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An analytical solution for compaction grouting problem considering exothermic temperature effect of slurry

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Abstract. In this paper, an analytical solution of large-strain cylindrical cavity expansion in compaction grouting problem under temperature field is given. Considering the stress increment caused by temperature, the analytical solution of cavity expansion under traditional isothermal conditions is improved by substituting the temperature stress increment into the cavity expansion analysis. Subsequently, combined with the first law of thermodynamics, the energy theory is also introduced into the cylindrical cavity expansion analysis, and the energy dissipation solution of cylindrical cavity expansion is derived. Finally, the validity and reliability of solution are proved by comparing the results of expansion pressure with those in published literatures. The results show that the dimensionless expansion pressure increases with the increase of temperature, and the thermal response increases with the increase of dilation angle. The higher the exothermic temperature of grouting slurry, the greater the plastic deformation energy of the surrounding soil, that is, the greater the influence on the surrounding soil deformation and the surrounding environment. The proposed solution not only enrich the theoretical system of cavity expansion, but also can be used as a theoretical tool for energy geotechnical engineering problems, such as CPT, nuclear waste disposal, energy pile and chemical grouting, etc.

Keywords: compaction grouting problem; cylindrical cavity expansion; exothermic temperature effect; slurry

1. Introduction

Since the theory of cavity expansion was firstly proposed in the field of metal engineering in the 1950s (Bishop 1945). In order to solve the practical engineering problem, many scholars have developed the theory of cavity expansion. Especially in the field of geotechnical engineering, a large number of theoretical, numerical and experimental results have been obtained based on the cavity expansion, aiming at the problems of soil squeezing of pile (Randolph *et al.* 1979, Manandhara and Yasufuku 2013, Castro *et al.* 2014),surrounding rock stability of tunnel (Marshall 2012, Yang and Pan 2015, Zou *et al.* 2017) and penetration test (Papanastasiou and Durban 1997, Salgado *et al.* 1997, Russell and Khalili 2002, Durban and Masri 2004, Shi *et al.* 2010, Tolooiyan and Gavin 2011, Yeung *et al.* 2012, Marchi *et al.* 2014, Sun *et al.* 2014, Mo *et al.* 2015, Niu *et al.* 2020, Yuan *et al.* 2020, Zhang *et al.* 2020, Zhang *et al.* 2021, Thiyyakkandi 2022, Patino-Ramirez *et al.* 2022).

For the first time, Vesic applied the cavity expansion theory used to interpret penetration problem in metal engineering to the soil squeezing of pile engineering (Vesic 1972). A series of papers on the elasto-plastic cavity expansion theory of isotropic rock and soil mass published by Collins, Carter and Yu (Yu and Houlsby 1991, Carter and Yeung 1985, Collins and Stimpson 1994, Collins and Yu 1996, Carter and Yu 2022), and their research of cavity expansion played a milestone role for solving the cavity expansion problem. Recently, Chen, Li and Sivasithamparam (Chen and Abousleiman 2013, Sivasithamparam and Castro 2018, Sivasithamparam and Castro 2020, Chen *et al.* 2020) proposed also a series of important semi-analytical solutions for drained and undrained soil mass based on the critical state models (MCC, K0-MCC and S-CLAY), focusing on the influence of soil anisotropy on the expansion of cylindrical cavity. In addition, Zhou and Gaaloul (Zhou *et al.* 2018, Gaaloul *et al.* 2021) conducted an in-depth study on the thermal field of soil mass for cavity expansion problem, and analyzed the influence of temperature on the theoretical calculation of cavity expansion. Gaaloul *et al*. (2021) studied the influence of temperature change on the ultimate expansion pressure of cylindrical cavity by incorporating temperature change into the theoretical framework of cavity expansion based on Mohr-Coulomb strength criterion. However, in this paper, small deformation theory was used to calculate the deformation of soil mass, it cannot reflect the large deformation characteristics of soil mass. Zhou *et al*. (2018) based on the ACMEG-T constitutive model and similar solution technique, considered the similar solution of cavity expansion under non-isothermal condition. The proposed solution quantifies for the first time the effect of temperature on cavity expansion. However, this solution is

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essentially a numerical solution, which requires the use of mathematical software for numerical solution of ODE. At the same time, this solution not consider the influence of intermediate principal stress characteristics of soil mass.

Since natural soils is composed of layered soils with different thickness and mechanical parameters, for the problem of cavity expansion in layered soils, the perceived distance of underlying layered soils and the properties of adjacent layered soils are related to the relative distance to the soil layer interface. The effect of thin soils also affects the calculation results of cavity expansion in multi-soil layers. Based on the completely elasto-plastic model and the theory of cavity expansion, Mo *et al.* (2015) proposed the elasto-plastic solutions of cylindrical and spherical cavity expansion in two layers of soil mass. In terms of energy dissipation analysis of cavity expansion of soil mass, Luo *et al.* (2022) solved the problem of cavity expansion in granular materials based on the principle of energy dissipation and the critical state line of granular geomaterials.

However, as far as the authors known, there are few theoretical solutions for energy dissipation analysis of large-strain cavity expansion in cohesive-frictional soils considering the effect of temperature. Based on the work of Yu and Houlsby (1991) and the theory of large deformation, and expands the work of Luo *et al.* (2022), which only considers the cavity expansion in granular materials. In this study, considering the stress increment caused by temperature, the temperature stress increment is substituted into the cavity expansion analysis, and the analytical solution of cavity expansion under traditional isothermal conditions is improved. Combined with the first law of thermodynamics, the energy theory is also introduced into the analysis of cavity expansion, and the energy dissipation solution of cavity expansion is derived. In other words, based on non-isothermal conditions, the analytical solution of energy dissipation for the large-strain cylindrical cavity expansion in cohesive-frictional soils under temperature field is given. The proposed solution not only enrich the theoretical system of cavity expansion, but also can be used as a theoretical tool for energy geotechnical engineering problems, such as CPT, nuclear waste disposal, energy pile and chemical grouting, etc.

2. Theory and methodology

In the following study, firstly, considering the influence of the intermediate principal stress characteristics of soil mass, the unified strength theory (UST) and strain increment representation method are substituted into the cavity expansion analysis to obtain an analytical solution of cavity expansion for considering the large-strain and intermediate principal stress characteristics of soil mass. Subsequently, combined with the first law of thermodynamics, the energy theory is also introduced into the cylindrical cavity expansion analysis, and the energy dissipation solution of cylindrical cavity expansion is derived. Finally, the validity and reliability of solution are proved by comparing the results of expansion pressure with those in published literatures.

2.1 Yield criterion (UST)

As the unified strength theory (UST) has a clear concept and simple failure criterion, and it is more general. Here, the UST yield criterion is used to simulate the constitutive relationship of elasto-plastic region. For the UST yield criterion, the mathematical expression can be expressed as

$$\begin{cases} \sigma_r - R\sigma_\theta = \sigma_0 \\ R = \dfrac{2(1+b)(1+\sin\varphi)+mb\left(\sin\varphi-1\right)}{\left[2(1+b)-mb\right]\left(1-\sin\varphi\right)} \\ \sigma_0 = \dfrac{4(1+b)c\cos\varphi}{\left[2(1+b)-mb\right]\left(1-\sin\varphi\right)} \end{cases} \tag{1}$$

where σ_r *and* σ_θ are the radial and tangential stresses, respectively, c and φ are cohesion and internal friction angle, respectively, b is the strength parameter reflecting the influence of middle principal stress on the yielding of geomaterial ($0 \leq b \leq 1$), m is the parameter, m→1 implies the geomaterial tends to be the plastic state, it is assumed in the following plastic analysis that $m \approx 1$ (Yu 1983, Zhao *et al.* 2017).

2.2 Problem definition

As shown in Fig. 1, the schematic diagram of compaction grouting in rock and soil mass is shown on the left, and the theoretical model of cylindrical cavity expansion is shown on the right. When the steel flower tube of lower grouting pipe begins to grouting, and the internal pressure increases to p, the radius of the corresponding cavity is a. Under continuous loading, when the pressure p increases to the yield strength p_y of rock and soil mass, the inner wall of cavity will yield first. With continuous increase, an elastio-plastic region (compaction region) is formed between the current radius a and the elastio-plastic boundary r_b. Outside the elastio-plastic boundary r_b is the elastic region, and the radial displacement of a and r_b is u_a and u_{rb}, respectively.

Here, the cylindrical cavity expansion problem can be assumed as a process of energy dissipation under non-isothermal conditions. When the cavity pressure p is greater than the critical plastic pressure p_{cr}, i.e., $p > p_{cr}$, the dissipated energy U_d increases with the increase of work done W by external force. The dissipated energy is mainly reflected in increase of plastic strain energy U_p and the quantity of heat Q in the plastic region until failure.

2.3 The first law of thermodynamics

According to the first law of thermodynamics, a part of work done W by the external force is converted into kinetic energy E_k, the quantity of heat Q, and the internal energy U, it can be expressed as

$$\frac{dW}{dt}+\frac{dQ}{dt}=\frac{dE_k}{dt}+\frac{dU}{dt} \tag{2}$$

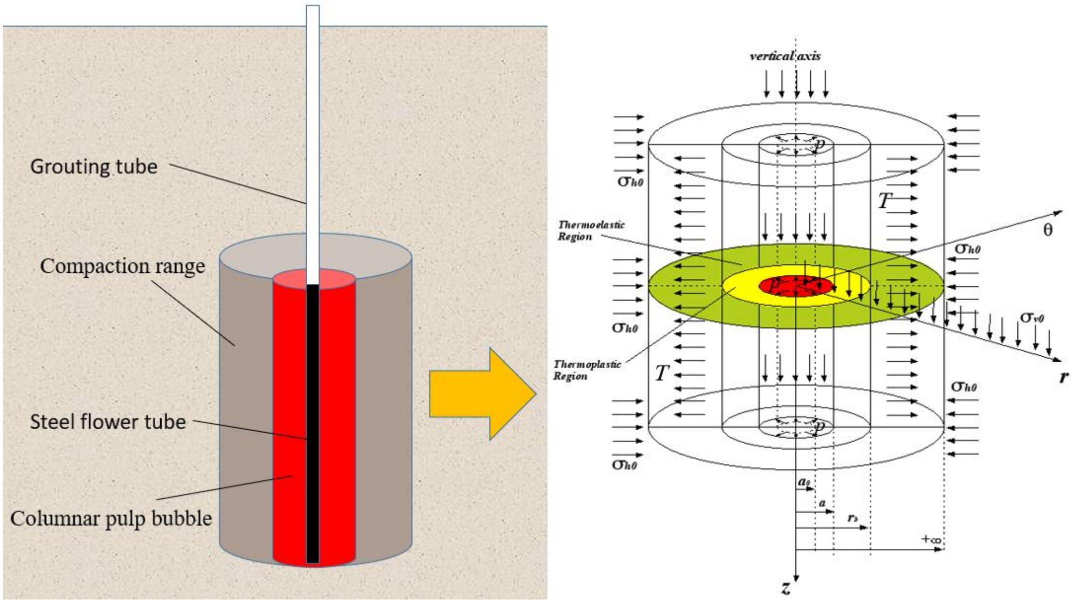


Fig. 1 The sketch of cylindrical cavity expansion of compaction grouting problem under temperature field

The expansion process can be assumed as a quasi-static cylindrical cavity expansion problem, the rate of external work will be equal to the rate of the internal energy and the quantity of heat, the Eq. (2) can be expressed as

$$\frac{dW}{dt}+\frac{dQ}{dt}=\frac{dU}{dt} \tag{3}$$

Integral Eq. (3) and assuming unit time, it can be obtained as

$$W+Q=U \tag{4}$$

The equilibrium equation for cylindrical cavity problem can be expressed as

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0 \tag{5}$$

2.4 Assumptions

Other assumptions can be made as:

(1) The large-strain can be given as (Chen and Abousleiman 2013)

$$\left.\begin{aligned}\varepsilon_r&=-\ln\left(\frac{dr}{dr_0}\right)\\ \varepsilon_\theta&=-\ln\left(\frac{r}{r_0}\right)\end{aligned}\right\} \tag{6}$$

(2) The boundary conditions can be given as

$$\begin{cases}\sigma_r\left(r=a\right)=\sigma_a=p\\ \sigma_r\left(r=r_b\right)=\sigma_{r_b}=p_y\end{cases} \tag{7}$$

2.5 Elastic region

The stress and displacement of soil mass around the cylindrical cavity can be expressed as (Gaaloul *et al*. 2021)

$$\begin{cases}\sigma_r=p_y\left(\frac{r_b}{r}\right)^2-\alpha\frac{E\Delta T}{1-2\nu}\left(\frac{r_b}{r}\right)^2+\alpha\frac{E\Delta T}{1-2\nu}\\ \sigma_\theta=-p_y\left(\frac{r_b}{r}\right)^2+\alpha\frac{E\Delta T}{1-2\nu}\left(\frac{r_b}{r}\right)^2+\alpha\frac{E\Delta T}{1-2\nu}\\ \sigma_z=\alpha\frac{E\Delta T}{1-2\nu}\\ u_r=\frac{1+\nu}{E}p_y\frac{r_b^2}{r}-\alpha\frac{(1+\nu)\Delta T}{1-2\nu}\frac{r_b^2}{r}\end{cases} \tag{8}$$

The radial displacement u_{rb} at the EP (Elasto-Plastic) boundary r_b can be given

$$\begin{cases}u_r=\frac{1+\nu}{E}p_y\frac{r_b^2}{r}-\alpha\frac{(1+\nu)\Delta T}{1-2\nu}\frac{r_b^2}{r}\\ u_r\big|_{r=rb}=\frac{1+\nu}{E}p_y r_b-\alpha\frac{(1+\nu)\Delta T}{1-2\nu}r_b\end{cases} \tag{9}$$

Thus, the strain at $r=r_b$ can be obtained

$$\varepsilon_{rb}=\frac{1+\nu}{E}p_y-\alpha\frac{(1+\nu)\Delta T}{1-2\nu} \tag{10}$$

The relationship between the displacement u_r and strain ε_{rb} can be obtained,

$$u_r=\varepsilon_{rb}\left(\frac{r_b}{r}\right)^2 r \tag{11}$$

2.6 Elasto-plastic region

2.6.1 Stress analysis

Combining Eqs. (1) and (8), the following equation can be obtained as

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-(1/R)(\sigma_r-\sigma_c)}{r}=0 \tag{12}$$

Integrating Eq. (12) along r ($a \le r \le r_b$) gives (Zhao *et al.* 2017)

$$\sigma_r = H\left(\frac{1}{r}\right)^{\frac{R-1}{R}}+\frac{\sigma_c}{1-R} \tag{13}$$

where H=constant of integration. Based on the boundary conditions

$$\begin{aligned} H &= \left(p_y+\frac{1}{R-1}\sigma_c\right)r_b^{\frac{R-1}{R}} \\ &= \left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)r_b^{\frac{R-1}{R}} \\ &= \left(p+\frac{1}{R-1}\sigma_c\right)a^{\frac{R-1}{R}} \end{aligned} \tag{14}$$

The equation can be deduced from Eq. (14)

$$\frac{r_b}{a}=\frac{\left[p_y+\frac{1}{R-1}\sigma_c\right]^{\frac{R}{R-1}}}{\left[p+\frac{1}{R-1}\sigma_c\right]^{\frac{R}{R-1}}} \tag{15}$$

Combining Eqs. (1), (14) and (15) gives

$$\begin{cases} \sigma_r = \left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)\left(\frac{r_b}{r}\right)^{\frac{R-1}{R}}+\frac{\sigma_c}{1-R} \\ \sigma_\theta = \frac{1}{R}\left(\left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)\left(\frac{r_b}{r}\right)^{\frac{R-1}{R}}+\frac{\sigma_c}{1-R}-\sigma_c\right) \end{cases} \tag{16}$$

The non-associated flow rule is adopted

$$\frac{d\varepsilon_r^p}{d\varepsilon_\theta^p}=-\frac{1}{\beta} \tag{17}$$

where $\beta=(1+\sin\psi)/(1-\sin\psi)$, ψ =the dilatancy angle of geomaterial, $d\varepsilon_r^p$ and $d\varepsilon_\theta^p$ are the radial and tangential plastic strain increments, respectively.

Based on the plastic increment theory

$$\begin{aligned} &\frac{d\varepsilon_r}{d\varepsilon_\theta}=\frac{d\varepsilon_r^e+d\varepsilon_r^p}{d\varepsilon_\theta^e+d\varepsilon_\theta^p} \\ &\Rightarrow \frac{d\varepsilon_r^p}{d\varepsilon_\theta^p}=\frac{d\varepsilon_r-d\varepsilon_r^e}{d\varepsilon_\theta-d\varepsilon_\theta^e}=-\frac{1}{\beta} \end{aligned} \tag{18}$$

where ε_r^p and ε_θ^p are radial and tangential plastic strains, respectively.

Substituting elastic strain equation (Hooke's law) into the above Eq. (18)

$$\begin{aligned} &\beta d\varepsilon_r + d\varepsilon_\theta \\ &= \frac{1-\nu^2}{E}\left[\beta-\frac{\nu}{1-\nu}\right]d\sigma_r+\frac{1-\nu^2}{E}\left[1-\frac{\nu\beta}{1-\nu}\right]d\sigma_\theta \end{aligned} \tag{19}$$

Based on the initial stress condition, substituting Eqs. (13) and (15) into Eq. (18) leads to (Chadwick 1959, Yu and Houlsby 1991),

$$\begin{aligned} \beta\varepsilon_r+\varepsilon_\theta &= \frac{1-\nu^2}{E}\left[\beta-\frac{\nu}{1-\nu}\right]\sigma_r \\ &+\frac{1-\nu^2}{E}\left[1-\frac{\nu\beta}{1-\nu}\right]\sigma_\theta+\frac{1-\nu^2}{E}\left[\beta-\frac{\nu}{1-\nu}+1-\frac{\nu\beta}{1-\nu}\right] \end{aligned} \tag{20}$$

The logarithmic strain Eq. (6) is used for reflect the large strain characteristic of soil mass (Chadwick 1959, Yu and Houlsby 1991)

$$\begin{cases} \ln\left[\left(\frac{r}{r_0}\right)^{1/\beta}\frac{dr}{dr_0}\right]=\ln\chi-\xi\ell \\ \chi=\exp\left\{\frac{(\beta+1)(1-2\nu)(1+\nu)\left[(R-1)\alpha\frac{E\Delta T}{1-2\nu}-\sigma_c\right]}{E(R-1)\beta}\right\} \\ \quad =\exp\left\{\frac{(\beta+1)(1+\nu)\left[\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)\right]}{E(R-1)\beta}\right\} \\ \xi=\frac{2(1-\nu^2)}{(1+\nu)(R-1)\beta}\frac{(R-1)\alpha\frac{E\Delta T}{1-2\nu}-\sigma_c}{2(1+R)G}\left[R\beta+1-\frac{\nu(R+\beta)}{1-\nu}\right] \\ \quad =\frac{2(1-\nu^2)}{(1+\nu)(R-1)\beta}\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G}\left[R\beta+1-\frac{\nu(R+\beta)}{1-\nu}\right] \\ \ell=(r_b/r)^{(R-1)/R} \end{cases} \tag{21}$$

The Eq. (21) can be integrated over the interval (r_b, r), leading to

$$\begin{cases} \frac{\chi}{\gamma}\left\{(1-\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G})^{(\beta+1)/\beta}-(r_0/r_b)^{(\beta+1)/\beta}\right\} \\ =\int_1^{(r_b/r)^{(R-1)/R}}\exp(\xi\ell)\ell^{-\gamma-1}d\ell \\ \gamma=\frac{R(\beta+1)}{(R-1)\beta} \end{cases} \tag{22}$$

Putting $r_0=a_0, r=a$ and making use of Eq. (15)

$$\begin{aligned} &\frac{\chi}{\gamma}\left\{\begin{aligned}&(1-\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G})^{(\beta+1)/\beta} \\ &-(r_b/a)^{-(R(\beta+1))/\beta}(a_0/a)^{(\beta+1)/\beta}\end{aligned}\right\} \\ &=\int_1^{(r_b/a)^{(R-1)/R}}\exp(\xi\ell)\ell^{-\gamma-1}d\ell \end{aligned} \tag{23}$$

With the aid of the series expansion

$$\exp(\xi\ell)=\sum_{n=0}^{\infty}\frac{(\xi\ell)^n}{n!} \tag{24}$$

The explicit expression is given

$$\frac{a}{a_0}=\left\{\frac{(r_b/a)^{-(\beta+1)/\beta}}{-(\gamma/\chi)\Lambda_1(H_1,\xi)+\left(1-\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G}\right)^{(\beta+1)/\beta}}\right\} \tag{25}$$

where

$$\Lambda_1(H_1,\xi)=\sum_{n=0}^{\infty}\mathrm{A}_n^1 \tag{26}$$

in which

$$\mathrm{A}_n^1=\begin{cases}\dfrac{\xi^n}{n!}\ln H_1 & if \quad n=\gamma\\ \dfrac{\xi^n}{n!(n-\gamma)}\ln\left[H_1^{n-\gamma}-1\right] & n\neq\gamma\end{cases} \tag{27}$$

The relation of initial and current radii can be given as

$$r_0=\begin{bmatrix}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}\\ +\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\end{bmatrix}^{\beta/(\beta+1)}\cdot r_b \tag{28}$$

2.6.2 The radial and tangential strains

Based on the Eq. (28), it can also be obtained as

$$\begin{cases}\dfrac{r}{r_0}=\dfrac{r}{\begin{bmatrix}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}\\ +\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\end{bmatrix}^{\beta/(\beta+1)}\cdot r_b}\\ \varepsilon_\theta=-\ln\left(\dfrac{r}{r_0}\right)\\ =-\ln\left\{\dfrac{r}{\begin{bmatrix}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}\\ +\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\end{bmatrix}^{\beta/(\beta+1)}\cdot r_b}\right\}\end{cases} \tag{29}$$

The elastic strain and displacement at the $r=r_b$ are obtained

$$\begin{cases}\left.\varepsilon_r^e\right|_{r=rb}=\dfrac{1-\nu^2}{E}\left(1+\dfrac{\nu}{1-\nu}\right)\left(p_y-p_0\right)\\ =\dfrac{1+\nu}{E}\left(p_y-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ \left.\varepsilon_\theta^e\right|_{r=rb}=-\dfrac{1-\nu^2}{E}\left(1+\dfrac{\nu}{1-\nu}\right)\left(p_y-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ =-\dfrac{1+\nu}{E}\left(p_y-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ \left.u_r\right|_{r=rb}=\dfrac{1-\nu^2}{E}\left(1+\dfrac{\nu}{1-\nu}\right)\left(p_y-p_0\right)r_b\\ =\dfrac{1+\nu}{E}\left(p_y-\alpha\dfrac{E\Delta T}{1-2\nu}\right)r_b\end{cases} \tag{30}$$

The elastic strains in the plastic region were assumed to be constant and equal to the elastic strain at the $r = r_b$, it can be obtained

$$\begin{cases}\varepsilon_\theta=-\ln\left(\dfrac{r}{r_0}\right)=-\ln\left\{\dfrac{r}{\begin{bmatrix}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}\\ +\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\end{bmatrix}^{\beta/(\beta+1)}\cdot r_b}\right\}\\ \varepsilon_\theta^e=\left.\varepsilon_\theta^e\right|_{r=rb}=-\dfrac{1+\nu}{E}\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ \varepsilon_\theta^p=\varepsilon_\theta-\varepsilon_\theta^e\\ \varepsilon_\theta^p=-\ln\left\{\dfrac{r}{\left[-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}+\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\right]^{\beta/(\beta+1)}\cdot r_b}\right\}\\ +\dfrac{1+\nu}{E}\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ \varepsilon_r^p=-\dfrac{1}{\beta}\varepsilon_\theta^p=-\dfrac{1}{\beta}\left(\varepsilon_\theta-\varepsilon_\theta^e\right)\\ \varepsilon_r^p=-\dfrac{1}{\beta}\left\{\begin{array}{l}-\ln\left\{\dfrac{r}{\begin{bmatrix}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}\\ +\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\end{bmatrix}^{\beta/(\beta+1)}\cdot r_b}\right\}\\ +\dfrac{1+\nu}{E}\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\end{array}\right\}\\ \varepsilon_r^e=\left.\varepsilon_r^e\right|_{r=rb}=\dfrac{1+\nu}{E}\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ \varepsilon_r=\varepsilon_r^e+\varepsilon_r^p\\ \varepsilon_r=\dfrac{1+\nu}{E}\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\\ -\dfrac{1}{\beta}\left\{\begin{array}{l}-\ln\left\{\dfrac{r}{\begin{bmatrix}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\dfrac{\gamma}{\chi}\\ +\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\end{bmatrix}^{\beta/(\beta+1)}\cdot r_b}\right\}\\ +\dfrac{1+\nu}{E}\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}-\alpha\dfrac{E\Delta T}{1-2\nu}\right)\end{array}\right\}\end{cases} \tag{31}$$

2.6.3 Limit expanding pressure

The stress at the $r=a$ can be expressed as,

$$\begin{cases}\sigma_r\left(r=a\right)=\sigma_a=p\\ p=\dfrac{\left(\dfrac{2p_0-\sigma_c}{1+R}\right)\left(R-1\right)+\sigma_c}{R-1}\left(\dfrac{a}{r_b}\right)^{R-1}-\dfrac{\sigma_c}{R-1}\\ p_u=\dfrac{\left(\dfrac{2p_0-\sigma_c}{1+R}\right)\left(R-1\right)+\sigma_c}{R-1}\left(\dfrac{a_u}{r_b}\right)^{R-1}-\dfrac{\sigma_c}{R-1}\end{cases} \tag{32}$$

Consider the pressure-expansion relation (Eq. (25)),

$$\begin{aligned}p&=\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}+\dfrac{1}{R-1}\sigma_c\right)\left(\dfrac{a}{r_b}\right)^{\frac{R-1}{R}}-\dfrac{1}{R-1}\sigma_c\\ &=\left(\dfrac{2R\alpha E\Delta T+\sigma_c\left(1-2\nu\right)}{\left(1-2\nu\right)\left(1+R\right)}+\dfrac{1}{R-1}\sigma_c\right)\cdot\\ &\left(\left\{\begin{bmatrix}\left(1-\dfrac{\alpha E\Delta T\left(R-1\right)-\sigma_c\left(1-2\nu\right)}{2(1+R)\left(1-2\nu\right)G}\right)^{(\beta+1)/\beta}\\ -\left(\gamma/\chi\right)\Lambda_1\left(H_1,\xi\right)\end{bmatrix}\left(\dfrac{a}{a_0}\right)^{(\beta+1)/\beta}\right\}^{\beta/(\beta+1)}\right)^{\frac{R-1}{R}}-\dfrac{1}{R-1}\sigma_c\end{aligned} \tag{33}$$

Combining Eq. (7) and (33) gives

$$p_u=\left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)\left(\frac{a_u}{r_b}\right)^{\frac{R-1}{R}}-\frac{1}{R-1}\sigma_c$$
$$=\left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)\cdot$$
$$\left(\left\{\left[\left(1-\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G}\right)^{(\beta+1)/\beta}-(\gamma/\chi)\Lambda_1(H_1,\xi)\right]\left(\frac{a_u}{a_0}\right)^{(\beta+1)/\beta}\right\}^{\beta/(\beta+1)}\right)^{\frac{R-1}{R}}-\frac{1}{R-1}\sigma_c \quad (34)$$

2.6.4 Energy dissipation analysis

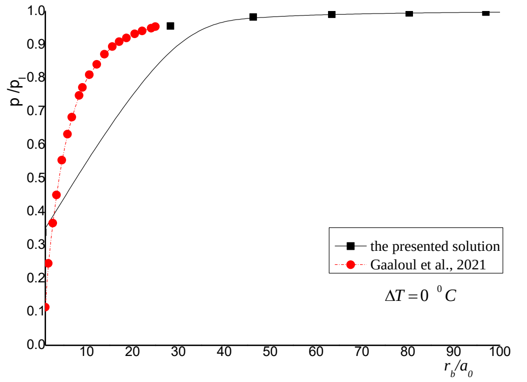
The dissipated energy in the plastic region is given as

$$U_p=\iiint_v\frac{1}{2}\left(\sigma_r^p\varepsilon_r^p+\sigma_\theta^p\varepsilon_\theta^p\right)dV-CM\Delta T$$
$$=2\pi\int_a^{rb}\frac{1}{2}\left(\begin{array}{l}\left(\left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)\left(\frac{r_b}{r}\right)^{\frac{R-1}{R}}+\frac{\sigma_c}{1-R}\right)\cdot\\ \left\{\frac{-1}{\beta}\left\{\begin{array}{l}-\ln\left\{\frac{r}{\left[\begin{array}{l}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\frac{\gamma}{\chi}\\ +\left(1-\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G}\right)^{(\beta+)/\beta}\end{array}\right]^{\beta/(\beta+1)}\cdot r_b}\right\}\\ +\frac{1-\nu^2}{E}\left(1+\frac{\nu}{1-\nu}\right)\left(p_y-\alpha\frac{E\Delta T}{1-2\nu}\right)\end{array}\right\}\right\}\\ +\left(\frac{1}{R}\left(\left(\frac{2R\alpha E\Delta T+\sigma_c(1-2\nu)}{(1-2\nu)(1+R)}+\frac{1}{R-1}\sigma_c\right)\left(\frac{r_b}{r}\right)^{\frac{R-1}{R}}+\frac{\sigma_c}{1-R}-\sigma_c\right)\right)\cdot\\ \left\{\begin{array}{l}-\ln\left\{\frac{r}{\left[\begin{array}{l}-\int_1^{\ell}\exp(\xi\ell)\ell^{-\gamma-1}d\ell\cdot\frac{\gamma}{\chi}\\ +\left(1-\frac{\alpha E\Delta T(R-1)-\sigma_c(1-2\nu)}{2(1+R)(1-2\nu)G}\right)^{(\beta+1)/\beta}\end{array}\right]^{\beta/(\beta+1)}\cdot r_b}\right\}\\ +\frac{1-\nu^2}{E}\left(1+\frac{\nu}{1-\nu}\right)\left(p_y-\alpha\frac{E\Delta T}{1-2\nu}\right)\end{array}\right\}\end{array}\right)rdr \quad (35)$$

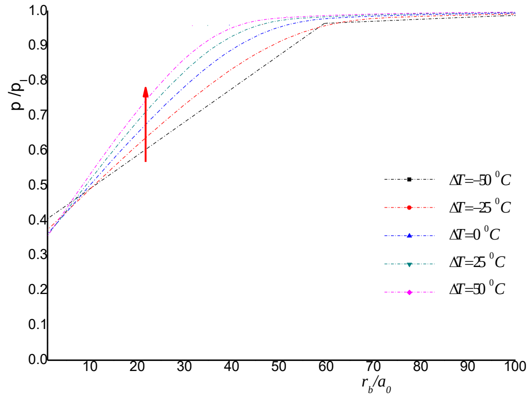
3. Validation and parametric study

3.1 Typical pressure-expansion curves

To verify the suitability of the theoretical solution, the presented theoretical solution is compared with Gaaloul *et al.* (2021), the value of parameters of soil mass are taken from Gaaloul *et al.* (2021), c = 10 *kPa*, the Poisson's ratio ν = 0.33, the internal friction angle φ = 12º and the dilation angle ψ = 12º, E = 18 *Mpa*, $\alpha = 10^{-5}\,{}^0C^{-1}$. Fig. 2 shows the relationship of the non-dimensional expansion pressure and the ratio r_b/a_0. As shown in Fig. 2(b), the non-dimensional expansion pressure increases with the increase of the temperature variation ΔT. In Fig. 2(a), with the increase of a/a_0, the non-dimensional expansion pressure increases quickly from the start, and afterward the expansion pressure is asymptotically close to the limit pressure. In addition, it is shown in Fig. 2(a) that the non-dimensional expansion pressure of the presented solution is approximately equal to the data from Gaaloul *et al.* (2021).



(a)



(b)

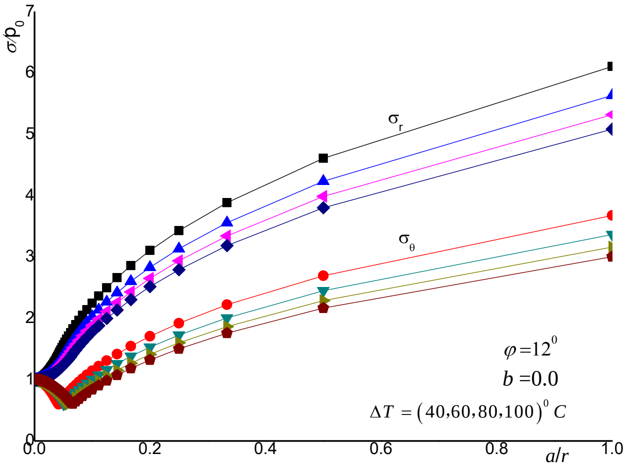
Fig. 2 (a) Dimensionless pressure versus dimensionless plastic radius and (b) Dimensionless pressure versus dimensionless plastic radius

3.2 Stress distribution around cavity

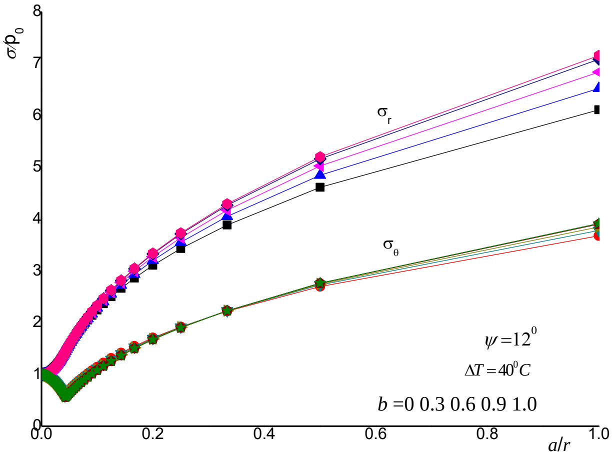
Fig. 3 shows the relationship of stress distribution (σ_r , σ_θ) around cylindrical cavity and the ratio a/r with different values of the coefficient b, the temperature variation and the dilation angle It can be seen from the Fig. 3 that the coefficient b, the temperature variation ΔT and the dilation angle have obvious influence on the stress distribution (σ_r , σ_θ) around cylindrical cavity. Among them, the dilation angle has the most obvious influence on the stress distribution (σ_r , σ_θ) around cylindrical cavity, while the coefficient b has the least influence on the stress distribution (σ_r , σ_θ) around cylindrical cavity. In addition, the influence of coefficient b on the σ_θ stress is limited, similar to the research trend in Zhao *et al.* (2017). At the same time, it can be seen that with the decrease of a/r, the influence of the coefficient b, temperature variation ΔT and dilation angle on radial and tangential stresses become smaller and smaller until it can be ignored. In conclusion, the effects of the coefficient b, temperature variation ΔT and dilation angle on the stress distribution (σ_r , σ_θ) around cylindrical cavity cannot be ignored.

3.3 Energy dissipation analysis in the plastic region

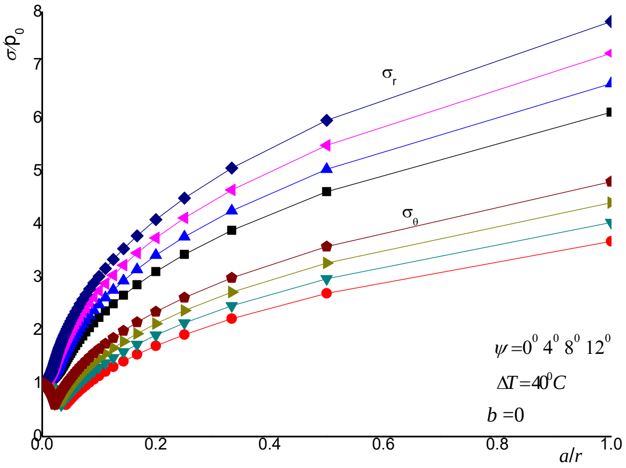
Fig. 4 shows the relationship of dissipated energy around cylindrical cavity and the ratio a/a_0 with different



(a)



(b)



(c)

Fig. 3 (a) Stress distribution around cylindrical cavity with different values of the temperature variation, (b) Stress distribution around cavity with different values of *b* and (c) Stress distribution around cavity with different values of the dilation angle

temperature variation ΔT, the dissipated energy in the plastic region increases with the increase of the coefficient temperature variation ΔT. It can be seen from Fig. 4 that the dissipated energy around the cylindrical cavity is

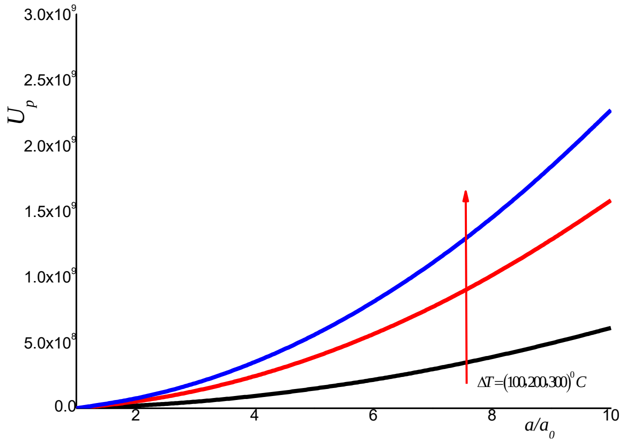


Fig. 4 The relationship of dissipated energy in the plastic region and the a/a_0 with different temperature variation

affected by the temperature variation ΔT and cannot be ignored. Without considering the kinetic energy, most of the work done by the external force is converted into energy and heat in the plastic region, and the similar to the research trend in Li and Mo (2022). From the obvious thermo-mechanical behavior of soil mass in above study, it can be seen that the influence of temperature should be given attention in civil engineering. In geotechnical engineering problems, it is increasingly important to consider the influence of thermal loads, especially in the applications of energetic geostructures and nuclear waste repositories, where the mechanical properties of soil mass are strongly influenced by temperature changes.

4. Conclusions

On the basis of the UST and the first law of thermodynamics, an energy dissipation analysis for large-strain cylindrical cavity expansion in compaction grouting problem under temperature field is described in this study. Compared with previous solutions, the following improvements have been achieved:

(1) Considering the stress increment caused by temperature, the analytical solution of cavity expansion under non-isothermal condition is improved by substituting the temperature stress increment into the cavity expansion analysis.

(2) Combined with the first law of thermodynamics, the energy theory is also introduced into the cylindrical cavity expansion analysis, and the energy dissipation solution of cylindrical cavity expansion is derived.

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Data availability statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

Conflicts of interests/Competing interests: No conflict of interest exits in the submission of this manuscript, and manuscript entitled "An analytical solution for compaction grouting problem considering exothermic temperature effect of slurry" is approved by all authors for publication. I would like to declare on behalf of my co-authors that the work described was original research that has not been published previously, and not under consideration for publication elsewhere, in whole or in part.

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