This relationship can be explained by the fact that as the power law index (p) increases, the critical buckling temperature becomes increasingly sensitive to variations in

Table 10 Comparison of buckling temperature $\Delta T_{cr}(K)$ for C–C of Si_3N_4/SUS304 beams with $L/h = 40$ under NTR

	Source	$p = 0$	0.5	1	2	5	10	∞
TID	Present	536.6508	422.7954	380.2322	346.0607	313.5091	293.7605	256.7068
	Chen *et al*. (2019)	534.7200	421.5600	379.2000	345.1500	312.6300	292.8700	255.8400
	Esfahani *et al*. (2013)	536.6200	422.1800	379.4700	345.2200	312.6400	292.8700	255.8100
	Majumdar and Das (2018b)	528.4000	419.2200	378.2200	345.1600	325.4900	305.4900	–
TD	Present	412.3013	377.8966	357.8082	337.1273	310.1282	291.6214	255.7068
	Chen *et al*. (2019)	411.3300	377.7700	357.8600	337.0100	310.1300	291.3600	255.8400
	Esfahani *et al*. (2013)	412.2400	377.9600	357.9400	337.0300	310.1200	291.3500	255.8100
	Majumdar and Das (2018b)	441.0000	381.5000	352.5800	335.0900	302.0000	281.0800	–

the volume fraction distribution. Thus, an increase in the index p leads to a more pronounced decrease in the critical buckling temperature. However, there is one notable exception: in the case of ZrO_2/SUS304 beams with a functional gradient following a power law, the relationship is reversed. For this specific system, the critical buckling temperature increases as the power law index (p) increases. This a typical trend is probably linked to the particular properties of the ZrO_2 and SUS304 materials and their microstructural characteristics. In conclusion, the general trend is for the critical buckling temperature to decrease as the power law index (p) increases, with the exception of the special case of ZrO_2/SUS304 beams, where the opposite behaviour is observed.

Table 10 studies the slender beam with (L/h = 40) under NTR temperature. The results of the critical thermal buckling are obtained from current TID and TD solutions. From the table and for all values of the inhomogeneity of the material p, it is confirmed again that the present model is in excellent agreement with the other solutions published in the refs. (Chen *et al*. 2019, Esfahani *et al*. 2013, Majumdar and Das 2018b).

Fig. 3 shows the critical buckling temperature variation (λ) of the Si_3N_4/SUS304 versus the slenderness ratio (L/h). The FG beam has an index material p=0.3, and the temperature rise is uniform. A clamped, simply, and clamped/simply boundary conditions are considered. We can note that non-dimensional thermal buckling (λ) increases with ratio a/h. The result of the critical buckling temperature of the simply supported FG beam is less than that of the clamped-clamped and clamped-simple FG beam. However, the values of the TID and TD solutions thermal load converge when the side-to-thickness ratio L/h increase and the difference increases. It is also observed that the TD solution has an important role in the critical buckling temperature for the thick beam than TID Solution. The maximum values of the results are for a minimum for the simply-simply beam, and the maximum are for the clamped-clamped one.

Fig. 4 illustrates the impact of the inhomogeneity index on the critical buckling temperature (λ) of the Si_3N_4/SUS3041D-FG-beam clamped in the edges and has the dimension L = 25h. It can be noted that the results of the

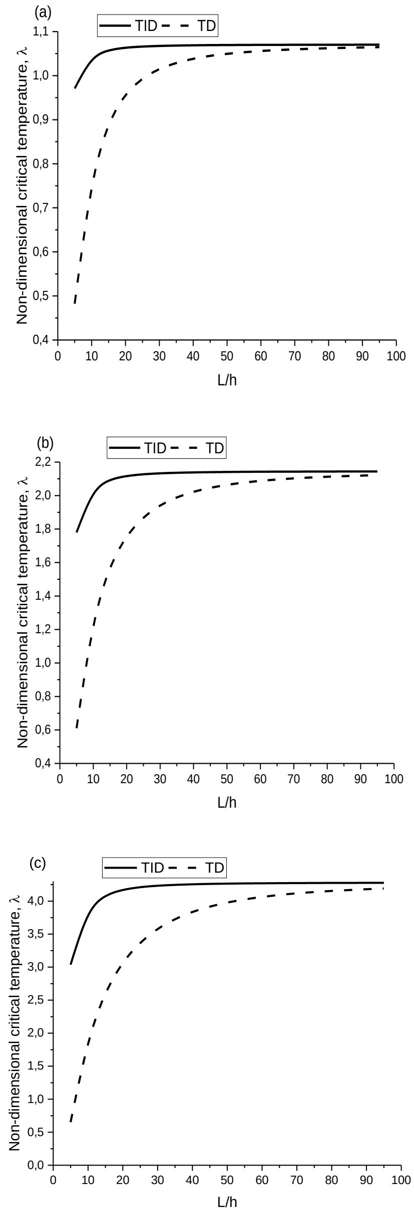

Fig. 3 Dimensionless thermal buckling temperature (λ) of beams made of Si3N4/SUS304 under UTR with p=0.3: (a) S-S, (b) C-S and (c) C-C

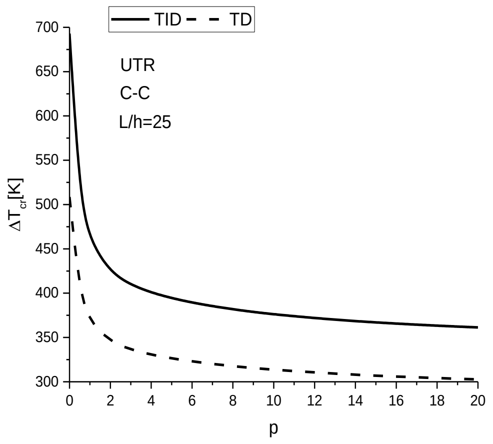

Fig. 4 Impact of the parameter of the material p and temperature dependency on the values of ΔT_{cr}

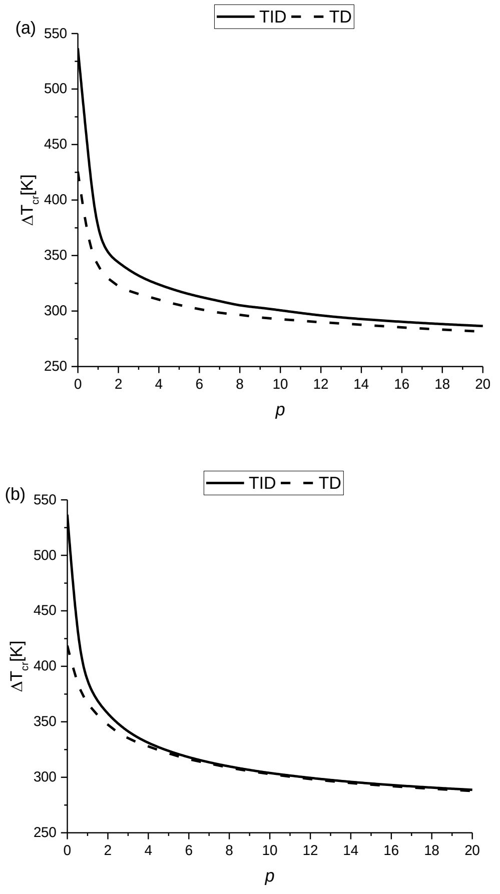

Fig. 5 Impact of index p on ΔT_{cr} for C–C of SUS304/Si_3N_4 beams under (a) LTR and (b) NLTR cases with $L/h = 40$

critical buckling temperature decrease continuously as a function of the parameter *p*. A decrement in the critical buckling temperature can be clearly observed when the parameter p increases. Also, we can conclude that the temperature independence overestimates the temperature result values. It is also clear that the divergence between TD and TID models is more pronounced at higher temperatures.

Fig. 5 presents the effects of the inhomogeneity index on the buckling temperature C–C of Si_3N_4/SUS304 beams using the current shear deformation theory. In this figure, two cases of thermal loadings, namely TID and TD, are compared to each other. The plotted graphs show an inverse relation between the buckling temperature and the index p.

The results of the TID and TD converge when the index p increases, which is valid for each linear/nonlinear temperature rise case.

5. Conclusions

In this research investigation, an analytical solution for the thermal stability of various models of beams with temperature-dependent and independent is proposed and developed on the basis of a novel logarithmic shear deformation theory (L-HSDT). This solution is based on the Ritz solution model to analyze the thermal stability of FGM 1D-structures (beams). The model of the power law function (P-FGM) is considered for the temperatures independent and dependent solutions of the beam under uniform, linear and nonlinear temperature rises, respectively, for simply, clamped, and both (C-S) supports. The Obtained results were presented in figures and tables for FG beams to include the effect of thermal conditions with various boundary conditions on the thermal buckling response of the material and compared with references. These demonstrate the accuracy of the present solution. The following conclusions may be drawn from the present analysis:

1) The results of thermal buckling obtained from an independent temperature solution are significantly overestimated in comparison to those obtained from a temperature-dependent solution, particularly for thick FG beams.
2) The critical temperature change decreases as the power law index increases, with the exception of the ZrO_2/SUS304 power law FG beam, where the relation is inversed.
3) The highest critical temperatures, irrespective of the L/h ratio, are observed for the C-C beam, followed by the C-S beam and the S-S beam. It can be observed that the critical temperatures for these boundary conditions are higher for higher L/h ratios.
4) A comparison of the buckling temperature of Al_2O_3/SUS304, Si_3N_4/SUS304 and ZrO_2/SUS304 beams subjected to a uniform temperature rise reveals that both TID and TD solutions indicate that the values for Al_2O_3/SUS304 are lower.

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