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Effect of the gravity on a nonlocal micropolar thermoelastic media with the multi-phase-lag model

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Abstract. Erigen's nonlocal thermoelasticity model is used to study the effect of viscosity on a micropolar thermoelastic solid in the context of the multi-phase-lag model. The harmonic wave analysis technique is employed to convert partial differential equations to ordinary differential equations to get the solution to the problem. The physical fields have been presented graphically for the nonlocal micropolar thermoelastic solid. Comparisons are made with the results of three theories different in the presence and absence of viscosity as well as the gravity field. Comparisons are made with the results of three theories different for different values of the nonlocal parameter. Numerical computations are carried out with the help of Matlab software.

Keywords: initial stress; micropolar; multi-phase-lags; nonlocal; viscosity

1. Introduction

The nonlocal theory of elasticity was used to study applications in nanomechanics including lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, fracture mechanics, surface tension fluids, etc. Eringen (1974) introduced the nonlocal thermoelasticity theory to discuss small-scale structure problems. Thermoelastic wave propagation in plates based on the nonlocal theory of thermoelasticity was discussed by Inan and Eringen (1991). Koutsoumaris *et al.* (2017) introduced a different approach to the nonlocal thermoelasticity theory with applications for beams. Liew *et al.* (2017) introduced a literature review of recent research studies on the applications of nonlocal elasticity theory in the modeling and simulation of graphene sheets. Balta and Shubi (1977) proposed the nonlocal thermoelasticity theory within the framework of continuum mechanics. They investigated the systematic use of constitutive relations and field equations in reference to the nonlocal elastic model for generalized thermoelasticity. The reflection of plane waves in a nonlocal micropolar thermoelastic solid under the effect of rotation was constructed by Kalkal *et al.* (2020). Sarkar and Tomar (2019) investigated the propagation of plane waves in a nonlocal thermoelastic medium with voids. Baljeet (2021) discussed the propagation of plane waves in a nonlocal generalized thermoelastic solid medium in the framework of Lord and Shulman generalization. Hobiny *et al.* (2022) discussed analytical solutions of nonlocal thermoelastic interaction on semi-infinite mediums induced by ramp-type heating. Said (2023) studied wave propagation in a nonlocal porous thermoelastic half-space with temperature-dependent properties.

Viscoelastic materials are very much efficient in constructing various structures, such as airplane structures, marine structures, and automotive parts. Furthermore, advanced viscous substances have a noteworthy contribution to improving rigidity and describe damping nature in various structures. Linear viscoelastic materials are rheological materials that exhibit time-temperature rate-of-loading dependence. When their response is not only a function of the current input but also the current and past input history, the characterization of the viscoelastic response can be expressed using the convolution (hereditary) integral. Tschoeg (1997) discussed a general overview of time-dependent material properties. Alfrey and Gurnee (1956) and Ferry (1980) investigated the mechanical model representation of linear viscoelastic behavior results. Bayones (2012) discussed the influence of diffusion on a homogenous isotropic magneto-thermo-viscoelastic medium. Mukhopadhyay (2000) investigated the effects of thermal relaxation times on thermo-viscoelastic interactions in an unbounded body with a spherical cavity subjected to periodic loading. Othman (2004) studied the problem of electromagneto-thermovisco-elasticity for a thermally and electrically conducting medium in the context of the Lord-Shulman theory. Abd-Alla and Abo-Dahab (2009) introduced an attempt to estimate the influence due to a time-harmonic normal point load or thermal source in a magneto-thermo-viscoelastic medium. Samanta and Maishal (2008) used a state space approach in a study of a magneto-thermo-viscoelastic solid subjected to a temperature pulse. Said (2020a) constructed a novel model of thermoelastic viscoelastic half-space under the effects of the gravity field, magnetic field, and variable thermal conductivity. Said (2022) discussed a viscoelastic-micropolar solid with voids and microtemperatures under the effect of the gravity field. Zenkour and El-Shahrany (2023) introduced the vibration of viscoelastic

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magnetostrictive plates embedded in viscoelastic foundations in hygrothermal environments.

It is well known that the usual theory of heat conduction based on Fourier's law predicts an infinite heat propagation speed. It is also known that heat transmission at low temperatures propagates by employing waves. These aspects have caused intense activity in the field of heat propagation. Extensive reviews on the second sound theories (hyperbolic heat conduction) are given in Hetnarski and Ignaczak (1999, 2000). A two-phase-lag model for both the heat flux vector and the temperature gradient was introduced by Tzou (1995). Choudhuri (2007) has proposed a theory with three-phase lag (3PHL). The purpose of the work of Choudhuri (2007) was to set up a mathematical model that includes 3PHL in the heat flux vector, the temperature gradient, and the thermal displacement gradient. Various vital approaches for determining the solutions to the governing equations of problems in thermodynamics, and thermoelasticity have been introduced by Marin *et al.* (2019, 2020a, b) and Aljadani *et al.* (2022a, 2022b). Recently, Zenkour (2020a, b, c, d) presented a refined multi-phase-lag (RPL) model that finds application in many problems. He investigated different versions of the RPL model to deal with thermoelastic responses of many structures as Zenkour and El-Mekawy (2020), and Zenkour *et al*. (2023).

A nonlocal micropolar thermoelastic solid under the effect of viscosity, gravity field, and initial stress in the context of the multi-phase-lag model was discussed. The partial differential equations convert to ordinary differential equations by using the harmonic wave analysis technique. Numerically simulated results are obtained and presented graphically to depict the effect of nonlocal parameter and viscosity on wave propagation in a thermoelastic micropolar solid. Comparison is made with the results for the physical fields in the absence and presence of the gravity field.

2. Formulation of the problem

A nonlocal micropolar viscoelastic-thermoelastic solid in a half-space $(x \geq 0)$ under the effect of the gravity field, and initial stress. The field equations and the constitutive relations are taken by Eringen (1973), Montanaro (1999), and Kumar *et al*. (2012), Zenkour (2018)

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ji,j} + F_i \tag{1}$$

$$\rho\, j\left(1-\varepsilon^2\nabla^2\right)\frac{\partial^2 \underline{\varphi}}{\partial t^2} = (\alpha^v+\beta^v+\gamma_1{}^v)\nabla(\nabla.\underline{\varphi}) - \gamma_1{}^v\nabla\wedge(\nabla\wedge\underline{\varphi}) + k^v(\nabla\wedge\underline{u}) - 2k^v\underline{\varphi}, \tag{2}$$

$$K\left(1+\sum_{r=1}^{N}\frac{\tau_\theta^r}{r!}\frac{\partial^r}{\partial t^r}\right)\nabla^2\Theta = \left(\delta+\tau_0\frac{\partial}{\partial t}+\sum_{r=1}^{N}\frac{\tau_q^{r+1}}{r+1!}\frac{\partial^r+1}{\partial t^{r+1}}\right)(\rho C_E\dot{\Theta}+\gamma^v T_0\dot{e}), \tag{3}$$

$$\left(1-\varepsilon^2\nabla^2\right)\sigma_{ij} = \lambda^v e_{kk}\delta_{ij} + 2\mu^v e_{ij} + k^v(u_{j,i}-\varepsilon_{ijr}\varphi_r) - \gamma^v\theta\delta_{ij} - P(\delta_{ij}+w_{ij}), \tag{4}$$

$$\left(1-\varepsilon^2\nabla^2\right)m_{ij} = \alpha^v\varphi_{r,r}\delta_{ij} + \beta^v\varphi_{i,j} + \gamma_1{}^v\varphi_{j,i}, \tag{5}$$

Where $\lambda^v = \lambda(1+\alpha_0\frac{\partial}{\partial t})$, $\mu^v = \mu(1+\alpha_1\frac{\partial}{\partial t})$,

$$k^v = k(1+k_0\frac{\partial}{\partial t}),\quad \beta^v = \beta(1+\beta_0\frac{\partial}{\partial t}),\quad \gamma^v = \gamma(1+\gamma_0\frac{\partial}{\partial t}),$$
$$\gamma_1{}^v = \gamma_1(1+\gamma^*\frac{\partial}{\partial t}),\quad F_1 = \rho g\frac{\partial w}{\partial x},\quad F_2 = 0,\quad F_3 = -\rho g\frac{\partial u}{\partial x}. \tag{6}$$

where, $\underline{u}=(u,0,w)$ is the displacement vector, $\underline{\varphi}=(0,\varphi_2,0)$ is the microrotation vector, $\varepsilon = a_0 e_0$ is the elastic nonlocal parameter having a dimension of length, a_0, e_0 are an internal characteristic length and a material constant respectively, σ_{ij} are the components of stress, e_{ij} are the components of strain, e_{kk} is the dilatation, λ, μ are elastic constants, $\alpha_0, \alpha_1, \gamma_0, k_0, \beta_0, \gamma^*$ are the viscoelastic parameters, $\theta = T - T_0$, where T is the temperature above the reference temperature T_0, δ_{ij} is the Kronecker's delta, $w_{ij} = \frac{1}{2}(u_{j,i}-u_{i,j})$ is the rotation vector, P is the initial stress, and $k, \alpha, \beta, \gamma_1$ are micropolar constants.

The equations of motion may be written as Said (2020a)

$$\rho(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2} = (\lambda^v+2\mu^v+k^v)\frac{\partial^2 u}{\partial x^2} + (\lambda^v+\mu^v+\frac{P}{2})\frac{\partial^2 w}{\partial x\partial z} + (k^v+\mu^v-\frac{P}{2})\frac{\partial^2 u}{\partial z^2} - k^v\frac{\partial\varphi_2}{\partial z} - \gamma^v\frac{\partial\theta}{\partial x} + \rho g(1-\varepsilon^2\nabla^2)\frac{\partial w}{\partial x}, \tag{7}$$

$$\rho(1-\varepsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2} = (k^v+\mu^v-\frac{P}{2})\frac{\partial^2 w}{\partial x^2} + (\lambda^v+\mu^v+\frac{P}{2})\frac{\partial^2 u}{\partial x\partial z} + (\lambda^v+2\mu^v+k^v)\frac{\partial^2 w}{\partial z^2} + k^v\frac{\partial\varphi_2}{\partial x} - \gamma^v\frac{\partial\theta}{\partial z} - \rho g(1-\varepsilon^2\nabla^2)\frac{\partial u}{\partial x}, \tag{8}$$

$$\rho\, j(1-\varepsilon^2\nabla^2)\frac{\partial^2\varphi_2}{\partial t^2} = \gamma_1^v\nabla^2\varphi_2 + k^v(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) - 2k^v\varphi_2 \tag{9}$$

Introduce the following non-dimensional variables:

$$(x', z', \varepsilon', u', w') = \frac{1}{l_0}(x, z, \varepsilon, u, w),\quad m'_{ij} = \frac{m_{ij}}{T_0},\quad \theta' = \frac{\theta}{T_0},$$
$$(t', \tau'_q, \tau'_\theta, \tau'_0) = \frac{c_0}{l_0}(t, \tau_q, \tau_\theta, \tau_0),\quad g' = \frac{l_0}{c_0^2}g,\quad \varphi'_2 = \varphi_2, \tag{10}$$
$$\sigma'_{ij} = \frac{\sigma_{ij}}{T_0},\quad l_0 = \sqrt{\frac{K}{\rho C_E T_0}},\quad c_0 = \sqrt{\frac{T_0}{\rho}}.$$

Using Eqs. (10), then Eqs. (7)- (9) and (3) will be as

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2} = \frac{(\lambda^v+2\mu^v+k^v)}{\rho c_0^2}\frac{\partial^2 u}{\partial x^2} + \frac{(\lambda^v+\mu^v+\frac{P}{2})}{\rho c_0^2}\frac{\partial^2 w}{\partial x\partial z} + \frac{(k^v+\mu^v-\frac{P}{2})}{\rho c_0^2}\frac{\partial^2 u}{\partial z^2} - \frac{k^v}{\rho c_0^2}\frac{\partial\varphi_2}{\partial z} - \frac{\gamma^v T_0}{\rho c_0^2}\frac{\partial\theta}{\partial x} + g(1-\varepsilon^2\nabla^2)\frac{\partial w}{\partial x}, \tag{11}$$

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2}=\frac{(k^v+\mu^v-\frac{P}{2})}{\rho c_0^2}\frac{\partial^2 w}{\partial x^2}+\frac{(\lambda^v+\mu^v+\frac{P}{2})}{\rho c_0^2}\frac{\partial^2 u}{\partial x\partial z}+\frac{(\lambda^v+2\mu^v+k^v)}{\rho c_0^2}\frac{\partial^2 w}{\partial z^2}+\frac{k^v}{\rho c_0^2}\frac{\partial \varphi_2}{\partial x}-\frac{\gamma^v T_0}{\rho c_0^2}\frac{\partial \theta}{\partial z}-g(1-\varepsilon^2\nabla^2)\frac{\partial u}{\partial x}, \tag{12}$$

$$\rho j c_0^2(1-\varepsilon^2\nabla^2)\frac{\partial^2\varphi_2}{\partial t^2}=\gamma_1^v\nabla^2\varphi_2+k^v l_0^2\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2k^v l_0^2\varphi_2, \tag{13}$$

$$\left(1+\sum_{r=1}^{N}\frac{\tau_\theta^r}{r!}\frac{\partial^r}{\partial t^r}\right)\nabla^2\theta=\left(\delta+\tau_0\frac{\partial}{\partial t}+\sum_{r=1}^{N}\frac{\tau_q^{r+1}}{r+1!}\frac{\partial^{r+1}}{\partial t^{r+1}}\right)\left(\frac{\rho C_E c_0 l_0}{K}\dot{\theta}+\frac{\gamma^v T_0 c_0 l_0}{K}\dot{e}\right). \tag{14}$$

We solve the problem by using the harmonic wave analysis technique

$$\left(u,w,\theta,\varphi_2,\sigma_{ij},m_{ij}\right)(x,z,t)=\left(\bar{u},\bar{w},\bar{\theta},\overline{\varphi_2},\overline{\sigma_{ij}},\overline{m_{ij}}\right)(z)\exp\left(mt+\mathrm{i}\,ax\right), \tag{15}$$

where $\bar{u}(z)$, etc. is the amplitude of the function $u(x,z,t)$ etc., i is the imaginary unit, m (complex) is the time constant, and a is the wavenumber in the $x-$ direction.

Using Eqs. (15) in Eqs. (11)- (14), we get

$$\left(N_1 D^2-N_2\right)\bar{u}+\mathrm{i}a\left(-g\varepsilon^2 D^2+A_2 D+N_0\right)\bar{w}-\mathrm{i}aA_4\bar{\theta}-A_5 D\overline{\varphi_2}=0, \tag{16}$$

$$\mathrm{i}\,a\left(g\varepsilon^2 D^2+A_2 D-N_0\right)\bar{u}+\left(N_3 D^2-N_4\right)\bar{w}-A_4 D\bar{\theta}+\mathrm{i}\,a A_5\overline{\varphi_2}=0, \tag{17}$$

$$A_7 D\bar{u}-\mathrm{i}a A_7\bar{w}+\left(N_5 D^2-N_6\right)\overline{\varphi_2}=0, \tag{18}$$

$$\mathrm{i}a N_7\bar{u}+N_7 D\bar{w}+\left(N_8-A_{10}D^2\right)\bar{\theta}=0, \tag{19}$$

where N_i are given in the appendix and $D=\frac{d}{dz}$.

The system of Eqs. (16)- (19) is solved to get

$$\left(D^8-C_1 D^6+C_2 D^4-C_3 D^2+C_4\right)\left\{\bar{u}(z),\bar{w}(z),\bar{\theta}(z),\overline{\varphi_2}(z)\right\}=0, \tag{20}$$

where C_i are given in the appendix.

Eq. (20) can be factored as

$$\left(D^2-k_1^2\right)\left(D^2-k_2^2\right)\left(D^2-k_3^2\right)\left(D^2-k_4^2\right)\bar{w}(z)=0, \tag{21}$$

where $k_n^2\ (n=1,2,3,4)$ are the roots of the characteristic equation: $k^8-C_1k^6+C_2k^4-C_3k^2+C_4=0.$

The bounded solution of Eq. (20) is

$$\bar{w}(z)=\sum_{n=1}^{4}M_n\exp(-k_n z), \tag{22}$$

$$\bar{\theta}(z)=\sum_{n=1}^{4}H_{1n}M_n\exp(-k_n z), \tag{23}$$

$$\bar{u}(z)=\sum_{n=1}^{4}H_{2n}M_n\exp(-k_n z), \tag{24}$$

$$\overline{\varphi_2}(z)=\sum_{n=1}^{4}H_{3n}M_n\exp(-k_n z). \tag{25}$$

Using the above equations, we get

$$\overline{\sigma_{zz}}(z)=\sum_{n=1}^{4}H_{4n}M_n\exp(-k_n z)-P^*, \tag{26}$$

$$\overline{\sigma_{xz}}(z)=\sum_{n=1}^{4}H_{5n}M_n\exp(-k_n z), \tag{27}$$

$$\overline{\sigma_{zx}}(z)=\sum_{n=1}^{4}H_{6n}M_n\exp(-k_n z), \tag{28}$$

$$\overline{m_{yz}}(z)=\sum_{n=1}^{4}H_{7n}M_n\exp(-k_n z), \tag{29}$$

where P^*, H_{ij} are given in the appendix.

3. Boundary conditions

In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. The constants $M_n(n=1,2,3,4)$ have to be chosen such that the boundary conditions on the surface at $z=0$ as Said (2020b)

$$u=f_1F(x,t),\qquad \sigma_{zz}=-f_0G(x,t)-\frac{P}{(1+\varepsilon^2a^2)},\qquad \sigma_{xz}=m_{yz}=0. \tag{30}$$

Where f_0, f_1 are constants and $F(x,t), G(x,t)$, are arbitraries functions. Applying the boundary conditions (30) at the surface $z=0$, we obtain a system of four equations. After applying the inverse of the matrix method, we have

$$\begin{pmatrix}M_1\\M_2\\M_3\\M_4\end{pmatrix}=\begin{pmatrix}H_{21}&H_{22}&H_{23}&H_{24}\\H_{41}&H_{42}&H_{43}&H_{44}\\H_{51}&H_{52}&H_{53}&H_{54}\\H_{71}&H_{72}&H_{73}&H_{74}\end{pmatrix}^{-1}\begin{pmatrix}f_1\\-f_0\\0\\0\end{pmatrix} \tag{31}$$

4. Particular cases

a) The refined phase-lag (RPL) model is given when $\tau_q=\tau_0\succ\tau_\theta\geq 0,\ \delta=1,$ and $N\geq 1$.
b) The Lord-Shulman (L-S) theory is given when $\tau_q=\tau_\theta=0,\ \delta=1,\ \tau_0\succ 0$.
c) The simple phase-lag (DuaL) model of Tzou (1995) is obtained when $\tau_q=\tau_0\succ\tau_\theta\geq 0,\ \delta=1$, $N=1$ and $\tau_q^2=0$.
d) A local micropolar viscoelastic-thermoelastic medium is given when $\varepsilon=0$.
e) A nonlocal micropolar thermoelastic medium is given when $\alpha_0,\alpha_1,\gamma_0,k_0,\beta_0,\gamma^*$ are vanishes.

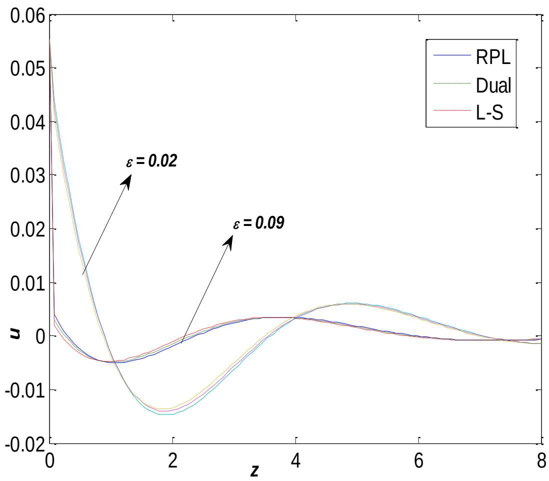

Fig. 1 Horizontal displacement distribution u for different values of local parameter

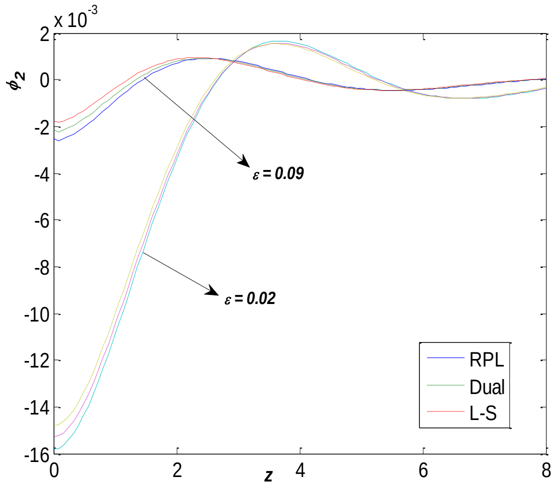

Fig. 2 Distribution of microrotation component φ_2 for different values of local parameter

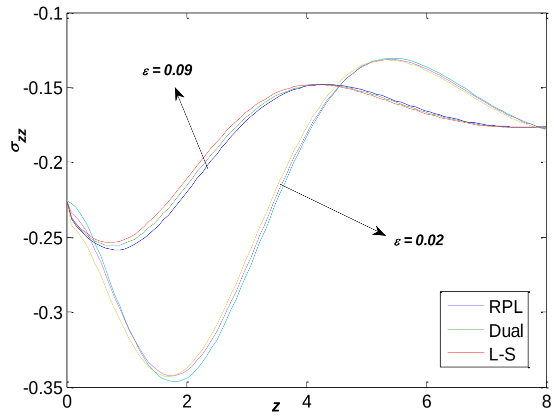

Fig. 3 Distribution of stress component σ_{zz} for different values of local parameter

5. Numerical calculations and discussion of the problem

To illustrate the theoretical results obtained in the preceding section, we consider an example where Magnesium Crystal-like material is modeled as an isotropic generalized magneto-micropolar thermoelastic solid with voids as Said (2020b)

$$\lambda = 3.76\text{x}10^9\ \text{N.m}^{-2},\ \mu = 5.86\text{x}10^9\ \text{N.m}^{-2},\ \rho = 8954\,\text{kg.m}^{-3},$$
$$a = 0.3,\quad T_0 = 293\text{K},\quad C_E = 383.1\ \text{J.kg}^{-1}.\text{K}^{-1},$$
$$\alpha_t = 1.78\text{x}10^{-3}K^{-1},\quad \beta = 1.1386\,\text{x}10^8\,\text{N.m}^{-2},\quad \tau_\theta = 5\text{x}10^{-7}\ \text{s},$$
$$\tau_q = 9\text{x}10^{-7}\ \text{s},\quad \tau_0 = 9\ \text{x}10^{-7}\ \text{s},\quad f_0 = 0.05,\ f_1 = 0.05,$$
$$K = 386\text{w.m}^{-1}.\text{K}^{-1}.\text{s}^{-1},\quad k = 75\text{x}10^3\text{N.m}^{-2},\quad j = 3\text{x}10^{-15}\text{N.m.kg}^{-1},$$
$$m = m_0 + i\xi,\quad m_0 = 0.5,\quad \xi = -0.5,\quad P = 50\text{N.m}^{-2},$$
$$\alpha_0 = 0.4\text{s},\quad \alpha_1 = 0.78\text{s},\quad k_0 = 0.5\text{s},\quad \gamma_0 = 0.3\text{s},.$$
$$\gamma^* = 0.2\text{s},\quad \beta_0 = 0.5\text{s},\quad \gamma_1 = 9\text{x}10^{-2}\ \text{N.m}^{-2}$$

The results are shown in Figs. 1-9. In these figures, the solid lines represent the solution in the refined-phase-lag model (RPL) model, the dashed lines represent the solution derived using the Lord-Shulman (L-S) theory and the dashed-dotted lines represent the solution derived using the dual-phase-lag (Dual) model.

Figs. 1-4 have been plotted to analyze the effect of the nonlocal parameter in the distribution of the displacement component u microrotation component φ_2 the stress components σ_{zz}, and the couple stress m_{yz} for the micropolar visco-thermoelastic solid. Fig. 1 shows the variations of horizontal displacement u against the distance z. It attains reaching its minimum value, then increases, and moves with a wave behavior. The increase of local parameter, values of u decrease then increase, and the last decrease. Fig. 2 shows that variations of microrotation component φ_2 starts from a negative value. It attains reaching its maximum value, then decreases, and moves with a wave behavior. The nonlocal parameter increase values of φ_2 after decrease and then increase. Fig. 3 shows that the variations of stress component σ_{zz} begin from a negative value and obey the boundary condition at z = 0. It attains its minimum value, then increases reach its maximum value, and then decrease. The nonlocal parameter increases values of σ_{zz} and then decrease. Fig. 4 shows that the variations of the couple stress m_{yz} begin from a zero value at z=0. It attains its maximum value, then decreases reach its minimum value, and then increase. The nonlocal parameter decreases values of m_{yz} then increase and last decrease.

Figs. 5-7 have been plotted to analyze the effect of the gravity field in the distribution of the displacement component u, microrotation component φ_2, and the couple stress m_{yz} for the micropolar viscoelastic-thermoelastic solid. Fig. 5 shows that the gravity field decreases the values of microrotation φ_2. It is clear from Fig. 6 that the gravity field decreases the values of horizontal displacement u. It clear from Fig. 7 that the gravity field increase the values of the couple stress m_{yz}.

Figs. 8 and 9 have been plotted to analyze the distribution of σ_{zz}, σ_{xz} with the displacement z for three different theories with and without viscosity. Fig. 8 clears

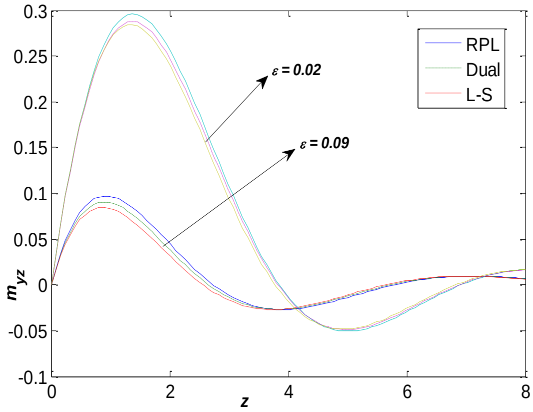

Fig. 4 Variation of the couple stress m_{yz} for different values of local parameter

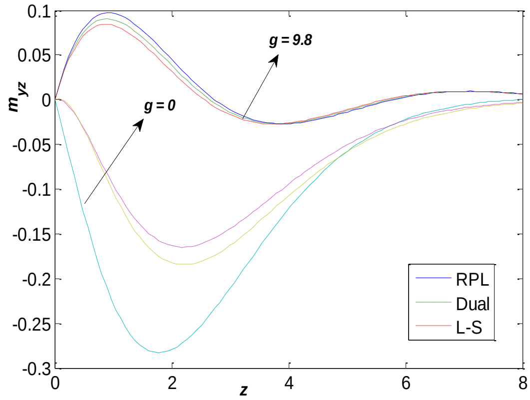

Fig. 7 Variation of the couple stress m_{yz} in the absence and presence of the gravity field

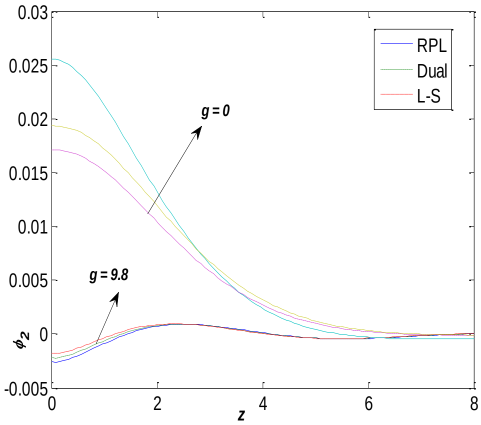

Fig. 5 Distribution of microrotation component φ_2 in the absence and presence of the gravity field

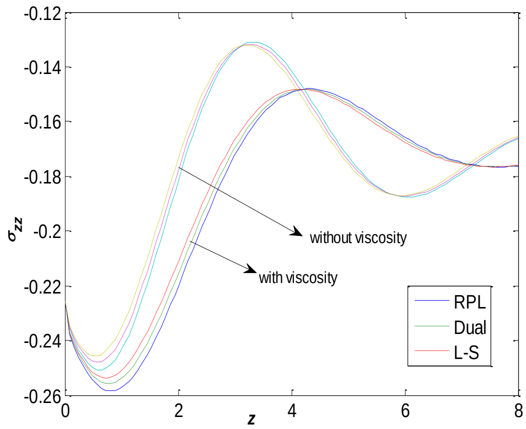

Fig. 8 Distribution of stress component σ_{zz} with and without viscosity

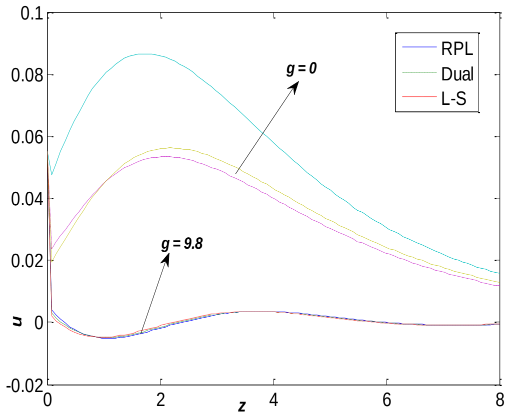

Fig. 6 Horizontal displacement distribution u in the absence and presence of the gravity field

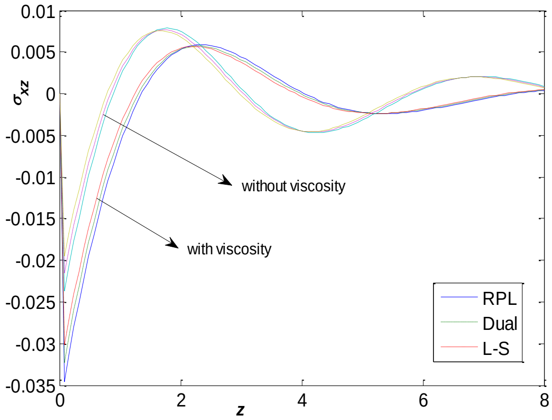

Fig. 9 Distribution of stress component σ_{xz} with and without viscosity

that σ_{zz} starts from negative values for the three different theories. It is noticed that the viscosity parameters act to decrease, then increase, and again decrease the values of σ_{zz}. It clears from Fig. 9 that the magnitudes of σ_{xz} decrease, then increase, and again decrease with the presence of the viscosity parameter.

Figs. 10 and 11 give 3D surface curves for the stress components σ_{xz} and the couple stress m_{yz} to study the nonlocal micropolar viscoelastic-thermoelastic isotropic half-space under the effect of the gravity field in the context of the refined phase-lags (RPL) model. These figures are very important to study the dependence of these physical

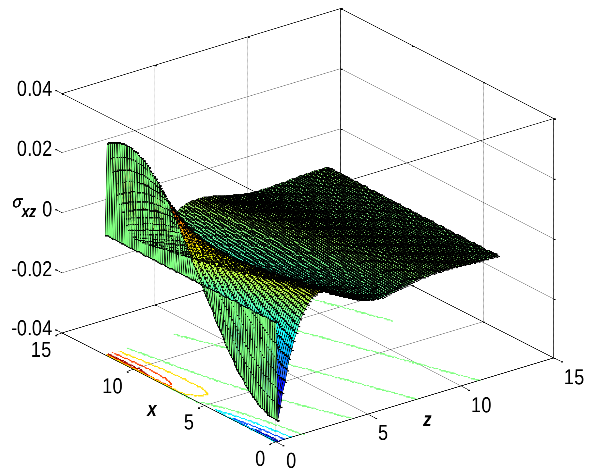

Fig. 10 Variation of the stress component σ_{xz} in the context of refined phase-lag model

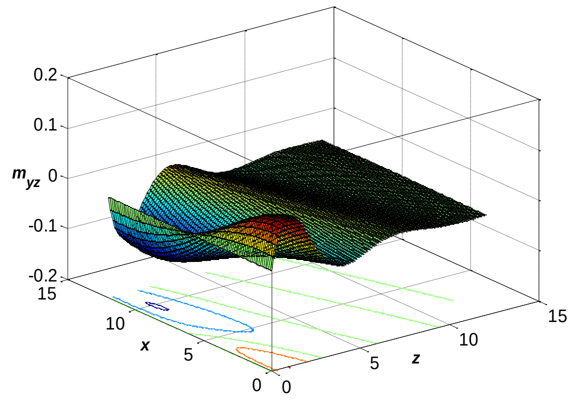

Fig. 11 Variation of the couple stress m_{yz} in the context of refined phase-lag model

fields on the horizontal component of distance. All the physical quantities satisfy the boundary condition and move with a wave behavior.

6. Conclusions

In this paper, a nonlocal micropolar viscoelastic-thermoelastic solid in the context of the multi-phase-lag model with harmonic wave analysis technique and subjected to the gravity field has been investigated. The following conclusions can be drawn from the analysis results.

- All the resulting physical fields are influenced by the nonlocal parameter and viscosity.
- The effect of the gravity field on all the studied fields is very significant.
- A significant oscillatory behavior is observed in the resulting physical fields.
- The multi-phase-lag model model has a more general problem. This is because the other models of thermoelasticity are obtained as unique cases from it.
- The vertical distance and the horizontal distance play important roles in the magnitudes of the physical fields.
- It has been observed that after achieving maximum and minimum values, thermoelastic damping becomes linear.

The present theoretical results may provide interesting information and a mathematical foundation for working on the subject, because the increasing interest in the theory of thermoelasticity has many applications in such diverse fields as geophysics, acoustic wave damping in a magnetic field, machine element design of such equipment as heat exchangers, boiler tubes, nuclear devices emitting electromagnetic radiations, the development of magnetometers that are high in sensitivity and are super-conducting, the engineering of electrical power, plasma physics, etc.

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Appendix

$$A_1 = \frac{\lambda(1+\alpha_0 m) + 2\mu(1+\alpha_1 m) + k(1+k_0 m)}{\rho c_0^2},$$

$$A_2 = \frac{\lambda(1+\alpha_0 m) + \mu(1+\alpha_1 m) + \frac{P}{2}}{\rho c_0^2},$$

$$A_3 = \frac{k(1+k_0 m) + \mu(1+\alpha_1 m) - \frac{P}{2}}{\rho c_0^2}, \qquad A_4 = \frac{\gamma(1+\gamma_0 m) T_0}{\rho c_0^2},$$

$$A_5 = \frac{k(1+k_0 m)}{\rho c_0^2}, \qquad A_6 = \gamma_1 (1+\gamma^* m), \qquad A_7 = l_0^2 k (1+k_0 m),$$

$$A_8 = 2A_7, \qquad A_9 = \rho j m^2 c_0^2, \qquad A_{10} = 1 + \sum_{r=1}^{N} \frac{\tau_\theta^r}{r!} m^r,$$

$$A_{11} = \delta + \tau_0 m + \sum_{r=1}^{N} \frac{\tau_q^{r+1}}{r+1!} m^{r+1}, \qquad A_{12} = \frac{\rho C_E c_0 l_0 m}{K},$$

$$A_{13} = \frac{\gamma(1+\gamma_0 m) c_0 l_0 m}{K}, \qquad N_0 = g(1+\varepsilon^2 a^2), \qquad N_1 = A_3 + \varepsilon^2 m^2,$$

$$N_2 = m^2(1+\varepsilon^2 a^2) + A_1 a^2, \qquad N_3 = A_1 + \varepsilon^2 m^2,$$

$$N_4 = A_3 a^2 + (1+\varepsilon^2 a^2) m^2, \qquad N_5 = A_6 + A_9 \varepsilon^2,$$

$$N_6 = A_6 a^2 + A_9 (1+\varepsilon^2 a^2) + A_8, \quad N_7 = A_{11} A_{13}, \quad N_8 = A_{11} A_{12} + A_{10} a^2,$$

$$N_9 = N_1 + A_2, \qquad N_{10} = N_4 + a^2 A_2, \qquad N_{11} = A_2 N_6 + A_5 A_7,$$

$$N_{12} = N_3 N_6 + N_5 N_4, \quad N_{13} = N_4 N_6 - a^2 A_5 A_7, \quad N_{14} = N_7 (N_3 - N_9),$$

$$N_{15} = N_7 (N_2 - N_{10}), \qquad N_{16} = A_4 N_7 + N_8 N_9 + N_2 A_{10},$$

$$N_{17} = N_2 N_8 + a^2 A_4 N_7, \qquad N_{18} = N_7 N_5 (N_3 - A_2),$$

$$N_{19} = N_7 (N_{11} - N_{12}), \qquad N_{20} = A_4 N_5 N_7 + N_8 N_5 A_2 + N_{11} A_{10},$$

$$N_{21} = g(N_6 A_{10} + N_8 N_5), \qquad N_{22} = N_7 N_6 A_4 + N_8 N_{11},$$

$$N_{23} = N_{18} N_{17} + N_7 N_9 N_{13} A_{10} + N_{20} N_{15} + N_0 N_6 N_7 N_{28} + N_0 N_8 N_7 N_{25},$$

$$N_{24} = N_{16} N_{19} + N_{22} N_{14} + N_{27} N_{30} + a^2 N_0 N_7 N_{29} + g\varepsilon^2 N_0 N_6 N_7 N_8,$$

$$N_{25} = g\varepsilon^2 N_6 + N_0 N_5, \qquad N_{26} = N_4 N_6 - a^2 A_5 A_7,$$

$$N_{27} = N_0 N_7 + g a^2 \varepsilon^2 N_7, \qquad N_{28} = A_{10} N_0 + g\varepsilon^2 N_8,$$

$$N_{29} = A_{10} N_{25} + g\varepsilon^2 N_5 N_8, \qquad N_{30} = N_8 N_{25} + A_{10} N_0 N_6,$$

$$N_{31} = g\varepsilon^2 N_5 N_7 N_0 N_8 + N_{18} N_{16} + N_7 N_{25} N_{28} + g\varepsilon^2 A_{10} N_0 N_6 N_7 - g\varepsilon^2 N_7 N_{30}$$

$$N_{32} = N_5 N_{15} A_2 A_{10} - N_9 N_{19} A_{10} - N_{14} N_{20} - N_{27} N_{29} - g\varepsilon^2 a^2 A_{10} N_0 N_5 N_7 A_{10},$$

$$C_0 = A_{10}(N_9 N_{18} - A_2 N_5 N_{14} + g\varepsilon^2 N_7 N_{25} - g\varepsilon^2 N_5 N_{27}) + g\varepsilon^2 N_5 N_7 N_{28} - g\varepsilon^2 N_7 N_{29},$$

$$C_1 = \frac{1}{C_0}\left(N_{31} + N_{32}\right) \qquad C_2 = \frac{1}{C_0}(N_{23} - N_{24}),$$

$$C_3 = \frac{1}{C_0}(N_{15} N_{22} + N_0^2 N_6 N_7 N_8 + N_7 N_{26} N_{16} - N_0 N_6 N_8 N_{27} - a^2 N_0 N_7 N_{30} - N_{19} N_{17}),$$

$$C_4 = \frac{N_7 N_{26} N_{17} - a^2 N_0^2 N_6 N_7 N_8}{C_0}, \qquad P^* = \frac{P \exp(-mt - \mathrm{i}ax)}{T_0 (1+\varepsilon^2 a^2)},$$

$$H_{1n} = \frac{g\varepsilon^2 N_7 k_n^4 + N_{14} k_n^3 - N_{27} k_n^2 + N_{15} k_n + a^2 N_0 N_7}{-g\varepsilon^2 A_{10} k_n^5 + N_9 A_{10} k_n^4 + N_{28} k_n^3 - N_{16} k_n^2 - N_0 N_8 k_n + N_{17}},$$

$$H_{2n} = \frac{N_7 k_n + \left(A_{10} k_n^2 - N_8\right) H_{1n}}{\mathrm{i} a N_7}, \qquad H_{3n} = \frac{A_7 k_n H_{2n} + \mathrm{i} a A_7}{N_5 k_n^2 - N_6},$$

$$H_{4n} = \frac{\mathrm{i}a\lambda(1+\alpha_0 m) H_{2n} - [\lambda(1+\alpha_0 m) + 2\mu(1+\alpha_1 m) + k(1+k_0 m)] k_n - \gamma T_0 (1+\gamma_0 m) H_{1n}}{T_0 (1+\varepsilon^2 a^2 - \varepsilon^2 k_n^2)},$$

$$H_{5n} = \frac{\mathrm{i}a[\mu(1+\alpha_1 m) + k(1+k_0 m) - \frac{P}{2}] - [\mu(1+\alpha_1 m) + \frac{P}{2}] k_n H_{2n} + k(1+k_0 m) H_{3n}}{T_0 (1+\varepsilon^2 a^2 - \varepsilon^2 k_n^2)},$$

$$H_{6n} = \frac{-[\mu(1+\alpha_1 m) + k(1+k_0 m) - \frac{P}{2}] k_n H_{2n} + \mathrm{i}a[\mu(1+\alpha_1 m) + \frac{P}{2}] - k(1+k_0 m) H_{3n}}{T_0 (1+\varepsilon^2 a^2 - \varepsilon^2 k_n^2)},$$

$$H_{7n} = \frac{-\beta(1+\beta_0 m) k_n H_{3n}}{l_0 T_0 (1+\varepsilon^2 a^2 - \varepsilon^2 k_n^2)}.$$