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Reliability analysis-based safety factor for stability of footings on frictional soils

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Abstract. The design of foundations based on a deterministic approach may not be safe and reliable occasionally, since soils sometimes show considerable spatial variability, and thus, significant uncertainties in turn affect the estimation of footing bearing capacity. The design of footing on cohesionless stratums on the basis of reliability analysis has not received much attention. This paper performs two-dimensional random finite difference analyses of shallow strip footings on a spatially variable frictional soil considering correlation structure. Friction angle (ϕ) is considered as a log-normally distributed random variable and Monte Carlo Simulation is then performed to determine the statistical response based on the random fields. A new approach reliability-based safety factor is defined based on various reliability levels by considering the coefficient of variation of ϕ and correlation length in both the horizontal and vertical directions. The comparison of the probabilistic safety factor and the conventional one illustrates the limitations of the deterministic safety factor and provides insight into how the heterogeneity of soils properties affects the required safety factor. Results show that the conventional safety factor of 3 can be conservative in some cases, especially for soil with low values of mean ϕ and COV_ϕ.

Keywords: footing bearing pressure; Monte Carlo Simulation; probabilistic analysis; reliability assessment; safety factor

1. Introduction

The calculation of the footing bearing capacity (BC) from conventional deterministic methods may lead to an uneconomical or unsafe design. This is because the safety factor (SF) of about 3-6 that was usually used in the deterministic analyses relies on experience. Besides, it does not reflect the inherent uncertainty involved in the BC (Shahin and Cheung 2011a, b). This weakness can be addressed using probabilistic analysis.

Due to depositional and post-depositional processes, soil strata exhibit noticeable heterogeneity (Baecher and Christian 2005, Lacasse and Nadim 1997, Vanmarcke 1977). The three main uncertainty sources associated with geotechnical engineering purposes were introduced as spatial variability, measurement error, and transformation uncertainties (Phoon and Kulhawy 1999). The spatial variability in soil properties and its effect on geotechnical design has been well investigated in the former studies. In particular, the investigation of the inherent random variations effect of strength properties on the footing BC was received considerable attention in recent years. Al-Bittar and Soubra (2017), Barakat *et al.* (2015), Fazeli Dehkordi *et al.* (2019), Pieczyńska *et al.* (2011), Shakir (2019), and Simões *et al.* (2020) made important contributions to the estimation of the probability distribution function (PDF) for footing BC. Their results revealed that appropriate PDF is a function of soil type and degree of soil characteristics variability.

Most of the probabilistic research studies conducted so far on the BC of shallow foundations, considered the undrained shear strength as a random variable (e.g., Griffiths and Fenton 2001, Griffiths *et al.* 2002, Griffiths *et al.* 2006, Halder and Chakraborty 2019, Halder and Chakraborty 2020, Kasama and Whittle 2011, Li *et al.* 2015, Li *et al.* 2021, Popescu *et al.* 2005, Shen *et al.* 2020, Shu *et al.* 2020, Shu *et al.* 2021, Wu *et al.* 2020, Wu *et al.* 2019, Ye *et al.* 2021) and some others obtained probabilistic BC for cohesive-frictional soils (e.g., Cherubini 2000, Fenton and Griffiths 2003, Ghazavi *et al.* 2021, Imanzadeh *et al.* 2020, Kawa and Puła 2020, Ranjbar Pouya *et al.* 2014). Their results indicate that soil heterogeneity leads to a considerable effect on the BC and the form of failure mechanism.

Very few studies have focused on reliability assessment of footing BC on frictional soils and examined the effect of statistical parameters of friction angle (ϕ) mainly on the mean, standard deviation, or coefficient of variation (COV) values of BC (Fazeli Dehkordi *et al.* 2019, Pieczyńska *et al.* 2011, Puła and Zaskórski 2015, Rezaie Soufi *et al.* 2020, Viviescas *et al.* 2021). However, ϕ that dominates the failure mechanism can play a more effective role in probabilistic analyses and contrasting behavior of cohesionless soils reported in earlier papers indicates the need for further research for this type of soil (Pieczyńska *et al.* 2011, Uncuoglu 2015).

Some past research studies calculated the SF with different approaches using statistical parameters of the probabilistic BC (e.g., Duncan 2000; Kasama and Whittle

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2011, Puła and Zaskórski 2015, Shahin and Cheung 2011a, b). These studies revealed that conventional values of SF usually can be overestimated and more precise estimates of SF can be obtained using reliability assessment.

The present paper aims to evaluate the reliability of the BC of strip footing rested on a spatially variable frictional soil due to more clarification on this type of soil in a probabilistic framework. The effect of ϕ variability on the BC at different reliability levels (RLs) is also investigated which to the best of the authors' knowledge, has not been evaluated up to the present. In addition, the main contribution of this research is introducing a new concept of SF which can be obtained by the appropriate target RL employing the random finite difference method (RFDM). This method is simple to implement and gives designers the opportunity to increase the accuracy of the design and makes a more rational decision by comparing probabilistic and deterministic SF values.

2. Problem definition

This study utilizes the Cholesky decomposition method to discretize the random field. Additionally, Monte Carlo Simulation (MCS) is combined with finite difference analysis through FLAC2D 7.0 (Itasca Group 2011), which incorporates the FISH programming language, to enable probabilistic analysis. The impact of various parameters such as the mean friction angle (μ_ϕ), coefficient of variation of friction angle (COV$_\phi$), correlation length in horizontal and vertical directions (θ_x, θ_y) on the SF and different statistical parameters of the BC is also studied.

3. Finite difference simulation

The numerical analysis model used in the present paper is illustrated in Fig. 1 where a 2D plane strain condition is modeled using the finite difference method. A strip footing with B=0.75 m width is assumed to be constructed on the surface of a cohesionless stratum and subjected to a controlled downward velocity with 10^{-6} m/step to reach the ultimate failure stage. The soil behavior is assumed to be the conventional elastic perfectly plastic model which obeys the Mohr-Coulomb failure criterion and a rough interface is considered between the foundation and supporting soil.

The soil domain has a 10 m width and 3 m depth. This domain is 13 times greater in the horizontal direction and 4 times greater in the vertical direction than the width of the footing, confirming the minimum required dimensions for the given footing (Popescu *et al*. 2005, Shu *et al*. 2020, Wu *et al*. 2019). The soil media is divided into 1920 uniform square grids. The nodes on the bottom boundary of the soil medium are fixed to have no vertical or horizontal movements. The right and left boundaries are constrained against horizontal displacement but are free to slide vertically. Several sensitivity analyses were performed on the model dimensions to deactivate the boundary conditions.

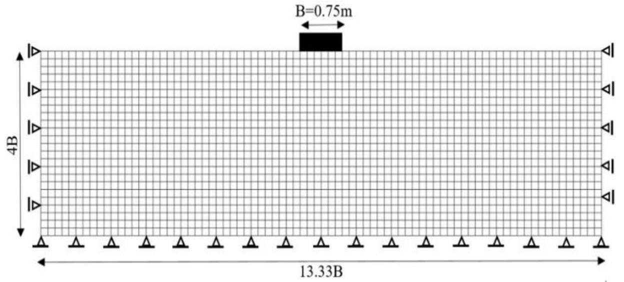


Fig. 1 Mesh with a total of 1920 four-nodded quadrilateral grids for strip footing

Table 1 Soil and foundation properties used in the present study

Parameter	Value
Soil	
Friction angle, ϕ (°)	Random variable
Cohesion, c (kPa)	1
Young's modulus, E (MPa)	65
Poisson's ratio, v	0.3
Dilation angle, ψ (°)	$(2/3)\mu_\phi$
Soil unit weight, γ (kN/m^3)	19
Foundation	
Young's modulus, E (GPa)	25
Poisson's ratio, v	0.2

The soil properties and foundation details are presented in Table 1. Friction angle is considered a non-Gaussian random variable because physical material properties cannot be negative and it is assumed to change from zone to zone representing the soil spatial variability in both horizontal and vertical directions (Ghazavi and Fazeli Dehkordi 2021, Ghazavi *et al*. 2023). Other parameters such as Young's modulus (E) and Poisson's ratio (v) are assumed to be spatially constant since past research studies have indicated that the contributions of E and v have a negligible effect on the random nature of the footing BC and have a greater influence on stochastic footing settlement (Puła and Zaskórski 2015, Wang *et al*. 2020). In the numerical analysis, more accurate results will be obtained when the dilation angle is considered to be equal to ψ=2/3ϕ (Erickson and Drescher 2002, Ghazavi and Lavasan 2008). Thereby, it is supposed that ψ=2/3μ_ϕ. To avoid numerical instability during analysis, and minimize computation time, sands sometimes are assigned a small cohesion value of 1 kPa (Luo *et al*. 2016).

4. Random field modeling

There are several methods for simulating random fields. In this study, the Cholesky decomposition technique is employed for random field generation because it is efficient and straightforward to implement for small fields (Fenton and Griffiths 2008). The 2D exponential correlation function used to generate anisotropic random fields is defined by

$$\rho = \exp\left(-2\sqrt{\frac{\Delta x^2}{\theta_1^2}+\frac{\Delta y^2}{\theta_2^2}}\right) \tag{1}$$

observations in the domain, Δx and Δy are the horizontal and vertical separation distances between two sample points in the space, and θ_1 and θ_2 are the major and minor correlation lengths.

The correlation length is the statistical parameter that describes the degree of correlation of a soil property throughout the random field domain, and it is defined as a length over which a significant correlation in a specific soil property is still observed. The covariance matrix "*A*" is a function of the correlation coefficient which is defined as follows

$$A = \begin{bmatrix} 1 & \rho_{1,2} & \rho_{1,3} & \cdots & \rho_{1,1920} \\ \rho_{2,1} & 1 & \rho_{2,3} & \cdots & \rho_{2,1920} \\ \rho_{3,1} & \rho_{3,2} & 1 & \cdots & \rho_{3,1920} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{1920,1} & \rho_{1920,2} & \rho_{1920,3} & \cdots & 1 \end{bmatrix} \tag{2}$$

In deterministic analysis, assuming uniform soil properties in the whole soil domain leads to design oversimplification. Therefore, random field theory is a useful tool to characterize spatial variation in the soil properties and a much more realistic design is achieved. The friction angle has been modeled by various distributions in former studies but the most frequent ones were Bounded and Log-normal probability distributions. This is because those distributions guarantee that always positive random variables will be achieved in analysis. Moreover, in this study due to the parametric study of the mean value of friction angle, considering log-normally distributed friction angle is more reasonable since in bounded distribution, mean values are related to lower and upper bound values which makes it difficult to examine the effect of mean friction angle on the BC. Therefore, similar to many former papers such as Barakat *et al.* (2015), Griffiths *et al.* (2011), Guo *et al.* (2019), Halder and Chakraborty (2019), and Pan and Dias (2017), ϕ is modeled by log-normal random field as follows

$$\emptyset(\tilde{x}) = \exp\left(\mu_{ln\emptyset} + \sigma_{ln\emptyset} G_i(ln\tilde{\emptyset})\right) \tag{3}$$

where, $\emptyset(\tilde{x})$ is ϕ at the spatial location $\tilde{x}$ at which ϕ is desire, $G_i\left(ln\,\tilde{\emptyset}\right)$ is a correlated standard normal random variable with spatial correlation lengths θ_1, θ_2. Parameters $\mu_{ln\emptyset}$ and $\sigma_{ln\emptyset}$ are obtained respectively from

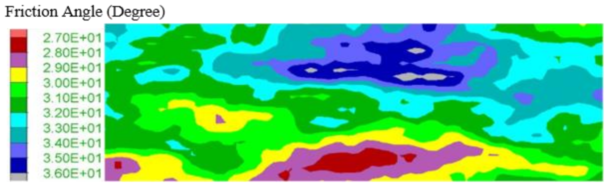
$$\sigma_{ln\emptyset}^2 = \ln\left(1+\left(\frac{\sigma_\emptyset}{\mu_\emptyset}\right)^2\right) = ln\left(1+COV_\emptyset^2\right) \tag{4}$$

$$\mu_{ln\emptyset} = ln(\mu_\emptyset) - \frac{1}{2}\sigma_{ln\emptyset}^2 \tag{5}$$

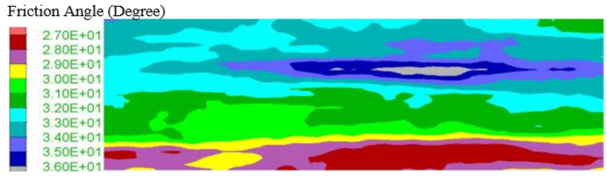
G_i is calculated by multiplying matrix L by a column vector of independent standard normal random variables $G_i(ln\emptyset)$ as follows

$$G_i(ln\tilde{\emptyset}) = \sum_{j=1}^{i} L_{ij} G_i(ln\emptyset) \qquad i = 1,2,3,\ldots \tag{6}$$

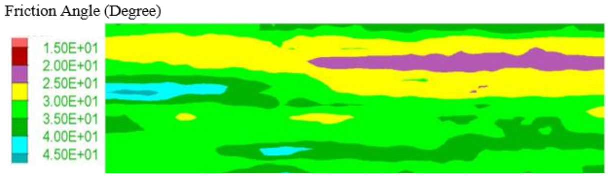
where L is a lower triangular matrix calculated by the Cholesky decomposition method as follows



(a) COV=5%, θ_x=5 m



(b) COV=5%, θ_x=20 m



(c) COV=20%, θ_x=20 m

Fig. 2 Typical realizations of ϕ random fields (θ_y=1 m)

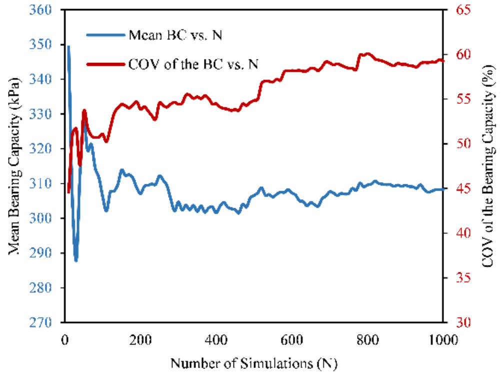


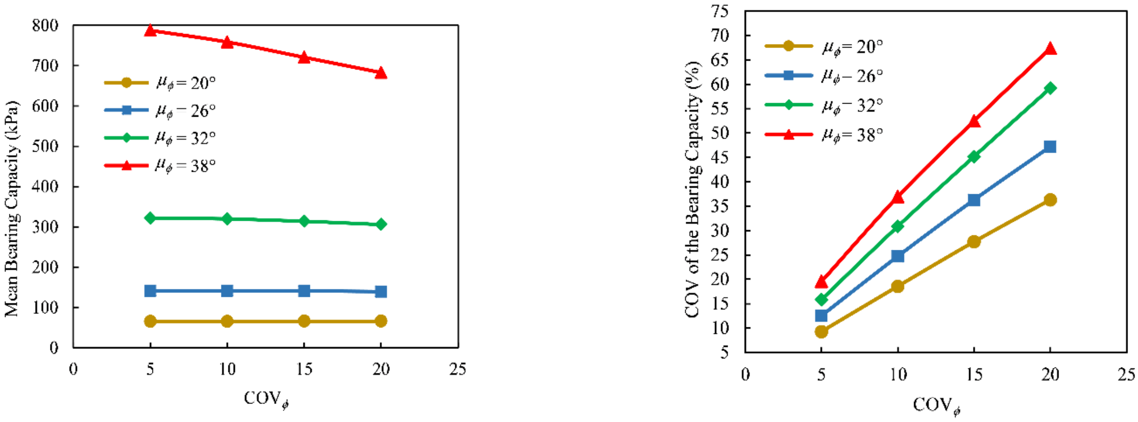
Fig. 3 Sensitivity analysis based on the number of MCS (μ_ϕ=32°, COV$_\phi$=20%, θ_x=10 m, θ_y=1 m)

$$A = L.L^T \tag{7}$$

Fig. 2 illustrates typical random fields of ϕ for μ_ϕ=32° that are generated with FLAC^{2D} which shows how the correlation length and COV$_\phi$ affect the random field.

5. Monte Carlo Simulation (MCS)

The MCS is a class of computational algorithms that relies upon repeated random sampling which calculates the response for each generated set by generating a series of random fields in a manner consistent with their probability distribution and correlation structure. To determine the number of efficient simulations, the influence of the number of MCS on the statistical parameters of the BC is investigated. Fig. 3 indicates that the mean and COV of the footing BC reaches a steady state after about 800 simulations. Therefore, N=1000 is adopted in the subsequent stochastic analyses. Increasing the value of COV$_\varphi$ results in more pronounced fluctuations of the mean



(a) mean BC (b) COV of the BC

Fig. 4 Effect of statistical parameters of ϕ on various statistical parameters of footing BC (θ_x=10 m, θ_y=1 m)

and COV of the footing BC as the number of simulations increases. Therefore, to conduct a sensitivity analysis, the most critical scenario is chosen when COV$_\varphi$ is at its maximum value of 20%. However, the fluctuation patterns of these parameters remain consistent as θ_x and θ_y increase.

6. Results and discussion

6.1 Effect of the statistical parameters of ϕ on the BC in mean and COV values

Fig. 4 shows the mean and COV of the BC at various mean and COV of ϕ. The corresponding values of COV$_\phi$ are considered based on typical values found in the literature as COV$_\phi$=5% to 20% (Meyerhof 1995; Phoon and Kulhawy 1999, Srivastava and Babu 2009). Results indicate that as the COV$_\phi$ increases, the average predicted BC decreases. Fenton and Griffiths (2003) indicated that increasing the soil variability increases the amount of loose and high-resistance zones in the soil equally but the tendency of failure line to pass through weak zones diminishes the BC of the spatially variable soil.

In the deterministic analysis, the BC values for μ_ϕ=20°, 26°, 32°, and 38° are estimated to be 65, 141, 326, and 837 kPa, respectively. In the previous studies, a substantial reduction was observed for the BC values considering the inherent spatial variability for cohesion averagely, compared to the corresponding deterministic analysis for homogeneous soil (Srivastava and Babu 2009, Fei 2021). While, results for cohesionless soil indicate that by considering spatial variability of ϕ, there is not much reduction in the average BC compared to that from the deterministic BC. This shows that the use of probabilistic methods for cohesive soils may be more necessary than for frictional soils.

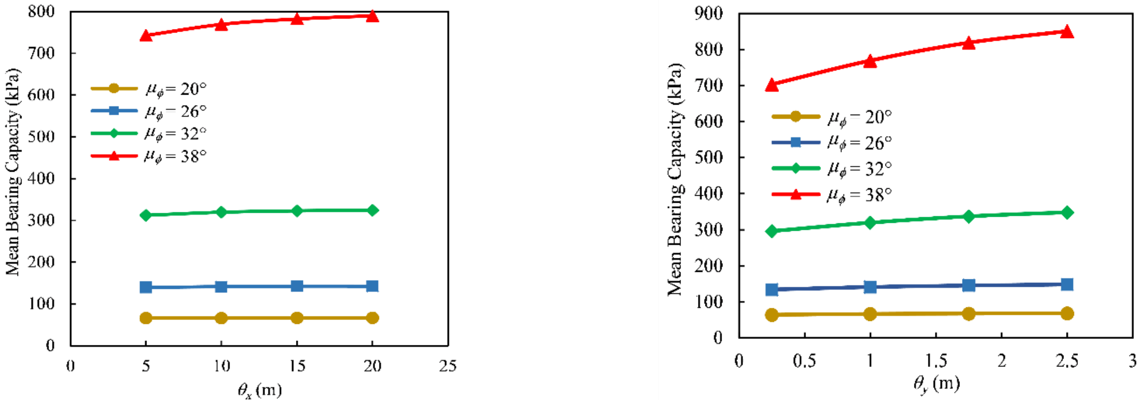
Results demonstrate that for low values of μ_ϕ, the mean BC variation with COV$_\phi$ is negligible. As μ_ϕ increases, the mean BC significantly varies with COV$_\phi$. For instance, for μ_ϕ=38° as the COV$_\phi$ increases from 5% to 20%, the BC value reduces by about 13%. Whereas, assuming a similar variation of COV$_\phi$ for μ_ϕ=20°, mean BC reduction is about 1%. Based on the definition of COV$_\phi$=σ_ϕ /μ_ϕ, it is obvious that enhancing the mean ϕ and COV$_\phi$ leads to producing a wider range of ϕ random values. Because weak areas are determinative factors in computing the BC, by increasing μ_ϕ, the BC shows greater sensitivity to COV$_\phi$.

Another observation from Fig. 4 is that the COV of the BC grows with an increase in μ_ϕ and COV$_\phi$. With increasing μ_ϕ, the standard deviation of ϕ increases while keeping COV$_\phi$ constant. On the other hand, the BC value is an ascending convex function of μ_ϕ and has a positive relationship with the ϕ (Fazeli Dehkordi *et al.* 2019, Rezaie Soufi *et al.* 2020). Therefore, an increase in the COV$_\phi$ results in a sharp change in the value of COV of the BC.

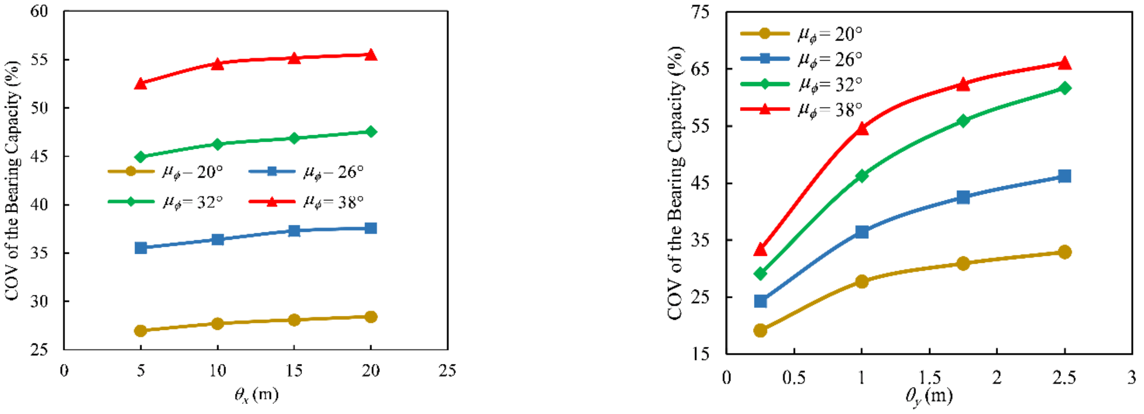
Fig. 5 plots the variations of the mean and COV of the footing BC in terms of vertical and horizontal correlation lengths (θ_x, θ_y) for μ_ϕ=20°, 26°, 32°, 38°. In the literature, a review of the values of correlation lengths indicates that depending on the geologic history, θ_x falls within the range of 5-40 m and θ_y varies within the range of 0.5-3 m (El-Ramly *et al.* 2002). It can be seen that a higher θ_x value gives a greater mean BC value. This is because a rising θ_x value means more horizontally uniform soil strata and therefore, shears bond forms through various stratums with distinctive shear strength. In these circumstances, high-strength stratums are determinants to compute the BC.

Fig. 5 also indicates that increasing the BC with θ_y is more pronounced than θ_x. For example, in μ_ϕ=38° as the θ_y increases from 0.25 m to 2.5 m, the BC value increases by 21%. This shows the importance of the value chosen for θ_y in geotechnical design. Generally, because θ_x is greater than the horizontal dimension of the failure wedge, in small-width strip foundations, θ_x does not have much effect on homogenizing the failure wedge. However, due to the small values of θ_y in nature, the failure wedge may consist of several deposits with different resistances. As a result, a higher θ_y value implies less soil heterogeneity and thus, the failure mechanism approaches the failure mechanism in deterministic conditions resulting in the enhancement of the BC.

As seen in Fig. 5, it is obvious from the results that the BC for cases with a larger mean value of ϕ shows greater sensitivity to θ_x and θ_y. As mentioned, a larger correlation



(a) Mean BC vs. θ_x, θ_y=1 m (c) Mean BC vs. θ_y, θ_x=10 m



(b) COV of BC vs. θ_x, θ_y=1 m (d) COV of BC vs. θ_y, θ_x=10 m

Fig. 5 Variation of statistical parameters of footing BC for μ_ϕ=20°, 26°, 32°, 38° (COV$_\phi$=15%)

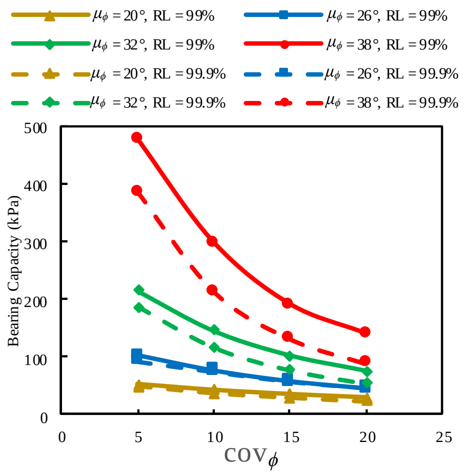
length results in smoother random field generation, therefore, high-strength deposits play a major role in determining the BC. On the other side of the spectrum, for a constant COV$_\phi$, greater μ_ϕ means a greater standard deviation of ϕ. Thus, by increasing μ_ϕ, wider dispersion in the probability distribution of ϕ is obtained and larger variation in the BC value occurs with the correlation length. In addition, similar to the findings of Ghazavi *et al.* (2021) comparing the results of COV of the BC indicates that θ_y has a much more pronounced effect on COV of the BC than θ_x.

6.2 Effect of statistical parameters of ϕ on the BC in 90%, 95%, 99%, and 99.9% RLs

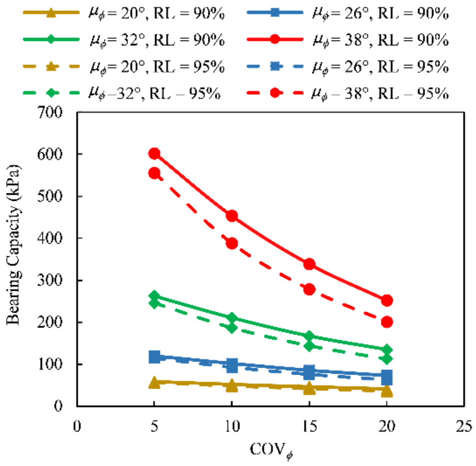
In Fig. 6, a set of probabilistic charts that assure target RLs of 90%, 95%, 99%, and 99.9% (or probability of failure = 10%, 5%, 1%, 0.1%) are developed. The empirical cumulative distribution function (ECDF) of values of BC for 1000 MCS is calculated for determining BC corresponding to each probability of failure. The ECDF is a step function that shows the proportion or percentage of data points that are less than or equal to a certain value. In this paper each analysis includes 1000 BC values, therefore, each step is equal to 1/1000 = 0.001. Then, the data value corresponding to the probability of failure of 10%, 5%, 1%, and 0.1% can be found by locating the point on the ECDF where the cumulative probability values are 0.1, 0.05, 0.01, and 0.001, respectively. For example, if BC corresponds to the reliability of 99% is 500 kPa, it implies that only 1% of BC values generated by MCS is less than or equal to 500 kPa.

A literature review of the probability of failure shows the acceptable range of this parameter for foundations is between 0.01% and 1% (Baecher and Christian 2005, Kamien 1997). Different levels of RLs may be considered based on the importance of the supported structure. Moreover, like factor of safety, the required probability of failure is a function of various factors such as type of structure, uncertainties in soil properties, and loading conditions, and can be estimated through the engineering judgment of designers. For example, for some structures like uninhabited storage warehouses or temporary structures, designers may decide to consider a higher value for the probability of failure. Therefore, designers are encouraged to adopt this method using RLs that are compatible with their specific projects.

Fig. 6 shows the variation of predicted BC corresponding to mentioned RLs in terms of COV$_\phi$ for μ_ϕ=20°, 26°, 32°, and 38°. Some previous studies showed that COV is the most influential statistical parameter of soil properties on the average value of the BC (Fazeli Dehkordi *et al.* 2019, Popescu *et al.* 2005, Ranjbar Pouya 2014). Results from this figure indicate that the variation in estimated BC at assumed RLs in terms of different COV$_\phi$s is much more significant than the variation in the average



(a) Predicted BC corresponding to RLs of 99% and 99.9%

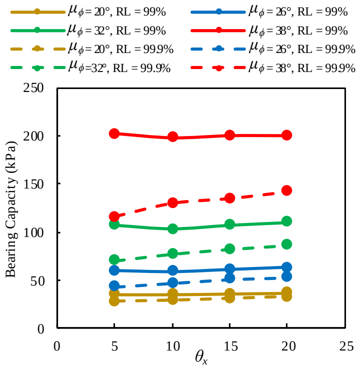


(b) Predicted BC corresponding to RLs of 90% and 95%

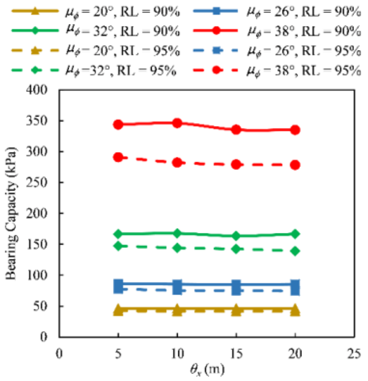
Fig. 6 Variation of probabilistic BC with COV$_\phi$ for μ_ϕ=20°, 26°, 32°, 38°

value of the BC. For instance, at an RL of 99%, by increasing the COV$_\phi$ from 5% to 20% for μ_ϕ=20° and 38°, the BC decreases by 43% and 70%, respectively, while as shown in Fig. 4, the largest decrease in the mean value of BC with the variation of COV$_\phi$ occurs for μ_ϕ=38°, which is 13%. Increasingly, this emphasizes the importance of considering the inherent soil variability in geotechnical design, especially when the design is based on RLs. Furthermore, the difference among BC values at different RLs widens when larger μ_ϕ is assumed due to enhancing the standard deviation of ϕ. For example, an appreciable variation in outcomes is observed when comparing the results obtained at 99% and 99.9% RLs for μ_ϕ=38°, with a difference of approximately 21%. However, when μ_ϕ=20°, the corresponding discrepancy is only approximately 8%. These show when dealing with sands that possess high strength parameters, a minor error in the estimation of required RL can result in a notable deviation in predicting the BC.

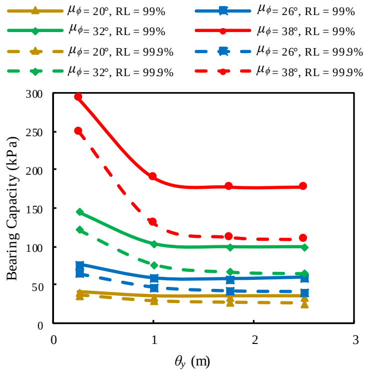
In Fig. 7, the effects of θ_x and θ_y on the BC corresponding to 90%, 95%, 99%, and 99.9% RLs are also



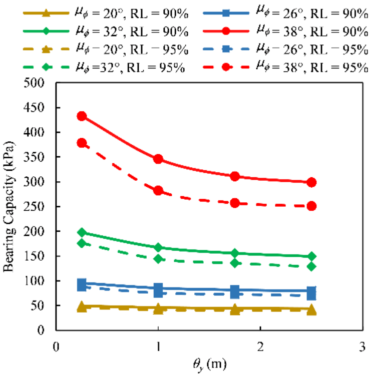
(a) Predicted BC corresponding to RLs of 99% and 99.9% vs. θ_x



(b) Predicted BC corresponding to RLs of 90% and 95% vs. θ_x



(c) Predicted BC corresponding to RLs of 99% and 99.9% vs. θ_y



(d) Predicted BC corresponding to RLs of 90% and 95% vs. θ_y

Fig. 7 Variation of probabilistic BC with correlation length for μ_ϕ=20°, 26°, 32°, 38°

investigated. Results indicate that increasing the θ_x value has no considerable effect on the BC at 90% and 95% RLs, and there is a slight increase in the amount of BC at 99% and 99.9% RLs as θ_x increases. However, as shown in Fig. 5 the effect of θ_x on the average value of the BC is relatively significant. Furthermore, in contrast to the mean BC value which shows an upward trend with θ_y, there exists a negative correlation between θ_y and BC at the studied RLs.

This can be explained by Fig. 8 which shows the cumulative probability distribution for the BC which is plotted for different cases and the BC corresponding to 90% and 95%, 99%, and 99.9% RLs are shown by ovals. By assuming COV$_\phi$=15% and $\theta_{x=}$10 m, heavy and thin dotted lines corresponding to θ_y=0.5 m θ_y=2.5 m intersect at about cumulative distribution function value of 40% (RL of 60%). On the other hand, for θ_y=0.5 m θ_y=2.5 m, the cumulative distribution function values corresponding to the mean BC is 53% (RL of 47%) and 62% (RL of 38%), which is displayed with horizontal dashed lines. This indicates that the BC at an RL below 60% is directly related to θ_y and otherwise inversely related.

6.3 Effect of the statistical parameters of ϕ on SF in 90% and 95%, 99%, and 99.9% RLs

Fig. 9 represents the SF corresponding to 90% and 95%, 99%, and 99.9% RLs for various μ_ϕ. Probabilistic SF can be derived using the concept of deterministic SF by employing probabilistic BC in target RL instead of allowable BC. This assumption can be reasonable because the BC at the appropriate RL includes design uncertainties and implies allowable BC that is safe for design. This section leads to a better understanding of the degree of accuracy of the SF adopted in conventional deterministic analyses. The probabilistic SF can be easily estimated as

$$SF_{Probabilistic} = \frac{q_u^{Determinsitic}}{q_{RL=90\%,95\%,99\%,99.9\%}} \tag{8}$$

where, $SF_{Probabilistic}$ represents the estimated SF at the desired RLs, $q_u^{Determinsitic}$ is the deterministic ultimate BC obtained using mean values of the soil properties through the same finite difference code, without taking into account the spatial variability of ϕ and $q_{RL=90\%,95\%,99\%,99.9\%}$ represents the predicted ultimate BC corresponding to mentioned RLs, calculated through probabilistic analyses.

Fig. 9 shows that adopting a single SF for soils with different properties and uncertainties cannot be logical. The results indicate that for a fixed value of COV$_\phi$, a higher ϕ value means a greater SF is required for the design, and increasing the COV$_\phi$ significantly increases the required SF. Therefore, determining the SF can depend on several factors such as statistical parameters of soil properties. For example, in this probabilistic analysis, SF=3 which is commonly used in the deterministic analysis is appropriate for a situation where μ_ϕ=32°, COV$_\phi$=15% and RL=99%. As a result, it is obvious that adopting SF=3 in many cases, especially for weak soils with low inherent random heterogeneity, can be uneconomical and conservative. On the other hand, in some cases, greater SF than 3 is needed. For example, for soil with μ_ϕ=38°, COV$_\phi$=20% and RL=99%, the necessary SF would be approximately 6. Assuming a SF of 3 would be considered unsafe and unreliable. Furthermore, results indicate that increasing θ_x value has no considerable effect on the SF at RLs of 90%, 95%, and 99%. However, there is a negative relationship between θ_x and SF at 99.9% RL.

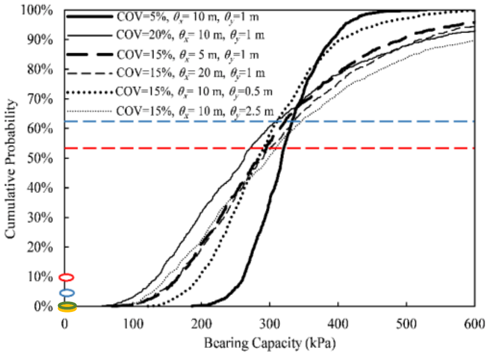


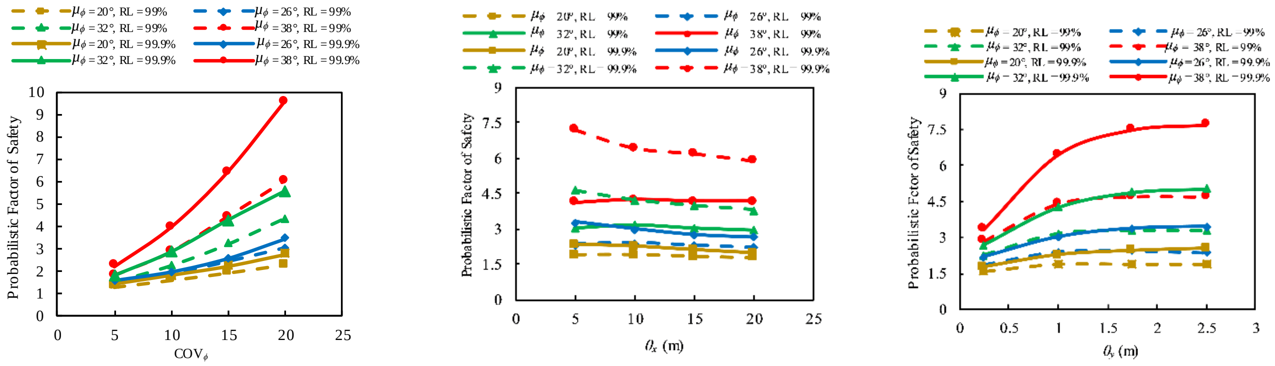
Fig. 8 Cumulative probability distribution for the BC (μ_ϕ=32°)

As θ_y increases, the SF increases and becomes stabilized at about θ_y=2.3B. Therefore, accurate measurement of this parameter is necessary for SF estimation. In addition, the impact of this parameter is greater for soils with higher values of mean friction angle.

7. Conclusions

In the present research, a reliability assessment of strip footings is carried out considering the spatial variability of ϕ using the random finite difference method. Unlike most previous studies that evaluated the mean and COV of the BC value, in this research, the BC analyses corresponding to RLs of 90%, 95%, 99%, and 99.9% are performed to apply them in the field. A new probabilistic SF is defined using a simple approach, which can be considered a supplement to SF obtained from deterministic analyses. Parametric studies are also conducted to show factors affecting BC and SF. Based on the performed study in the current research, the following concluding remarks may be extracted:

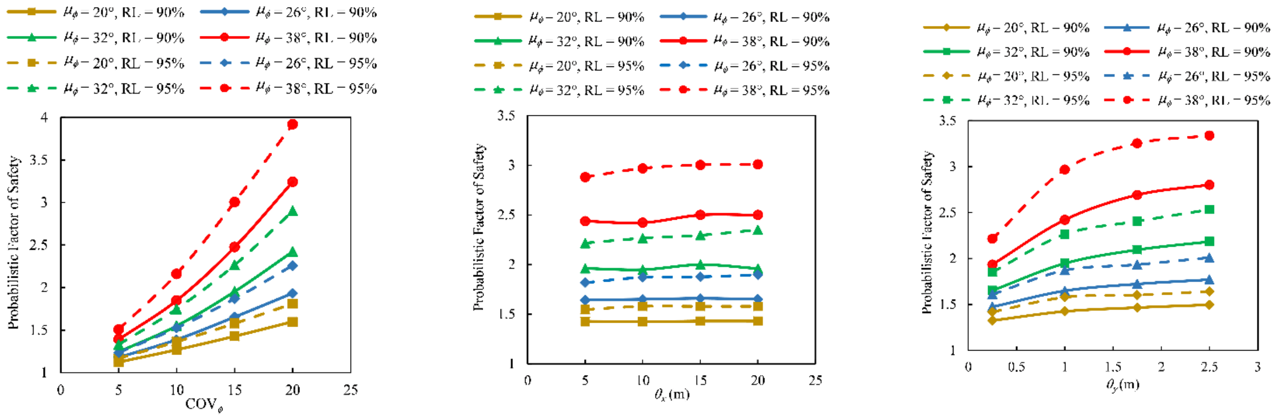
- The difference between the mean BC assuming inherent variability in strength characteristics and those calculated from deterministic analysis for cohesionless soil is negligible, and it widens as ϕ values increase. In other words, the effect of inherent spatial variability of ϕ on BC is much more pronounced for larger ϕ values. Similarly, as the mean ϕ decreases, the impact of considering COV$_\phi$ on the footing BC is reduced. Therefore, for weak frictional soils, the probabilistic analysis does not have much positive effect on the accuracy of BC estimation and only increases computational costs.
- The COV$_\phi$ influences footing bearing pressure substantially at 90%, 95%, 99%, and 99.9% RLs. However, it has a relatively smaller effect on the mean value of the BC. Because most of the former studies investigate the effect of inherent variability on mean BC, the results of



(a) Predicted SF corresponding to RLs of 99% and 99.9% vs. COV_ϕ

(c) Predict SF corresponding to RLs of 99% and 99.9% vs. θ_x

(e) Predict SF corresponding to RLs of 99% and 99.9% vs. θ_y



(b) Predicted SF corresponding to RLs of 90% and 95% vs. COV_ϕ

(d) Predict SF corresponding to RLs of 90% and 95% vs. θ_x

(f) Predict SF corresponding to RLs of 90% and 95% vs. θ_y

Fig. 9 Variation of probabilistic SF for $\mu\phi$=20°, 26°, 32°, 38°

current study about the BC in defined RLs emphasize that COV_ϕ can be more determinative in BC estimation than previously thought. This demonstrates the importance of accurate estimation of COV_ϕ as the most influential statistical parameter in design for practical purposes. In addition, considering appropriate RL as a function of different factors such as the degree of uncertainties of soil properties and the importance of the supported structure is very important for foundation design because BC is dramatically affected by the adopted RL.

- The effect of the θ_x value on the footing BC with assumed RLs is negligible in most cases, while θ_x has a relatively considerable impact on the mean value of footing BC, particularly for higher μ_ϕ values. A higher θ_y value gives greater mean BC but decreases the BC at considered RLs. Therefore, correlation length cannot always be considered a parameter that contributes to increasing the BC.

- Conventional deterministic SF does not reflect the inherent uncertainty of contributing parameters in computing the footing BC and thus, according to reliability-based SF results, the conventional SF=3 usually used in deterministic analyses sometimes does not guarantee economical yet safe design. Therefore, choosing the SF based on statistical parameters of soil properties can lead to a more rational design for footings. The value of probabilistic SF is more strongly dependent on adopting COV_ϕ than the other parameters. This means that a small change in the adopted value for variability of soil properties substantially influences the obtained SF.

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