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Probabilistic optimization of nailing system for soil walls in uncertain condition

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Abstract. One of the applicable methods for the stabilization of soil walls is the nailing system which consists of tensile struts. The stability and safety of soil nail wall systems are influenced by the geometrical parameters of the nailing system. Generally, the determination of nailing parameters in order to achieve optimal performance of the nailing system for the safety of soil walls is defined in the framework of optimization problems. Also, according to the various uncertainty in the mechanical parameters of soil structures, it is necessary to evaluate the reliability of the system as a probabilistic problem. In this paper, the optimal design of the nailing system is carried out in deterministic and probabilistic cases using meta-heuristic and reliability-based design optimization methods. The colliding body optimization algorithm and first-order reliability method are used for optimization and reliability analysis problems, respectively. The objective function is defined based on the total cost of nails and safety factors and reliability index are selected as constraints. The mechanical properties of the nailing system are selected as design variables and the mechanical properties of the soil are selected as random variables. The results show that the reliability of the optimally designed soil nail system is very sensitive to uncertainty in soil mechanical parameters. Also, the design results are affected by uncertainties in soil mechanical parameters due to the values of safety factors. Reliability-based design optimization results show that a nailing system can be designed for the expected level of reliability and failure probability.

Keywords: nailing system; optimization; reliability; uncertainty

1. Introduction

There are different uncertainties in all steps of the design, construction, and maintenance of structures. These uncertainties can be evaluated based on probabilistic models, and reliability analysis can assess the safety levels of structural systems using the probability of failure (Ditlevsen 1982). In general, for estimating the probability of failure based on the probabilistic model and reliability analysis in the structure, various analytical methods such as the first-order reliability method (FORM) (Liu and Der Kiureghian 1991, Hadidi *et al.* 2019), second-order reliability method (SORM) (Der Kiureghian and Stefano 1991), optimization algorithms (Kaveh *et al.* 2014), simulation methods (Rashki *et al.* 2012, Azar *et al.* 2015), response surfaces (Chakraborty and Chowdhury 2016, Goswami *et al.* 2016) and neural networks (Vazirizade *et al.* 2017) are used.

Recent years have seen many developments in methods for design under uncertainty. The reliability-based design optimization is defined as the minimization of the probability of failure considering the model's parameter uncertainties. Several studies have been proposed for reliability-based optimization (Tu *et al.* 1999, Doan *et al.* 2018, Keshtegar and Hao 2018, and Lehky *et al.* 2018). In order to achieve a reliable and economical design, the RBDO is widely used in structural optimization problems (Liu and Paavola 2015, Huu *et al.* 2016, Zhao *et al.* 2016, Le *et al.* 2017, Shirgir *et al.* 2023, Shamsaddinlou *et al.* 2023).

One of the engineering problems that have various uncertainties is the geotechnical engineering problem. In this regard, studies on safety factors and reliability have been conducted in geotechnical engineering (Duncan 2000, Johari and Golkarfard 2018, Johari *et al.* 2021). Also, various studies carried out for slope stability from different points of view (Johari *et al.* 2022, Johari and Fooladi 2022, Kalantari *et al.* 2023). One of the practical methods in the stabilization of soil slopes and walls is the nailing method. So far, various studies have been conducted in the field of the soil nail system (Ghareh 2015, Pak *et al.* 2019, Villalobos 2020, Kalehsar *et al.* 2021, Kalantari and Johari 2022). Also, due to the uncertainty in a nailed soil wall system, various studies have been performed on the reliability of the nailing system (Babu and Singh 2011, Johari *et al.* 2020). Another group of problems in geotechnical engineering is design optimization problems in soil structures (Manahiloh *et al.* 2015, Hosseini *et al.* 2016, Yuan *et al.* 2019, Arama *et al.* 2021). Many studies have attempted to design a nailing system with optimization concepts (Patra and Basudhar 2005, Fan and Luo 2008, Seo *et al.* 2014, Sharma and Ramakrishnan 2020, Benayoun *et al.* 2021).

In this paper, according to various uncertainty in soil nail walls, the design optimization of the nailing system in deterministic and uncertain conditions is examined. In other words, the main purpose is to design the geometry and layout of the nails, considering the uncertainty in the

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mechanical parameters of the soil. In the deterministic case, the optimal design of the nailing system is done using the meta-heuristic algorithm. Then, by using the reliability analysis method, the reliability index for this system is calculated, and sensitivity analysis is performed for the safety factors based on the soil parameters. Also, in uncertainty cases, the optimum design of the nailing system is done with a reliability-based design optimization method. In reliability-based design optimization, the optimum design is performed based on desired target reliability.

2. Formulation of soil nail system

The analysis and design of soil nail walls were done in two different Strength Limit States and Service Limit States. Strength limit states evaluate the conditions that the whole system or single components fail in which the strength is less than the stresses of applied loads. The design of a soil nail wall should be reliable under all of the potential failure conditions that the strength limit states identified. The potential failure conditions of a soil nail wall according to strength limit states are classified as the external failure mode, internal failure mode, and facing failure mode (Rawat and Gupta 2016).

External failure modes investigate the potential failure surfaces on soil mass. In external failure modes, the soil nail wall body is usually assumed as a block. Stability analysis calculates all of the resisting forces that act along the failure surfaces caused to the equilibrium of this block. If there are one or more soil nails on the failure surface, the external stabilizing force of the nails must be accounted for the equilibrium of the block. As a significant consequence of external failure modes, the evaluation of external stability is very important in the design of soil nail walls. So two general modes including global failure mode and sliding failure mode are considered in the analysis of soil nail wall systems.

The two-dimensional limit equilibrium equations are used for slope stability analyses to calculate the factors of safety in soil nail walls. According to this principle, the resisting forces and driving forces are determined on the failure surface. The factor of safety for external failure modes is expressed as (FWHA. 2003):

a) Global failure mode:

$$FS_G = \frac{\sum R}{\sum D} = \frac{cL_F + T_{eq}\cos(\psi - i) + \left[(W + Q_T - F_v)\cos\psi + T_{eq}\sin(\psi - i) - F_h\sin\psi\right]\tan\phi}{(W + Q_T - F_v)\sin\psi + F_h\cos\psi} \tag{1}$$

$$\psi = 45 + \left(\frac{\varphi}{2}\right) \tag{2}$$

$$T_{eq}\ [kN/m] = \frac{1}{S_h}\sum (T_{all})_j \tag{3}$$

$$T_{all} = min\{R_T \quad , \quad R_P\} \tag{4}$$

$$R_P = \pi D L_P q_u \tag{5}$$

$$R_T = \left(\frac{\pi d^2}{4}\right) F_y \tag{6}$$

where ϕ is soil angle of internal friction, c is soil cohesion, ψ is the inclination of failure plane, i is nail inclination, S_h is nail horizontal spacing, q_u is bond strength of nails, D is the diameter of the drill hole, L_P is pullout length, d is the diameter of bar and F_y is the yield stress of steel.

b) Sliding failure mode:

Sliding stability analysis evaluates the effect of the nails to resist sliding along the base of the excavation body against the lateral earth pressures behind the soil nails. To assess the sliding stability of a soil nail wall system the Rankine or Coulomb theories of lateral earth pressures are used. The soil nail wall system is modeled as a rigid block that resists the lateral earth forces applied behind the retained body. The factor of safety against sliding (FS_{sl}) is calculated as the ratio of horizontal resisting forces $(\sum R)$ to the applied driving horizontal forces $(\sum D)$ as follows

$$FS_{sl} = \frac{\sum R}{\sum D} = \frac{c_b B_L + \left(W + Q_T - F_v + P\sin\beta_{eq}\right)\tan\phi_b}{F_h + P\cos\beta_{eq}} \tag{7}$$

$$P = \frac{\gamma H_1^2}{2} K(1 - k_v)\left\{1 + \frac{2q_s}{\gamma H_1}\left[\frac{\cos\alpha}{\cos(\beta-\alpha)}\right]\right\} \tag{8}$$

$$K = \frac{\cos^2(\phi-\alpha-\omega)}{\cos\omega\cos^2\alpha\cos(\alpha+\beta+\omega)\left[1+\sqrt{\frac{\sin(\phi+\beta)\sin(\phi-\beta-\omega)}{\cos(\alpha+\beta+\omega)\cos(\beta-\alpha)}}\right]^2} \tag{9}$$

$$\omega = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) \tag{10}$$

soil cohesion strength along the base, α is face batter angle and β is backslope angle.

Internal failure modes occur when load transfer mechanisms fail between soil, nails, and grout fail. Since the soil nail wall system deforms during excavation, the bond strength between the grout and the surrounding soil mobilizes for nails. The bond strength is distributed along the nail according to distribution that is affected by different parameters. When the bond strength is activated, tensile forces are transferred from the soil to the nail. Vary and different internal failure modes are identifiable based on the nail Length, nail tensile strength, bond strength, and bond stress distributions. the two most common internal failure modes include nail pullout failure and nail tensile failure.

c) Nail Pullout Failure:

Nail pullout failure may be occurred on the soil-grout interface because of weak cohesion between soil and grout.

$$(FS_P)_z = \frac{(R_P)_z}{(T_{max})_z} = \frac{(Q_u L_P)_z}{(T_{max})_z} \tag{11}$$

$$(T_{max})_z = K(q_s + \gamma z)S_H S_V \tag{12}$$

$$(R_P)_z = (Q_u L_P)_z \tag{13}$$

$$Q_u = \pi q_u D_{DH} \tag{14}$$

$$(L_P)_z = L - \left[\frac{(H-z)\cos(\psi+\alpha)}{\cos\alpha\sin(\psi+i)}\right] \tag{15}$$

d) Tensile Failure of the Nail:

If the transferred force to the nail is more than the tensile strength of nails, the tensile failure mode can appear.

$$(FS_T)_z = \frac{(R_T)_z}{(T_{max})_z} = \frac{(Q_u L_P)_z}{(T_{max})_z} \tag{16}$$

$$R_T = A_t F_y \tag{17}$$

e) Facing Failure Mode

The most probable modes of failure of the soil nail wall system in the facing component are flexure failure and punching shear failure. Flexure failure is a failure mode in which the flexural capacity of facing is less than the bending beyond. Also punching shear failure mode occurs around the head nails on the facing layer. Each two failure mode should be evaluated for both temporary and permanent facings.

$$FS_{FP} = \frac{R_{FP}}{T_o} \tag{18}$$

$$R_{FP} = C_P V_F \tag{19}$$

$$C_P = 1 \tag{20}$$

$$V_F = 330\sqrt{f_c'}\pi D_c' h_c \tag{21}$$

$$D_c' = L_{BP} + h \tag{22}$$

$$T_o = T_{max}.\,[0.6 + 0.2(S_{max} - 1)] \tag{23}$$

Also

$$FS_{FF} = \frac{R_{FF}}{T_o} \tag{24}$$

$$R_{FF} = \frac{C_F}{265} \times (a_{vn} + a_{vm}) \times \left(\frac{S_H h}{S_V}\right) \times F_y \tag{25}$$

$$R_{FF} = \frac{C_F}{265} \times (a_{hn} + a_{hm}) \times \left(\frac{S_V h}{S_H}\right) \times F_y \tag{26}$$

$$C_F = 1 \tag{27}$$

$$T_o = T_{max}.\,[0.6 + 0.2(S_{max} - 1)] \tag{28}$$

For the design of soil nail walls, the minimum safety factors recommended being used for each of the various failure modes. These recommended factors of safety are given for static and seismic loads, and for temporary and permanent structures (Table 1). The recommended factors of safety are only applicable to the ASD method.

Table1 Minimum recommended factors of safety for the design of soil nail walls (FHWA. 2003)

Failure Mode	Resisting Component	Static Loading	Seismic Loading
External Stability	Global Stability	1.35	1.1
	Sliding Stability	1.5	1.1
	Bearing Capacity	3.0	2.3
Internal Stability	Pullout Resistance	2.0	1.5
	Nail Tensile Strength	1.8	1.35
Facing Strength	Facing Flexure	1.5	1.1
	Facing Punching Shear	1.5	1.1

3. Reliability analysis

Reliability is defined as the probability of a limit state function g(X) greater than zero, P{g(X)>0} (Du 2005). In other words, reliability equals to the probability of random variables X, falling into the safe region, defined by g(X)>0.

The probability of failure is defined as the probability P{g(X)<0}, and equals to the probability of random variables X, existed in the fracture region defined by g(X)<0. If the probability distribution function of the random variables X is $f_x(x)$, then the probability of failure can be calculated using the following integral (Du 2005)

$$P_f = P\{g(X) < 0\} = \int_{g(X)<0} f_x(X)dx \tag{29}$$

and reliability can be calculated as follows

$$Reliability = 1 - P_f = P\{g(X) > 0\} = \int_{g(X)>0} f_x(X)dx \tag{30}$$

The probability of failure can be determined approximately according to the reliability index (β) in first order reliability method (FORM) as follows (Der Kiureghian 2005)

$$P_f = \int_{g(X)\le 0} \cdots \int f_X(x)dX \approx \Phi(-\beta) \tag{31}$$

where P_f is failure probability, $f_x(X)$ is the probability distribution function and $g(X)$ is the limit state function that divides design regions into failure and safe as $g(X) < 0$ and $g(X) > 0$, respectively, by using the basic random variables X. In many engineering problems, the limit state function g(X) is a complex and implicit function. Also, $\Phi(.)$ is the cumulative distribution function.

In order to simplification of the calculation, all random variables $X = (x_1, x_2, \dots, x_n)$ with desired distributions are transferred to $U = (u_1, u_2, \dots, u_n)$ variables with standard normal distribution. So, the probability integral can be written as follows

$$p_f = P\{g(U) < 0\} = \int_{g(U)<0} \phi_u(u)du \tag{32}$$

where $\phi_u(u)$ is probability distribution function (PDF) in U space.

Also, in the FORM analysis Taylor's first expansion is used to linearization of limit state function $g(U)$ as follows (Der Kiureghian 2005)

$$g(U) \approx g(u^*) + \nabla g(u^*)(U - u^*)^T \tag{33}$$

where u^* is the expansion point and $\nabla g(u^*)$ is the gradient of the g function at u^*.

In the standard normal space which the random variables are statistically independent and the limit state function $g(\mathbf{u})$ is linear, the reliability index (β), can be defined as the shortest distance from the failure surface ($g(\mathbf{u}) = 0$) to the origin. So, the probability of failure (P_f) is calculated by

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{34}$$

where

$$\Phi(\beta) = \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}u^2\right) du \tag{35}$$

Thus, the reliability index may be described by (Lee *et al.* 2002)

find **u**; which minimizes $\beta = |u| = \sqrt{u^T u}$
subjected to $g(\mathbf{u}) = 0$

The primary goal in the FORM is to search for the most probable point (MPP, i.e., U^*), that is a point located closest to the origin in transformed standard normal space, consequently, $\beta = \|U^*\|$ (Lee *et al.* 2002).

Hasofer and Lind introduced an iterative algorithm to find the MPP (Hasofer and Lind 1974). This method is used for variables with normal distribution, then Rackwitz and Flessler (1978) developed this algorithm for random variables with any desired distribution. Liu and Der Kiureghian (1991) improved the HL-RF method to enhance the convergence rate using a merit function. Santosh *et al.* (2006) improved the HL-RF method based on Armijo rule. Recently for searching the MPP, there are various FORM algorithms such as finite-step length (Gong and Yi 2011), non-gradient-based algorithm (Gong *et al.* 2014), conjugate gradient (Farsani and Keshtegar 2015), chaotic conjugate search direction (Keshtegar 2016) and stability transformation method (Meng *et al.* 2017). The improved HL-RF method is formularized by using the steepest descent search direction to find the MPP.

3.1 Modified HL-RF method

The iterative equation of HL-RF algorithm for FORM can be defined according to

$$U_{k+1} = U_k + s_k d_k \tag{36}$$

where s_k is step size. In standard HL-RF method, the step size is assumpted as 1. d_k is search direction vector, that can be calculated as follows (Makhduomi *et al.* 2017)

$$d_k = \frac{\nabla^T g(U_k) U_k - g(U_k)}{\nabla^T g(U_k) \nabla g(U_k)} \nabla g(U_k) - U_k \tag{37}$$

in which $\nabla g(U_k)$ is gradient vector of the limit state function $g()$ at point U_k, and for random variables with normal distribution

$$\nabla g(U_k) = \left\{\frac{\partial g}{\partial u_1}, \frac{\partial g}{\partial u_2}, \ldots, \frac{\partial g}{\partial u_n}\right\} = \left\{\sigma_1 \frac{\partial g}{\partial x_1}, \sigma_2 \frac{\partial g}{\partial x_2}, \ldots, \sigma_n \frac{\partial g}{\partial x_n}\right\} \tag{38}$$

According to Eq. (36), the step size and search direction are two effective components in the iterative HL-RF equation. This iterative equation can be controlled based on the step size to find MPP. Therefore, the iterative equation of improved HL-RF (iHL-RF) can be obtained from Eq. (36), where α_k is the adjusted step size. The step size is regulated using the merit function as follows

$$m(U_k) = \left\| U_k - \frac{\nabla^T g(U_k) U_k}{\nabla^T g(U_k) \nabla g(U_k)} \nabla g(U_k) \right\|^2 + \frac{g(U_k)^2}{g(U_0)^2} \tag{39}$$

As it is known, the merit function has a positive value $m(U_k) \geq 0$, and it is calculated based on the previous results as well as the HL-RF method. Therefore, the step size can be calculated as follows (Makhduomi *et al.* 2017)

$$s_{k+1} = \begin{cases} \dfrac{m(U_{k-1})}{m(U_k)} s_k & m(U_k) \geq m(U_{k-1}) \\ s_k & m(U_k) < m(U_{k-1}) \end{cases} \tag{40}$$

The first step size is assumpted as 1.5 (i.e., $s_0 = 1.5$). According to the adaptive step size in Eq. (40), it can be concluded that, $s_{k+1} \leq s_k$.

In the HL-RF algorithm, similar to other optimization methods, the convergence criterion is used. First, the design point should be placed close to the limit state surface (Du 2005)

$$\left|\frac{g(U^*)}{g_0}\right| \leq e_1 \tag{41}$$

That, g_0 is a scale factor, usually the initial step value of the limit state function, and e_1 is a threshold that is assumed to be about 0.0001. Secondly, the design point should be the closest point to the origin on the limit state surface. For this case, this should be the gradient projection point. For example, the gradient vector of the limit state function must has to pass the origin. This convergence criterion is defined as

$$\|U^* - (\alpha^T U^*)\alpha\| \leq e_2 \tag{42}$$

that, e_2 is a threshold of about 0.001. α is the importance vector that is a unit vector, so by a scaling to $\|U^*\|$ this criterion is expressed as

$$1 - \frac{\alpha^T U^*}{\|U^*\|} \leq e_2 \tag{43}$$

In FORM analysis, the α vector can be defined by the negative and normalized version of the gradient vector

$$\alpha = -\frac{\nabla g}{\|\nabla g\|} \tag{44}$$

As, the reliability index equals to $\beta = \alpha^T U^*$. Thus

$$\alpha = \frac{\partial \beta}{\partial U^*} \tag{45}$$

This vector is used to display the relative importance of variables.This is the primary importance vector for the variables that define in the *U*-space. The higher absolute value of α is the most important variable.

4. Colliding Bodies Optimization algorithm (CBO)

The Colliding bodies optimization algorithm is defined based on a collision between some pair of bodies, and after the collision, these bodies move toward minimum energy level (Kaveh and Mahdavi 2014). Consider two moving bodies with masses of m_1 and m_2 and velocities of v_1 and v_2, in which one object collides with another object. According to the laws of physics, the total momentum and energy of the system after and before the collision are conserved. It can be expressed as

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{46}$$

and

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2 + Q \tag{47}$$

where v_1, v_2, v_1' and v_2' are the velocities of the bodies before and after the collision respectively. m_1 and m_2 are the masses of the first and second body, respectively; also Q is the kinetic energy loss due to collision.

After a collision, the velocities of bodies pair can be determined as

$$v_1' = \frac{(m_1 - \varepsilon m_2)v_1 + (m_2 + \varepsilon m_2)v_2}{m_1 + m_2} \tag{48}$$

$$v_2' = \frac{(m_2 - \varepsilon m_1)v_2 + (m_1 + \varepsilon m_1)v_1}{m_1 + m_2} \tag{49}$$

where ε is coefficient of restitution (COR) of two colliding bodies, which defined as

$$\varepsilon = \frac{|v_2' - v_1'|}{|v_2 - v_1|} = \frac{v'}{v} \tag{50}$$

for realistic objects, ε is in the range of 0 and 1.

In the CBO algorithm, the agents X_i are assumed to be the Colliding Bodies (CB). The particles set are divided into two same groups including stationary and moving particles, where the particles move towards each other, and a collision occurs between pairs of particles. After this event, the positions of the particles are updated according to their new velocities.

A summary of the steps in the CBO algorithm is as follows

Step 1. The initial positions of CBs are generated randomly in the search space

$$X_i^0 = X_{min} + rand(X_{max} - X_{min}) \quad i = 1,2,\ldots,2n \tag{51}$$

where X_i^0 is the initial value of the *i*th CB; X_{min} and X_{max} are the lower and upper bound of the variables; *rand* is a random value in the range of 0 to 1; and *2n* is the number of CBs.

Step 2. The mass of the body for each CB is determined as

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}} \quad k = 1,2,\ldots,2n \tag{52}$$

where *fit(i)* illustrates the cost of the ith agent. It is clear that a CB with a large mass displays a good performance than the lightest ones.

Step 3. The CBs sorted based on the cost in increasing order. The sorted CBs are divided into two same groups. The good CBs are stationary bodies with zero velocity before the collision. So

$$v_i = 0 \qquad i = 1,2,\ldots,n \tag{53}$$

The bad CBs are moving agents, that move toward the stationary agents. These moving bodies have a velocity of before collision as follows

$$v_i = x_i - x_{i-n} \qquad i = n+1,\ldots,2n \tag{54}$$

where x_i and x_{i-n} are the position vectors of the *i*-th CB in the moving group and its pair in the stationary group, respectively.

Step 4. After the collision, the velocity of particles in each group is determined as follows

After the collision the moving CBs have velocity as follows

$$v_i' = \frac{(m_i - \varepsilon m_{i-n})v_i}{m_i + m_{i-n}} \quad i = n+1,\ldots,2n \tag{55}$$

and, the velocity of stationary CBs is

$$v_i' = \frac{(m_{i+n} + \varepsilon m_{i+n})v_{i+n}}{m_i + m_{i+n}} \quad i = 1,\ldots,n \tag{56}$$

where ε is COR that controls the exploration and exploitation rates. So The COR decreases linearly from unit value to zero and is defined as

$$\varepsilon = 1 - \frac{iter}{iter_{max}} \tag{57}$$

where $iter$ and $iter_{max}$ are the numbers of current and maximum iterations respectively.

Step 5. The new positions of CBs are determined based on the velocities of CBs after the collision. So, the new positions of moving CBs are

$$X_i^{new} = X_{i-n} + rand^{o} v_i' \quad i = n+1,\ldots,2n \tag{58}$$

Also, the new position of each stationary CB is

$$X_i^{new} = X_i + rand^{o} v_i' \quad i = 1,\ldots,n \tag{59}$$

where X_i^{new} is the new position of the *i*-th moving and stationary CB after the collision.

Step 6. The solution route is repeated from Step 2 until the termination criterion is satisfied.

5. Efficient reliability-based optimization

The meta-heuristic methods in solving an optimization problem have a high convergence rate, However, the reliability of the system is not considered in the optimal design. Also, the inverse reliability methods are gradient-based and need to calculate the gradient relative to random variables and unknown parameters, so with an increase in the unknowns, the complexity of the problem increases. As a result, combining these two methods can increase the efficiency of the strategy.

This method can define as follow

Find: Design Variables=$\{L_i, d_i, D_i, \alpha_i, S_v \text{ and } S_h\}$

To minimize: Objective Function= Total Cost

$$\text{Subject to:} \begin{cases} FS_d \geq FS_{all} \\ \big|\|u^*\| - \beta_t\big| \approx 0 \\ \alpha \leq 0 \end{cases} \tag{60}$$

where u is the vector of random variables, and u^* is the MPP.

To solve this optimization problem, the combination of two CBO and FORM methods is used as a double loop. Table 2 illustrates the problem solving pseudo code for this strategy. Also, considering that, the value of the reliability index is calculated using the FORM for each particles of CBO, Constraints of inverse reliability problem in Eq. (60) will be satisfied.

Random variables can be selected as the most influencing load and resistance parameters for the given failure mode. In-situ soil is one of the materials that mechanical properties changes with location and depth. Therefore, uncertainty in these parameters causes them to be considered random variables. However, the approach can be extended to a higher number and variety of random variables. The influence of the variability of in-situ soil parameters, namely, cohesion (c), angle of internal friction (φ), unit weight of the soil (γ) and bond strength of nails, has been studied by performing reliability analysis for different failures modes. In-situ soil cohesion (c), angle of internal friction (φ), unit weight of the soil (γ) and bond

Table 2 Double loop pseudo of efficient RBDO

```
Start
a- Initialize N agent for the design variables
while iter<itermax
b- For i=1,…,N
c- Calculate the objective function for agents
        While error < threshold error
        c-1 Select the start point for random variables
        c-2 Evaluate the limit state function
        c-3 Evaluate the gradient of limit state function
        c-4 Compute the search direction and step size
        c-5 Determine the position of new point
        c-6 Calculate the reliability index
        end
        c-7 Report the reliability index
d- Sort the agents in an ascending order
e- The particles are divided into two Good
and Bad particles groups
f- Determine new position of each particle
g- Collecting the particles
h- Repeat from c till g
k- Iter=iter+1
end
Report the best solution
Stop
```

strength of nails are considered as normally distributed uncorrelated random variables with mean value and standard deviation. Soil and nail strength parameters are adopted as random variables for global stability, sliding stability, soil-nail pullout failure, tensile strength and facing flexure failure, and facing punching shear failure modes.

6. Numerical study

The main purpose of this research is to the optimal design of the soil nailing system under uncertain conditions. Therefore, first, using the meta-heuristic optimization method, an optimal design for the soil nailing system is performed for a numerical example of deep excavation under certain conditions, and the optimal parameters are determined for the geometric details of the nailing. Then, the reliability analysis is performed on an optimally designed soil nail wall system under uncertainty in the mechanical parameters of the soil mass and excavation. Also, a sensitivity analysis is used to evaluate the effect of the uncertainty of the soil nail system parameters on the factors of safety. Finally, the optimal design of the soil nail wall for deep excavation under uncertainties is performed using the reliability-based design optimization method with meta-heuristic methods.

Some accepted procedures are presented for the design of soil nail walls as guidelines. The guidelines have been compiled based on experimental results and fundamental theories. The FHWA manual is one of the guidelines used for the design and construction of soil nail wall details. In this study, the design procedure and checks were done considering the Allowable Stress Design approach (ASD) as presented in FHWA guidelines.

Due to sufficient accuracy and ease of simulation, finite element analysis has been developed. In order to validate the applied analysis results and design procedure for soil nail walls, the numerical analysis was done using finite element and limit equilibrium techniques. The ABAQUS software was used for Finite Element Analysis (FEA) and MATLAB script was used for Limit Equilibrium Analysis (LEA). For both numerical analyses (FEA and LEA), the effective stress condition or drained parameters were used. The factors of safety were determined from the combination of the FEA and LEA results.

Top-down construction was simulated in ABAQUS by staged construction technique. The soil was modelled using Mohr–Coulomb soil model, which is dependent on soil cohesion and internal friction angle. The Plain Strain model of soil excavation was considered for the model analysis.
The axial tension force of the nails and deformation of the facing wall was calculated from the Finite Element Analysis in ABAQUS. The nails were modelled using an embedded region model in which their stiffness was determined considering the grouted nails. Also, the facing of the soil nail wall was modelled using plate elements. To simulate a relatively larger area, the end boundary conditions were considered to be at roller condition on the sides (Ux = 0) and fixed at the bottom (Ux = Uy = 0). The finite element model that includes soil, nail and a facing wall is depicted in Fig. 2.

In order to evaluate the performance and efficiency of the proposed method for reliability-based design optimization with meta-heuristic methods for the optimal design of soil nail walls, realistic numerical studies are used. These numerical examples include deep excavations that have been stabilized using soil nail walls (Sharma and Ramakrishnan 2020). Based on the strength limit and service limit the global stability, sliding check, nail tensile resistance, pull-out resistance, face bending, and face-punching are used to control the internal and external failure modes. The geometric details of the numerical

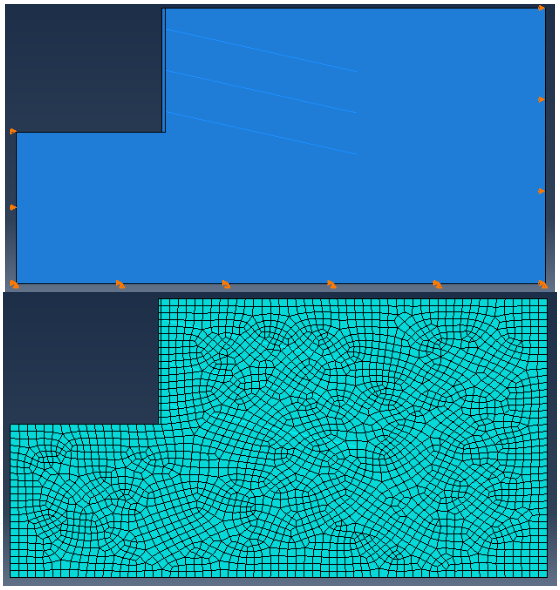


Fig. 2 Finite element model of soil nail wall system

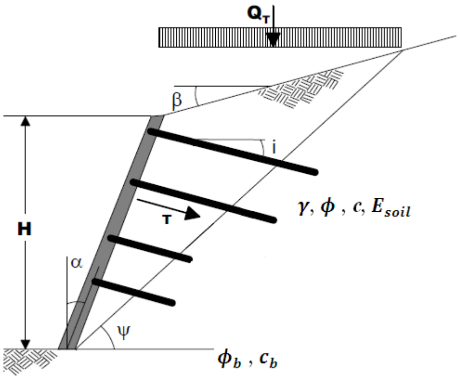


Fig. 3 Schematic of the geometric details of the numerical example

example are shown schematically in Fig. 3. The Required parameters for introducing the geometrical and mechanical details of the soil nail wall system are presented in Table 3.

As mentioned, initially an optimum soil nail wall system designed using meta-heuristic optimization algorithm. This optimum design lead to nailig system with minimum construction cost and constraints of safety factores. The CBO algorithm is used to solve the optimization problem of soil nail wall design. In order to design the nailing system, the geometric parameters of the soil nail wall such as the length of nails, the angle of nails, the size of bars, the vertical distance of nails, and the diameter of nail holes are selected as the design variable. The objective function to solve the design optimization problem is defined based on the cost function of the soil nail wall system. The cost of the nailing system can be calculated as a function of the drilling and grouting volume and the weight of the nail bars on the scale of wall surface. So the objective function can be written as

$$Objective\ function = \frac{\sum_{i=1}^{n}\left(\gamma_{st}\left(\frac{\pi d_i^2}{4}\right)L_i C_S + \left(\frac{\pi D_i^2}{4}\right)L_i C_c\right)}{S_H.H} \qquad (61)$$

Table 3 Geometrical and Mechanical Parameters of Problem

Part	Parameter	Unit	Value
Geometry of Excavation	Height of Wall (H)	M	8
	Wall Face Angle (α)	-	5
	Backslope Angle (β)	-	0
	Dead Load Surcharge (Q_T)	kN/m^2	10
	Live Load Surcharge (Q_T)	kN/m^2	20
Mechanical Parameters of Soil	Soil Friction Angle (ϕ)	-	30
	Soil Cohesion (c)	kN/m^2	3
	Unit Weight of Soil (γ)	kN/m^3	20
	Soil Module of Elasticity (E_{soil})	kN/m^2	20000
	Base Friction Angle (ϕ_b)	-	30
	Base Cohesion (c_b)	kN/m^2	5
Mechanical Parameters of Nail	Number of Nails Row (n)	-	3, 5
	Bond Strenght (q_u)	kPa	125
	Grout Module of Elasticity (E_{gr})	GPa	2
	Yield Stress of Steel (F_y)	Mpa	415
	Steel Module of Elasticity (E_{st})	Gpa	200
Mechanical Parameters of Concrete	Strength of Concrete (f_c)	Mpa	21
	Concrete Module of Elasticity (E_c)	GPa	20
	Tickness of Facing (t_c)	m	0.2

The constraints of this problem are defined based on the safety factors of global stability, sliding, soil nail pull out, nail tension strength, faceing punching and faceing flextural failure.

For reliability analysis, the first-order reliability method based on the HL-RF algorithm is used. Due to the uncertainty in the mechanical parameters of soil and nails, the components of soil adhesion coefficient, internal friction angle of soil, soil specific gravity, soil modulus of elasticity, and bond strength of soil nails are selected as random variables. The random variables are modeled using the lognormal probability distribution function. The limit state function is defined based on the probability that the system safety factors exceed the allowable values. So the limit state function can be selected as (Babu and Singh 2011)

$$g(x) = (Factor\ of\ Safety)_i - 1 \qquad (62)$$

For reliability-based design optimization of the soil nail wall system, a combination of reliability analysis and CBO optimization algorithm is used. In order to design the nailing system, the geometric parameters of the soil nail wall such as the length of nails, the angle of nails, the size of bars, the vertical distance of nails, and the diameter of nail holes are selected as the design variable. the search domain of design variables such as lower and upper bound are presented in Table 4. Also, the mechanical parameters of soil and nails, such as the soil cohesion coefficient, internal friction angle of soil, soil specific gravity, soil modulus of elasticity, and bond strength of soil nails are selected as random variables. The distribution parameters of the

Table 4 Design Variables in Optimization Problem

Parameter	Unit	Search Domain	
		Lower Bond	Upper Bond
Length of Nail	m	1	20
Bar Size	mm	10	30
Angle of Nail	-	0	30
Vertical Distances of Nails	m	0.5	3
Horizontal Distances of Nails	m	0.5	3
Diameter of Nail	mm	50	200

Table 5 Random Variables in reliability analysis

Parameter	Unit	Normal Distribution	
		Mean (μ)	Standard Deviation (σ)
Soil Friction Angle (ϕ)	-	30	6
Soil Cohesion (c)	kN/m^2	3	0.6
Unit Weight of Soil (γ)	kN/m^3	20	4
Soil Module of Elasticity (E_{soil})	kN/m^2	20000	4000
Base Friction Angle (ϕ_b)	-	30	6
Base Cohesion (c_b)	kN/m^2	5	1
Bond Strenght	kPa	125	25

random variables are presented in Table 5. The cost of the nailing system can be calculated as a function of the drilling and grouting volume and the weight of the nail bars on the scale of wall surface. So the objective function can be written as

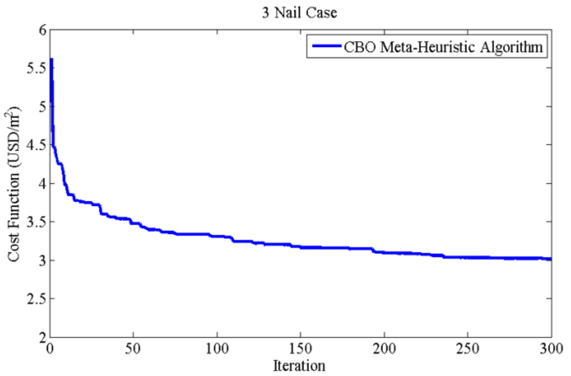
$$Objective\ function = \frac{\sum_{i=1}^{n}\left(\gamma_{st}\left(\frac{\pi d_i^2}{4}\right)L_i C_S + \left(\frac{\pi D_i^2}{4}\right)L_i C_c\right)}{S_H.H} \quad (63)$$

The constraints of this problem are defined based on the set of target reliability index and safety factors.

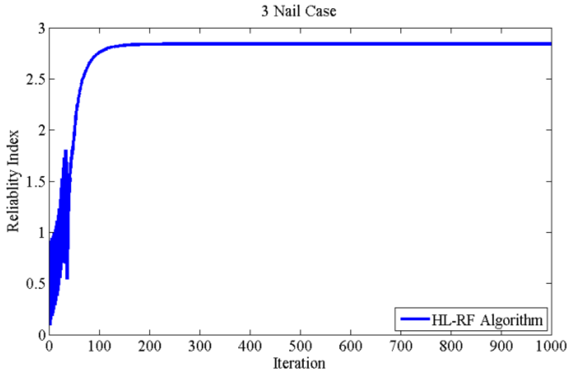
$$Constraints: \begin{cases} \beta = \beta_{target} \\ (FS)_i \geq (FS)_{i-Allowable} \end{cases} \quad (64)$$

7. Results and discussion

In the first case, the problem of the optimum design of soil nail walls using the CBO algorithm and reliability analysis of walls with a nailing system is presented. These problems are solved in stabilization cases with 3 nails and 5 nails. So, the convergence history of the objective function based on the total cost of nails and grout using the CBO algorithm and the convergence history of the reliability index using FORM are given in Figs. 4 and 5. For a stabilization case with 3 nails, the minimum cost is 3.0181 \$/m^2, and the reliability index for this optimum designed soil nailing system with 20% COV uncertainty of soil parameters and limit state functions based on safety factors is 2.840. Also, for a stabilization case with 5 nails, the minimum cost is 3.2637 \$/m^2, and the reliability index for

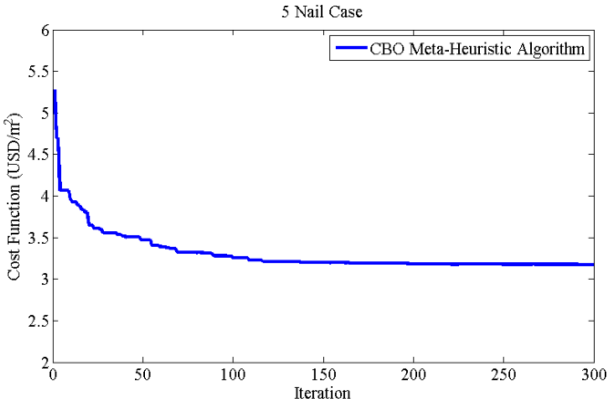


(a) Cost function

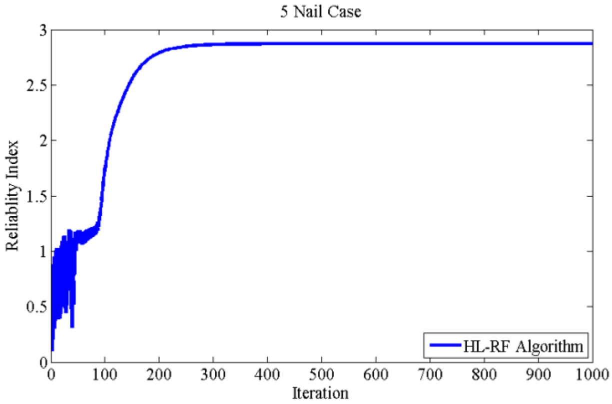


(b) Reliability index

Fig. 4 Convergence history for case of 3 Nail



(a) Cost function



(b) Reliability index

Fig. 5 Convergence history for case of 5 Nail

Table 6. Optimum designed parameters and constraints of problem

	Parameters	3 Nail	5 Nail
Objective Function	Cost ($\$/m^2$)	3.0181	3.2637
Design Variables	Diameter of Nail (mm)	110 110 110	110 110 110 110 100
	Diameter of Strud (mm)	18 16 16	16 16 16 16 16
	Length of Nail (m)	4.925 3.771 3.117	4.408 4.378 4.127 3.116 3.089
	Angle of Inc (°)	24.40 20.35 18.97	19.61 21.07 13.11 24.19 8.90
	Vertical Distance (m)	2.715 1.613 1.241	1.276 1.497 1.287 0.813 0.745
	Horizontal Distance (m)	0.623	0.878
Constraints	Global Stability	1.355	1.379
	Sliding Stability	1.565	1.505
	Pullout Resistance	2	2.221
	Tension Resistance	1.934	1.800
	Facing Punch	5.039	8.195

this optimum designed soil nailing system with 20% COV uncertainty of soil parameters and limit state functions based on safety factors is 2.871.

The minimum cost, the optimum designed nailing parameters, and safety factor constraints are presented in Table 6. According to Table 6, the optimum design of a soil nail wall with 3 nails is affected by pull-out strength. Also, the optimum design of a soil nail wall with 5 nails is affected by the tension strength of the nails.

As mentioned, the behavior of a soil nail wall is influenced by the geometrical and mechanical parameters of soil and nails. Using the importance vector, it is possible to determine the relative importance and effect of each parameter. The importance vector is calculated by reliability analysis based on the limit state functions of the safety factors. The problem is solved for 3 and 5 nail cases. In this reliability analysis, the mechanical parameters of soil were selected as random variables. The reliability index and importance vector are evaluated for various uncertainty with 10, 20, and 30 % COV.

Table 7 Reliability index and importance vector

	% COV	Case	3 Nail	5 Nail
β	COV =%10		5.680	5.743
	COV =%20		2.840	2.871
	COV =%30		1.893	1.914
Importance Vector (α)	c		-0.402	-0.447
	φ		-0.246	-0.230
	γ		-0.590	-0.553
	c_b		-0.417	-0.467
	φ_b		-0.361	-0.320
	q_s		-0.354	-0.347

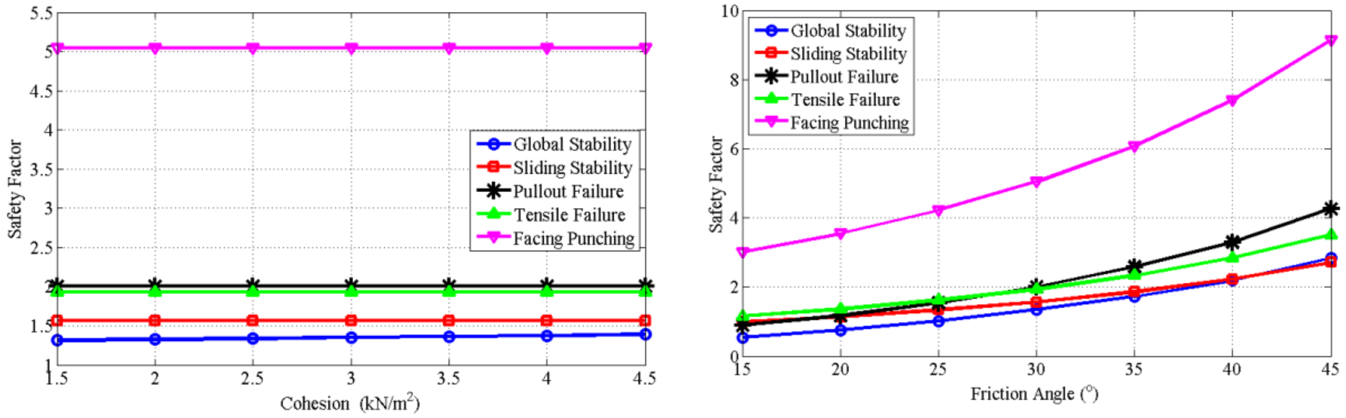
Table 7 shows the reliability index and components of the importance vector for the soil parameters. According to these results, for both 3 nails and 5 nails cases, increasing uncertainty in soil parameters causes an extreme decrease in the reliability of the soil nail walls. Also, the unit weight of the soil is the most effective parameter and friction angle has the least effect on the reliability of the soil nail wall.

Also, a sensitivity analysis has been carried out by using different values of mechanical parameters of soil, and the variation of safety factors has been investigated. In other words, relations between variations of soil mechanical parameters and safety factors are revealed. Figure 6 presents the sensitivity of the safety factors of soil nail walls relative to the soil mechanical parameters. From Figure 6 one can notice that the safety factors of a soil nail wall have different sensitivity to the soil mechanical parameters.

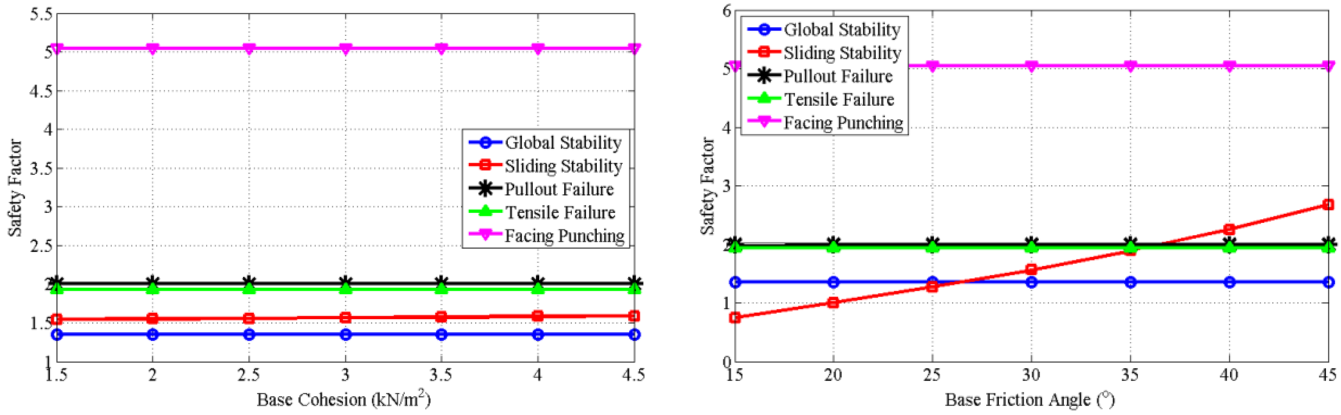
As can be seen in Figs. 6(a), only global stability is slightly sensitive to cohesion, and other safety factors are not sensitive to cohesion. According to Fig. 6(b), all safety factors are sensitive to friction angle. Increasing the internal friction angle causes a significant increase in all safety factors. As shown in Fig. 6(c), only sliding stability is slightly sensitive to base cohesion, and other safety factors are not sensitive to base cohesion. Also according to Fig. 6(d), only sliding stability is extremely sensitive to base friction, and other safety factors are not sensitive to base friction. According to the results in Fig. 6(e), All safety factors are affected by the specific unit weight parameter.

The sliding stability safety factor increases slightly with increasing specific unit weight. But other safety factors decrease significantly with increasing specific unit weight. As can be seen in Fig. 6(f), just two global stability and pullout failure safety factors are sensitive relative to the bond strength of nails.

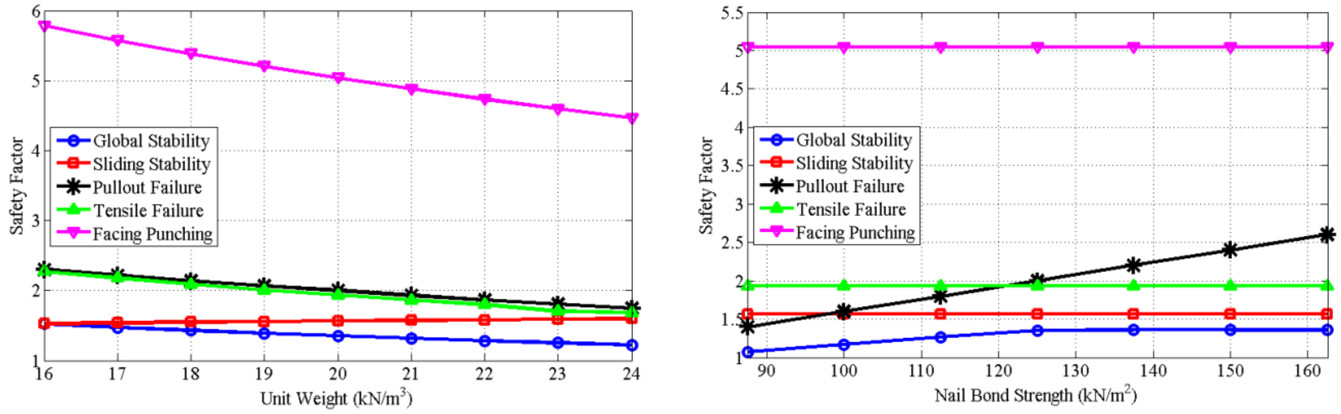
In the second part, the optimal design of the nailing system is carried out using reliability-based design optimization. As mentioned in section 4, the reliability-based design optimization (RBDO) includes the reliability analysis and CBO meta-heuristic algorithm. For the case of stabilization with 3 nails, the designed nailing system based



(a) Cohision (b) Friction angles



(c) Base cohision (d) Base friction



(e) Specified unit weight (f) Nail bond streng

Fig. 6 Sensitivity of safety factors for random variables

on a meta-heuristic algorithm and safety factor constraints has a reliability index of 2.840 for uncertainty of 20% COV.

This reliability index is equivalent to the probability of failure of 0.0022, which indicates the stability in situations between below-average and above-average (Babu and Singh 2011). In order to improve the stability of geotechnical structures in good situations, a reliability index of 4 and a probability of failure of 0.00003 is required (Babu and Singh 2011). In the case of reliability-based optimal design, this value of the reliability index is added to the constraints of the problem as the target reliability index.

The results obtained from the reliability-based optimal design for the nailing with 3 nails are presented in Table 8.

According to the results obtained from the optimal design of soil nail wall based on reliability, it is observed that this method increases the cost of the nailing system by 18.7% (((3.5823-3.0181)/3.0181)×100) but increases the level of safety and reliability of the system.

As it is known, the safety factors of nailed walls are affected by the nails' tensile force. Also, the deformation constraint for the soil wall is a function of the lateral displacement of the wall. In this regard, according to the introduction of the numerical study, to calculate the tensile strength of nails and determine the deformation of the soil wall, the finite element method was used in ABAQUS. Therefore, during the optimum design process of the nailing

Table 8. Optimum designed parameters and constraints of RBDO problem

	Parameters	3 Nail
Objective Function	Cost ($\$/m^2$)	3.5823
Design Variables	Diameter of Nail (mm)	120
		120
		120
	Diameter of Strud (mm)	20
		20
		20
	Length of Nail (m)	5.004
		3.671
		3.642
	Angle of Inc (°)	19.66
		19.90
		19.62
	Vertical Distance (m)	2.606
		1.394
		1.402
	Horizontal Distance (m)	0.773
Constraints	Reliability Index	4
	Global Stability	1.578
	Sliding Stability	1.635
	Pullout Resistance	2.071
	Tension Resistance	2.140
	Facing Punch	6.771

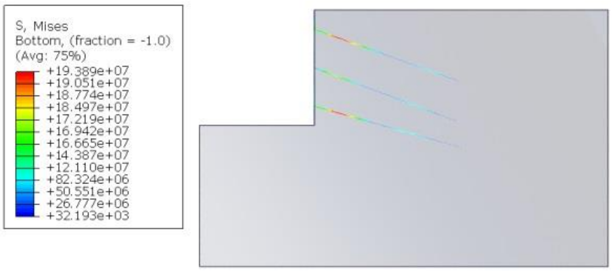


Fig. 8 The results of finite element model for the stress of 3 nails

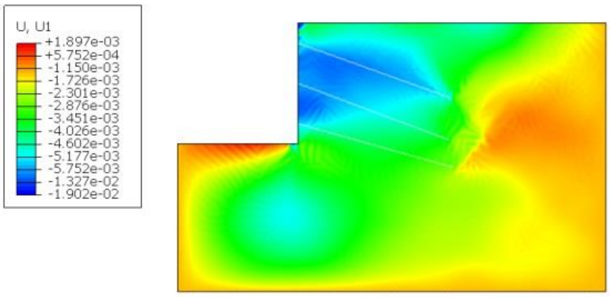


Fig. 9 The results of finite element model for lateral displacement

system, the results of soil nail wall analysis were used to determine the nail forces and lateral wall displacement. In the following, the results of finite element analysis of the soil nail wall system for the optimum design of the nailed system are presented. Fig. 8 shows the stress distribution contours in the nails. Fig. 9 also shows the lateral displacement contours of the soil wall. According to the displacement distribution in the soil wall, it is observed that the lateral deformation in the wall with 3 nails is distributed uniformly.

8. Conclusions

The overall objective of this paper was to study the effect of uncertainty in the mechanical properties of soil on the safety of soil nail walls. Also, the optimum parameters of the nailing systems were calculated so that results cause a reduction in the cost and failure probability in uncertainty conditions. Also, the optimum parameters of the nailing systems were calculated subjected to the cost, safety factors, reliability, and failure probability in uncertain conditions. Colliding bodies optimization, a good operation optimization algorithm in engineering problems, is used to estimate optimum parameters of nailing systems in deterministic conditions, and a reliability-based design optimization method is used in uncertainty conditions. The objective function used in this study is based on the total cost of nail bars and grout. To design the soil nail wall for an expected level of reliability the target reliability index was used.

As a numerical study, an 8-meter height soil wall is considered where stabilized in 2 cases with 3 and 5 nails. To calculate the nails' tensile forces and safety factors limited equilibrium analysis and finite element analysis methods are used. In the first case study, nailing system parameters were optimized using the CBO algorithm. As a result, the value of mechanical parameters of the nailing system is determined for both 3 and 5 nail cases. Then reliability analysis using the FORM was performed to optimally design soil nail wall and reliability index is calculated for different uncertainty levels. The results show that the cost and reliability of both 3 and 5 nails cases are almost similar, although the cost and reliability of the 5 nails case are a little higher. Also, increasing the degree of uncertainty in the mechanical parameters of the soil significantly reduces the reliability of the soil nail wall.

To investigate the importance of each mechanical parameter of soil in the reliability of the nailing system with limit state functions based on safety factors, importance vector analysis was used. The results show that the specified unit weight and friction angle respectively have the most and least importance in the reliability of soil nail wall. Also, a sensitivity analysis is conducted to evaluate the effect of uncertainty in soil mechanical parameters on safety factors such as Global Stability, Sliding Stability, Pullout Resistance, Tension Resistance, and Facing Punch. The results show that none of the safety factors in the soil nail wall are sensitive to soil and base cohesion. While all of the safety factors are sensitive to the soil friction angle and specified unit weight.

In the second case study, the reliability-based design optimization of the soil nail wall was performed based on the target reliability. In order for the soil structure to stay in a good stability situation, the target reliability index of 4 is used. The design results based on the RBDO method show that the values calculated for the design variables increase the cost of the nailing system by 18.7%. But for this design, the reliability of the system increases by 40.85%, and the probability of failure decreases by 98.6%.

References

Arama, Z.A., Kayabekir, A.E., Bekdaş, G., Kim, S. and Geem Z.W., (2021), "The usage of the harmony search algorithm for the optimal design problem of reinforced concrete retaining walls" *Appl. Sci.*, **11**(3), 1343. https://doi.org/10.3390/app11031343.

Azar, B.F., Hadidi, A. and Rafiee, A. (2015), "An efficient simulation method for reliability analysis of systems with expensive-to-evaluate performance functions", *Struct. Eng. Mech.*, **55**(5), 979-999. https://doi.org/10.12989/sem.2015.55.5.979.

Babu, G.L.S. and Singh, V.P. (2011), "Reliability-based load and resistance factors for soil-nail walls", *Can. Geotech. J.*, **48**(6), 915- 930. https://doi.org/10.1139/t11-005.

Benayoun, F., Boumezerane, D., Bekkouche, S.R. and Ismail, F. (2021), "Optimization of geometric parameters of soil nailing using response surface methodology", *Arabian J. Geosci.*, **14**, 1965. https://doi.org/10.1007/s12517-021-08280-z.

Chakraborty, S. and Rajib Chowdhury, R. (2016), "Assessment of polynomial correlated function expansion for high-fidelity structural reliability analysis", *Struct. Saf.*, **59**, 9-19. https://doi.org/10.1016/j.strusafe.2015.10.002.

Der Kiureghian, A.D. and Stefano, M.D. (1991), "Efficient algorithm for second-order reliability analysis", *J. Eng. Mech.*, **117**(12), 2904-2923. https://doi.org/10.1061/(ASCE)0733-9399(1991)117:12(2904).

Der Kiureghian, A. (2005), Engineering Design Reliability Handbook, CRC Press, Boca Raton, FL, USA.

Ditlevsen, O. (1982), "Model uncertainty in structural reliability", *Struct. Saf.*, **1**(1), 73-86. https://doi.org/10.1016/0167-4730(82)90016-9.

Doan, B.Q., Liu, G. and Xu, C. (2018), "An efficient approach for reliability-based design optimization combined sequential optimization with approximate models", *Int. J. Comput. Methods*, **15**(1), 1850018. https://doi.org/10.1142/S0219876218500184.

Du, X. (2005), *First-Order and Second-Reliability Methods, in Probabilistic Engineering Design*, Missouri S&T, Rolla, ME, USA.

Duncan, J.M. (2000), "Factors of safety and reliability in geotechnical engineering". *J. Geotech. Geoenviron. Eng.*, **126**(4), 307-316. https://doi.org/10.1061/(ASCE)1090-0241(2000)126:4(307).

Fan, C.C. and Luo, J.H. (2008), "Numerical study on the optimum layout of soil–nailed slopes", *Comput. Geotech.*, **35**(4), 585-599. https://doi.org/10.1016/j.compgeo.2007.09.002.

Farsani, A.M. and Keshtegar, B. (2015), "Reliability analysis of corroded reinforced concrete beams using enhanced HL-RF method", *Civil Eng. Infrastruct. J.*, **48**(2), 297-304. https://doi.org/10.7508/CEIJ.2015.02.006.

FHWA. (2003), *Geotechnical engineering circular No. 7 soil nail walls*. Federal Highway Administration, Washington, D.C. Report FHWA0-IF-03-017.

Ghareh, S. (2015), "Parametric assessment of soil-nailing retaining structures in cohesive and cohesionless soils" *Measurement*, **73**, 341-351. https://doi.org/10.1016/j.measurement.2015.05.043

Gong, J.X. and Yi, P. (2011), "A robust iterative algorithm for structural reliability analysis", *Struct. Multidiscip. O.*, **43**(4), 519-527. https://doi.org/10.1007/s00158-010-0582-y.

Gong, J.X., Yi, P. and Zhao, N. (2014), "Non-gradient-based algorithm for structural reliability analysis", *J. Eng. Mech.*, **140**(6), 04014029. https://doi.org/10.1061/(ASCE)EM.1943-7889.0000722.

Goswami, S., Ghosh, S. and Chakraborty, S. (2016), "Reliability analysis of structures by iterative improved response surface method", *Struct. Saf.*, **60**, 56-66. https://doi.org/10.1016/j.strusafe.2016.02.002

Hadidi, A., Azar, B.F. and Shirgir, S. (2019), "Reliability assessment of semi-active control of structures with MR damper", *Earthq. Structures*, **17**(2), 131-141. https://doi.org/10.12989/eas.2019.17.2.131.

Hasofer, A.M. and Lind, N.C. (1974), "Exact and invariant second-moment code format", *J. Eng. Mech. Division*, **100**(1), 111-121. https://doi.org/10.1061/JMCEA3.0001848.

Hosseini, M., Naeini, S.A.M., Dehghani, A.A. and Khaledian, Y. (2016), "Estimation of soil mechanical resistance parameter by using particle swarm optimization, genetic algorithm and multiple regression methods", *Soil Tillage Res.*, **157**, 32-42. https://doi.org/10.1016/j.still.2015.11.004.

Huu, V.H., Thoi, T.N., Anh, L.L. and Trang, T.N. (2016), "An effective reliability-based improved constrained differential evolution for reliability-based design optimization of truss structures", *Adv. Eng. Softw.*, **92**, 48-56. https://doi.org/10.1016/j.advengsoft.2015.11.001.

Johari, A. and Fooladi, H. (2022), "Simulation of the conditional models of borehole's characteristics for slope reliability assessment", *Transport. Geotech.*, **35**, 100778. https://doi.org/10.1016/j.trgeo.2022.100778.

Johari, A. and Golkarfard, H. (2018), "Reliability analysis of unsaturated soil sites based on fundamental period throughout Shiraz, Iran", *Soil Dyn. Earthq. Eng.*, **115**, 183-197. https://doi.org/10.1016/j.soildyn.2018.08.012.

Johari, A., Golkarfard, H. and Mesbahi, M. (2022), "The effect of nano-clay stabilizing treatment on the real excavation wall failure: A case study", *Scientia Iranica*, **29**(3), 1006-1023. https://doi.org/10.24200/SCI.2022.56364.4690.

Johari, A., Vali, B. and Golkarfard, H. (2021), "System reliability analysis of ground response based on peak ground acceleration considering soil layers cross-correlation", *Soil Dyn. Earthq. Eng.*, **141**, 106475. https://doi.org/10.1016/j.soildyn.2020.106475.

Johari, A., Hajivand A.K. and Binesh, S.M. (2020), "System reliability analysis of soil nail wall using random finite element method", *Bull. Eng. Geol. Environ.*, **79**, 2777-2798. https://doi.org/10.1007/s10064-020-01740-y.

Kalantari, A.R. and Johari, A. (2022), "System reliability analysis for seismic stability of the soldier pile wall using the conditional random finite-element method", *Int. J. Geomech.*, **22**(10), 04022159. https://doi.org/10.1061/(ASCE)GM.1943-5622.0002534.

Kalantari, A.R., Johari, A., Zandpour, M. and Kalantari, M. (2023), "Effect of spatial variability of soil properties and geostatistical conditional simulation on reliability characteristics and critical slip Surfaces of Soil Slopes", *Transport. Geotech.*, 100933. https://doi.org/10.1016/j.trgeo.2023.100933.

Kalehsar, R.I., Khodaei, M., Dehghan, A.N. and Najafi, N., (2021), "Numerical modeling of effect of surcharge load on the stability of nailed soil slopes", *Modeling Earth Syst. Environ.*, https://doi.org/10.1007/s40808-021-01087-7.

Kaveh, A. and Mahdavi, V.R. (2014), "Colliding bodies optimization: A novel meta-heuristic method", *Comput. Struct.*, **139**, 18-27. https://doi.org/10.1016/j.compstruc.2014.04.005.

Kaveh, A. Massoudi, M.S. and Ghanooni Bagha, M. (2014), "Structural reliability analysis using charged system search algorithm", *Iranian J. Sci. Tech T. Civil Eng.*, **38**(2), 439-448. https://doi.org/10.22099/IJSTC.2014.2420.

Keshtegar, B. (2016), "Chaotic conjugate stability transformation method for structural reliability analysis", *Comput. Method. Appl. M.*, **310**, 866-885. https://doi.org/10.1016/j.cma.2016.07.046.

Keshtegar. B. and Hao. P. (2018), "Enhanced single-loop method for efficient reliability-based design optimization with complex constraints", *Struct. Multidiscip. O.*, **57**(4), 1731-1747. https://doi.org/10.1007/s00158-017-1842-x.

Le, L.A., Vinh, T.B., Huu, V.H. and Thoi, T.N. (2017), "An efficient coupled numerical method for reliability-based design optimization of steel frames", *J. Constr. Steel Res.*, **138**, 389-400. https://doi.org/10.1016/j.jcsr.2017.08.002.

Lee, J.O., Yang, Y.S. and Ruy, W.S. (2002), "A comparative study on reliability-index and target-performance-based probabilistic structural design optimization", *Comput. Struct.*, **80**(3-4), 257-269. https://doi.org/10.1016/S0045-7949(02)00006-8.

Lehky. D., Slowik. O. and Novak. D. (2018), "Reliability-based design: Artificial neural networks and double-loop reliability-based optimization approaches", *Adv. Eng. Softw.*, **117**, 123-135. https://doi.org/10.1016/j.advengsoft.2017.06.013.

Liu, P.L. and Der Kiureghian, A. (1991), "Optimization algorithms for structural reliability", *Struct. Saf.*, **9**(3), 161-177. https://doi.org/10.1016/0167-4730(91)90041-7.

Liu, Q. and Paavola, J. (2015), "Drift reliability-based optimization method of frames subjected to stochastic earthquake ground motion", *Appl. Math. Model.*, **39**, 982-999. https://doi.org/10.1016/j.apm.2014.07.021.

Makhduomi, H., Keshtegar, B. and Shahraki, M. (2017), "A comparative study of first-order reliability method-based steepest descent search directions for reliability analysis of steel structures", *Adv. Civil Eng.*, 8643801. https://doi.org/10.1155/2017/8643801.

Manahiloh, K.N., Nejad, M.M. and Momeni, M.S., (2015), "Optimization of design parameters and cost of geosynthetic-reinforced earth walls using harmony search algorithm", *Int. J. Geosynth. Ground Eng.*, **1**, 15. https://doi.org/10.1007/s40891-015-0017-3.

Meng, Z., Li, G., Yang, D. and Zhan, L. (2017), "A new directional stability transformation method of chaos control for first-order reliability analysis", *Struct. Multidiscip. Optim.*, **55**(2), 601-612. https://doi.org/10.1007/s00158-016-1525-z.

Pak, A., Maleki, J., Aghakhani, N. and Yousefi, M., (2019), "Numerical investigation of stability of deep excavations supported by soil-nailing method", *Geomech. Geoeng.*, **16**(6), 434-451. https://doi.org/10.1080/17486025.2019.1680878.

Patra, C.R. and Basudhar, P.K. (2005), "Optimum design of nailed soil slopes", *Geotech. Geol. Eng.*, **23**, 273-296. https://doi.org/10.1007/s10706-004-2146-7.

Rackwitz, R. and Flessler, B. (1978), "Structural reliability under combined random load sequences", *Comput. Struct.*, **9**(5), 489-494. https://doi.org/10.1016/0045-7949(78)90046-9.

Rashki, M., Miri, M. and Moghaddam, M.A. (2012), "A new efficient simulation method to approximate the probability of failure and most probable point", *Struct. Saf.*, **39**, 22-29. https://doi.org/10.1016/j.strusafe.2012.06.003.

Rawat, S. and Gupta, A.K., (2016), "Analysis of a nailed soil slope using limit equilibrium and finite element methods", *Int. J. Geosynth. Ground Eng.*, **2**, 34. https://doi.org/10.1007/s40891-016-0076-0.

Santosh, T., Saraf, R., Ghosh, A. and Kushwaha, H. (2006), "Optimum step length selection rule in modified HL–RF method for structural reliability", *Int. J. Pressure Vessels Piping*, **83**(10), 742-748. https://doi.org/10.1016/j.ijpvp.2006.07.004

Seo, H.J., Lee, I. and Lee, S.W. (2014), "Optimization of soil nailing design considering three failure modes" *KSCE J. Civil Eng. Geotech. Eng.*, **18**(2), 488-496. https://doi.org/10.1007/s12205-014-0552-9.

Shamsaddinlou, A., Shirgir, S., Hadidi, A. and Azar, B.F. (2023), "An efficient reliability-based design of TMD & MTMD in nonlinear structures under uncertainty", *Structures*, **51**, 258-274. https://doi.org/10.1016/j.istruc.2023.03.053.

Sharma, A. and Ramkrishnan, R. (2020), "Parametric optimization and multi-regression analysis for soil nailing using numerical approaches", *Geotech. Geol. Eng.*, **38**, 3505-3523. https://doi.org/10.1007/s10706-020-01230-8.

Shirgir, S., Shamsaddinlou, A., Zare, R.N., Zehtabiyan, S. and Bonab, M.H. (2023), "An efficient double-loop reliability-based optimization with metaheuristic algorithms to design soil nail walls under uncertain condition", *Reliab. Eng. Syst. Safe.*, **232**, 109077. https://doi.org/10.1016/j.ress.2022.109077.

Tu. J. and Choi. K.K. and Park, Y.H. (1999), "A new study on reliability based design optimization", *ASME J. Mech.Design*, **121**(4), 557-564. https://doi.org/10.1115/1.2829499.

Vazirizade, S.M., Nozhati, S. and Zadeh, M.A. (2017), "Seismic reliability assessment of structures using artificial neural network", *J. Build. Eng.*, **11**, 230-235. https://doi.org/10.1016/j.jobe.2017.04.001.

Villalobos, S.A. and Villalobos, F.A. (2021), "Effect of nail spacing on the global stability of soil nailed walls using limit equilibrium and finite element methods", *Transport. Geotech.*, **26**, 100454. https://doi.org/10.1016/j.trgeo.2020.100454.

Zhao, Q., Chen, X., Ma, Z. and Lin, Y. (2016), "A comparison of deterministic, reliability-based topology optimization under uncertainties", *Acta Mech. Solida Sinica*, **29**(1), 31-45. https://doi.org/10.1016/S0894-9166(16)60005-8.

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