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# Nonlinear forced vibration of axially moving functionally graded cylindrical shells under hygro-thermal loads

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**Abstract.** Studying the dynamic behavior of axially moving cylindrical shells in hygro-thermal environments has important theoretical and engineering value for aircraft design. Therefore, in this paper, considering hygro-thermal effect, the nonlinear forced vibration of an axially moving cylindrical shell made of functionally graded materials (FGM) is studied. It is assumed that the material properties vary continuously along the thickness and contain pores. The Donnell thin shell theory is used to derive the motion equations of FGM cylindrical shells with hygro-thermal loads. Under the four sides clamped (CCCC) boundary conditions, the Gallekin method and multi-scale method are used for nonlinear analysis. The effects of power law index, porosity coefficient, temperature rise, moisture concentration, axial velocity, prestress, damping and external excitation amplitude on nonlinear forced vibration are explored through parametric research. It can be found that, the changes in temperature and humidity have a significant effect. Increasing in temperature and humidity will cause the resonance position to shift to the left and increase the resonance amplitude.

**Keywords:** axial motion; FGM cylindrical shell; Hygro-thermal load; nonlinear forced vibration; the multi-scale method

## 1. Introduction

The cylindrical shell is widely used in many engineering fields. It is of great importance both in practice and theoretically to investigate the structural vibration and sound radiation of cylindrical shell. FGM is usually composed of ceramic materials and metal materials and often contains pores due to manufacturing problems. Because of its light weight, high strength, high thermal resistance and other excellent mechanical properties, FGM and relevant structures have been widely studied (Abazid *et al.* 2020, Al Mukahal and Sobhy 2021, Belarouci and Fekrar 2021, Beli *et al.* 2022, Belkhodja *et al.* 2022, Hadji and Tounsi 2021, Hashemi-Nejad *et al.* 2022, Sobhy and Al Mukahal 2022, Daikh *et al.* 2021a, 2021b, Esen *et al.* 2022a, 2022b, Hamed *et al.* 2019, Chen *et al.* 2022a, 2022b, Ding *et al.* 2021, 2023a, 2023b, Ding *et al.* 2023a, 2023b, 2023c, 2022a, 2022b, Gan and She 2023, Gan *et al.* 2023, Li *et al.* 2023, She 2021, She and Ding 2023, She and Li 2022, She *et al.* 2021, 2022, Wu and She 2023, Xu and She 2022, 2023, Xu *et al.* 2023a, 2023b, 2023c, 2024, Zhang and She 2022, 2023a, 2023b, 2024, Zhang *et al.* 2021, 2022, 2023a, 2023b, 2023c, 2023d, 2023e, Zhao *et al.* 2022a, 2022b, Ding and She 2023c, Ding *et al.* 2023d, Gan and She 2024, Shan and She 2023, Dong *et al.* 2024, Zhao *et al.* 2017, 2021, Xie *et al.* 2023a, 2023b). Madand *et al.* (2023) studied the ultimate elastic velocity of a FGM porous rotating disk by means of the variational principle. Zghal *et al.* (2022) employed the improved hybrid finite element beam model to explore the impact of porosity on the static bending analysis of FGM beams. Considering the thickness stretch, Zenkour (2020) studied the static problem of rectangular FGM porous thick plates. Taking the uniform, linear and nonlinear temperature changes into account, Barati and his partners (2018) deliberated the electro-thermal-mechanical vibration behavior of FGM plates containing pores. Mirjavadi *et al.* (2018) discussed the vibration of porous FGM beam in a thermal environment. Daikh *et al.* (2021) studied the free vibration of FGM rectangular sandwich plate with Navier's solutions.

Though the variational iteration method, the vibration characteristics and natural frequencies of tapered beams made of FGM were investigated by chen *et al*. (2021). Pan *et al.* (2021) analyzed the probabilistic research results of hybrid mode fracture problems of internal inclined cracks in FGM strips, in which random microstructure properties of FGM were considered. Belalia (2019) discussed the nonlinear free vibration of FGM sandwich plates. Rezaiee-Pajand et al. analyzed the hygro-thermo-elastic nonlinear analysis of FGM porous composite thin and moderately thick shallow panels. Rezaiee-Pajand et al. (2019) discussed the nonlinear vibration analysis of carbon nanotube reinforced composite plane structures. Rezaiee-Pajand and Masoodi (2019) examined FGM shells with large deformations and finite rotations. Masoodi and Arabi (2018) discussed the geometrically nonlinear thermomechanical behavior of shell-like structures.

There are a large number of axially moving structures in engineering fields such as aerospace and high-speed rail. Ali and Hawwab (2023) studied the nonlinear vibration response of axially moving beams using finite difference method. Wang *et al.* (2023) researched the nonlinear dynamic characteristics of FGM axially moving beam using

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interpolation matrix method. Zhang *et al.* (2023) applied Floquet theory to study the influence of material properties with periodic distribution on dynamic stability of axially moving plates. Raj *et al.* (2022) established the Kelvin-Voigt model to explore the nonlinear dynamics of axially moving viscoelastic beams. Wu *et al.* (2022) studied the nonlinear dynamic stability analysis of FGM axially moving rectangular plate. Fasihi *et al.* (2022) used Euler-Bernoulli beam theory (E-EBT) to analyze the nonlinear vibration behaviors of axially moving pipes. Zhang *et al.* (2022) employed E-EBT to discuss the effect of elastic coefficient on the internal resonance behaviors of axially moving beams. Chen *et al.* (2022) used the finite element method to discuss the effects of boundary velocity on the nonlinear stability analysis of axially moving beams. Liu (2022) investigated the nonlinear dynamic behaviors of axially moving composite cantilever beams. Luo and Zhang (2022) employed Runge-Kutta method to illustrate the influence of fluid velocity on the dynamic behaviors of axially moving fluid-conveying pipes. Qiao and Yao (2022) researched the nonlinear dynamic stability of axially moving plates in magnetic field. Shakouri *et al.* (2022) studied the effect of constant velocity on the free vibration behaviors of axially moving cylindrical nano-shells using Galerkin method.

The change of temperature and moisture content in the environment usually causes the change of internal stress of the structure, reduces the stiffness of the structure, and thus causes the structure deformation failure and leads to major accidents. Such as aircraft, ships, submarines and other important equipment are usually directly exposed to the environment, so it attracts a large number of scholars to study the influence of moisture and heat effect on structure. Li and Tang (2022) studied the effect of the variation of hygro-thermal loads in three directions on the nonlinear bending response of a three-direction FGM beam. Tang and Ding (2019) studied the nonlinear hygrothermal dynamics of FGM beams considering uniform and sinusoidal distributed damp and heat load. In humid and thermal environment, Karimiasl *et al.* (2019) investigated the post-buckling behavior of a doubly curved shell using perturbation method. Liu *et al.* (2022) investigated the nonlinear forced vibration of graphene toroidal plates based on the HSDT. Under the action of external loads, Lal and Markad (2021) studied the nonlinear progressive failure of shell panels under hygro-thermal environment. Using SPAM semi-analytic method, Dastjerdi and his co-workers (2020) studied the nonlinear dynamics of an FGM torus and cylindrical shell. Monge and his partners (2022) used the Fourier's heat conduction equation and Fick's moisture diffusion law to study the bending of hyperbolic shells subjected to external loads in hot and humid environments. Rezaiee-Pajand and Masoodi (2022) explore the nonlinear mechanical properties of FGM porous plates, including curved thin plates and medium thickness shallow plates. Penna *et al.* (2021) applied the stress gradient elasticity theory to analyze the dynamic response of porous FG nano-beams in humid and hot environment. Ebrahimi *et al.* (2020) employed the nonlocal elastic theory to study the bending problem of nanobeams under hygro-thermal loading. Akbas (2019) used Newton-Raphson method to analyze the nonlinear behavior of FGM cantilever beam in hygro-thermal environment.

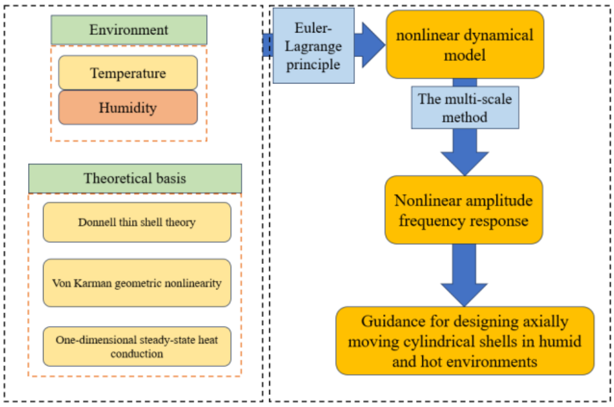


Fig. 1 The flowchart of all steps for this paper

Through reading the literature, it is found that changes in humidity and temperature have a great influence on the strength and stiffness of the material, which can damage the structural properties. Inspired by this, the nonlinear dynamic behavior of FGM cylindrical shell subjected to hygro-thermal loading in uniform temperature field and heat conduction field is studied. By using Galerkin method and multi-scale method, the forced vibration amplitude-frequency response curves of FGM cylindrical shells are obtained. Then the parameter analysis was carried out. The flowchart of all steps for this paper is given in Fig. 1.

## 2. Material parameters

Fig. 2 shows an FGM cylindrical shell. The geometric size of the cylindrical shell and the position of external load are shown in the figure. The FGM cylindrical shell is made of two materials, and we suppose that the volume content of ceramics has the following form (Wang 2018)

$$V_c = \left(\frac{2z+h}{2h}\right)^N, \quad V_m = 1 - V_c \tag{1}$$

where $N$ represents the volume fraction index of the ceramic. Considering the distribution of foams in the material, using the mixing rule, the physical property parameters of FGM can be calculated by (Wang 2018, Zhao *et al.* 2022b)

$$\begin{aligned}
E(z) &= (E_c - E_m)\left(\frac{2z+h}{2h}\right)^N + E_m - \frac{\beta}{2}(E_c + E_m) \\
\alpha(z) &= (\alpha_c - \alpha_m)\left(\frac{2z+h}{2h}\right)^N + \alpha_m - \frac{\beta}{2}(\alpha_c + \alpha_m) \\
\rho(z) &= (\rho_c - \rho_m)\left(\frac{2z+h}{2h}\right)^N + \rho_m - \frac{\beta}{2}(\rho_c + \rho_m) \\
\lambda(z) &= (\lambda_c - \lambda_m)\left(\frac{2z+h}{2h}\right)^N + \lambda_m - \frac{\beta}{2}(\lambda_c + \lambda_m) \\
\chi(z) &= (\chi_c - \chi_m)\left(\frac{2z+h}{2h}\right)^N + \chi_m - \frac{\beta}{2}(\chi_c + \chi_m)
\end{aligned} \tag{2}$$

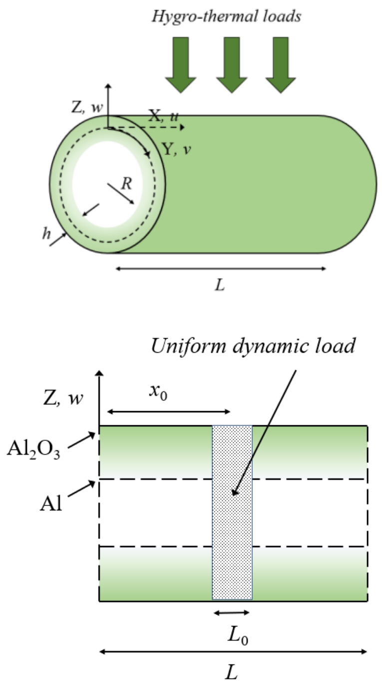



Fig. 2 Schematic diagram

Where [ $E,\nu,\alpha,\rho,\lambda,\chi$ ] respectively represents [Young's modulus, Poisson's ratio, coefficient of thermal expansion, density, thermal conductivity, humidity coefficient]. In this paper, both the uniform temperature field and the heat transfer along the shell thickness direction were considered. Humidity variation and uniform temperature field temperature can be determined by

$$\begin{aligned}\Delta T &= T - T_0\\ \Delta C &= C - C_0\end{aligned} \tag{3}$$

here, T and C are denoted as the current temperature and humidity, $T_0$ = 300K and $C_0$ = 0. For heat conduction, the temperature field is given by (Zhang and She 2023b)

$$T(z) = T_m + (T_c - T_m)\zeta(z) \tag{4}$$

in which, $T_m$ and $T_c$ are the internal and external surface temperatures of the cylindrical shell, and (Zhang and She 2023b)

$$\zeta(z) = \frac{1}{\vartheta}\left[\begin{aligned}&\left(\frac{2z+h}{2h}\right) - \frac{\lambda_c-\lambda_m}{(N+1)\lambda_m}\left(\frac{2z+h}{2h}\right)^{N+1}\\ &+\frac{(\lambda_c-\lambda_m)^2}{(2N+1)\lambda_m^2}\left(\frac{2z+h}{2h}\right)^{2N+1} - \frac{(\lambda_c-\lambda_m)^3}{(3N+1)\lambda_m^3}\left(\frac{2z+h}{2h}\right)^{3N+1}\\ &+\frac{(\lambda_c-\lambda_m)^4}{(4N+1)\lambda_m^4}\left(\frac{2z+h}{2h}\right)^{4N+1} - \frac{(\lambda_c-\lambda_m)^5}{(5N+1)\lambda_m^5}\left(\frac{2z+h}{2h}\right)^{5N+1}\end{aligned}\right] \tag{5}$$

Here (Zhang and She 2023b)

$$\begin{aligned}\vartheta = 1 &- \frac{\lambda_c-\lambda_m}{(N+1)\lambda_m} + \frac{(\lambda_c-\lambda_m)^2}{(2N+1)\lambda_m^2}\\ &- \frac{(\lambda_c-\lambda_m)^3}{(3N+1)\lambda_m^3} + \frac{(\lambda_c-\lambda_m)^4}{(4N+1)\lambda_m^4} - \frac{(\lambda_c-\lambda_m)^5}{(5N+1)\lambda_m^5}\end{aligned} \tag{6}$$

In addition, in this paper, we assume that Poisson's ratio is constant, i.e., $\nu(z)$ = 0.3..

## 3. Governing equation

Considering only thin shells and ignoring the influence of shear deformation, according to Donnell shell theory, the constitutive relation can be written as

$$\begin{bmatrix}\sigma_x\\ \sigma_\theta\\ \tau_{x\theta}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\cdot\left(\begin{bmatrix}\varepsilon_x(z)\\ \varepsilon_\theta(z)\\ \gamma_{x\theta}(z)\end{bmatrix} - \begin{bmatrix}\alpha(z)\\ \alpha(z)\\ 0\end{bmatrix}\Delta T - \begin{bmatrix}\chi(z)\\ \chi(z)\\ 0\end{bmatrix}\Delta C\right) \tag{7}$$

Therefore, the following internal forces and moments can be obtained

$$\begin{bmatrix}N_x\\ N_\theta\\ N_{x\theta}\\ M_x\\ M_\theta\\ M_{x\theta}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0\\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0\\ 0 & 0 & A_{66} & 0 & 0 & B_{66}\\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0\\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0\\ 0 & 0 & B_{66} & 0 & 0 & D_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x^0\\ \varepsilon_\theta^0\\ \gamma_{x\theta}^0\\ \delta_x\\ \delta_\theta\\ \delta_{x\theta}\end{bmatrix} - \begin{bmatrix}N_x^T + N_x^C\\ N_\theta^T + N_\theta^C\\ 0\\ 0\\ 0\\ 0\end{bmatrix} \tag{8}$$

Herein, $[A_{ij}, B_{ij}, D_{ij}] = \int_{-\frac{h}{2}}^{\frac{h}{2}}[1, z, z^2]Q_{ij}(z,)dz,\ (i, j = 1, 2, 6)$ and

$$\begin{aligned}&[N_x^T, N_\theta^T] = \int_{-h/2}^{h/2}\begin{bmatrix}A_x^T\\ A_\theta^T\end{bmatrix}[1, z]\Delta T dz\\ &[N_x^C, N_\theta^C] = \int_{-h/2}^{h/2}\begin{bmatrix}A_x^C\\ A_\theta^C\end{bmatrix}[1, z]\Delta T dz\\ &\begin{bmatrix}A_x^T\\ A_\theta^T\end{bmatrix} = -\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12} & Q_{22}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\alpha\\ \alpha\end{bmatrix}\\ &\begin{bmatrix}A_x^C\\ A_\theta^C\end{bmatrix} = -\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{12} & Q_{22}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\chi\\ \chi\end{bmatrix}\end{aligned} \tag{9}$$

Based on Euler-Lagrange equation, the nonlinear dynamic equation can be deduced as (Zhang and She 2023b)

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta} = \int_{-h/2}^{h/2}\rho(z)dz\times\left(\frac{\partial^2 u}{\partial t^2} + 2V\frac{\partial^2 u}{\partial x\partial t} + V^2\frac{\partial^2 u}{\partial x^2}\right), \tag{10}$$

$$\begin{aligned}&\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial\theta} + \frac{1}{R}\left(\frac{2\partial M_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial M_\theta}{\partial\theta}\right) =\\ &\int_{-h/2}^{h/2}\rho(z)dz\times\left(\frac{\partial^2 v}{\partial t^2} + 2V\frac{\partial^2 v}{\partial x\partial t} + V^2\frac{\partial^2 v}{\partial x^2}\right),\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial x\partial\theta} + \frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial\theta^2} - \frac{N_\theta}{R} + N_x\frac{\partial^2 w}{\partial x^2}\\ &+\frac{2N_{x\theta}}{R}\frac{\partial^2 w}{\partial x\partial\theta} - P\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2}N_\theta\frac{\partial^2 w}{\partial\theta^2} + q\cos(\Omega t) =\\ &\int_{-h/2}^{h/2}\rho(z)dz\times\left(\frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x\partial t} + V^2\frac{\partial^2 w}{\partial x^2}\right)\\ &+C_t\left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right).\end{aligned} \tag{12}$$

Where, $V$, $C_t$ and $P$ represent the axial movement velocity, damping coefficient and prestress.

## 4. Solution method

Due to the fact that four sides clamped (CCCC) can provide the couples to counteract thermal bending moments, the stretching-bending coupling effect do not exist. For SSSS, the stretching-bending coupling effect exists, which will make the problem more complex. Therefore, in this paper, CCCC will be adopted. For CCCC, the displacement function has the following form (Soedel 2004, Hirano 1988, Zhang and She 2023b).

$$u(x,\theta,t)=\sum_m^{\infty}\sum_n^{\infty}U_{mn}(t)\left(\begin{array}{l}-\sin\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\\+k\sinh\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\end{array}\right)\cos\left(\frac{\pi n\theta}{\gamma}\right) \quad (13)$$

$$v(x,\theta,t)=\sum_m^{\infty}\sum_n^{\infty}V_{mn}(t)\left(\begin{array}{l}-\cos\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\\+k\cosh\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\end{array}\right)\sin\left(\frac{\pi n\theta}{\gamma}\right) \quad (14)$$

$$w(x,\theta,t)=\sum_m^{\infty}\sum_n^{\infty}W_{mn}(t)\left(\begin{array}{l}-\cos\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\\+k\cosh\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\end{array}\right)\cos\left(\frac{\pi n\theta}{\gamma}\right) \quad (15)$$

in which, $U_{min}(t)$, $V_{min}(t)$ and $W_{min}(t)$ are the displacement amplitudes in three directions, and (m, n) represents the models, $k$ and $\tau$ can be determined using the same steps as our previous works (Zang and She 2023b).

Suppose that the load can be expanded as

$$q(x,\theta,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(-\cos\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)+k\cosh\left(\frac{\tau}{R}\left(\frac{L}{2}-x\right)\right)\right)\Upsilon \quad (16)$$

In which, $\Upsilon=\cos\left(\frac{\pi n\theta}{\gamma}\right)\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{4}\right)q$ .

Using Eqs. (13)-(16) and Galerkin method, we can obtain the classical Duffing equation, as (Zhang and She 2023b)

$$\begin{array}{l}\bar{I}_0\frac{d^2W(t)}{dt^2}+\left(\frac{C_tLR\pi}{2}\right)\frac{dW(t)}{dt}+F_1W(t)\\+F_2W^2(t)+F_3W^3(t)=\left(\frac{LR\pi q}{2}\right)\cos(\Omega t)\end{array} \quad (17)$$

In addition, Eq. (17) can be solved by using the multiple scales method, which has the same steps as our previous works (Zhang and She 2023b).

## 5. Numerical analyses

In this chapter, various parameters are studied in detail to explore the nonlinear dynamic characteristics of the FGM cylindrical shell under hygro-thermal loading. First of all, to ensure the validity and credibility of this paper, the frequency parameter $\Omega=\omega R(\rho_1/E_1)$ of a homogeneous cylindrical shell is calculated by using the methods proposed in this paper. And through comparing with (Zhang *et al.* 2000, Loy *et al.* 1997), we can find from Fig. 3 that the results of the three studies are highly in good agreement.

The properties of materials used in this study are shown in Table 1. In this paper, the nonlinear response of homogeneous temperature field and heat conduction is considered. In order to obtain the fundamental frequency, the natural frequencies corresponding to different circular wave numbers are studied in Table 2 under the conditions of ΔT=100 K, P=0, β =0.1, N=1 and ΔC=0.0005. As seen, when m=1 and n=6, the fundamental frequency can be arrived at, this result is also valid for the following analysis.

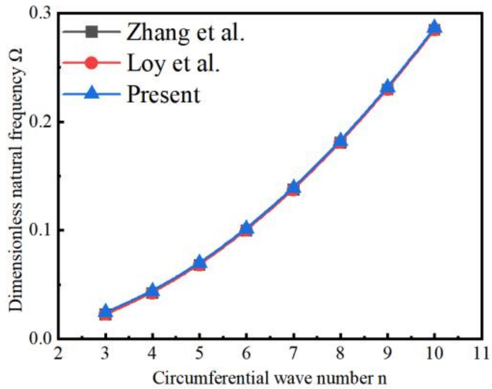


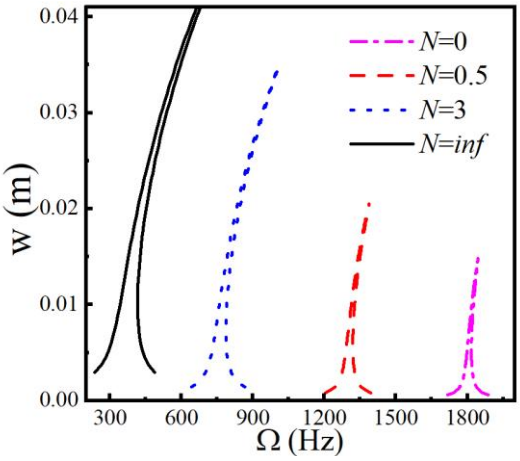

Fig. 3 Comparison of dimensionless natural frequencies $\Omega=\omega R(\rho_1/E_1)$ (m=1, $L/R$ = 20, $h/R$ = 0.01, $v$ = 0.3)

Table 1 Physical parameters of materials

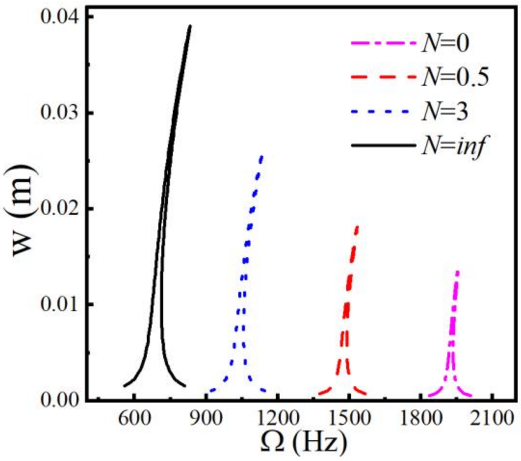
| Alumina | Aluminum |
|---|---|
| $E_c$=380 GPa | $E_m$=70 GPa |
| $\rho_c$=3800 kg/m$^3$ | $\rho_m$=2707 kg/m$^3$ |
| $v_c$=0.3 | $v_m$=0.3 |
| $\alpha_c$=7×10$^{-6}$ 1/°C | $\alpha_m$=23×10$^{-6}$ 1/°C |
| $\lambda_c$=29.3 W/(mK) | $\lambda_m$=237 W/(mK) |
| $\chi_c$=0.001 (wt%$H_2O$)$^{-1}$ | $\chi_m$=0.44 (wt%$H_2O$)$^{-1}$ |

Table 2 Natural frequencies for different modes at Δ$T$=100 K, $P$=0, $\beta$ =0.1, $N$=1, Δ$C$=0.0005

| Modal shape ($m$, $n$) | Fundamental frequency Ω (Hz) | |
|---|---|---|
| | Uniform temperature field | Heat conduction |
| (1, 1) | 3744 | 3760 |
| (1, 2) | 2914 | 2940 |
| (1, 3) | 2105 | 2153 |
| (1, 4) | 1495 | 1586 |
| (1, 5) | 1116 | 1274 |
| (1, 6) | 1005 | 1227 |
| (1, 7) | 1164 | 1411 |
| (1, 8) | 1505 | 1751 |
| (1, 9) | 1953 | 2191 |
| (1, 10) | 2478 | 2707 |

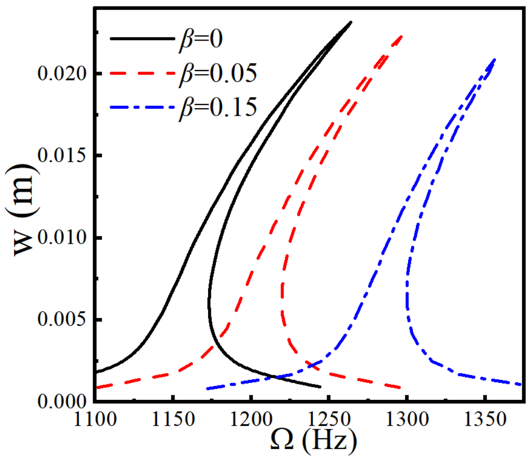



(a) UTF

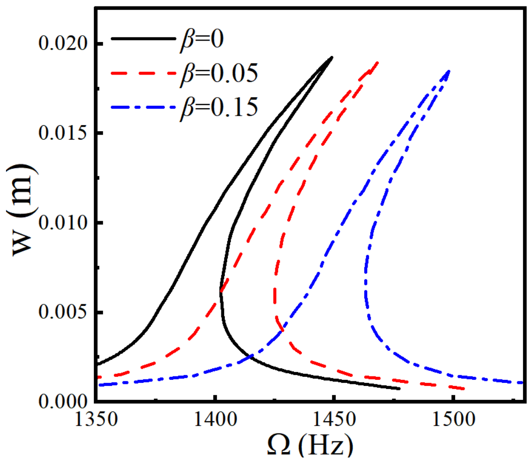



(b) HC

Fig. 4 Influence of volume fraction index at $\beta$=0.1, $P$=5 MPa, $C_t$=1.5×10$^4$, $V$=10 m/s, Δ$T$=100K, Δ$C$=0.0005, $q$=40 MPa.




(a) UTF




(b) HC

Fig. 5 Porosity coefficients influence at N=1, $P$=30 MPa, $C_t$=1.5×10$^4$, $V$=10 m/s, Δ$T$=100K, Δ$C$=0.0005, $q$=40 MPa.

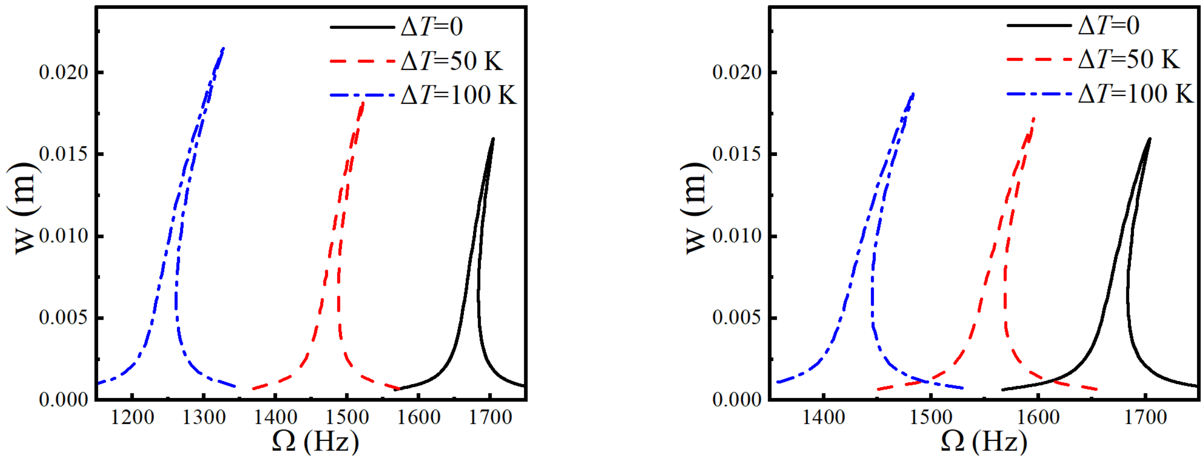
In the following research, the symbols "UTF" stands for Uniform temperature field, and "HC" for heat conduction, $L/R$ = 2, $h/R$ = 0.025.

Fig. 4 shows the effect of volume fraction index on the forced vibration amplitude-frequency response curve of FGM cylindrical shell. It can be seen that as the volume fraction index of the ceramic decreases, the natural frequency of the response curve increases. This means that adding ceramics can enhance the stiffness and delay the resonance position. In addition, the amplitude of the FGM cylindrical shell decreases with the increase of the ceramic volume fraction. This shows that the addition of ceramics can suppress resonance and has the effect of strengthening damping. Therefore, proper introduction of ceramics can improve the stiffness and stability of composite materials.

In Fig. 5, the effects of different porosity coefficients on forced vibration of FGM cylindrical shells are studied. Obviously, cylindrical shells without pores have the lowest natural frequency, and the larger the porosity coefficient, the greater the frequency of resonance. This is because the increase of the porosity coefficient will reduce the moment of inertia. At the same time, the natural frequency of heat conduction is larger than that in UTF, which changes less for different porosity coefficients. So, heat conduction can delay the location of resonance compared to the case of UTF.

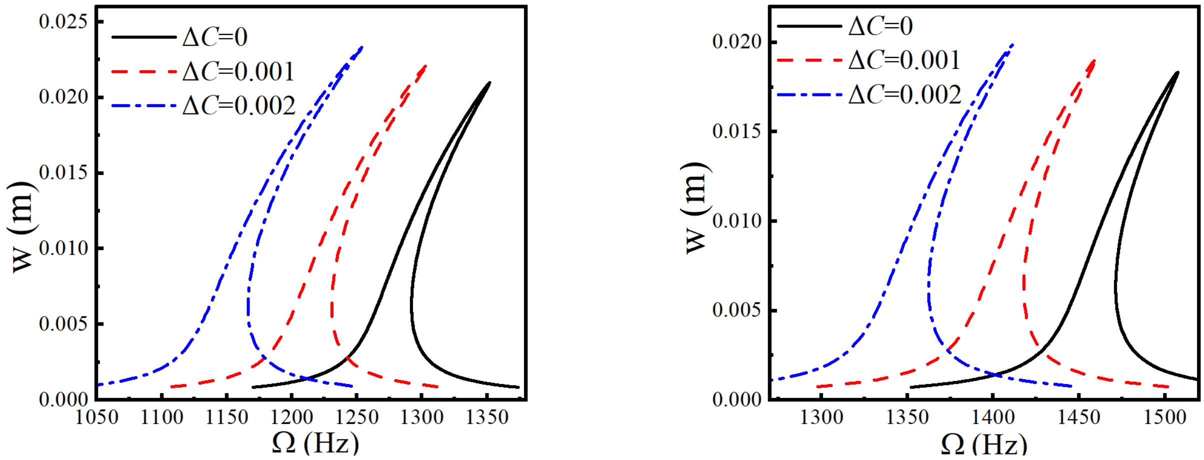
The effect of temperature on the forced vibration is shown in Fig. 6. As shown, an increase in temperature not only leads to a decrease in the natural frequency, but also an increase in the amplitude of the response curve. That is to say, with the increase of temperature, the rigid body of the shell is reduced, that is to say, with the increase of temperature, the rigid body of the shell reduces, so as to lead the advance of the resonance position. This is due to that the increase of temperature will weaken damping and strengthen resonance.

Fig. 7 investigates the effect of moisture content change on forced vibration of FGM cylindrical shell. As seen, the increase of moisture content will lead to the decrease of natural frequency of FGM cylindrical shell. This is due to the fact that higher moisture content reduces the bending stiffness of the shell, causing the forced resonance curve to shift to the left. Therefore, a dry environment can improve the vibration resistance compared to a humid environment.



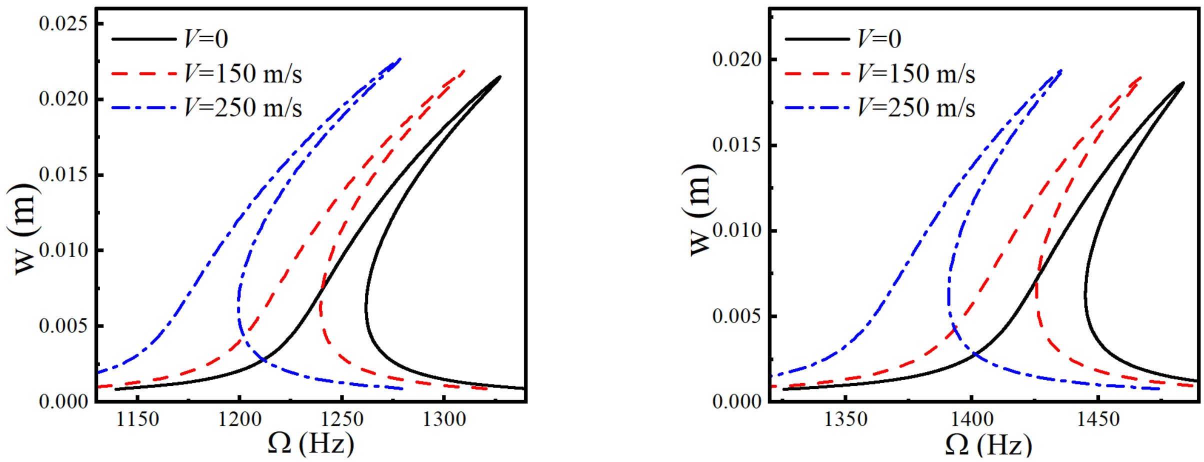

(a) UTF (b) HC

Fig. 6 Temperature influence at N=1, $\beta$=0.1, $P$=30 MPa, $C_t$=1.5×10$^4$, $V$=10 m/s, Δ$C$=0.0005, $q$=40 MPa




(a) UTF (b) HC

Fig. 7 The influence of humidity variations at N=1, $\beta$=0.1, $P$=30 MPa, $C_t$=1.5×10$^4$, $V$=10 m/s, Δ$T$=100K, $q$=40 MPa

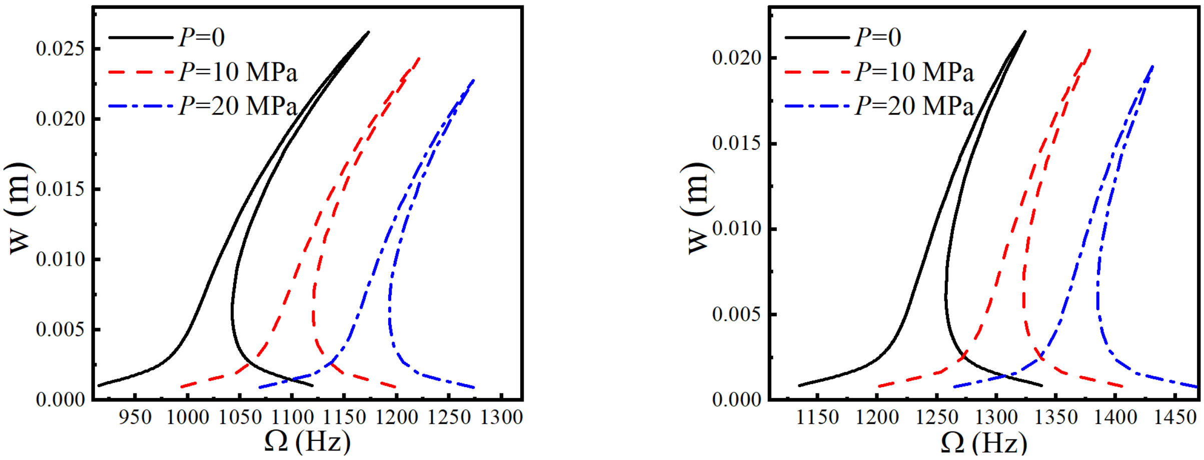



(a) UTF (b) HC

Fig. 8 The influence of axially moving velocity at N=1, $\beta$=0.1, $P$=30 MPa, $C_t$=1.5×10$^4$, Δ$C$=0.0005, Δ$T$=100K, $q$=40 MPa

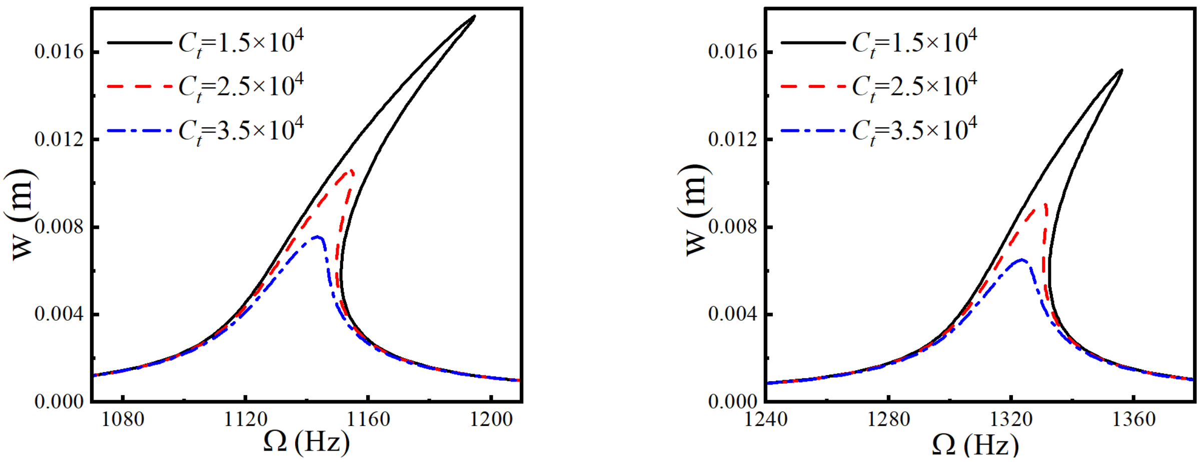
The effect of axial velocity is discussed in Fig. 8. As shown, the increase of the axial movement velocity will reduce the natural frequency, causing the resonance position to shift to the left. The reason is that increasing axial velocity weakens the stiffness of the shell. In addition, the influence of the axial velocity change on the amplitude of the shell in UTF is greater than that in the heat conduction.

The effect of prestress is illustrated in Fig. 9. It is



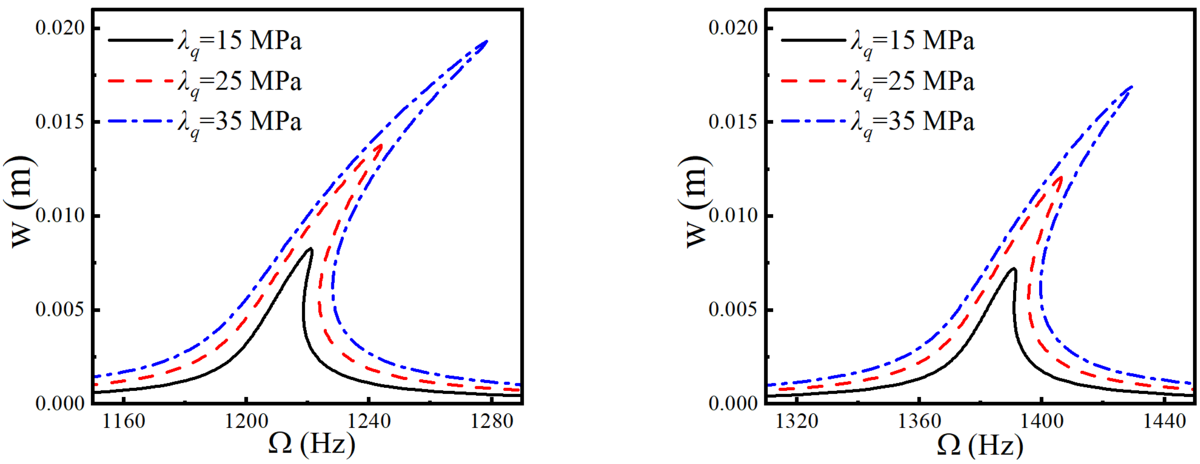

(a) UTF (b) HC

Fig. 9 The influence of prestress at N=1, $\beta$=0.1, $C_t$=1.5×$10^4$, $V$=10 m/s, $\Delta T$=100K, $\Delta C$=0.0005, $q$=40 MPa




(a) UTF (b) HC

Fig. 10 The influence of damping at N=1, $\beta$=0.15, $P$=10 MPa, $V$=10 m/s, $\Delta T$=100K, $\Delta C$=0.0005, $q$=30 MPa




(a) UTF (b) HC

Fig. 11 The influence of external excitation at N=1, $\beta$=0.15, $C_t$=1.5×$10^4$, $P$=20 MPa, $V$=10 m/s, $\Delta T$=100K, $\Delta C$=0.0005

noteworthy that applying prestress can greatly increase the resonance frequency and reduce the resonance amplitude compared with the prestressed P=0. And this phenomenon becomes more obvious with the increase of prestress. The results shows that proper application of prestress can enhance the bending stiffness and strengthen the damping of the shell.

Fig. 10 shows the influence of different damping coefficients Ct in the condition of UTF and HC. According to the amplitude-frequency response curve corresponding to different Ct values given in the figure, we find the amplitude decreases significantly with the increase of Ct value, and the degree of right-bending of the peak value weakens. In other words, the existence of damping will inhibit the increase of the response amplitude and affect the nonlinear term of the system, resulting in the reduction of the bending degree of the amplitude-frequency curve. It is important to note that the resonance location is not affected by Ct.

In Fig. 11, different external excitation amplitudes are applied to the outer surface of the shell to study its influence on forced vibration. Obviously, a larger external excitation will cause a larger magnitude of the main resonance, and the resonance area increases correspondingly. Moreover, the main resonance peak bends to the right as $\lambda_q$ increases. Therefore, it can be concluded that the external excitation will affect the resonance amplitude, the size of the resonance region, and the degree of bending of the amplitude-frequency curve.

## 6. Conclusions

In this article, the nonlinear dynamic equations are obtained by using Euler-Lagrange principle. Combined with Galerkin method and multi-scale method, the influence of different parameters on the response of nonlinear forced vibration under uniform temperature field and heat conduction is discussed. The main results are:

1. By appropriately reducing the volume fraction index and increasing the porosity coefficient, the natural frequency increases and the response amplitude suppresses.
2. Changes in temperature and humidity have a significant effect on the forced vibration of the FGM cylindrical shell. Increases in temperature and humidity cause the resonance position to shift to the left and increase the resonance amplitude.
3. An increase in axial velocity will reduce the natural frequency, but applying a larger prestressing force can delay the resonance position and reduce the vibration amplitude.
4. The resonant frequency is not affected by damping and external excitation amplitude. The increase of damping will suppress the resonance and weaken the right-bending degree of the peak value of the vibration curve. On the contrary, the increase of external excitation amplitude will increase the resonance amplitude and the peak value will bend to the right.
5. Under the same conditions, the natural frequency of the shell in the condition of uniform temperature field is smaller than that in the heat conduction, while the vibration amplitude has the opposite trend. In addition, the uniform temperature field is more affected by the variation of various parameters.

It should be pointed out that, in this study, we assume that the axial motion velocity is constant. In practical engineering, the axial velocity may not necessarily be constant. Therefore, in future research, we will investigate the influence of variable velocity on the dynamic behavior of cylindrical shells. In addition, viscoelastic problems are also a focus of future research.

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