

Maximum shear modulus of rigid-soft mixtures subjected to overconsolidation stress history

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Abstract. The use of sand-tire chip mixtures in construction industry is a sustainable and environmentally friendly approach that addresses both waste tire disposal and soil improvement needs. However, the addition of tire chip particles to natural soils decreases maximum shear modulus (G_{max}), but increases compressibility, which can be potential drawbacks. This study examines the effect of overconsolidation stress history on the maximum shear modulus (G_{max}) of rigid-soft mixtures with varying size ratios (SR) and tire chip contents (TC) by measuring the wave velocity through a 1-D compression test during loading and unloading. The results demonstrate that the G_{max} of tested mixtures in the normally consolidated state increased with increasing SR and decreasing TC . However, the tested mixtures with a smaller SR exhibited a greater increase in G_{max} during unloading because of the active pore-filling behavior of the smaller rubber particles and the consequent increased connectivity between sand particles. The SR -dependent impact of the overconsolidation stress history on G_{max} was verified using the ratio between the swelling and compression indices. Most importantly, this study reveals that the excessive settlement and lower G_{max} of rigid-soft mixtures can be overcome by introducing an overconsolidated state in sand-tire chip mixtures with low TC .

Keywords: compressibility; maximum shear modulus; overconsolidation; rigid-soft mixtures; tire chip

1. Introduction

Approximately 4.4 million tons of scrap tires are generated annually in the United States, 3.4 million tons in the European Union, 5.7 million tons in the rest of the world, and a cumulative total of 5000 million tons are expected to be discarded worldwide in 2030 (Liu *et al.* 2018, Yadav and Tiwari 2019). Because scrap tires are commonly disposed of in landfills or burned off, investigations on the beneficial utilization of scrap tires to achieve environmental sustainability have attracted the attention of many researchers. The inherent material properties of scrap tires or tire chips, such as their high durability, high permeability, and low bulk density, allow them to be mixed with soil to enhance the engineering properties of the mixtures (Tafreshi and Norouzi 2015, Lopera Perez *et al.* 2016, Arefnia *et al.* 2020, Li *et al.* 2020, Lajevardi and Enamia 2021), leading to the possible utilization of sand-tire chip (or rubber) mixtures as construction materials.

The use of sand-rubber (or sand-tire chip) mixtures in geotechnical engineering has several advantages, including improved mechanical properties (i.e., an increase in the friction angle owing to an increase in interlocking) (Terzi *et al.* 2015, Arachchige *et al.* 2022), reduced environmental

impact (Humphrey and Manion 1992, Edinçliler *et al.* 2010), improved structural performance (i.e., a reduction in vibration owing to the high damping capacity of rubber particles) (Hing-Ho *et al.* 2012, Brunet *et al.* 2016), and economic benefits (Tasalloti *et al.* 2021). Therefore, this composite material has become increasingly popular in the construction industry. However, previous studies have generally reported that sand-rubber mixtures exhibit a smaller shear wave velocity or maximum shear modulus than pure sand because of the very low elastic stiffness of rubber (Feng and Sutter 2000, Lee *et al.* 2010, Anastasiadis *et al.* 2012, Ghazavi and Kavandi 2022). Additionally, sand-rubber mixtures show significant compressibility compared with pure sand (Hataf and Rahimi 2006, Sheikh *et al.* 2013, Ryu *et al.* 2022). Thus, the decrease in shear wave velocity (or maximum shear modulus) and the increase in compressibility of sand-tire chip mixtures compared to unmixed natural geomaterials may be the biggest obstacles to overcome for the active reuse of scrap tires as beneficial construction materials.

Previous studies (Ryu *et al.* 2022, Won *et al.* 2023) demonstrated that the rubber (tire chip) content and size ratio between rubber and sand particles determine the small strain stiffness and compressibility of sand-rubber mixtures. Because the rubber content and size ratio determine the connectivity between rigid sand particles (Evans and Valdes 2011, Pistolas *et al.* 2017, Ryu *et al.* 2022), the change in shear wave velocity (or maximum shear modulus) and compression index of sand-rubber mixtures can be explained based on the change in connectivity between sand particles: 1) with increasing rubber or tire chip fraction, the

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connectivity between sand particles decreases; therefore, the small strain stiffness decreases, but the compression index increases; 2) with decreasing size ratio (smaller rubber and larger sand sizes) at a given rubber content, the connectivity between sand particles decreases; therefore, the small strain stiffness decreases, but the compression index increases.

The stress history controls the strength, stiffness, and deformation behavior of soil deposits and is generally given by the overconsolidation ratio. Compared to normally consolidated soils, overconsolidated soils exhibit distinctive mechanical properties, including reduced compressibility and increased strength and stiffness (Bo and Chu 2006, Choo and Burns 2014). Accordingly, the limitations of sand-rubber mixtures can be overcome by introducing preloading or compaction techniques to induce an overconsolidation state in rigid-soft mixtures (Rao and Dutta 2006, Sheikh *et al.* 2013, Tafreshi and Norouzi 2015, Ryu *et al.* 2022). However, studies evaluating the effect of overconsolidation stress history on the small-strain stiffness of sand-rubber mixtures have rarely been conducted, although the accurate estimation of small-strain stiffness is essential for informed decision-making in geotechnical design and analysis (Wang *et al.* 2021, Cheng *et al.* 2023).

This study aims to explore the effect of overconsolidation stress history on the small strain stiffness (or maximum shear modulus) of rigid-soft mixtures. Sand-tire chip mixtures of three different size ratios ($SR = d_{TC}/d_s = 8.0, 0.8, \text{ and } 0.2$, where d_{TC} median particle size of tire chips and d_s mean particle size of sand) were prepared with varying tire chip contents ($TC = 0, 10, 20, 40, 60, 80, \text{ and } 100\%$). A one-dimensional compression test was conducted, and the shear wave velocity of the mixtures was investigated at each loading and unloading stage. Because this study employed a modified oedometer cell to measure the shear waves, a change in compressibility was also reported.

2. Experimental study

2.1 Materials and sample preparations

Two silica sands (Kyung-In, South Korea) with different median particle sizes (d_{50}) of 1.69 mm (K3 sand) and 0.163 mm (K7 sand) and two tire chips (Samjin Company, South Korea) with different d_{50} of 1.3 mm (T1 tire chip) and 0.3 mm (T2 tire chip) were used to prepare sand-rubber mixtures with different size ratio (SR). Fig. 1 shows the particle size distribution of the unmixed materials used in this study. Table 1 summarizes the physical properties of unmixed materials.

The rigid-soft mixture was prepared by mixing sand and tire chips to satisfy three different SR s, $SR = 8.0$ (K7–T1), 0.8 (K3–T1), and 0.2 (K3–T2). The tire chip content (TC) in each mixture varied by 0, 10, 20, 40, 60, 80, and 100% (by weight) to investigate the effect of TC on the mechanical behavior of rigid-soft mixtures. The weights of each rigid and soft particle corresponding to the target TC at each SR were mixed in a container for at least 30 min to homogenize

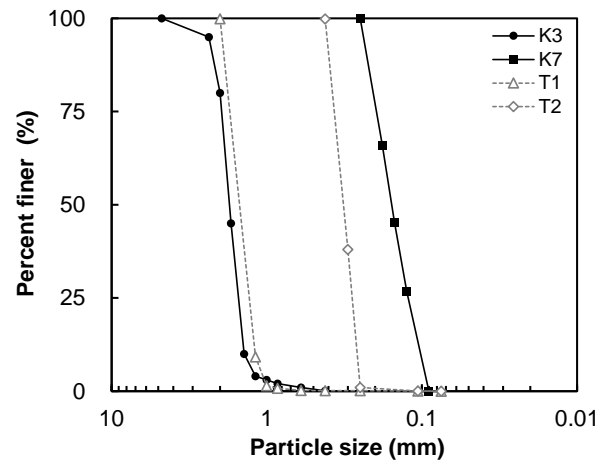


Fig. 1 Particle size distribution of tested unmixed materials

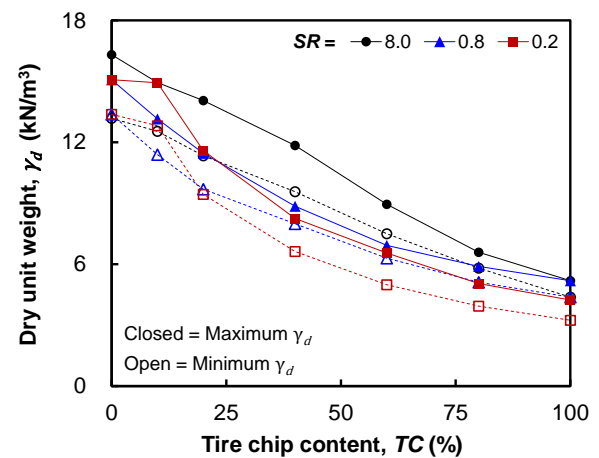


Fig. 2 Maximum and minimum dry unit weights of tested rigid-soft mixtures at various SR s according to TC

the mixtures. The maximum (ASTM D4254) and minimum void ratios (JGS 2008) of rigid-soft mixtures at varying TC s and SR s were measured (Fig. 2), and the samples were prepared at an initial relative density of 50%. Table 2 summarizes the test matrix.

2.2 Test procedure

The mixtures were placed in an oedometer cell with an inner diameter of 100 mm and a height of 65 mm using the dry funnel deposition method. The falling height was maintained close to zero during dry funnel deposition to achieve uniform mixtures without particle segregation (Lade and Yamamuro 1997, Choo and Burns 2015). The vertical stress was loaded to approximately 440 kPa and then unloaded to approximately 6.9 kPa, maintaining a load increment ratio of 1 (Table 2). The bender elements were installed at the top cap and bottom plate of the oedometer cell, and the generated square wave from the signal generator (Keysight 33500 B) was propagated through the rigid-soft mixtures (Fig. 3). The incoming shear wave was detected by an oscilloscope (Keysight DSOX2014A) after passing through a filter-amplifier (Krohn-Hite 3364). The shear wave velocity (V_s) was calculated from the recorded

Table 1 Physical properties of unmixed materials used in this study

Properties	Sand		Tire chip		Method
	K3	K7	T1	T2	
d_{50} (mm)	1.69	0.16	1.30	0.34	ASTM D422
C_u	1.32	1.71	1.27	1.28	
Soil classification	SP	SP	-	-	USCS
G_s	2.66	2.66	1.14	1.14	ASTM D854
e_{max}	0.95	0.98	1.55	2.45	ASTM D4264
e_{min}	0.73	0.60	1.16	1.64	JGS 0161

Note: d_{50} = median particle size; C_u = uniformity coefficient; G_s = specific gravity; e_{min} and e_{max} = minimum and maximum void ratios, respectively

Table 2 Test matrix

Host material	K7 sand	K3 sand	K3 sand
Added material	T1 rubber	T1 rubber	T2 rubber
Size ratio ($SR, d_{rc}/d_s$)	8.0	0.8	0.2
Initial relative density (%)	50%		
Tire chip content (TC in %)	0, 10, 20, 40, 60, 80, 100		
Applied vertical stress (kPa)	6.9 – 440 (load increment ratio = 1)		

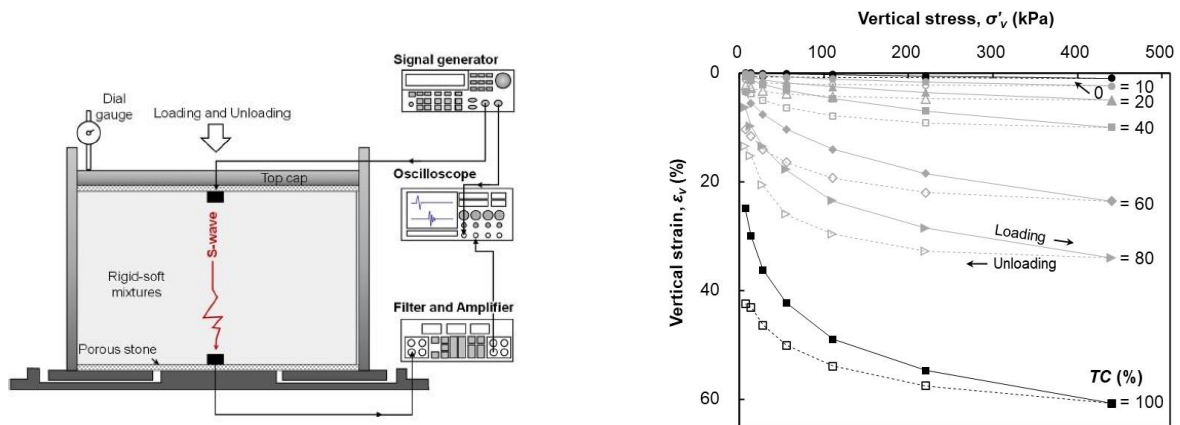


Fig. 3 Test setup for measuring shear waves

travel time and travel distance, considering the specimen settlement (Lee and Santamarina 2005). The maximum shear modulus (G_{max}) of each mixture was calculated using the following Eq. (1)

$$G_{max} = \rho(V_s)^2 \quad (1)$$

where ρ = total density.

3. Results and analyses

3.1 Stress-strain response

Fig. 4(a) shows the variation of vertical (or volumetric) strain according to applied vertical stress (σ'_v) for the tested mixtures with size ratio (SR) = 8.0 at varying tire chip contents (TC). The deformation of rigid-soft mixtures is caused by the rearrangement of particles, which is a function of the interparticle interactions between soft and

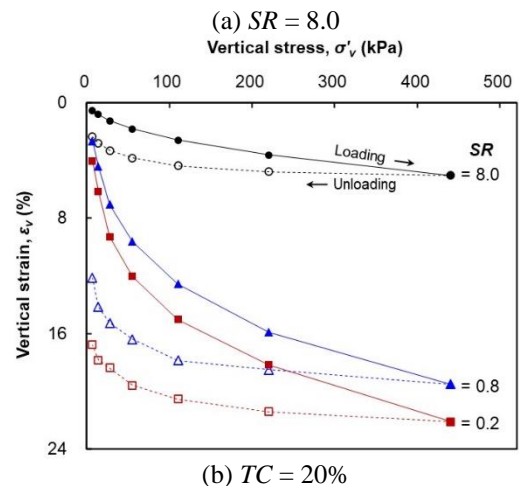


Fig. 4 Stress-strain response of selected rigid-soft mixtures

rigid particles (Ryu *et al.* 2022). The particle rearrangement is accompanied by an interparticle shearing process in which the particle interface properties have a critical

influence (Liu and De Lo 2001). Because the sand-tire chip interface has a lower tangential stiffness than the sand-sand interface, the decrease in sand-to-sand contacts (i.e., connectivity between sand particles) leads to easier particle rearrangements (Edil and Bosscher 1994, Li *et al.* 2019). Additionally, the inclusion of tire chips into the sand matrix resulted in a decrease in the packing density (Fig. 2), leading to a greater possibility of sliding or slip between particles. Consequently, the vertical strain at a given applied vertical stress (σ'_v) increased with increasing TC (Fig. 4(a)).

Fig. 4(b) shows the variation of vertical strain according to σ'_v for the tested mixtures with $TC = 20\%$ at varying SR s. At a fixed TC , an increase in SR increases the connectivity between rigid sand particles (Evans and Valdes 2011, Lopera Perez *et al.* 2016, Ryu *et al.* 2022). Accordingly, the vertical strain at a given σ'_v decreased as SR increases (Fig. 4(b)).

Fig. 4 also shows the stress-strain relationship during the unloading step. Because the total strain in soils is composed of recoverable elastic and irrecoverable plastic strains, the stress-strain relationships during the loading and unloading steps did not match, reflecting the formation of permanent fabric changes during unloading. A comparison between the compressibility during the loading step (normally consolidated state) and the unloading step (overconsolidated state) is presented in the next section.

3.2 Compression index and swelling index

The compression and swell indices (C_c and C_s , respectively) calculated from the gradient of the linear portion of the stress-void ratio curve during the loading and unloading stages of the tested materials with varying size ratios (SR) are shown in Fig. 5 according to the tire chip content (TC). As expected, the C_c and C_s of the tested mixtures increased with TC because of the significant deformability and recoverability of the tire chip particles (Ryu *et al.* 2022). However, the increasing trend of C_c in the tested mixtures with increasing TC varied according to SR . For the tested mixtures with $SR = 8.0$, the C_c showed exponential growth with TC . In contrast, in the case of mixtures with $SR = 0.8$ and 0.2 , C_c showed logarithmic growth with TC . It is evident that the dominant structural skeleton shifts from the contacts between sand particles to those between sand-to-rubber or rubber-to-rubber contacts with increasing TC . Thus, the exponential increase in C_c with TC reflects a delay in the transition of the structural skeleton. Stated another way, the tested mixture with a higher SR at a low TC shows sand-like behavior because the contacts between sand particles can be preserved. In contrast, in mixtures with low SR , C_c rapidly increased with increasing amounts of tire chip particles, indicating that the contact between sand particles can be easily disrupted by the presence of smaller rubber particles in the pore space between larger sand particles (Mark 1981). This change in connectivity between sand particles according to SR also influenced the magnitude of C_c at a given TC (Fig. 5) though the difference in C_c between the specimen with $SR = 0.8$ (K3 sand–T1 tire chip) and that with $SR = 0.2$ (K3 sand–T2 tire chip) was not clear because of the difference in

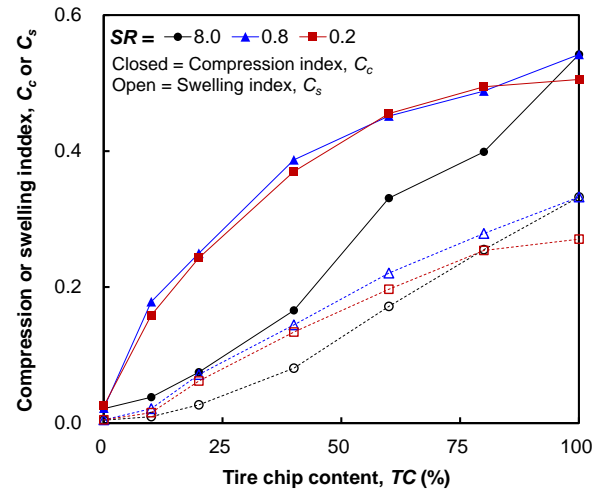


Fig. 5 Measured compression and swell indices of rigid-soft mixtures

C_c of pure tire chips. Note that the calculated C_c and C_s of larger tire chips (i.e., T1) were higher than those of smaller tire chips (i.e., T2) because an increased tire chip size increases the flexibility and curvature of the tire chip (Grayson *et al.* 2013, Tasaloti *et al.* 2021).

The variation in C_s of all tested materials, regardless of SR s, showed exponential growth with increasing TC , indicating that all tested mixtures in the overconsolidated state can exhibit sand-like behavior at low TC . As previously stated, the connectivity between sand particles decreased with decreasing SR ; thus, soft tire chip particles in mixtures with lower SR actively participate in the load-carrying skeleton (Ryu *et al.* 2022). Consequently, smaller rubber particles located in the pore space between larger sand particles can be distorted and squeezed with an increase in vertical stress (Kim and Santamarina 2008, Evans and Valdes 2011, Choo and Burns 2014). Thus, rigid-soft mixtures with smaller SR can exhibit greater increases in sand-to-sand contact with increasing applied stress. Because the volume change during unloading was insignificant owing to the prevalence of elastic strain (Fig. 4), it can be postulated that the contact modes at the maximum vertical stress can be preserved during the unloading step. Consequently, the overconsolidated sand-rubber mixtures at low TC ($TC < 20\%$) exhibited sand-like behavior, regardless of the SR s between the rubber and sand particles.

3.3 Maximum shear modulus, G_{max}

Fig. 6 shows the variation of maximum shear modulus (G_{max}) of tested rigid-soft mixtures with size ratio (SR) = (a) 8.0 and (b) 0.2 at varying tire chip contents (TC) according to applied vertical stress (σ'_v). As previously stated, the direct contact between rigid sand particles can be disrupted by the presence of tire chip particles, leading to a continuous decrease in the connectivity between rigid sand particles with increasing TC . Therefore, the G_{max} of rigid-soft mixtures at a specified σ'_v decreased with TC (Feng and

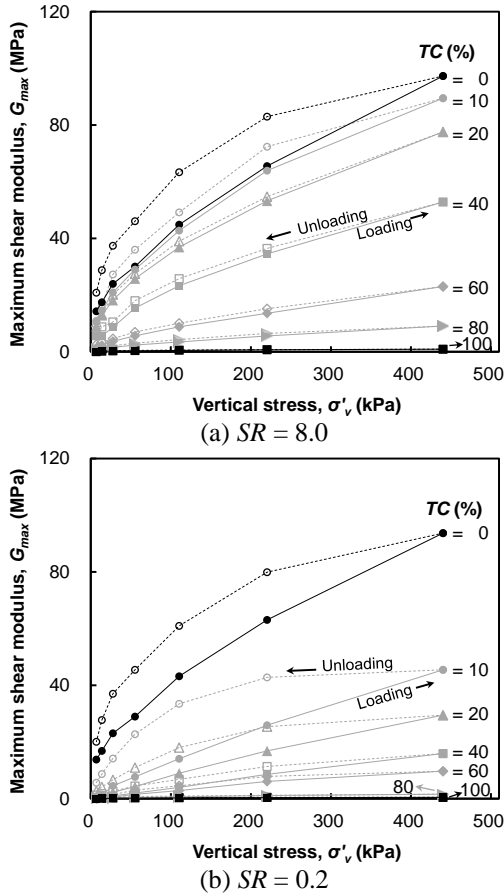


Fig. 6 G_{max} of selected rigid-soft mixtures

Sutter 2000, Kim and Santamarina 2008, Anastasiadis *et al.* 2012). Also, comparing Figs. 6(a) and 6(b) indicates that G_{max} at a specific TC increased with increasing SR , which can be attributed to the increase in sand particle connectivity (Lee *et al.* 2010, Ehsani *et al.* 2015, Pistolas *et al.* 2017).

Because the quality of the interparticle contact (i.e., the coordination number and contact area) increased with increasing σ'_v , the determined G_{max} increased with σ'_v (Fig. 6). Additionally, the larger value of G_{max} during the unloading step than that during loading indicates that lateral stress locking and permanent fabric changes caused the mixtures to become stiffer (Santamarina *et al.* 2001, Choo and Burns 2014). Note that the tested mixture with a smaller SR showed a greater increase in G_{max} during the unloading step (Fig. 6(b)), whereas the difference in G_{max} during the loading and unloading steps for the mixtures with $SR = 8.0$ was insignificant (Fig. 6(a)). Thus, Fig. 6 demonstrates the SR -dependent impact of the overconsolidation stress history on G_{max} . A detailed comparison between the G_{max} during the loading step (normally consolidated state) and unloading step (overconsolidated state) is presented in the next section.

3.4 A-factor and n -exponent in G_{max} estimating equation

The maximum shear modulus (G_{max}) of soils can be incorporated into the power function of the applied vertical

effective stress (σ'_v), as summarized in Eq. (2) (Santamarina *et al.* 2001)

$$G_{max} = A \left(\frac{\sigma'_v}{1 \text{ kPa}} \right)^n \quad (2)$$

where $A = G_{max}$ at $\sigma'_v = 1$ kPa (experimentally determined); and $n =$ stress exponent (experimentally determined).

Fig. 7 summarizes the variation in the A -factor and n -exponent of all tested mixtures during loading and unloading. Note the interparticle contact stiffness between the sand particles is greater than that of the sand-to-rubber contact or rubber-to-rubber contact. Because the A -factor in Equation (2) describes the initial stiffness of soils, the A -factor of the tested rigid-soft mixtures was determined by the connectivity between sand particles. Fig. 7(a) clearly demonstrates that the A -factor during the loading step decreased with increasing tire chip content (TC) because of the decrease in the number of sand-to-sand contacts. Additionally, Fig. 7(a) highlights the effect of size ratio (SR) on the variation of the A -factor during loading according to TC : (1) in the case where the $SR > 1$ (i.e., tested mixture with $SR = 8.0$), A -factor decreased gradually with increasing TC when $TC \leq 20\%$, demonstrating that the sand-rubber mixtures with larger SR s at low TC exhibited sand-like behavior; (2) in the case where the $SR \leq 1$ (i.e., tested mixture with $SR = 0.8$ and 0.2), the A -factor decreased rapidly with increasing TC , demonstrating the active disruption of direct contact between sand particles for these mixtures despite the small TC .

A higher A -factor was observed during unloading than during loading because of the stress history effect (Fig. 7(a)). In particular, mixtures with smaller SR s exhibited greater increases in the A -factor. For example, for the mixture with $SR = 8.0$, the A -factor increased by 78% at $TC = 10\%$, whereas for the mixture with $SR = 0.2$, the A -factor increased by 2,276% at $TC = 10\%$. However, the variation of the A -factor during unloading still showed a similar trend based on TC and SR , with the exception of the tested specimen with $SR = 0.2$ at $TC = 10\%$. Because the smaller tire chip particles in mixtures with $SR = 0.2$, which disrupt the direct contact between sand particles at low σ'_v , can be easily distorted/squeezed during loading at low TC s, the direct contact between sand particles can be developed with increasing applied stress. Thus, the mixture with $SR = 0.2$ showed a very comparable A -factor to that of the mixture with $SR = 8.0$ at $TC = 10\%$ during unloading. Note that this change in the dominant contact mode in rigid-soft mixtures has already been implicated in the variation of the n -exponent, which is discussed in the following.

The n -exponent reflects the sensitivity of the G_{max} of the soils to the applied stress. Thus, if rigid-soft mixtures experience significant changes in the dominant contact mode (i.e., a large increase in the number of sand-to-sand contacts) during loading, a greater n -exponent can be observed (Won *et al.* 2023). Fig. 7(b) clearly demonstrates that the n -exponent increased with increasing TC and decreasing SR during the loading step, indicating that the tested rigid-soft mixtures with smaller SR experienced greater increases in the number of sand-to-sand contacts. When $TC \geq 60\%$, the determined n -exponent started to

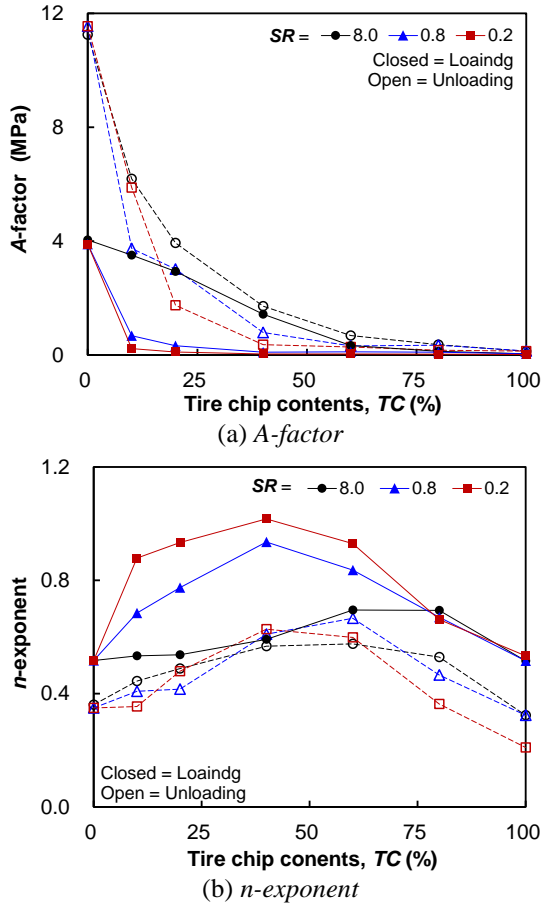


Fig. 7 Variation of parameters during loading and unloading

decrease, approaching the n -exponent value of pure tire chip because the rubber-to-rubber contacts determine the engineering behavior of the rigid-soft mixture at high TC (Lee *et al.* 2010).

A lower n -exponent during unloading than during loading can be observed in Fig. 7(b) because of the prevalence of elastic strain (Choo and Burns 2014). At low TC ($\leq 20\%$), the variation of n -exponent according to TC s and SR s during unloading was insignificant compared to that during loading (Fig. 7(b)), reflecting the limited change in dominant contact mode for these mixtures. This in turn reinforces that the sand-to-sand contacts developed during the loading step can be preserved during the unloading step for mixtures with a low TC . When $TC > 20\%$, the n -exponent during unloading changed consistently with the n -exponent during loading, indicating that the tested rigid-soft mixtures in the overconsolidated state also experienced a change in dominant contact mode at high TC .

4. Discussions

The preceding section demonstrated that the rigid-soft mixture in the overconsolidated state can show a significant increase in the maximum shear modulus (G_{max}) and a decrease in compressibility compared to the normally consolidated state. However, when the tire chip content

(TC) was high, the swelling index was much greater than that of pure sand, and the A -factor in Eq. (2) was much smaller than that of pure sand. Thus, the following section discusses the overconsolidation stress history effect on G_{max} only for mixtures with a low TC .

4.1 Overconsolidation stress history effect on G_{max}

The G_{max} of overconsolidated soils can be expressed as a function of the overconsolidation ratio ($OCR = \sigma'_p / \sigma'_v$, where σ'_p = past maximum vertical effective stress and σ'_v = applied vertical stress)

$$G_{max} = A \left(\frac{\sigma'_v}{1 \text{ kPa}} \right)^n \times OCR^m \quad (3)$$

where m -exponent = experimentally determined fitting parameter (Jovičić and Coop 1997, Atkinson 2000). Because Equation (3) is expressed in terms of σ'_v , the m -exponent in Eq. (3) accounts for both fabric changes and lateral stress locking, which increases the stiffness of the mixtures during the unloading stage (Choo *et al.* 2015). It is known that the m -exponent of pure sand is close to zero and that of clay ranges from 0.15 to 0.5 as a function of plasticity index. Accordingly, the ratio between the G_{max} of the overconsolidated soil ($G_{max}(OC)$, Eq. (3)) and that of the normally consolidated soil ($G_{max}(NC)$, Eq. (2)) can be expressed as a power function of OCR

$$\frac{G_{max}(OC)}{G_{max}(NC)} = OCR^m \quad (4)$$

As the m -exponent in Eq. (4) directly determines the sensitivity of the G_{max} of tested soils to the stress history effect, determining m -exponent is very important. In this context, Choo and Burns (2014) suggested that the m -exponent can be expressed as the difference between the n -exponents during loading (n_l) and unloading (n_u) (Eq. (5)) based on the geometrical trend of $\log(G_{max}) - \log(\sigma'_v)$ and the stress history model of the G_{max} of Hryciw and Thomaan (1993) (derivation of Eq. (5) can be found in Choo and Burns (2014)). Because the variation of n_u showed an insignificant change according to TC and SR owing to the prevalence of elastic strain (Fig. 7(b)), Eq. (5) highlights that soils with greater n_l will have a greater m -exponent and the consequent greater impact of overconsolidation stress history on G_{max} .

$$\frac{G_{max}(OC)}{G_{max}(NC)} = OCR^m = OCR^{n_l - n_u} \quad (5)$$

Fig. 8 shows the variations in the measured m -exponent and n -exponent difference ($= n_l - n_u$) of the tested mixtures with varying SR s according to TC . The similar variation trends of these two parameters reinforced the validity of Eq. (5). Most notably, Fig. 8 reveals that the m -exponent increased with decreasing SR at a given TC , indicating a higher stress history effect for mixtures with a smaller SR . In the case of mixtures with $SR = 0.2$ and 0.8 , the maximum m -exponent can be observed at $TC = 10 - 20\%$ because of the maximum difference in n_l and n_u at this TC , which can be attributed to the significant change in the contact mode

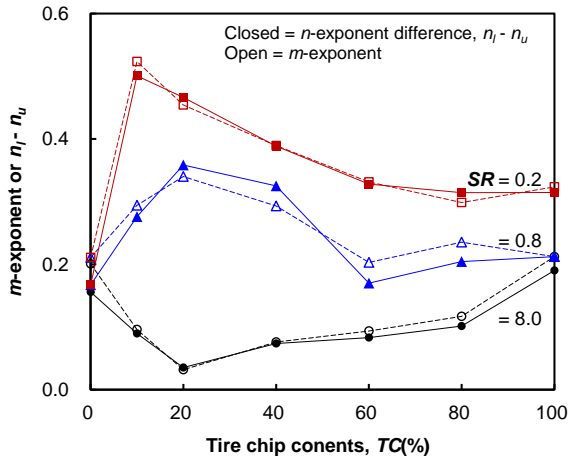


Fig. 8 Variation of m -exponent and n -exponent difference ($n_l - n_u$) of mixtures with various SR s according to TC

between particles (i.e., a huge increase in sand-to-sand connectivity). In contrast, the m -exponent for mixtures with $SR = 8.0$ was even smaller than that of pure sand or pure tire chips, reflecting the negligible effect of stress history on the mixtures with a larger SR because of the comparable variations in n_l and n_u according to TC (Fig. 7(b)).

The theoretical maximum rubber size that can be retained in the pore space between rigid sand particles with a particle size of D ranges from $0.155 \cdot D$ to $0.414 \cdot D$ as a function of packing density (Choo and Burns 2015). Thus, the rubber particles in the mixtures with $SR = 0.2$ at low TC can be easily located in the pore space between sand particles once they are squeezed owing to the applied stress (Fig. 9). This results in a significant increase in the number of sand-to-sand contacts and consequent large increases in the n -exponent during loading (n_l) and the m -exponent. In the case of the mixture with $SR = 0.8$, the sizes of sand and rubber particles were comparable; thus, the applied stress-induced increase in sand-to-sand connectivity was relatively limited even at low TC , leading to a smaller m -exponent than that of the mixture with $SR = 0.2$. In the case of mixtures with $SR = 8.0$ at low TC , a limited increase in sand-to-sand contact during loading was expected because the sand particles in these mixtures already actively participate in the load-carrying skeleton. However, a lower stress history effect than that of pure sand or tire chips was not expected. In this respect, the smaller sand particles in the mixtures with large SR can penetrate into the larger tire chip particles with increasing applied stress (Xiao *et al.* 2019) (Fig. 9). This resulted in less development of direct contact between the sand particles during unloading. Thus, the increase in G_{max} for the tested mixtures with $SR = 8.0$ during unloading, compared to loading, may be limited.

4.2 Evaluation of permanent fabric change

Because the permanent fabric change of soils is one of the main causes of the stress history effect on the G_{max} of soils, the parameter $(1 - C_s/C_c)$, where C_s = swelling index and C_c = compression index) was employed to represent the permanent fabric changes within mixtures (Choo *et al.* 2015)

(Fig. 10). Theoretically, $C_s/C_c = 0$ for purely plastic materials, while $C_s/C_c = 1$ for purely elastic materials, respectively. Therefore, an increase of $(1 - C_s/C_c)$ indicates an increase in permanent fabric change within mixtures, which results in a higher stress history effect. Additionally, it is notable that lateral stress locking, or locked-in lateral stress, which is the other main cause of the stress history effect on G_{max} , is the result of irrecoverable plastic strain (Han *et al.* 2023). Thus, $(1 - C_s/C_c)$ can also reflect the increase in lateral stress during unloading.

Fig.10 shows that $(1 - C_s/C_c)$ generally decreased with increasing TC because the tire chips are highly elastic material. Most importantly, Fig. 10 demonstrates that $(1 - C_s/C_c)$ increased with decreasing SR , indicating that the mixtures with smaller SR exhibited greater permanent fabric change (or lateral stress locking) and, subsequently, greater stress history effect. Thus, Fig. 10 confirms the findings of this study, which indicate the rigid-soft mixtures with smaller SR experienced a greater stress history effect on G_{max} (Fig. 8).

In summary, the results of this study demonstrate that the excessive settlement and lower small strain stiffness of rigid-soft mixtures can be overcome by introducing an overconsolidated state in sand-tire chip mixtures with $TC \leq 10 - 20\%$. In particular, mixtures with a smaller SR (sand size > rubber size) exhibited a greater increase in stiffness during unloading owing to the active pore-filling behavior of the small tire chip particles. Thus, the sand-rubber mixtures used in geotechnical projects should be sufficiently compacted to induce an overconsolidated state in mixed soils.

5. Conclusions

This study evaluated the effect of the overconsolidation stress history on the maximum shear modulus (G_{max}) of sand-rubber mixtures with different size ratios (SR s = 0.2, 0.8 and 8.0) and tire chip contents (TC s = 0, 10, 20, 40, 60, 80 and 100%). The shear wave velocity and settlement were measured during the loading and unloading stages of 1-D compression tests. The key observations of this study are as follows:

- The swelling index (C_s) increases exponentially with TC for all SR s, indicating that the tested mixtures at low TC in the overconsolidated state can exhibit sand-like behavior regardless of SR .
- The A -factor in the G_{max} relationship during loading decreases with increasing TC and decreasing SR because these two factors determine the connectivity between rigid sand particles. However, the mixtures with smaller SR s exhibited a greater increase in A -factor during unloading, indicating that the impact of the overconsolidation stress history on G_{max} is SR -dependent.
- The parameter $(1 - C_s/C_c)$ increases with decreasing SR , indicating that the mixtures with smaller SR experience greater permanent fabric change (or lateral stress locking) and, subsequently, greater stress history effect.
- Examination of the n -exponents in the G_{max} relationship during the loading and unloading stages confirmed the

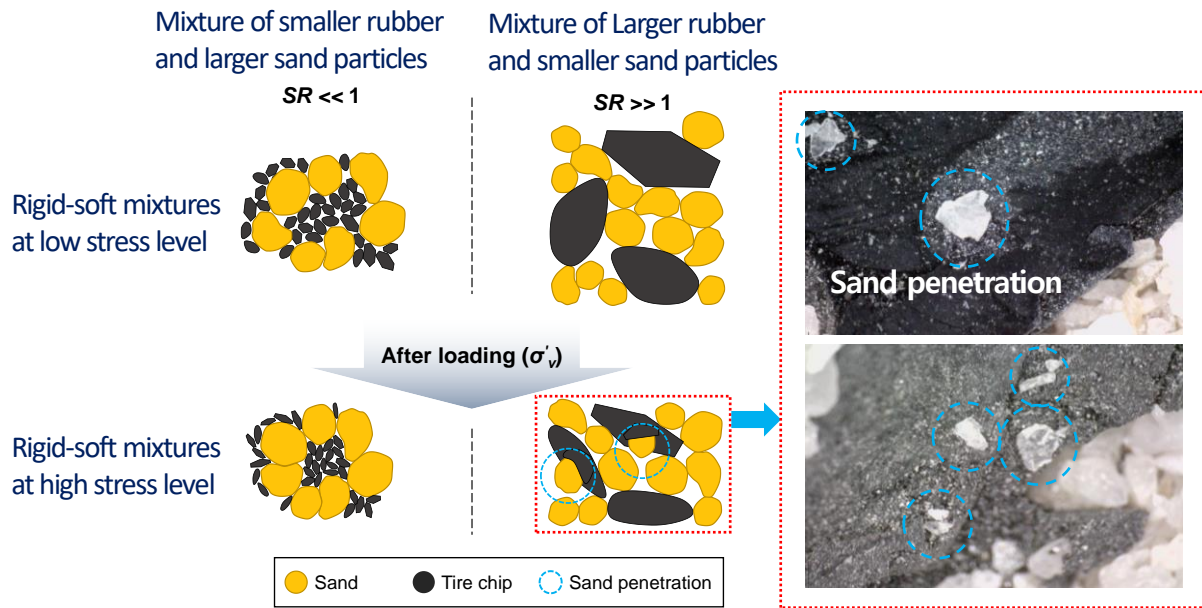


Fig. 9 Schematic diagrams of fabric changes with increasing applied stress and picture of sand particles penetrating into larger rubber particles.

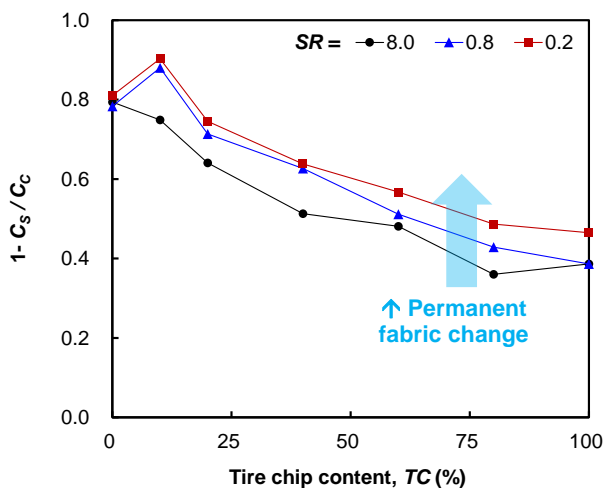


Fig. 10 Variation of $(1 - C_s/C_c)$ according to TC at various SRs

impact of the overconsolidation ratio on G_{max} . Owing to the significant increase in the number of sand-to-sand contacts with increasing applied stress, mixtures with smaller SR exhibited significant stress history effects. In contrast, the tested mixtures with $SR = 8.0$ show a negligible stress history effect on G_{max} because of the limited change in the structural skeleton and possible penetration of sand particles into the larger rubber particles.

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