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Unidirectional cyclic shearing of sands: Evaluation of three different constitutive models

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Abstract. Advanced nonlinear effective stress constitutive models are started to be frequently used in one-dimensional (1D) and two-dimensional (2D) site response analysis for assessment of porewater generation and liquefaction potential in soft soil deposits. The emphasis of this research is on the assessment of the implementation of this category of models at the element stage. Initially, the performance of a coupled porewater pressure (PWP) and constitutive models were evaluated employing a catalogue of 40 unidirectional cyclic simple shear tests with a variety of relative densities between 35% and 80% and effective vertical stresses between 40 and 80 kPa. The authors evaluated three coupled constitutive models (PDMY02, PM4SAND and PDMY03) using cyclic direct simple shear tests and for decide input parameters used in the model, procedures are recommended. The ability of the coupled model to capture dilation as strength is valuable because the studied models reasonably capture the cyclic performance noted in the experiments and should be utilized to conduct effective stress-based 1D and 2D site response analysis. Sandy soils may become softer and liquefy during earthquakes as a result of pore-water pressure (PWP) development, which may have an impact on seismic design and site response. The tested constitutive models are mathematically coupled with a cyclic strain-based PWP generation model and can capture small-strain stiffness and large-strain shear strength. Results show that there are minor discrepancies between measured and computed excess PWP ratios, indicating that the tested constitutive models provide reasonable estimations of PWP increase during cyclic shear (r_u) and the banana shape is reproduced in a proper way indicating that dilation and shear- strain behavior is well captured by the models.

Keywords: calibration parameters; coupled constitutive models; element test simulation

1. Introduction

Liquefaction has been caused considerable losses during earthquakes and continues generating significant effects to the developed construction environment. Substantial improvements have been made during last four decades trying to understand and predict liquefaction triggering and the produced effects for this phenomenon using different resources such as laboratory tests, advanced numerical modeling, and case history observations (Ramirez *et al.* 2018). Entirely coupled nonlinear elastoplastic site response analyses performs consistent numerical estimation of liquefaction triggering and effects of the liquefaction incidents. The proposed advanced models have the advantage to catch soil comportment under significant shear strains (e.g., predisposition for excess porewater pressure generation followed by process of dilation), redistribution and dissipation of excess pore pressure that one dimensional site response analysis using semi coupled PWP generation and hyperbolic constitutive model based on [Moreno-Torres *et al.* (2010), Moreno-Torres *et al.* (2018)A, Moreno-Torres *et al*. (2018), Mendoza-Bolaños *et al.* (2023), DEEPSOIL (Hashash 2017) and DMOD2000 (Matasovic and Ordonez 2007)] does not capture in a proper manner.

Coupled PWP generation and nonlinear elastoplastic soil constitutive models have been established in the last 20+ years to replicate the seismic response of saturated sand under dynamic loading (e.g., Prevost 1985, Finn 1986, Byrne 1991, Jefferies 1993, Beaty and Byrne 1998, Elgamal *et al.* 1998, Kramer and Arduino 1999, Yang *et al.* 2003, Dafalias and Manzari 2004, Ziotopoulou and Boulanger 2015). The advanced coupled constitutive model ability and its predictions have often been tested using one element-level laboratory tests and/or centrifuge model results in a developed software, using a unique uniform sand layer with different geometries and compositions. Ziotopoulou and Boulanger (2015) stated that at the element level the examined constitutive models are able to reach outcomes contradictory with identified soil behavior for many practical loading circumstances. Ramirez *et al*. (2018) stated that a logical estimation of the competences and restrictions of different computer programs and soil constitutive models used in describing different characteristics of site response performance can be analyzed using excess pore pressures, accelerations, and settlements considering similar conditions that are valued in growing recent understanding of the precision of the numerical models, and the knowledge that can be learned from the

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numerical models and software in view of stress – strain behavior. Various advanced constitutive models based on mechanics that accurately define cyclic soil performance have been suggested (e.g., Elgamal *et al*. 2002, Dafalias and Manzari 2004, Iai and Ozutsumi 2005, Yang *et al.* 2008, Beaty and Byrne 2011, Boulanger and Ziotopoulou 2012, Wang *et al*. 2014, Wichtmann *et al.* 2019, as well as others). Although these sophisticated models allow for the replication of universal soil behavior (including dilatation) and are more adaptable than a hyperbolic constitutive model. Due to the difficulty of estimation and adjustment of the model considerations and the potential necessity for more sophisticated laboratory testing, there is frequently a significant dependence on default settings. These models require precise estimation and adjustment of various parameters to match specific soil behaviors under different conditions, often necessitating sophisticated laboratory testing. However, due to the complexity of this process and the potential need for advanced testing, practitioners sometimes resort to default settings provided with the models. This reliance on defaults can compromise the accuracy of predictions for soil behavior in specific scenarios, reflecting the delicate balance between the sophistication of the models and the practical challenges of calibration in geotechnical engineering and related fields.

It is compared and evaluated several advanced constitutive models used in earthquake engineering applications. These models have been developed to simulate the behavior of sandy soils during seismic events, including liquefaction triggering, postliquefaction response, and large shear deformation. The models under consideration are: PDMY03, PM4Sand, CycLiqCPSP, SANISAND, Constitutive model for cyclic mobility (PDMY02), UBCSand, Nor – Sand and ISA (Intergranular strain anisotropy).

The discussion will focus on comparing the models based on their complexity, parameters considered, and simulation capabilities. It is assessed the strengths and limitations of each model, highlighting their unique features and applications. By understanding the characteristics of these models, we can make informed decisions about their suitability for specific engineering analyses. The comparison is based on several key aspects, including model complexity, considered parameters, and simulation capabilities.

1. Nor-Sand is a simple critical state model designed to simulate the behavior of sandy soils:

- It considers parameters such as effective stress, relative density, and cyclic loading history.
- The model is based on the critical state theory of soils, which aims to predict the soil's behavior under seismic loads.
- Nor-Sand is particularly useful for evaluating soil stability and bearing capacity during and after earthquakes.
- As a simple critical state model, Nor-Sand may not capture the complex behavior of sandy soils under all loading conditions. It may lack the ability to represent more intricate phenomena, leading to limitations in accurately predicting soil responses.

2. UBCSand constitutive model:

- Widely employed model for simulating the behavior of sandy soil under cyclic loading.
- Considers parameters such as effective stress, shear strength, relative density, and cyclic loading history.
- Accounts for the influence of pore pressure generated during cyclic loading on soil behavior.
- Capable of simulating both soil liquefaction and post-liquefaction deformation, making it suitable for assessing soil response during and after earthquakes.
- While the UBCSand model may provide reasonable predictions for some sandy soil behaviors, it might not fully account for other aspects such as cyclic mobility and post-liquefaction response, limiting its applicability in certain scenarios.

3. Constitutive model for cyclic mobility:

- Focuses on simulating cyclic mobility of soil during seismic events, a critical phenomenon in seismic engineering.
- Parameters considered include effective stress, shear strength, relative density, and cyclic loading history.
- Provides insights into soil displacements and deformations during seismic events, enabling assessment of structural safety and stability.
- This model may focus mainly on cyclic mobility and might not adequately address other important aspects of sandy soil behavior, potentially leading to incomplete predictions and limited practical use.

4. SANISAND: Simple plasticity sand model accounting for fabric change:

- Simple yet effective plasticity model employed for simulating sandy soil behavior.
- Parameters considered include effective stress, cyclic loading, variation of shear strength with relative density, and changes in the soil's internal structure (fabric change).
- Versatile model capable of simulating both soil liquefaction and post-liquefaction deformation, making it valuable for studying various aspects of soil behavior during and after earthquakes.
- While SANISAND is a simple plasticity model designed to account for fabric change in sandy soils, it might oversimplify the complexities of soil behavior, potentially resulting in less accurate predictions for certain conditions.

5. CycLiqCPSP: A unified plasticity model for large post-liquefaction shear deformation of sand:

- Focuses on significant shear deformation occurring after soil liquefaction.
- Utilizes a unified plasticity formulation considering shear strength, dilatancy, compressibility, and cyclic

loading.

- Incorporates cyclic loading history and pore pressure generation during liquefaction, allowing more accurate simulation of post-liquefaction deformation.
- While CycLiqCPSP offers a unified plasticity model for large post-liquefaction shear deformation of sand, it might not fully capture the subtleties of other soil responses and could be limited in its ability to predict certain behaviors.

6. PM4Sand: Sand plasticity model for earthquake engineering applications:

- Widely utilized model capable of simulating the behavior of sandy soils during seismic events.
- Considered parameters encompass effective stress, relative density, cyclic loading history, and variation of shear strength with relative density.
- Accounts for stress redistribution in the soil due to seismic loading, enabling more precise simulation of soil behavior during earthquakes.
- Although the PM4Sand model is specifically designed for earthquake engineering applications, it may have limitations in representing the full range of complex soil behaviors, leading to potential inaccuracies in certain scenarios.

7. PDMY 03: A 3D model for earthquake-induced liquefaction triggering and postliquefaction response:

- Highly sophisticated model employing a three-dimensional formulation to simulate soil liquefaction during earthquakes and subsequent response.
- Considered parameters include effective stress, shear strength, relative density, and cyclic loading history.
- Three-dimensional formulation enables a more realistic simulation of soil response to liquefaction.
- While PDMY03 is a 3D model for earthquake-induced liquefaction triggering and post-liquefaction response, it may have computational complexity and data requirements that could be challenging for some engineering applications.

8. ISA: Intergranular strain anisotropy:

- The model considers the anisotropic behavior of granular soils, which is crucial for accurately capturing the deformation response of soils under different stress paths. Anisotropy is particularly important in geotechnical engineering, where soils can experience varying stress conditions.
- The model's incorporation of anisotropy provides an advantage over conventional models that assume isotropic behavior, especially when the stress state significantly differs from a triaxial test condition. This can lead to more accurate predictions of soil responses.
- The consideration of intergranular strain anisotropy adds complexity to the model formulation and implementation. This complexity may require a deeper understanding of soil mechanics and constitutive modeling, making it potentially more challenging to apply compared to simpler models.

In summary, each model discussed in this analysis offers unique features and applications. The selection of the appropriate model depends on factors such as the desired level of complexity, the parameters to be considered, and the specific objectives of the analysis. Evaluating the advantages and limitations of each model is crucial for selecting the most suitable one for a given study.

Several porewater pressure models have been proposed to predict increases in porewater pore pressure (PWP) during cyclic loading. In many models, excess PWP is separated into two components: transient and residual. Transient PWP is approximately equal to the changes in applied mean normal stresses resulting from dynamic loading (Scott 1963, Lambe and Whitman 1991), and therefore have little influence on the effective stresses acting on the soil. Residual excess PWP, on the other hand, results from the progressive collapse of the soil skeleton (i.e., plastic deformations) (Green 2001), and greatly influence effective stress and soil response. PWP generation models such as those proposed by Seed *et al.* (1975), Dobry *et al.* (1985), Vucetic (1986), Green *et al.* (2000), Ivsic (2006), Polito *et al.* (2008), Park and Ahn (2013) and Chiaradonna *et al*. (2018) incorporate both components.

The discussion of porewater pressure models is concentrated on performance to reproduce PWP buildup. The comparison is based on simulation capabilities and accuracy of the model.

1. Seed *et al.* (1975) Porewater Pressure Model is specifically used to predict the build-up of excess pore water pressure during cyclic loading events, which can lead to soil liquefaction and associated ground failures, such as settlement, tilting, or even sliding:

- The PWP model is essential for assessing the liquefaction potential of soils, which is critical for the safe design of foundations and other structures in seismically active regions.
- It provides a predictive capability for assessing the vulnerability of soils to liquefaction during seismic events. This allows engineers to identify high-risk areas and take appropriate mitigation measures.
- The model incorporates essential soil parameters, such as the cyclic stress ratio (CSR), the initial effective stress state of the soil, and the number of loading cycles. These parameters are measurable and can be determined through laboratory testing and site investigation.
- The accuracy of predictions using the PWP model depends on the quality and reliability of input data and parameters. Errors in these inputs can lead to inaccurate assessments of liquefaction potential.
- The PWP model is primarily applicable to loose, saturated, and non-cohesive soils (typically sands and silts). It may not be suitable for cohesive soils (clays) or other soil types.

2. Dobry *et al*. (1985) Porewater Pressure Model is specifically used to predict the build-up of excess pore water pressure during cyclic loading events, which can lead to soil liquefaction and associated ground failures:

   - The PWP model is considered an improvement over earlier models, as it provides a more accurate and refined prediction of pore water pressure generation during cyclic loading. It is based on extensive laboratory and field data, enhancing its predictive capability.
   - The model can be used for site-specific analysis, allowing engineers to assess liquefaction hazards at specific locations, considering unique soil properties, earthquake characteristics, and the geometry of the problem.
   - The PWP model incorporates essential soil parameters, such as the cyclic stress ratio (CSR), the initial effective stress state of the soil, and the number of loading cycles. These parameters can be determined through laboratory testing and site investigation, making it applicable in practice.
   - Similar to other liquefaction models, the PWP model is based on simplifying assumptions about soil behavior during cyclic loading. These assumptions may not fully capture the complexity of real-world soil behavior.
   - The accuracy of predictions using the PWP model is sensitive to the quality and reliability of input data and parameters. Errors in these inputs can lead to inaccurate assessments of liquefaction potential.

3. Vucetic *et al*. (1986) Porewater Pressure Model focuses on the evaluation of liquefaction potential in sandy soils during seismic events:

   - The Vucetic and Dobry (1986) method provides a well-established and widely used approach for assessing liquefaction potential in sandy soils. It has been applied to numerous geotechnical engineering projects and has contributed to our understanding of soil behavior during earthquakes.
   - The method offers a relatively simple and practical way to estimate liquefaction potential using commonly available soil and seismic data. This makes it accessible for engineers and researchers.
   - The methodology is based on extensive empirical data and field observations, which lend credibility to its use in practice. It has been calibrated and validated against real-world liquefaction events.
   - The method is primarily applicable to sandy soils and may not be suitable for other types of soils, such as silts or clays. Different methodologies are required for those soil types.
   - It may not account for all variations in soil properties and seismic conditions within a specific site, leading to potential inaccuracies in some cases.

4. Green *et al*. (2000) Porewater Pressure Model aims to provide a more physically meaningful representation of excess pore pressure generation during seismic events:

   - Energy-based models are grounded in fundamental principles of soil mechanics and wave propagation. They provide a physically sound representation of how seismic energy is converted into excess pore pressure within cohesionless soils.
   - These models can offer more accurate predictions of excess pore pressures compared to empirical or simplified methods. They consider the energy imparted by seismic waves to the soil, which can lead to more reliable results.
   - Energy-based models are generally consistent with established principles of soil behavior and do not rely heavily on site-specific calibration. This consistency can make them applicable to a wider range of scenarios and soil types.
   - Accurate application of energy-based models requires extensive geotechnical data, including soil properties, seismic data, and site-specific information. Data availability and quality can be limitations, particularly in some regions.
   - Proper calibration and validation of energy-based models against real-world observations are essential to ensure their accuracy. This process can be time-consuming and may require field testing.

5. Ivsic (2006) Porewater Pressure Model is a simple empirical model that relates the pore pressure ratio to a damage parameter, which represents the accumulated shear stress normalized by the initial effective stress:

   - The model can capture the cumulative nature and dependence on strain level of the pore pressure generation.
   - The model can be used to present data of different cyclic soil tests in a comparable manner.
   - The model can be calibrated using the CSR-N curve measured from a stress-controlled test, which is widely available and easy to obtain.
   - The model does not account for the effects of soil type, relative density, fines content, confining pressure, particle crushing, thermal effects, and strain rate on the pore pressure generation, which may be significant in some cases.
   - The model may not be applicable to soils with different characteristics, such as clayey or cemented soils, or soils with different stress histories.

6. Polito *et al*. (2008) Porewater Pressure Model is based on the concept of energy dissipation within the soil:

   - The models can capture the effect of irregular loading cycles on the pore pressure generation, which may not be sufficiently taken into account by the conventional stress-based models.
   - The models can be applied to both triaxial and simple shear loading conditions, as well as to different types of sands and silty soils.
   - The models can be calibrated using simple parameters

that can be obtained from laboratory tests or empirical correlations.

- The models may not be able to account for the effects of cyclic shear strain, particle crushing, thermal effects, and strain rate on the pore pressure generation, which may be significant in some cases.
- The models may require additional validation and refinement using more experimental data and field observations.

7. Park and Ahn (2013) Porewater Pressure Model is a simple empirical model that predicts the build-up of residual pore pressure in sands and silts during seismic loading:

- The model can be easily implemented in a time-domain analysis program since it does not require a priori definition of the damage parameter.
- The model can accurately capture the pore pressure generation under various loading conditions, such as different cyclic stress ratios, number of cycles, and strain levels.
- The model can account for the effects of relative density, fines content, and confining pressure on the pore pressure generation.
- The model does not consider the effects of cyclic shear strain, which has been shown to be a more accurate predictor of pore pressure generation than cyclic shear stress.
- The model does not account for the effects of particle crushing, thermal effects, and strain rate on the pore pressure generation, which may be significant in some cases.

8. Chiaradonna *et al.* (2018) Porewater Pressure Model is used to simulate the seismic response of well-documented reclaimed sites where widespread liquefaction occurred, showing that it leads to a good estimate of the site response:

- The model is based on empirical correlations with in-situ tests, which makes it easy to calibrate.
- The calibration procedure is based on the results of in-situ tests, which makes it more accurate.
- The model has been used to simulate the seismic response of well-documented reclaimed sites where widespread liquefaction occurred, showing that it leads to a good estimate of the site response.
- The model is only suitable for predicting pore water pressure build-up in 1D effective stress dynamic analyses.
- The model is based on empirical correlations with in-situ tests, which may not be applicable to all soil types.

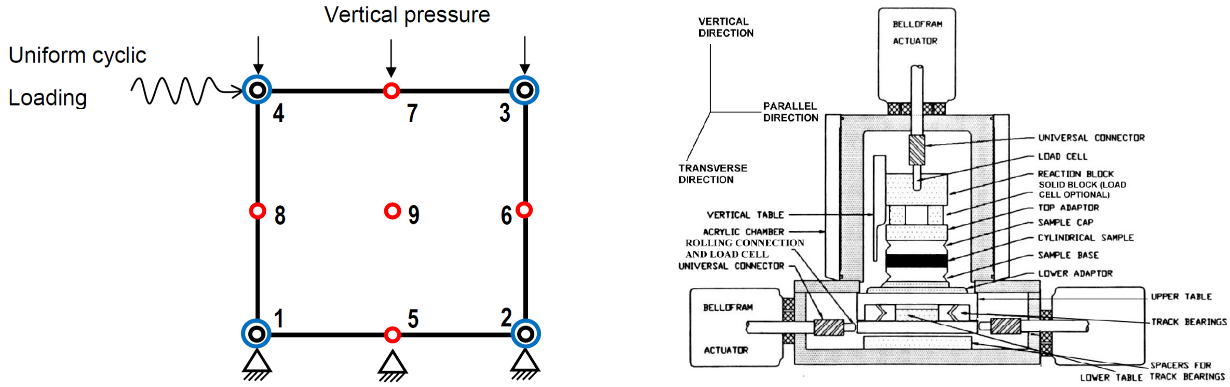
One of the tests that has been used in laboratory to study dynamic soil behavior is the direct simple shear (DSS) test (Bulent-Sonmezer 2019, Bulent-Sonmezer 2019b, Bulent-Sonmezer 2020, Nong *et al*. 2021). The DSS has been selected when a near arrangement models the plane strain situation and the rotation of principal stresses in soils (Boulanger *et al*. 1993). The mentioned test, one and only component of horizontal shear stress with harmonic cycles was utilized on the specimen during the cyclic system. It can be use or not static horizontal shear stress superimposed in the same direction as the initial compression shear stress, the unidirectional shear mode is utilized to mimic the outcome of soils exposed to one-dimensional propagation of shear wave. Nevertheless, in the field, the shear wave propagation is in multidirectional fashion and the DSS test was modified to consider multidirectional loading (Kammerer 2004, Rutherford 2012, Matsuda *et al*. 2016).

The results from simple shear testing and cyclic triaxial testing for pore pressure generation are not always comparable, as they may depend on several factors such as soil type, density, stress level, loading condition, and testing device. Different models have been proposed to predict the pore pressure ratio based on different parameters such as cyclic shear stress, cyclic shear strain, or energy dissipation. However, these models may not be always compatible with each other or with the experimental data, especially when comparing the results from simple shear testing and cyclic triaxial testing. Therefore, some correction or calibration may be needed to make the results from different testing methods comparable, as suggested by some recent studies such as Khashila *et al*. (2021), Nong *et al*. (2021), Viana da Fonseca *et al.* (2023).

Nonlinear soil-fluid, fully-coupled effective stress dynamic analyses were carried out in this study using the finite-element platform Open-Source Earthquake Engineering Simulator (Open Sees, version 3.0.3). The pressure - dependent - multi yield version 02 (PDMY02) soil model developed and implemented in OpenSees by Yang *et al*. (2003) (PDMY02) was used with the 9-4 quad UP element; the improved bounding surface constitutive model for sand (PM4SAND) plasticity model developed and implemented in OpenSees by Chen & Arduino (2018) was used with the 9-4 quad UP element, Khosravifar et al. (2018) developed and implemented in OpenSees a 3D model for earthquake-induced liquefaction triggering and post-liquefaction response (PDMY03) with 9-4 quad UP element.

40 cyclic simple shear stress controlled (cySS) tests, for which the raw data is available, were gathered for this evaluation from Wu *et al.* (2002). To define the replication of the PWP and Stress-Strain behavior from the test findings, the calibration of the primary parameters was completed in order to streamline evaluations. In order to determine the most advanced coupled PWP generation and constitutive model in terms of PWP, the coupled PWP generation and constitutive models were evaluated using the laboratory test data and statistics (residuals).

Finally, the combined PWP generation and constitutive models contrasted the shear stress-shear strain behavior anticipated with the behavior seen in 40 cyclic simple shear laboratory experiments. The most efficient linked PWP generation and constitutive model was determined using statistics (residuals) comparing measured and anticipated shear stresses (compared when shear strain = 0). In conclusion, this work examines and quantifies how well



(a) (b)

Fig. 1 (a) 9-4 Quad UP element configuration for stress – controlled cyclic simple shear simulation. (Mandokhail *et al.* 2017) and (b) Cross-section of the bi-directional simple shear apparatus where the samples were tested (Wu *et al*. 2002)

three coupled PWP generation and constitutive models capture the dynamic response of an element in terms of excess pore pressures and stress-strain behavior. The study's primary contribution is the calibration and application of three advanced constitutive models to evaluate the liquefaction resistance of sands under simple shear loading.

2. Coupled porewater pressure generation and constitutive models

A brief description of the selected models is provided in the following lines, but a more detailed discussion of them can be found in the references listed in each model description for a more thorough understanding of the considered coupled PWP generation and constitutive model used in this study.

- PDMY02: Pressure Dependent Multi Yield 02 is an elastic - plastic material model designed to reproduce the response of granular materials that are pressure sensitive during the loading process. PDMY02 was initially developed to replicate liquefaction and cyclic mobility as described by Yang *et al.* (2003). To simulate cyclic loading, the PDMY02 constitutive material model employs a series of nested yield surfaces. The model is distinguished by the absence of the critical state soil mechanic context formulation. As a result, different parameters must be calculated for different initial void ratio conditions (Carey and Kutter 2017). Yang *et al.* coded the model in OpenSees. The cySS tests are simulated using a single 9-4 Quad UP (Fig. 1(a)) element with stress controlled loading and the actual arrangement used in the DSS laboratory tests (Fig. 1(b)).
- PM4Sand: The critical state and stress - ratio - controlled structure developed by Dafalias and Manzari (2004) was used as a starting point for the model. This framework was modified to improve model stress-dilatancy relationships, empirical modulus reduction curves, and fabric development characterization during the liquefaction process (Boulanger and Ziotopoulou 2017). PM4Sand can only reproduce 2-D stress conditions, and the model's creator has no intention of reproducing solicitations that require 3-D stress conditions. The developed formulation was created to represent a 2D stress tensor condition (Carey and Kutter 2017). The cySS tests are simulated using a single 9-4 Quad UP (Fig. 1(a)) element with stress controlled loading and the actual arrangement used in the DSS laboratory tests (Fig. 1(b)).
- PDMY03: Pressure Dependent Multi Yield 03 is an updated version of the PDMY02 model that includes additional equations to comply with recognized procedures on the dependence of liquefaction triggering on the number of loading cycles, effective overburden stress (K_σ), and static shear stress (K_α) because those factors are controlled by input parameters that allow the constitutive model to produce realistic relationships between CRR and the number of uniform loading cycles (Khosravifar *et al*. 2018). Khosravifar *et al*. (2018) coded the model in OpenSees. The cySS tests are simulated using a single 9-4 Quad UP (Fig. 1(a)) element with stress controlled loading and the actual arrangement used in the DSS laboratory tests (Fig. 1(b)).

To simulate a cyclic direct simple shear test in the software OpenSees, it was followed the next steps: a.) Define the material properties of the soil element using the model specific parameters. b.) Define the soil element using the element command. c.) Define the boundary conditions for the soil element using the fix and equalDOF commands. d.) Define the cyclic loading protocol using the timeSeries, pattern, and loadPattern commands. e.) Define the analysis parameters using the constraints, system, and numberer commands and f.) Run the analysis using the analyze command.

3. Evaluation of advanced coupled PWP generation and constitutive models

The evaluation process involves: a-compiling cyclic laboratory tests; b-calibrating constitutive model parameters; and c-validating the constitutive model using model parameters calibrated directly.

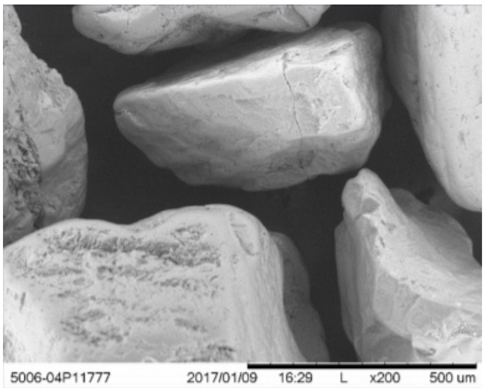


Fig. 2 SEM images of Monterey 0/30 sand reported by Beyzaei *et al*. (2015)

3.1 Compiled cyclic direct simple shear laboratory tests

Compiled cyclic unidirectional loading direct simple shear laboratory tests (40 cySS test, Table 1) performed sitting on Monterey 0/30 sand (Wu et al., 2002) were collected. These tests were chosen because entire computational stress-strain-porewater pressure time histories were accessible.

With medium- to fine-grained sand that ranges in shape from subangular to subrounded and 98% by weight held in the middle of the No. 20 and No. 100 sieves, Monterey 0/30 sand is practical and attainable (Figs. 2 and 3). The mineralogy is quite similar to Monterey No. 0 sand, which is clean quartz beach sand with trace amounts of feldspar and mica and has a gradation similar to that used in earlier liquefaction analyses (Wu *et al*. 2002). The employed sand had a specific gravity of 2.64, a minimum and maximum void ratio of 0.541 and 0.855, and all samples were set up using the moist tamping technique (Wu *et al*. 2002).

The calibration process of a constitutive model usually needs concessions because those constitutive models have restrictions and cannot replicate all the features of sand response perceived in the laboratory or field tests. The customer of the constitutive model should decide what responses want to take in account for the considered problem in order to select the responses that must be calibrated in the constitutive model. The PWP history time, the shear-strain accumulation rate after liquefaction triggering and the cyclic resistance Ratio (CRR) against the number of cycles to trigger liquefaction are significant responses for liquefaction tests and evaluations. For this type of cases, the principal objective of the calibration is to reproduce in a proper way as closely to the real behavior, these three features as observed in the cySS laboratory tests.

Cyclic simple shear tests (cySS) are frequently utilized to learning the behavior of porewater pressure generation and liquefaction, with one of the first research that was conducted by Seed and Lee (1966). This type of experiments has well-known restrictions when field conditions are trying to be simulated during seismic loading. Simple shear experiments provide one illustration of this circumstance when there are no corresponding shear stresses on the vertical planes. Additionally, corner and edge

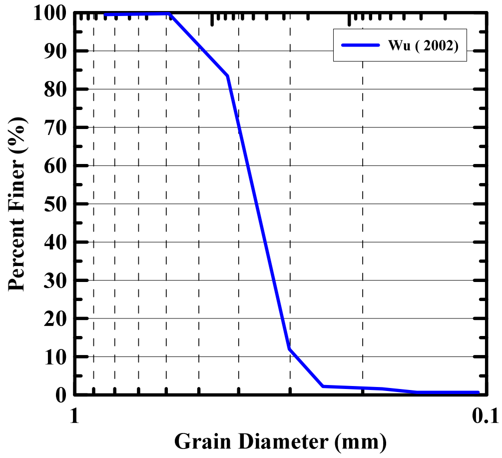


Fig. 3 Grain size distribution of Monterey 0/30 sand

Table 1 Summary of cySS tests used to evaluate constitutive models

Bin	Sub-bin	Test [1]	e_o[2]	$D_{r,c}$ (%)	σ'_c (kPa)	CSR[3]	D_{50} (mm)
CDS1	A	M50	0.745	35	40	0.150	0.36
		M49	0.745	35	40	0.181	0.36
		M80	0.745	35	40	0.162	0.36
		M63	0.745	35	40	0.202	0.36
	B	M48	0.745	35	40	0.244	0.36
CDS2	A	M17	0.714	45	40	0.170	0.36
		M72	0.714	45	40	0.190	0.36
		M62	0.714	45	40	0.214	0.36
		M73	0.714	45	40	0.225	0.36
		M82	0.714	45	40	0.233	0.36
	B	M47	0.714	45	40	0.303	0.36
CDS3	A	M61	0.667	60	40	0.263	0.36
		M78	0.667	60	40	0.303	0.36
		M60	0.667	60	40	0.292	0.36
	B	M7	0.667	60	40	0.347	0.36
		M66	0.667	60	40	0.390	0.36
CDS4	A	M70	0.604	80	40	0.453	0.36
	B	M71	0.604	80	40	0.528	0.36
CDS5	-	M21	0.745	35	80	0.177	0.36
CDS6	A	M24	0.714	45	80	0.140	0.36
		M23	0.714	45	80	0.205	0.36
	B	M20	0.714	45	80	0.264	0.36
		M124	0.714	45	80	0.328	0.36
CDS7	A	M16	0.667	60	80	0.170	0.36
		M15	0.667	60	80	0.200	0.36
		M59	0.667	60	80	0.238	0.36
		M79	0.667	60	80	0.243	0.36
		M28	0.667	60	80	0.266	0.36
	B	M19	0.667	60	80	0.354	0.36
		M25	0.667	60	80	0.362	0.36
	C	M125	0.667	60	80	0.438	0.36
CDS8	A	M36	0.604	80	80	0.241	0.36
	B	M34	0.604	80	80	0.310	0.36
		M105	0.604	80	80	0.389	0.36
		M33	0.604	80	80	0.389	0.36
		M30	0.604	80	80	0.401	0.36
	C	M110	0.604	80	80	0.430	0.36
		M31	0.604	80	80	0.454	0.36
		M32	0.604	80	80	0.457	0.36
		M35	0.604	80	80	0.488	0.36

[1]M = Monterey sand. [2]Initial void ratio. [3]Cyclic stress

effects have been observed, along with the sample's propensity to rock and pinch, which result in non-uniform stress and strain distributions. However, this test represents one of the most excellent ways to assess existing PWP generation models. Additionally, comparing cySS experiments permit the authors to assess potential mode of shear effects on PWP generation.

Important to mention is that load bias is not considered important in the performed analysis because Wu *et al.* (2002) reported the application of small load bias almost close to cero that do not cause interference in calculation for unidirectional loading, and it is not to be accounted. Utilizing consolidation stress (σ'_c) and relative density (D_r), the cySS were isolated into 9 vital bins based on comparable PWP and stress-strain reactions. The consolidation normal stress, σ'_c, is used in cySS tests. 17 cySS sub-bins were created by further subdividing these bins based on the various kinds of cyclic stress ratio (CSR = τ_{cyclic}/σ'_c, where τ_{cyclic} is the cyclic stress and σ'_c is the overburden pressure) that were used. As anticipated, an upgrading CSR created speedier PWP generation ratios and lowered the number of cycles needed to obtain the excess P WP ratio, $r_u = 1$ ($r_u = u_x/\sigma'_{vo}$, where u_x = excess PWP and σ'_{vo} = initial effective vertical stress) for a displayed σ'_c and D_r.

For tests in Bin CDS1A, Fig. 4 displays the stress path, stress-strain, stress, and r_u-strain and r_u time histories. The relative density after consolidation (D_{rc}) loss in these tests was set at 35%. The samples experienced cyclic mobility-type failure after an intermediate number of cycles, as shown by the PWP and stress-strain reactions. The rate of PWP production increases as the employed cyclic stress ratio (CSR) increases. The PWP amplifies largely under control until the soil liquefies after the soil yields (often when $r_u \sim 0.80$).

For medium-dense ($D_{rc} \sim 60\%$) sand trials in Bin CDS7A, Fig. 5 depicts the stress route, stress-strain, r_u-strain, and stress and r_u time records. The medium-dense sand tests in these studies require many more cycles to liquefy as the tests in Bin CDS1A; nonetheless, the CSRs are comparable with only a slight increase in quantity compared to Fig. 4. A cyclic mobility-type failure typically exhibits the stress-strain behavior described above, where small-scale shear resistance is triggered, and large stresses appear as the test cycles approach the stress track beginning point. While straining continues, the sample grows and increases shear resistance, while the excess PWP quickly decreases.

Fig. 6 shows how dense sand samples ($D_{rc} \sim 80\%$) responded in Bin CDS8C. These samples exhibit some cyclic mobility compared to the medium-dense samples in Fig. 5, but the higher relative densities result in much milder increases in PWP generation and an increase in the number of cycles needed to approach liquefaction. Because of the prolonged use of a higher CSR and the strain and r_u amplitudes being clearly related to the mode of shear, r_u moves closer to unity in these trials. In other words, strain and r_u amplitudes deviate from cySS amplitudes throughout the loading cycle. The responses of each bin also clearly show how the CSR (i.e., the FS_{liq}) used impacts the ratio of PWP production.

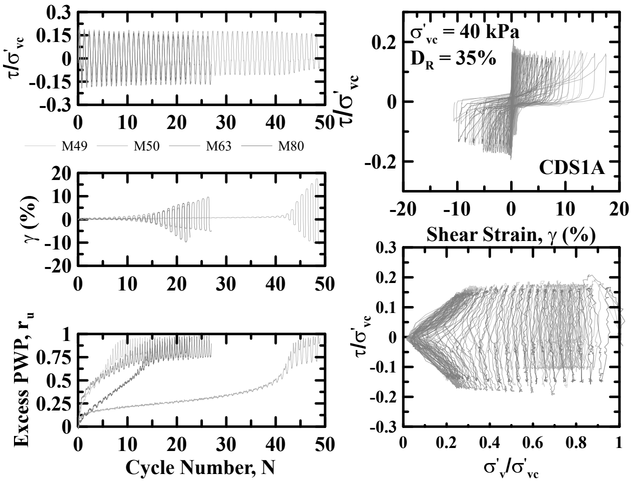


Fig. 4 Example of cySS test for loose sand.

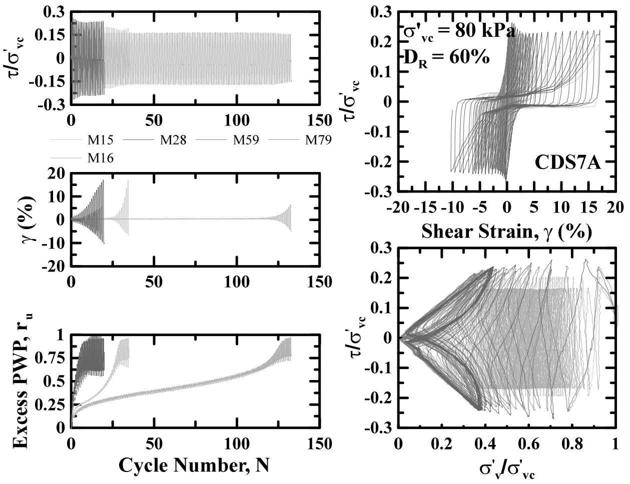


Fig. 5 Example of cySS test for medium dense sand.

The analysis of liquefaction risk in medium-dense and dense sands, particularly at a density exceeding 60%, is crucial. It is equally important to consider the occurrence of liquefaction in loose, sandy, saturated soils. The focus lies on comprehending the overall impact of liquefaction risk in site response analysis for dense, medium-dense, and loose sands. While the relevance of nearby dense sand conditions and potential transitional aspects should not be disregarded, it is worth noting that denser soils generally exhibit greater resistance to liquefaction due to reduced pore space, which limits water pore pressure accumulation.

Liquefaction risk assessment in medium-dense and dense sands is essential to ensure the safety and stability of structures built on such soils. By analyzing the potential for liquefaction in these soils, engineers and geologists can identify areas of concern and take appropriate measures to mitigate the risk. The occurrence of liquefaction in loose sandy soils is well-documented, and understanding the factors that contribute to this phenomenon is crucial for accurate site response analysis.

In site response analysis, the focus is on evaluating the behavior of soils under seismic loading. By considering the overall influence of liquefaction risk, engineers can design structures that are better equipped to withstand earthquakes.

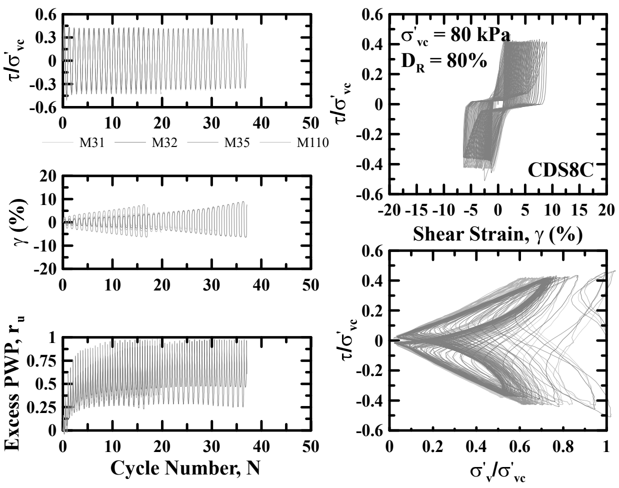


Fig. 6 Example of cySS test for dense sand.

The presence of nearby dense sand conditions and transitional aspects should not be overlooked, as they can significantly affect the response of soils to seismic events. Denser soils, with their reduced pore space, tend to exhibit greater resistance to liquefaction, making them more suitable for construction in seismically active regions.

The analysis of liquefaction risk in medium-dense and dense sands, alongside the acknowledged occurrence of liquefaction in loose sandy saturated soils, is crucial for site response analysis. By understanding the overall influence of liquefaction risk and considering the relevance of nearby dense sand conditions and potential transitional aspects, engineers and geologists can make informed decisions regarding the design and construction of structures in seismically active areas.

3.2 Calibration of constitutive model parameters

To accurately represent the actual material behavior, advanced constitutive models typically use a large number of parameters. Generally speaking, the major goal of these constitutive models is to replicate complex loading scenarios (stress path) that are beyond the scope of standard laboratory studies. Additionally, whereas constitutive models require the calibration of constants that may not make physical sense but are crucial for the model description, they must be calibrated through trial and error. Conventional parameters are also easily identified (Ghofrani *et al*. 2016). Another factor is that while a set of parameters is accurately calibrated for a particular soil based on the results of experimental tests, these calibrated parameters are only valid for the particular soil under consideration and cannot be applied to another soil. This reality necessitates parameter calibration for each case and for the particular examined soil, and the aim for researchers is to employ constrained knowledge of the real material properties to create trustworthy true predictions.

In practice, for analyses of geotechnical constructions numerous of these responses have to be obtained employing empirical relationships to in-situ test and/or index test information. During the process of calibration, all the considered models are changed by key parameters to fit the CRR to catch a peak shear strain of 3% in a single amplitude at 15 of loading cycles with the CRR curves from cySS experiments at a consolidation stress of 100 kPa. Reproductions utilizing this process of calibration are evaluated to cyclic strengths, cyclic stress – strain reactions, and PWP reactions at different consolidation stresses.

For Monterey 0/30 sand, the model parameters should be calibrated to reproduce the response observed on experimental results. Regarding the complexity of the considered models and input parameters, each soil model calibration for the considered models is developed such that the input parameters are based on limited information like relative density (D_R) or SPT $(N_1)_{60}$ (corrected SPT blow counts normalized for overburden stress of 1 atmosphere).

Relative density (D_R) can be estimated in practice by correlation to penetration resistance. The present study considers the following equation developed by Wu et al. (2002) that was used to correlate D_R and $(N_1)_{60}$ values where the D_R is expressed as a percentage (from 0% to 100%) with a minimum value of 10%.

$$(N_1)_{60} = 0.0048 * {D_R}^2 + 0.008 * D_R - 0.56 \qquad (1)$$

V_{s1} (shear wave velocity) may be estimated using the correlation by Andrus and Stokoe (2000) which is used with a slight modification performed by Boulanger and Ziotopoulou (2017) that limits the extrapolation to very small $(N_1)_{60}$ values as

$$V_{s1} = 85 * [(N_1)_{60} + 2.5]^{0.25} \qquad (2)$$

The elastic shear modulus (G_{max1}) can be calibrated to fit in-situ V_{s1} measurements, according to

$$G_{max1} = \rho * {V_{s1}}^2 V_{s1} = 85 * [(N_1)_{60} + 2.5]^{0.25} \qquad (3)$$

The target CRR values is based on the liquefaction triggering correlation proposed by Idriss and Boulanger (2008) (Sukkarak *et al*. 2021). This formulation produces target CRR values for an effective overburden stress of 1 atmosphere and an earthquake magnitude of M = 7.5 for the corresponding SPT $(N_1)_{60}$ as

$$CRR_{\sigma'=1\,atm, M=7.5} = exp\left(\frac{(N_1)_{60}}{14.1} + \left(\frac{(N_1)_{60}}{126}\right)^2 - \left(\frac{(N_1)_{60}}{23.6}\right)^3 + \left(\frac{(N_1)_{60}}{25.4}\right)^4 - 2.8\right) \qquad (4)$$

It is unrealistic to expect the CRR correlations from laboratory tests on clean Monterey No. 0/30 sand samples to be identical to the CRR correlations regressed from field performance studies. The differences between these CRR relationships provide important information that may prove useful for relating laboratory cyclic resistance ratio (CRR) and cyclic stress ratio (CSR) to observed field performance.

The difference between laboratory - measured cyclic resistance ratios (CRR) can be explained considering numerous factors such as fines content, aging, cementation, pore pressure migration, void redistribution and previous stress history. The influence of those factors implies the requirement to adjust the laboratory CSR values to reflect the effects of these factors.

Table 2 PDMY02 Primary Input Model Parameters

Parameter	Comments
rho	Material density.
refShearModu	Reference shear modulus.
refBulkModu	Reference bulk modulus.
frictionAng	Friction angle.
PTAng	Phase transformation angle.
peakShearStra	Peak shear strain.
refPress	Reference pressure (80 kPa).
PressDependCoe	Pressure dependent coefficient.
contrac1	Contraction parameter.
contrac3	A contraction non-negative constant reflecting K_σ effect.
dilant1	Dilation parameter.
dilant3	A dilation non-negative constant reflecting K_σ effect.
contrac2	A non-negative constant reflecting dilation history on contraction tendency.
liquefac1	Damage parameter to define accumulated permanent shear strain as a function of dilation history.
liquefac2	Damage parameter to define biased accumulation of permanent shear strain as a function of load reversal history.
e	Void ratio.
c	Numerical constant (0.1 kPa).

In the present study, the unidirectional laboratory test data will be converted to "equivalent" field CSR values by considering the following adjustment for multi-directional (field) loading effects and another important testing issue which may lead to artificial reduction of laboratory measured CRR-values is membrane compliance. The two CSR adjustment factors are taken from the study presented by Wu *et al.* (2002) where the correction for membrane compliance and multi-directional loading correction are considered.

PDMY02 Constitutive Model

The model is calibrated for various relative densities (D_R) ranging from 35% to 80% (D_R = 35%, 45%, 60%, and 80%). Understanding the distinctions between the stress and strain invariants used to define the yield surfaces in the constitutive model and the stress and strain terms frequently employed in the correlations of engineering design practice is necessary for calibrating the PDMY02 model (Table 2).

The number of liquefaction cycles at a cyclic shear stress level (3%) with a consolidation stress of 100 kPa, as well as the stress time, the acquired strain histories, and excess pore pressure, were highlighted in order to calibrate the PDMY02 model's parameters. The program generated the yield surfaces because the hyperbolic curve it produced was ideal for simulating the liquefaction of soil under cyclical conditions without drainage. The hyperbolic curve produced by the code serves as the basis for the shear modulus reduction curves.

For tests with overburden loads of 40 kPa, 80 kPa, and 100 kPa, the numerical prediction of cyclic strength curves

Table 3 PM4Sand Input Model Parameters

Parameter	Comments
Dr	Relative density.
G0	Shear modulus constant.
hpo	Contraction rate parameter.
Den	Mass density of the material.
P_atm	Atmospheric pressure.
h0	Variable that adjusts the ratio of plastic modulus to elastic modulus.
emax and emin	Maximum and minimum void ratios.
nb	Bounding surface parameter, $n^b \geq 0$.
nd	Dilatancy surface parameter $n^d \geq 0$.
Ado	Dilatancy parameter, will be computed at the time of initialization if input value is negative.
z_max	Fabric-dilatancy tensor parameter.
cz	Fabric-dilatancy tensor parameter.
ce	Variable that adjusts the rate of strain accumulation in cyclic loading.
phic	Critical state effective friction angle.
nu	Poisson's ratio
cgd	Variable that adjusts degradation of elastic modulus with accumulation of fabric.
cdr	Variable that controls the rotated dilatancy surface.
ckaf	Variable that controls the effect that sustained static shear stresses have on plastic modulus.
Q	Critical state line parameter.
R	Critical state line parameter.
m	Yield surface constant (radius of yield surface in stress ratio space).
Fsed_min	Variable that controls the minimum value the reduction factor of the elastic moduli can get during reconsolidation.
p_sedo	Mean effective stress up to which reconsolidation strains are enhanced.

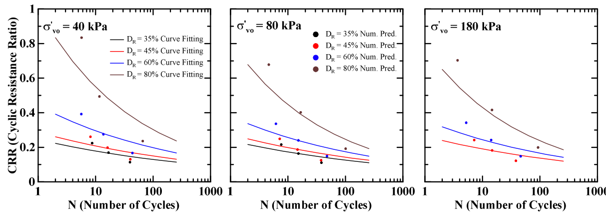


Fig. 7 CRR curves from CySS tests and PDMY02 simulations for specimens consolidated to 40 kPa, 80 kPa

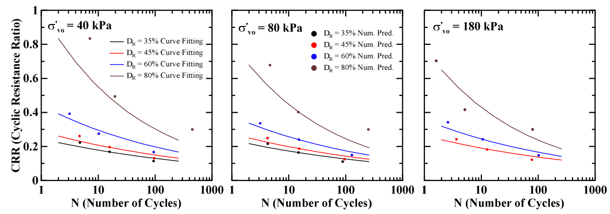


Fig. 8 CRR curves from CySS tests and PM4Sand simulations for specimens consolidated to 40 kPa, 80 kPa and 100 kPa

(CRR) is compared to the best curve fitting of data acquired by Wu *et al.* (2002) in Fig. 7. The CRR curves from the numerical prediction simulations are steeper than the CRR

Table 4 PDMY03 Input Model Parameters

Parameter	Comments
Rho	Material density.
refShearModu	Reference shear modulus.
refBulkModu	Reference bulk modulus.
frictionAng	Friction angle.
PTAng	Phase transformation angle.
peakShearStra	Peak shear strain.
refPress	Reference pressure (80 kPa).
PressDependCoe	Pressure dependent coefficient.
Ca	This parameter is the main input parameter controlling the contraction rate and subsequently the porewater pressure generation rate.
Cb	This parameter accounts for fabric damage.
Cc	This parameter accounts for the overburden stress effect (i.e., K_s effect).
Cd	A new parameter introduced in the updated model to increase (decrease) the rate of contraction for large (small) shear stress ratios.
Ce	A new parameter introduced in the updated model to control the dependency of contraction rate to static shear stress ratio and achieve desired K_a.
Da	This parameter, combined with the difference between ϕ and ϕ_{PT}, are the primary parameters to control the dilation tendency after crossing the PT surface.
Db	This parameter accounts for fabric damage in the dilation equation.
Dc	This parameter accounts for the effects of overburden stress on the dilation rate (i.e., K_s effect).
S_o	Shear strength at zero mean effective pressure.

curves from the cySS testing. For all confinement studied pressures, this effect is clear. These discrepancies should be attributed to the formulation's inability to produce CRR dependence on consolidation stress (i.e., K_σ behavior), as the model's primary focus is on recreating cyclic mobility rather than liquefaction triggering as seen in the experimental data. However, overall, there is a good correlation between the experimental and numerical curves.

PM4Sand Constitutive Model

In order to test the model's ability to accurately recreate the acquired cycle strength curve and cyclic stress-strainresponse, the calibration process was carried out using the parameter h_{po} and the initial circumstances supplied for the PDMY02 constitutive model (Table 3). The necessary G_o parameter was acquired by manipulating the G_{max} calculation algebraically utilizing the link between

$$G_{max} = G_o p_A \left(\frac{p}{p_A}\right)^{1/2} \quad (5)$$

For tests with overburden loads of 40 kPa, 80 kPa, and 100 kPa, Wu *et al.*'s best curve fitting of the data from their 2002 study is again compared to the numerical prediction of

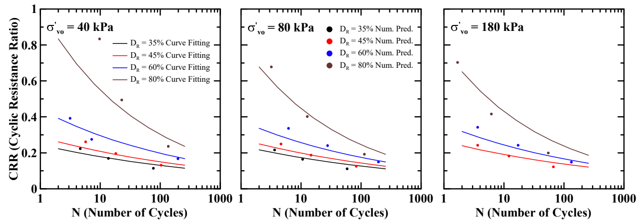


Fig. 9 CRR curves from CySS tests and PDMY03 simulations for specimens consolidated to 40 kPa, 80 kPa and 100 kPa

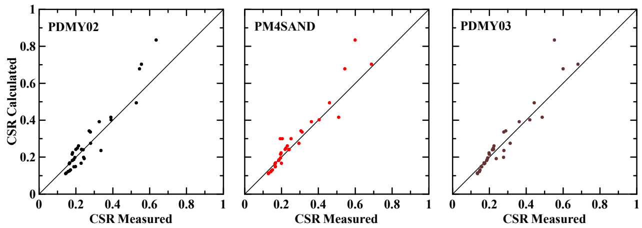


Fig. 10 Comparison CSR Measured vs CSR Calculated

cyclic strength curves in Fig. 8. For relative densities ranging from 35% to 60%, the numerical prediction simulations replicate CRR curves that are marginally steeper than the CRR curves from cySS tests, but for extremely dense samples, the curve of CRR considering low confinement pressures is overestimated, resulting in a high number of cycles predicted compared to the results of laboratory tests. Since the PM4Sand model's formulation was designed to produce a stronger dependence of CRR on consolidation stress (i.e., K_σ behavior) than is seen in the actual data, these variations are acknowledged (Parra – Bastidas 2016).

PDMY03 Constitutive Model

The relative density (D_R) values for which the model is calibrated range from 35% to 80%. In order to accurately capture the time histories of stress, strain, and excess pore pressure, the PDMY03 model parameters (Table 4) were calibrated using the same initial circumstances described for the PDMY02 and PM4SAND constitutive models. Because the code-produced hyperbolic curve is sufficient for modeling liquefaction where the soil behaves in undrained cyclic conditions as PDMY02 model, yield surfaces were generated.

In Fig. 9, the best curve fitting of the data obtained by Wu *et al.* (2002) for tests with overburden stresses of 40 kPa, 80 kPa, and 100 kPa is compared to the numerical prediction of cyclic strength curves. The CRR curves from the numerical prediction simulations are slightly steeper than the CRR curves from the cySS testing. Compared to the earlier taken into account models, the CRR curves are better recorded. This is explained by the fact that the model accounts for the relationship between the number of loading cycles, effective overburden stress (K_σ), and static shear stress (K_α), all of which are controlled variables in laboratory tests. In general, the model's predictions closely match the experimental and numerical curves.

Fig. 10 compares the CSR measured and CSR calculated values for three models: PDMY02, PM4SAND, and PDMY03. CSR is a measure of the liquefaction resistance of soils under seismic loading. The models are constitutive relations that describe the stress-strain behavior of sands under cyclic loading. The linear regression line shows the degree of agreement between the CSR measured and CSR calculated values. A slope of 1 and an intercept of 0 indicate a perfect match. The plots show that the PDMY03 and PM4SAND models have a good agreement, while the PDMY02 model has a modest agreement. The data points are also scattered, indicating some variability and uncertainty. The results suggest that the PDMY02 and PM4SAND models are more reliable and accurate than the PDMY02 model for predicting the liquefaction resistance of sands.

3.2 Validation of PWP results using calibrated model parameters

Fig. 11 through Fig. 13 compare the PWP generation time histories computed using the PDMY02, PM4SAND, and PDMY03 models, respectively, to the measured responses of tests in bins CDS1A, CDS7A, and CDS8C using the Coupled PWP and constitutive model parameters calibrated from each individual test (as described above).

The PDMY02, PM4SAND, and PDMY03 models, in general, qualitatively captured the reaction of loose specimens [Fig. 11 through Fig. 13(a) to 13(c)], demonstrating the model's ability to capture variances in response corresponding to minor variations in applied CSRs (i.e., small differences in FS_{liq}). In a similar way, Fig. 11 through Figs. 13(a) to 13(c) show that, with the exception of tests in which there is a high amount of dilatation, the PDMY02, PM4SAND, and PDMY03 models likewise qualitatively reproduced the measured PWP response of medium-dense specimens in bins S7A.

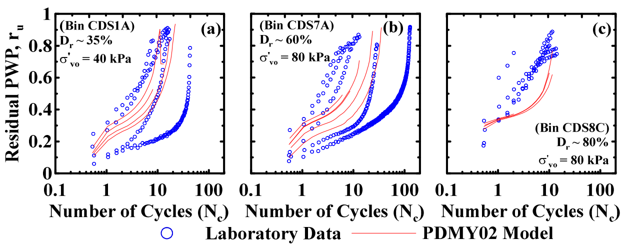


Fig. 11 Validation of PDMY02 model: (a) cySS tests on loose specimens; (b) cySS tests on medium-dense specimens; and (c) cySS tests on dense specimens

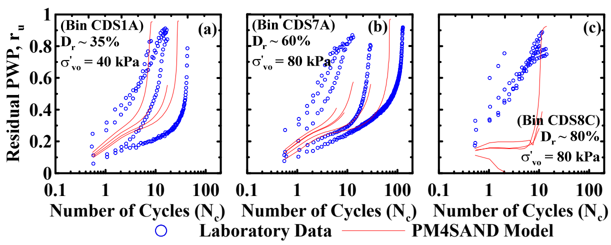


Fig. 12 Validation of PM4Sand model: (a) cySS tests on loose specimens, (b) cySS tests on medium-dense specimens; and (c) cySS tests on dense specimens

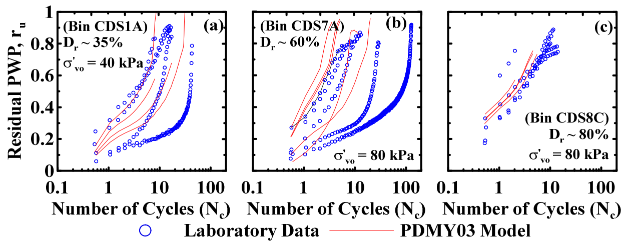


Fig. 13 Validation of PDMY03 model: (a) cySS tests on loose specimens, (b) cySS tests on medium-dense specimens; and (c) cySS tests on dense specimens

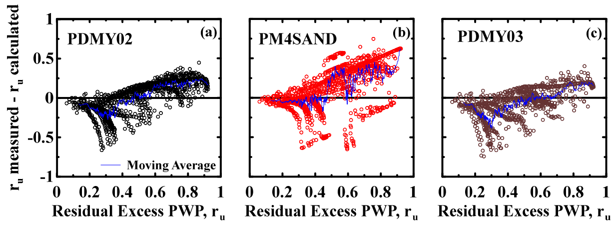


Fig. 14 Residuals computed from ru time histories for tests listed in Table 1 using calibration model parameters for: (a) PDMY02 model, (b) PM4SAND model; and (c) PDMY03 model

Fig. 11 through Fig.13 compare the PWP generation time histories computed using the PDMY02, PM4SAND, and PDMY03 models, respectively, to the measured responses of tests in bins CDS1A, CDS7A, and CDS8C using the Coupled PWP and constitutive model parameters calibrated from each individual test (as described above). The PDMY02, PM4SAND, and PDMY03 models, in general, qualitatively captured the reaction of loose specimens [Fig. 11 through Figs. 13(a) to 13(c)], demonstrating the model's ability to capture variances in response corresponding to minor variations in applied CSRs (i.e., small differences in FS_{liq}). In a similar way, Fig.11 through Figs. 13(a) to 13(c) show that, with the exception of tests in which there is a high amount of dilatation, the PDMY02, PM4SAND, and PDMY03 models likewise qualitatively reproduced the measured PWP response of medium-dense specimens in bins S7A.

With reference to the previously indicated validation dataset of cySS tests (17 bins), Fig. 14 shows the residual statistics corresponding to residual excess PWP, ru, time histories. These results show that the PDMY03 model predicted low to medium PWP overly and medium to high PWP underly, with the lowest residuals (-0.35 to 0.23) and almost no bias (as indicated by the moving average trend). The PDMY02 model has a minor bias, less performance than the PDMY02 model, and low residuals (almost 0.27). The PM4SAND model produced bias for medium to high PWP and relatively high residuals (almost +0.50).

In general, the validation dataset (17 bins) shows that when model parameters are validated using individual test results, the PDMY02, PM4SAND, and PDMY03 models reasonably capture PWP generation for individual cyclic testing.

4. Evaluation of predicted stress-strain behavior

The connected PWP and constitutive models' stress-strain behavior was calculated using the same cySS bins that were taken into account for the PWP evaluation implementation. When the measured shear strain, γ_c = 0%, residual models were developed utilizing computed and measured shear stresses.

The stress-strain and excess r_u-strain behavior computed using the PDMY02, PM4SAND, and PDMY03 models are compared to the measured responses of the cySS tests in the bins CDS1A, CDS7AB, and CDS8C in Fig. 15 through 17. For strains less than γ_c = 3%, the three models accurately predict the stress-strain and r_u-strain behavior in loose to dense objects. When dilatation becomes more evident (γ_c = 3% and $r_u > 0.75$) and considerable modulus deterioration occurs, the models predict stress-strain and r_u-strain performance. The tested constitutive models reveals that the simulation of stress-strain loops concurs in a proper way the measured loops for all r_u, suggesting the capture of decrease in soil strength and variation of cyclic shear stress which is replicated relatively well. It means that soil dilation spikes are captured by the constitutive models which constitutes a great advantage over conventional hyperbolic constitutive model like GQ/H+u. Mei *et al.* (2019) stated that when $r_u \geq 0.8$, the shear strain suffers large increments reflected in soil dilation and GQ/H+u cannot replicate this behavior.

Residuals are calculated by subtracting the predicted value of τ/σ'_{vo} of a data point from observed value of τ/σ'_{vo} at the same level of strain and residuals are used to measure how well a model fits the data and to check the assumptions of the model. The residuals can be represented by a moving average because they depend on the previous residuals that captures this dependence and smooths out the fluctuations in the residuals. Fig. 18 shows the discrepancy between the measured and estimated values of CSRs (τ/σ'_{vo}) for the 17 cycles of 40 trials in the forecasting data set. The combined PWP and constitutive models using the PDMY02, PM4SAND, and PDMY03 models appear to have quite low residuals (< ±0.15) for all cycles. The PM4SAND model seems to overestimate the values of cycles in the mid range (0.5 to 0.65), while the PDMY03 model does better as the moving-average trend of the residuals is close to zero.

The most accurate match between the observed and calculated loop was ascertained by utilizing the highest value of the concordance correlation coefficient (CCC; Lin 1989 where CCC = $\frac{2S_{12}}{S_1{}^2+S_2{}^2+(\bar{Y}_2-\bar{Y}_1)^2}$ where Y_1 = mean of computed values of r_u or stresses; Y_2 = mean of target values of r_u or stresses; S_1 = variance of computed values of r_u or stresses; S_2 = variance of target values of r_u or stresses; S_{12} = covariance between computed and target values). The

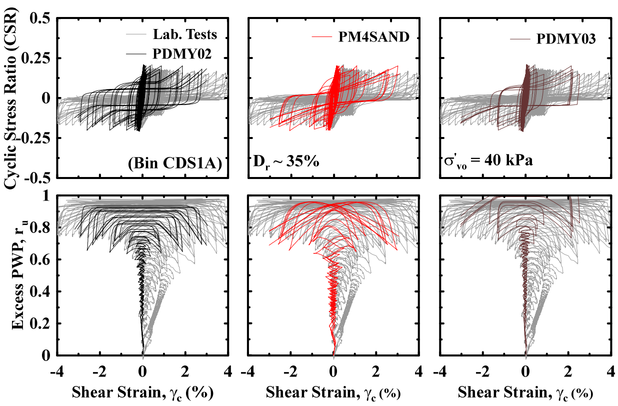


Fig. 15 Comparison of calculated PDMY02, PM4SAND and PDMY03 models and measured PWP ratios for cySS test loose specimens

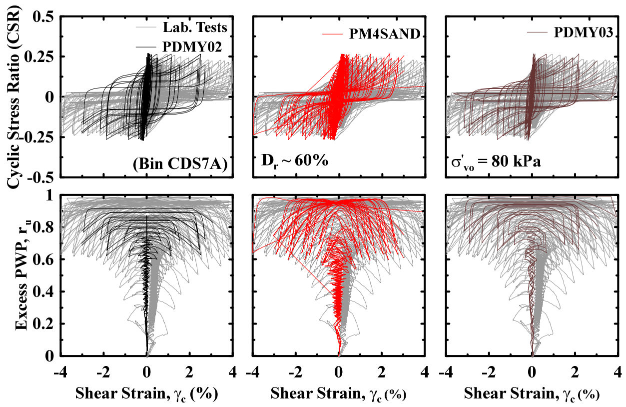


Fig. 16 Comparison of calculated PDMY02, PM4SAND and PDMY03 models and measured PWP ratios for cySS test loose specimens

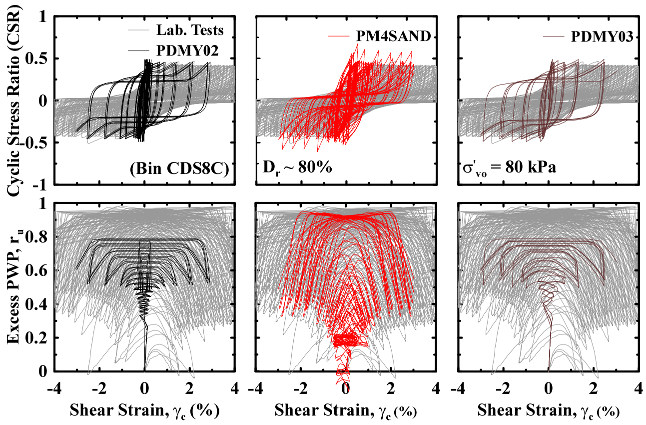


Fig. 17 Comparison of calculated PDMY02, PM4SAND and PDMY03 models and measured PWP ratios for cySS test dense specimens

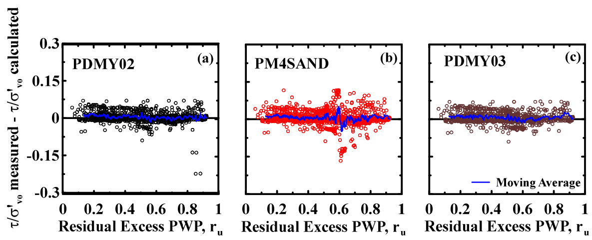


Fig. 18 Residuals computed from measured and computed CSR values (τ/σ'_{vo}) for tests listed in Table 1 using correlation-based model parameters for: (a) PDMY02 model, (b) PM4SAND and (c) PDMY03 model

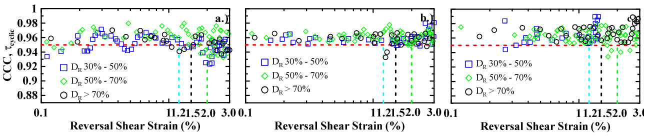


Fig. 19 Computed CCC values for (a) PDMY02, (b) PM4SAND and (c) PDMY03 models with respect to reversal shear

Table 5 Qualitative ranking of the performance of the models on different observations based on the simulations (xxx is the best score while x is the lowest one)

Description	PDMY02	PM4SAND	PDMY03
Maximum stress ration under undrained conditions	xxx	xxx	xxx
Stress – Strain behavior reproduction	xx	xxx	xxx
Pore pressure reproduction under undrained cyclic conditions	xx	xxx	xxx
Cyclic mobility effects	x	x	x
Accumulation rate under stress controlled cycles	xx	xx	xxx

CCC quantifies the level of similarity between processed and examined data. CCC values lie on a range from −1 to 1, in which 1 is a perfect match and zero implies no agreement between the compared datasets. McBride (2005) presented a scale to judge the level of agreement when employing CCC where CCC > 0.95 is considered as substantial concordance, CCC < 0.9 is deemed inadequate concordance, and CCC between 0.95 and 0.90 is judged as moderate agreement.

Analyzing Fig. 19, it is evident that the calculated and observed cyclic shear stress CCC values are in agreement and the outcomes of those calculations are reliable. Specifically, for dense, medium-dense, and loose sand, the CCC values surpass 0.96 at shear stresses of 1.2%, 1.5%, and 2%, respectively. These shear strains, also referred to as limit shear strains, are the lowest at which the dilation behavior is evident and the stress-strain hysteresis loops can be accurately described as proposed by Mei *et al.* (2019). This illustrates that the models can successfully depict the entire stress-strain hysteresis loop for all spectra. The limitations and strengths of the models have been qualitatively ranked in Table 5.

5. Conclusions

The present study evaluated three coupled constitutive models (PDMY02, PM4SAND, and PDMY03) through a database of 40 stress-controlled cyclic tests. Each of these models was found to be accurate in predicting stress-strain and pore-water pressure (PWP) behavior across a wide range of relative densities, displaying very modest residuals and minimal bias for all D_R levels. The improved models were also able to simulate dilation at high r_u values (greater than 0.75), thus making them suitable for use in engineering simulations. This study thus confirms the viability of using these coupled constitutive models for site response analysis.

The PDMY02, PM4SAND, and PDMY03 models all showed good representation of the key features of PWP production during the validation phase, when the initial liquefaction was close to the starting vertical effective stress. The residuals for PWP comparison and stress-strain comparison were all very small (< ± 0.27 and < ± 0.15 respectively), and the bias for all D_R levels was minimal. Over a broad range of relative densities, these models were able to accurately replicate the stress-strain and PWP response, as well as predict the dilation due to excess PWP. This can be used to predict when dilation spikes will occur with a high degree of accuracy in simulations of centrifuge tests involving accelerated time histories.

The statistical results obtained of three different models (PDMY02, PM4SAND, and PDMY03) for predicting the pore water pressure (PWP) and the cyclic shear stress ratio (CSR) of sand under cyclic simple shear (cySS) tests indicates that the PDMY03 model has the best performance among the three models, as it has the lowest residuals, the least bias, and the highest CCC values for all cycles and spectra. Also, the results demonstrate that the models can successfully capture the stress-strain behavior of sand under cySS tests.

The PDMY02, PM4SAND, and PDMY03 models have been shown to be able to accurately predict the production of PWP and the stress-strain behavior for all r_u values, and the upgraded constitutive model's capacity to reproduce dilation at r_u values higher than 0.75 has improved the accuracy of these coupling models. This shows that it is possible to use the advanced constitutive models to perform site response analysis in engineering practice.

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