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Static stress analysis of multi-layered soils with twin tunnels by using finite and infinite elements

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Abstract. The aim of this paper is to investigate stress analysis of semi-infinite soils consisting of two layers with twin rectangular tunnels under static loads. The region close to the ground surface and tunnel modelled within finite elements. In order to use a more realistic model, the far region is modelled within infinite elements. The material model of the layered soil is considered as elastic and isotropic. In the finite element solution of the problem, two dimensional (2D) plane solid elements are used with sixteen-nodes rectangular finite and eight-nodes infinite shapes. Finite and infinite elements are ordered to be suitable for the tunnel and the soils. The governing equations of the problem are obtained by using the virtual work principle. In the numerical process, the five-point Gauss rule is used for the calculation of the integrations. In order to validate using methods, comparison studies are performed. In the numerical results, the stress distributions of the two layered soils containing twin rectangular tunnels presented. In the presented results, effects of the location of the tunnels on the stress distributions along soil depth are obtained and discussed in detail. The obtained results show that the locations of the tunnels are very effective on the stress distribution on the soils.

Keywords: finite-Infinite elements; multi-layered soils; stress analysis; tunnels

1. Introduction

Tunnels are important underground structures which used for many applications such as traffic mining, military facilities, underground cables. In urban areas, twin tunnels are frequently used in close proximity to each other at shallow depth. The geometric structure of a tunnel seriously affects its stability. Especially circular tunnels are stable and preferred more in terms of excavation. Square tunnels are more difficult to construct and do not have the same stability. However, it can be preferred because it maximizes the volume of use while minimizing the amount of excavation.

Twin tunnels interaction, shape of tunnels, cavity, locations in the ground and layered soils which reveal different stiffness in the layered soil, create various problems for the ground and the structure. In order to examine this situation, semi-infinite soil is modeled.

The behavior of a finite structure in a semi-infinite soil under static loading is a subject that needs to be well studied in terms of geotechnical engineering. In static calculations; various parameters such as loads on the tunnel, laminated soil of different thickness, possible slip, and tunnel cover thickness should be considered. Depending on the usage area, the static effects of the twin tunnels between the ground and each other should be well known in terms of the long service life of the tunnels. In square section tunnels, which are generally preferred in highway and submarine tunnels, different stress problems occur depending on the section geometry. Sensitivity is important for accurate results in the static analysis of the structure in the ground under different effects. The most correct solution to the interaction problem between the structure and the ground is to take into account the joint movement of the structure and the ground. In this direction, a finite element model, which takes into account the joint movement of the structure and the ground. The static reactions on the infinite ground will be damped after a certain point in the infinite direction. Taking this situation into account, the structure and the area close to the ground surface is modeled with finite elements, and the far region is modeled with infinite elements.

In the literature, Saini *et al*. (1978), used finite and infinite elements for dam-soil-hydraulic dynamic interaction analysis. Booker and Small (1981), performed static and dynamic analysis of unbounded space. Lynn and Hadid (1981) studied static responses semi-infinite soil under ring load by using infinite element with $1/rn$ decay function. Dasgupta (1982), produced a new algorithm for solving semi-infinite vibration problems. Zienkiewicz et. al. (1983) performed vertical direction static analysis of semi-infinite soil by using different type of infinite element. Curnier (1983) produced two dimensional infinite elements for static semi-infinite problems. Pissanetzky (1984) proposed a new simple infinite element for static semi-infinite media problems. Bettess and Bettess (1984) developed infinite element with decay function for static unbounded medium problems. Rajapakse and Karasudhi (1985), studied the elasto-static displacement behavior of a

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layered semi-infinite medium. Kumar (1985) proposed three different type of infinite element for static semi-infinite medium problems. Tzong and Penzien (1986), performed a hybrid modeling for dynamic interaction analysis in a semi-infinite medium. Karpurapu (1988), Ahmet modeled the two-layered semi-infinite medium using composite infinite elements. Karpurapu and Bathurst (1988) performed stress analysis of a tunnel in unbounded medium by using infinite element with 5 nodes. Selvadurai and Karpurapu (1989) analyzed semi-infinite saturated soil medium by using composite finite-infinite element. Liou (1989), harmonic soil-structure interaction behaviors in soil with multi-layered cylindrical hole are investigated. Liu and Novak (1991) analyzed static responses of pile-soil-cap interaction using direct method. Assadi and Sloan (1991) numerically investigated the stability of a square tunnel under surcharge load. Bettess (1992) investigated mapped, decay function, decay periodic function and unbounded wave problems for two and three dimensional unbounded media. Brunotte *et al*. (1992), suggested simple and realistic finite element formulations for infinite-medium problems. Yun *et al.* (1995), carried out an analysis that takes into account the dynamic movement of the ground and the structure on the multi-layered soil. El-Esnawy *et al*. (1995) examined using a new infinite element horizontal displacements of the infinite medium problem. Yang *et al.* (1996) developed infinite element for dynamic solutions of semi-unbounded media. Wolf and Song (1996), modeled the infinite region with finite elements for interaction analysis. Yerli *et al*. (1998) studied time-dependent displacements in a semi-infinite medium with the direct method. Yang *et al*. (2001) investigated the dynamic responses of viscoelastic soils under the effect of moving load. Yang *et al*. (2003) studied the vibration behavior of the ground as a result of the movements of the trains. Nakamura *et al*. (2003) analyzed the rectangular shaped transportation tunnels. Mori and Abe (2004) applied a new method in the construction of rectangular tunnels. Houmat (2008), proposed a p-element model for solving the infinite element problem. Wilson *et al*. (2008) investigated the stability of square tunnels used for transportation. Savadatti and Guddati (2010), optimized the finite-infinite element model by using finite elements compatible with infinite elements. Hung and Yang (2010) investigated the vibration behavior of underground trains on the ground. Fang *et al*. (2011) analysis of twin tube tunnels carried out with a different method. Liu (2012) investigated the static stresses in the tunnel in case of explosion. Wilson *et al*. (2012) investigated the effects of shear caused by increasing soil depth in square tunnels. Xie *et al*. (2013) modeled three dimensional railway foundation on semi-infinite media. Abbo *et al*. (2013) examined the stability problems of large rectangular tunnels. Lu *et al*. (2014) carried out geometric optimization studies on tunnels. Erkal *et al*. (2014) examined unbounded medium problem under uniform cylindrical load. Demidem *et al*. (2014) applied finite-infinite element formulation for structural and non – structural applications. Kargar *et al*. (2014) semi-analytical method used for the static analysis of non-circular tunnels. Liu *et al*. (2015) performed static analysis of asphalt pavement structures by using finite-infinite elements. Fu *et al*. (2015) compared the behavior of single and twin tunnels in the ground under static load. Lin *et al*. (2016) investigated dynamic behaviour of underground tunnels using finite-infinite elements. Fang *et al*. (2016) performed the stress analysis of two circular tunnels of different sizes. Lin *et al*. (2017) performed vibration analysis of two dimensional twin tunnels using finite-infinite element method. Wang *et al*. (2017) studied the stress and displacement behavior of two circular tunnels of different sizes. Wang *et al*. (2017) investigated the behavior of twin circular tunnels of different sizes under static loading in the ground. Wen *et al*. (2018) proposed different mapping functions for two dimensional semi-infinite problems. Wang *et al*. (2018) performed the static analysis of non-round tunnels in semi-infinite soil. Haji *et al*. (2019) examined the ground environment with a tunnel in 3D by constructing the gravity model using the finite-infinite element method. Ahn *et al*. (2022) investigated the settlement conditions of twin tunnels in the ground. Shi *et al*. (2022) performed a 3-D parametric study to examine the tunneling effects of pipes in the soil. Kim *et al*. (2022) modeled transportation tunnels for performance analysis. Oi *et al*. (2022) examined the static behavior of twin tunnels that overlap as a result of excavation. Liu *et al*. (2022) parallel tunnels studied their harmony within the ground. Yüksel and Akbaş (2022a, b, c) studied the static analysis of semi-infinite medium with cavity and different soil layers. Yang *et al*. (2023) studied the dynamic responses due to wave propagation in a visco-elastic semi-infinite soil.

The modeling and solution of the semi-infinite soil problems can be obtained with classical finite elements by limiting a semi-infinite environment to a region at a sufficient distance. The Modeling with finite elements by limiting long distances causes the banded structure and symmetry properties of the system matrices to deteriorate. In this case, realistic results may not be obtained. In addition, solving a large number of equations will cause large computational costs and programming of this model is quite difficult as it becomes complex. Solving these equations requires serious time and effort. Instead, realistic solutions can be obtained by using finite elements near the source and infinite elements in the far region. So, the far region of elements should be modelled as infinite elements in order to get realistic solution and reduce the computational cost. In this study, the far region of soil medium is modelled infinite elements. Primary objective of this investigation is to analyze stress distribution of semi-infinite soils consisting of two layers with twin rectangular tunnels under static loads by using finite and infinite elements. Effects of the location of the tunnels on the stress distributions along soil depth are obtained and discussed in detail.

2. Theory and formulations

Consider a two layered semi-infinite soil with two identical tunnels under the uniform distributed load Fig. 1. The finite region soil dimensions are shown as L_X in the horizontal

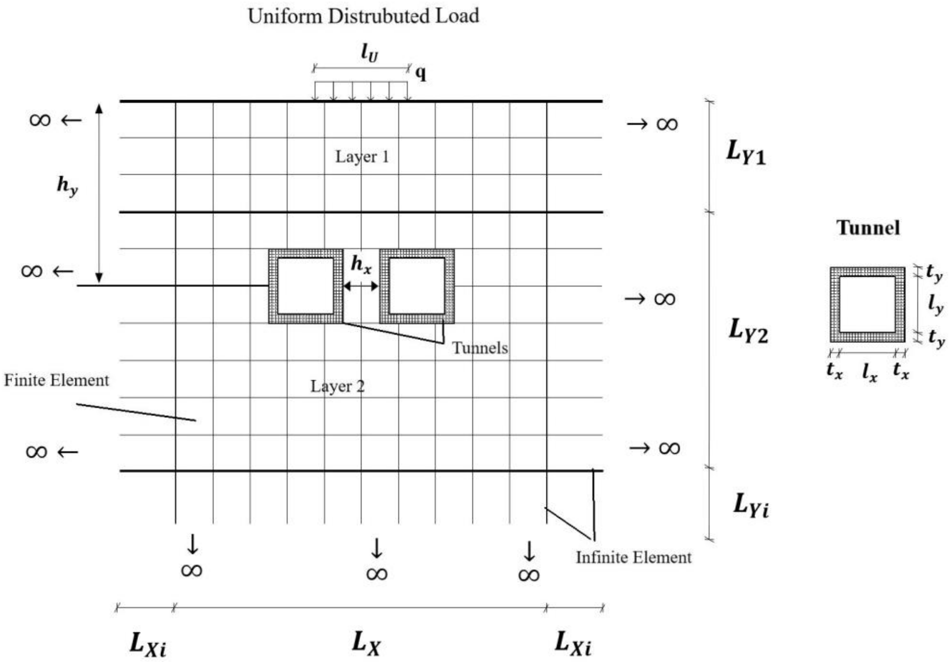


Fig. 1 The layered semi-infinite soil with twin tunnels under uniform load

direction with layer 1 L_{Y1} in the vertical direction and layer L_{Y2} in the vertical directions. The infinite region dimensions are expressed as L_{Xi} and L_{Yi} in the horizontal and vertical directions, respectively. Each rectangular tunnel lengths are specified as $l_x = l_y$ and tunnel thicknesses as $t_x = t_y$.

Distance between tunnels shown as h_x and distance of tunnels from ground surface as h_y. The uniform distributed load q of length l_U is given as the loading from the upper middle surface of the ground.

In the solution of the soil-structure interaction problem, the plane model is used in order to obtain more realistic results. The far region of soil which is infinite medium is modelled as infinite elements in order to get realistic solution and reduce the computational cost.

Equations of motion in plane elasticity for a semi-infinite medium region are expressed as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x = 0 \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0 \tag{1}$$

Where, f_x and f_y are the volume forces in the x and y directions. Normal and shear stresses, σ_x , σ_y and τ_{xy} represent respectively. The strain and displacements relations are given

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{2}$$

where, u and v are represented as the x, y directions displacements. The semi-infinite medium is assumed to be linear elastic, homogeneous and isotropic. The constitutive equations of the body are shown.

$$\{\sigma\} = [D]\{\varepsilon\} \tag{3}$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad \{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad [D] = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix} \tag{4}$$

The components of [D] for each layer are presented as follows

$$D_{11} = D_{22} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \qquad D_{12} = D_{21} = \frac{\nu E}{(1+\nu)(1-2\nu)} \qquad D_{33} = G = \frac{(\frac{1}{2}-\nu)E}{2(1+\nu)} \tag{5}$$

where, E and G are the modulus of elasticity and shear, and ν is the Poisson's ratio for each layer or tunnel element. The equations of motion in terms of displacements are given as follows

$$\frac{\partial}{\partial x}\left(D_{11}\frac{\partial u}{\partial x} + D_{12}\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial y}\left(D_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right) = -f_x \qquad \frac{\partial}{\partial y}\left(D_{12}\frac{\partial u}{\partial x} + D_{22}\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial x}\left(D_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right) = -f_y \tag{6}$$

Boundary conditions are given in terms of displacements as follows

$$t_x = \left(D_{11}\frac{\partial u}{\partial x} + D_{12}\frac{\partial v}{\partial y}\right)n_x + D_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)n_y$$
$$t_y = \left(D_{12}\frac{\partial u}{\partial x} + D_{22}\frac{\partial v}{\partial y}\right)n_y + D_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)n_x \quad (7)$$

Equations of motion converted to integral form with virtual work principle as follows

$$\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix} = \begin{bmatrix}\frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix}\begin{Bmatrix}u \\ v\end{Bmatrix} \quad (8)$$

The finite-infinite element formulation for plane elasticity problems is summarized. The interpolation shape functions are given

$$u = \sum_{i=1}^{n} N_i u_i \qquad v = \sum_{i=1}^{n} N_i v_i \quad (9)$$

$$\begin{Bmatrix}u \\ v\end{Bmatrix} = \begin{bmatrix}N_1 & 0 & N_2 & 0 & \dots N_n & 0 \\ 0 & N_1 & 0 & N_2 & \dots 0 & \dots N_n\end{bmatrix}\begin{bmatrix}u_1 \\ v_1 \\ u_2 \\ v_2 \\ \vdots \\ u_n \\ v_n\end{bmatrix} \quad (10)$$

The variation of node displacements is expressed.

$$\{\delta u\} = [N]\{\delta u_d\} \quad (11)$$

Strain and stresses in terms of node displacements

$$\{\varepsilon\} = [B]\{\Delta\}, \quad \{\sigma\} = [D][B]\{\Delta\} \quad (12)$$

where [B] is the strain matrix, $\{\Delta\}$ is the displacement.

$$[B] = [L]^T[N] \quad (13)$$

where, the differential operators [L] are given.

$$\begin{Bmatrix}\delta u \\ \delta v\end{Bmatrix} = [N]\{\delta\Delta\}, \quad \{\delta\varepsilon\} = [B]\{\delta\Delta\} \quad (14)$$

Based on the virtual work, the equilibrium equation can be depicted as

$$h_e \int\int_{A_e} \{\delta\Delta\}^T([B]^T[D][B]\{\Delta\})dxdy = h_e \int\int_{A_e} \{\delta\Delta\}^T[N]^T\begin{Bmatrix}f_x \\ f_y\end{Bmatrix}dxdy \quad (15)$$

The element stiffness matrices and element load vectors were obtained, respectively.

$$[k] = h_e \int\int_{A_e} [B]^T[D][B]dxdy$$
$$[f] = h_e \int\int_{A_e} [N]^T\begin{Bmatrix}f_x \\ f_y\end{Bmatrix}dxdy \quad (16)$$

$$[k]\{\Delta\} = [f] \quad (17)$$

The obtained integration form solved by the 5-point Gauss Legendre integration method.

$$I = \int_a^b f(x)dx = \sum_{i=1}^{n} w_i f_i(x) \quad (18)$$

Finite element interpolation shape functions are shown (Table 1)

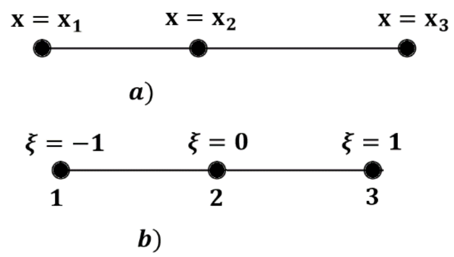
Infinite element shape functions with one-dimensional 1/r type decay are shown. By using the one-dimensional shape functions shown. Two-dimensional shape functions with 8 nodes are obtained depending on the direction (Fig. 3).

$$N_1 = \frac{-2\xi}{1-\xi} \qquad N_2 = \frac{1+\xi}{1-\xi} \quad (19)$$

Coordinate transformation expressions are given

$$x = N_1 x_1 + N_2 x_2 \quad (20)$$

By using the one-dimensional finite element cubic shape functions are shown in Table 2. Two dimensional shape functions with 16 nodes were generated using 1 dimensional cubic shape functions (Fig. 4).



(a) Real element (b) Reference element

Fig. 2 One-dimensional infinite element

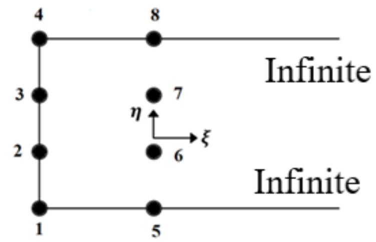


Fig. 3 One-dimensional infinite element

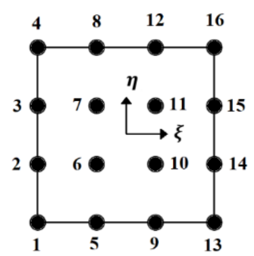


Fig. 4 Finite element with sixteen nodes

Table 2 Interpolation shape functions for a two-dimensional 16-node cubic finite element

Number of nodes	Shape function
M1	$\left(\frac{9}{16}\right)\left(\frac{-2\xi}{1-\xi}\right)(1-\eta)\left(\eta^2-\frac{1}{9}\right)$
M2	$\left(\frac{27}{16}\right)\left(\frac{-2\xi}{1-\xi}\right)(\eta^2-1)\left(\eta-\frac{1}{3}\right)$
M3	$\left(\frac{27}{16}\right)\left(\frac{-2\xi}{1-\xi}\right)(1-\eta^2)\left(\eta+\frac{1}{3}\right)$
M4	$\left(\frac{9}{16}\right)\left(\frac{-2\xi}{1-\xi}\right)(1+\eta)\left(\eta^2-\frac{1}{9}\right)$
M5	$\left(\frac{9}{16}\right)\left(\frac{1+\xi}{1-\xi}\right)(1-\eta)\left(\eta^2-\frac{1}{9}\right)$
M6	$\left(\frac{27}{16}\right)\left(\frac{1+\xi}{1-\xi}\right)(\eta^2-1)\left(\eta-\frac{1}{3}\right)$
M7	$\left(\frac{27}{16}\right)\left(\frac{1+\xi}{1-\xi}\right)(1-\eta^2)\left(\eta+\frac{1}{3}\right)$
M8	$\left(\frac{9}{16}\right)\left(\frac{1+\xi}{1-\xi}\right)(1+\eta)\left(\eta^2-\frac{1}{9}\right)$

Table 2 Interpolation shape functions for a two-dimensional 16-node cubic finite element

Number of nodes	Shape function
1	$\frac{81}{256}(1-\xi)(1-\eta)(\frac{1}{9}-\xi^2)(\frac{1}{9}-\eta^2)$
2	$\frac{243}{256}(1-\eta^2)(\xi^2-\frac{1}{9})(\frac{1}{3}-\eta)(1-\xi)$
3	$\frac{243}{256}(1-\eta^2)(\xi^2-\frac{1}{9})(\frac{1}{3}+\eta)(1-\xi)$
4	$\frac{81}{256}(1-\xi)(1+\eta)(\frac{1}{9}-\xi^2)(\frac{1}{9}-\eta^2)$
5	$\frac{243}{256}(1-\xi^2)(\eta^2-\frac{1}{9})(\frac{1}{3}-\xi)(1-\eta)$
6	$\frac{729}{256}(1-\xi^2)(1-\eta^2)(\frac{1}{3}-\xi)(\frac{1}{3}-\eta)$
7	$\frac{729}{256}(1-\xi^2)(1-\eta^2)(\frac{1}{3}-\xi)(\frac{1}{3}+\eta)$
8	$\frac{243}{256}(1-\xi^2)(\eta^2-\frac{1}{9})(\frac{1}{3}-\xi)(1+\eta)$
9	$\frac{243}{256}(1-\xi^2)(\eta^2-\frac{1}{9})(\frac{1}{3}-\xi)(1-\eta)$
10	$\frac{729}{256}(1-\xi^2)(1-\eta^2)(\frac{1}{3}+\xi)(\frac{1}{3}-\eta)$
11	$\frac{729}{256}(1-\xi^2)(1-\eta^2)(\frac{1}{3}+\xi)(\frac{1}{3}+\eta)$
12	$\frac{243}{256}(1-\xi^2)(\eta^2-\frac{1}{9})(\frac{1}{3}+\xi)(1+\eta)$
13	$\frac{81}{256}(1+\xi)(1-\eta)(\frac{1}{9}-\xi^2)(\frac{1}{9}-\eta^2)$
14	$\frac{243}{256}(1-\eta^2)(\xi^2-\frac{1}{9})(\frac{1}{3}-\eta)(1+\xi)$
15	$\frac{243}{256}(1-\eta^2)(\xi^2-\frac{1}{9})(\frac{1}{3}+\eta)(1+\xi)$
16	$\frac{81}{256}(1+\xi)(1+\eta)(\frac{1}{9}-\xi^2)(\frac{1}{9}-\eta^2)$

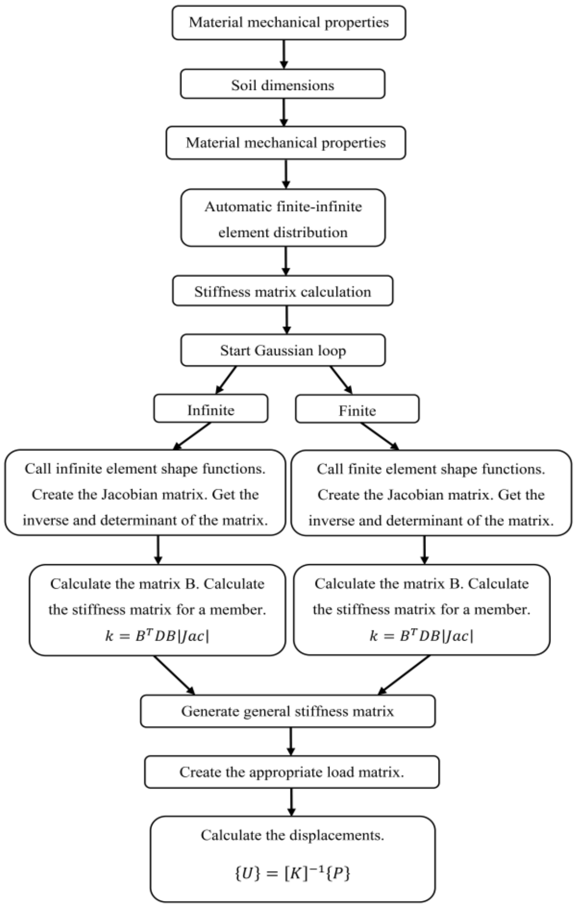


Fig. 5 The algorithm scheme of the solution

Implementing assembly procedure for the finite elements, the system stiffness and load vectors are obtained from the element stiffness and load vectors. By solving Eq. (17) in the system element model, the displacement vector obtained and after, the stresses values can be obtained by using Eqs. (2) and (3). It is noted that each layer and tunnels are modelled and defined their own material properties in the finite elements. In the finite element model and solution of the problem, the algorithm scheme is presented in Fig. 5.

3. Numerical results and discussion

In this study, the results obtained from the used model are verified by the Boussinesq theory and the static results in the literature (Timoshenko and Goodier 1951, Curnier 1983) in Fig. 6. The Boussinesq analytical formulation are expressed in eaution 21. The three-dimensional problem of a point load (P=2) acting on an elastic half-space analyzed. The soil parameters are E=1, ν=1. The two dimensions' elements used are 16-node finite elements and 8-node infinite elements. The mesh distribution is compatible with the ground and tunnels are made (Fig. 6).

In the numerical solutions, $n_x = n_y = 28$ finite elements and 86 infinite elements are used. The results in the presented study converged with the literature (Curnier

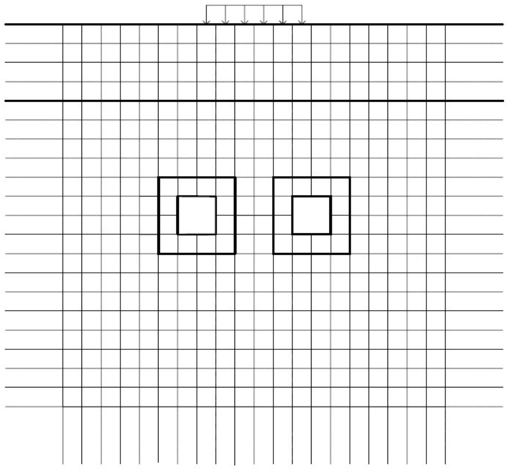
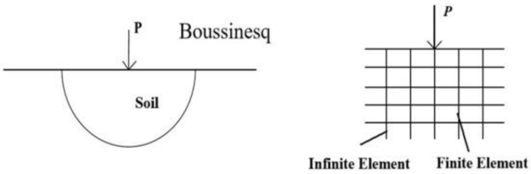


Fig. 6 Compatible mesh distribution with tunnel and ground



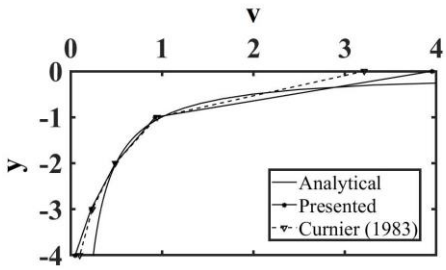


Fig. 7 Point load acting on three-dimensional elastic half-space Boussinesq (Timoshenko and Goodier 1951) and literature (Curnier 1983)

1983) and theoretical results Boussinesq (Timoshenko and Goodier 1951).

$$w_{r=0} = \frac{(1+\nu)(3-2\nu)}{2\pi E}\frac{P}{z} \tag{21}$$

For the convergence study, the static responses of layered soil with twin cavity under uniform load are obtained with finite-infinite elements (Fig. 8) in Table 3.

The used model is examined by using a different number of finite and infinite elements. The dimensions of the semi-infinite medium were considered as follows; $L_X = 20$ m, layer 1 $L_{Y1} = 5\,m$, layer 2 $L_{Y2} = 15\,m$, infinite elements $L_{Xi} = L_{Yi} = 10$ m, $h_y = 10$ meters and cavity dimensions $l_x = l_y = 10$ m. The soil medium parameters are layer 1; $E_1 = 1.26$ GPa, $\nu_1 = 0.4$ (Hatzigeorgioua and Beskosbc 2010) and layer 2; $E_2 = 22.4$ GPa, $\nu_2 = 0.33$ (Løkke and Chopra (2017)) Uniform load is taken as 2000 kN/m and 5 m. It is shown from Table 3 that, the static responses converge after the number of finite elements 32x32. So, the number of finite elements are used as 40x40 in all numerical studies.

Table 3 Convergence study on a layered semi-infinite soil with two cavity

Finite element with infinite element		8×8	16×16	32×32	40×40
Point 3	v (10^{-4}) m	-33.0000	-30.0000	-29.0000	-29.0000
	$\sigma_x(10^5)Pa$	-126.5600	-2.7406	-3.9354	-4.0726
	$\sigma_y(10^6)Pa$	-22.3080	-5.0477	-5.0470	-5.0439
	$\tau_{xy}(10^4)Pa$	601.6700	3.7678	4.3651	4.0282

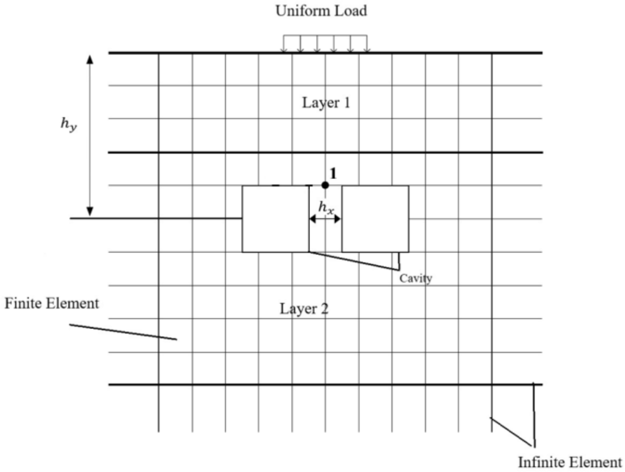
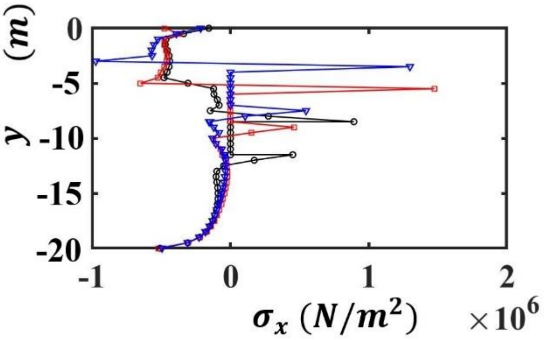


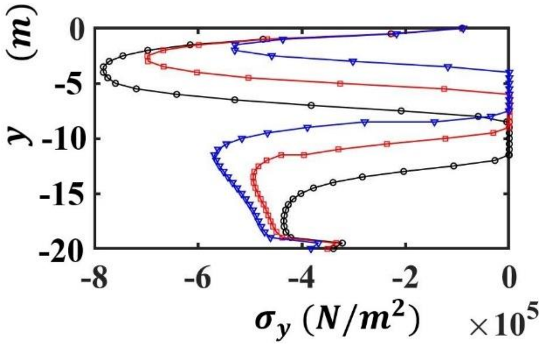
Fig. 8 Semi-infinite soil with two cavities under uniform distributed load

In the numerical solutions, layered soil medium with twin tunnel under uniform load is modeled with finite-infinite elements (Fig. 1). Stress along semi-infinite medium are obtained. The dimensions of the semi-infinite medium were considered as follows; $L_X = 20$ m, layer 1 $L_{Y1} = 5\,m$, layer 2 $L_{Y2} = 15\,m$, infinite elements $L_{Xi} = L_{Yi} = 10$ m, $h_y = 10$ m. and square tunnel inner width $l_x = l_y = 3$ m tunnel thickness $t_x = t_y =$0.5 m. The soil medium parameters are layer 1; $E_1 = 1.26$ GPa, $\nu_1 = 0.4$ (Hatzigeorgioua and Beskosbc 2010) and layer 2; $E_2 = 22.4$ GPa, $\nu_2 = 0.33$ (Løkke and Chopra 2017). The same tunnels materials parameters are $E_t = 34.5$ GPa and $\nu_t = 0.20$. Uniform load is 2000 kN/m and 5 m.

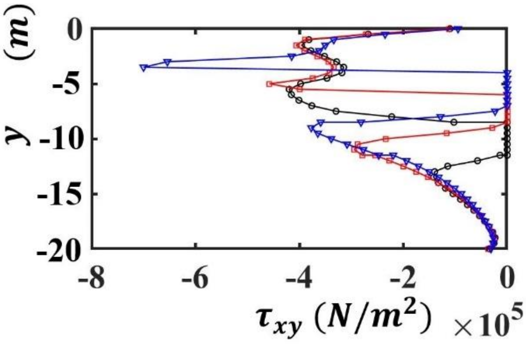
In the presented results, effects of the location of the tunnels on the stress distributions along soil depth are obtained and discussed in detail. In Figs. 9-11, effects of the vertical positions of the tunnels from ground surface (h_y) on the stress distribution of along soil depth are investigated under uniform distributed load. It is noted that (h_y) defines the distance between ground surface and middle point of the tunnels as shown Figs. 9-11. In these figures, the stress distributions are obtained different sections of the soil-depth, for example I-I, II-II and III-III sections are displayed Figs. 9-11, respectively. The observations and results from Figs. 9-11 are presented as; the stresses are zero naturally in the inner tunnel as seen from sections I-I and II-II (Figs. 9 and 10). The all stresses increase suddenly in the boundary of tunnels, and it cause the stress concentration in these regions as seen from Figs. 9 and 10. In III-III section, the

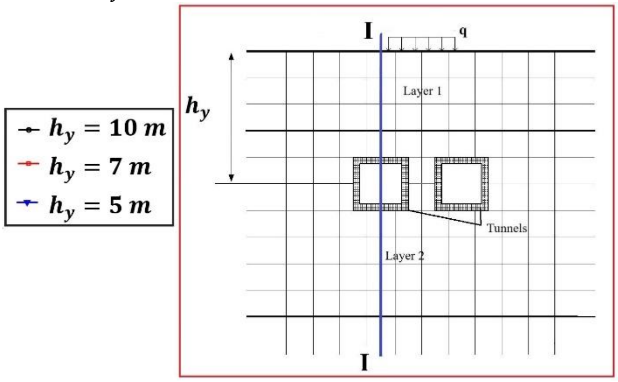


(a) Normal Stress distribution along soil depth for X Direction (σ_x)



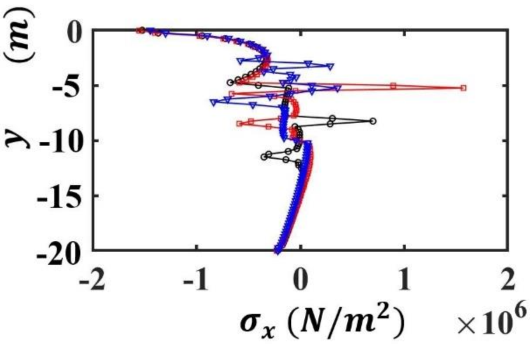
(b) Normal Stress distribution along soil depth for Y Direction(σ_y)



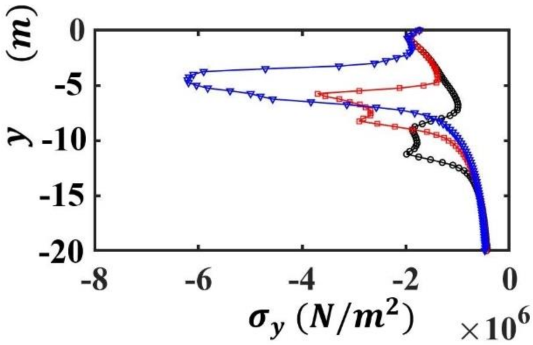


(c) Shear Stress distribution along soil depth(τ_{xy})

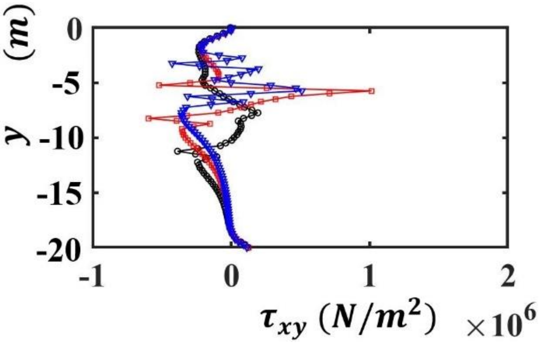
Fig. 9 Effects of the vertical positions of the tunnels from ground surface (h_y) on the stress distribution along for soil-depth for section I-I under uniform distributed load

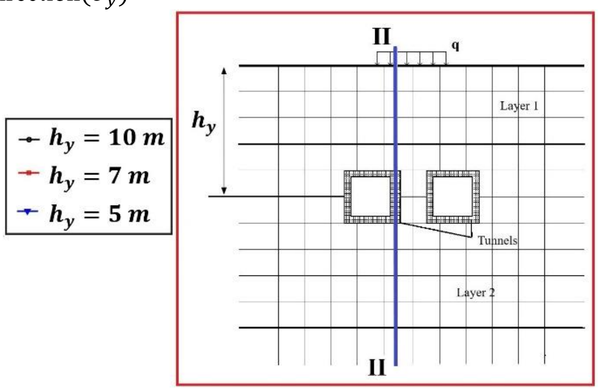


(a) Normal Stress distribution along soil depth for X Direction (σ_x)



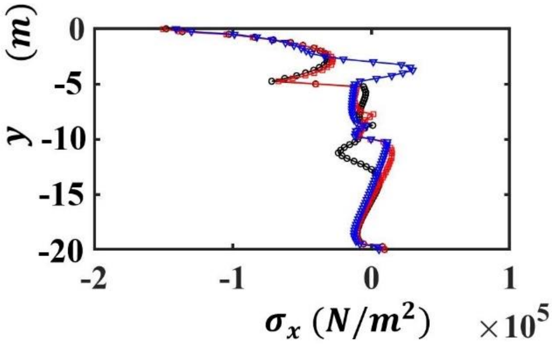
(b) Normal Stress distribution along soil depth for Y Direction(σ_y)



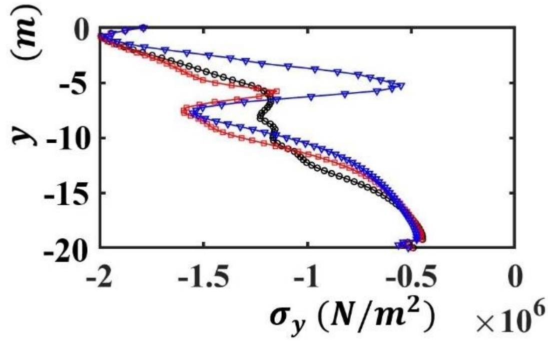


(c) Shear Stress distribution along soil depth(τ_{xy})

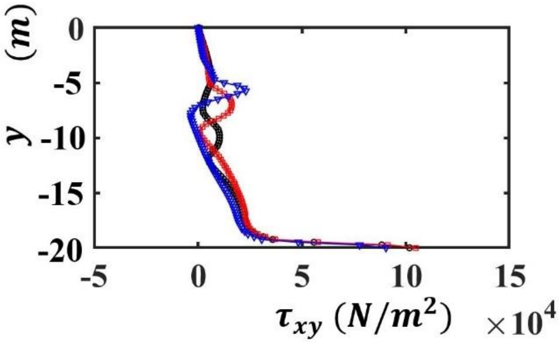
Fig. 10 Effects of the vertical positions of the tunnels from ground surface (h_y) on the stress distribution along for soil-depth for section II-II under uniform distributed load

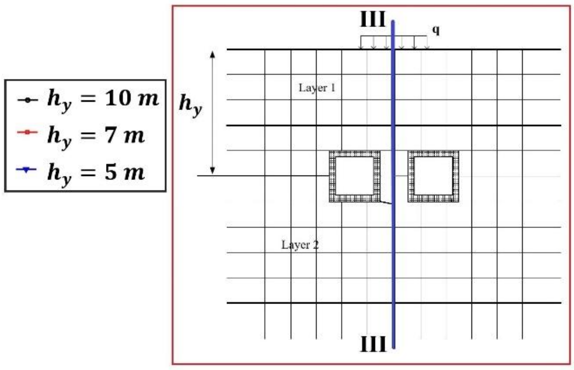


(a) Normal Stress distribution along soil depth for X Direction (σ_x)



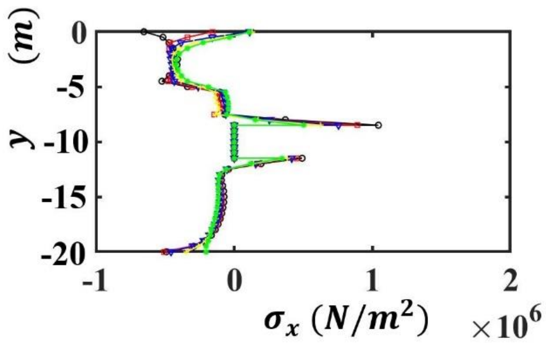
(b) Normal Stress distribution along soil depth for Y Direction(σ_y)



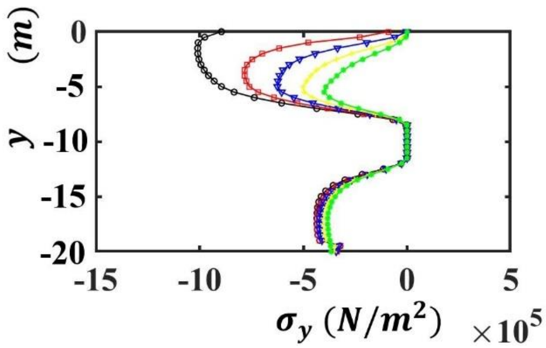


(c) Shear Stress distribution along soil depth(τ_{xy})

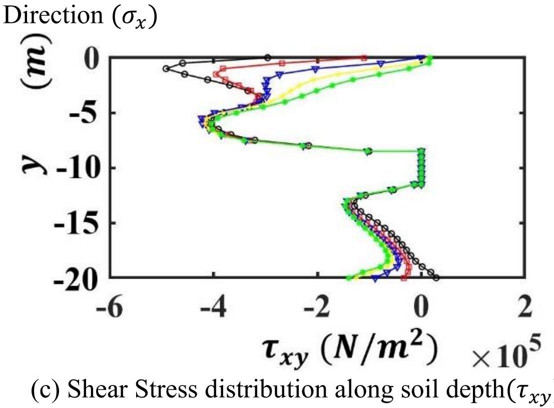
Fig. 11 Effects of the vertical positions of the tunnels from ground surface (h_y) on the stress distribution along for soil-depth for section III-III under uniform distributed load

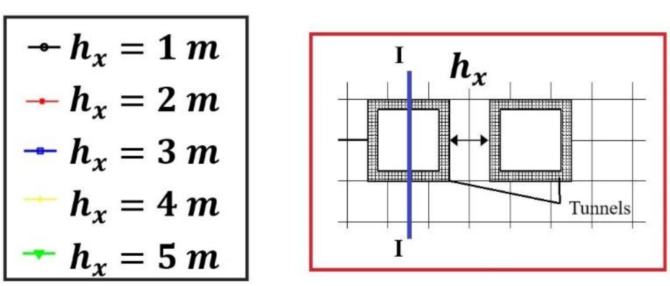


(a) Normal Stress distribution along soil depth for X Direction (σ_x)



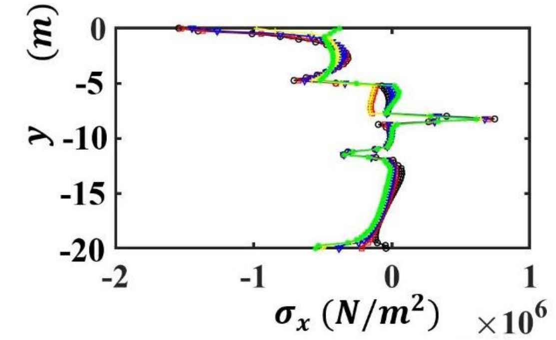
(b) Normal Stress distribution along soil depth for Y Direction(σ_y)



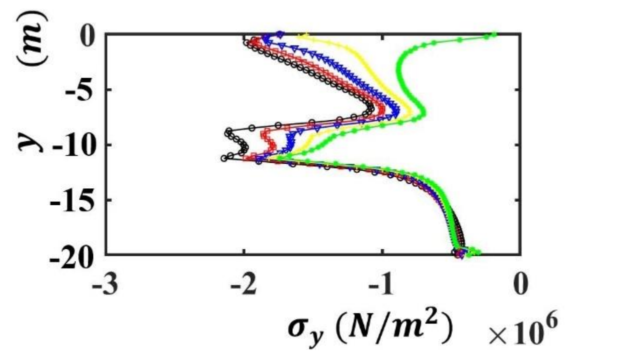


(c) Shear Stress distribution along soil depth(τ_{xy})

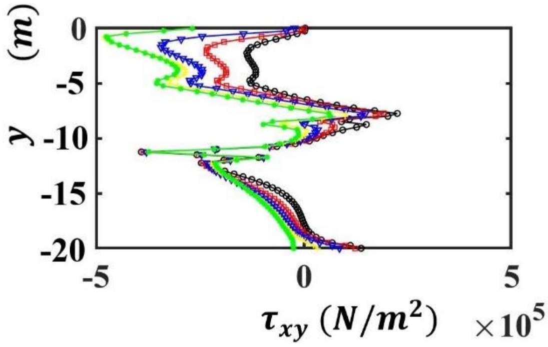
Fig. 12 Effects of the distance between twin tunnels (h_x) on the stress distribution along for soil-depth for section I-I under uniform distributed load



(a) Normal Stress distribution along soil depth for X Direction (σ_x)



(b) Normal Stress distribution along soil depth for Y Direction(σ_y)



(c) Shear Stress distribution along soil depth(τ_{xy})

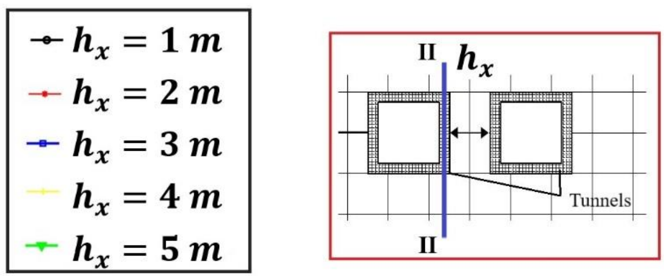
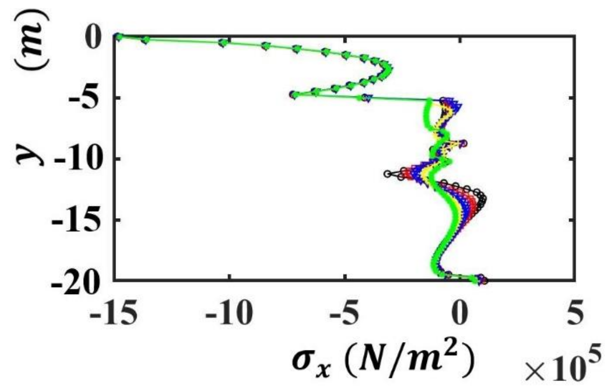
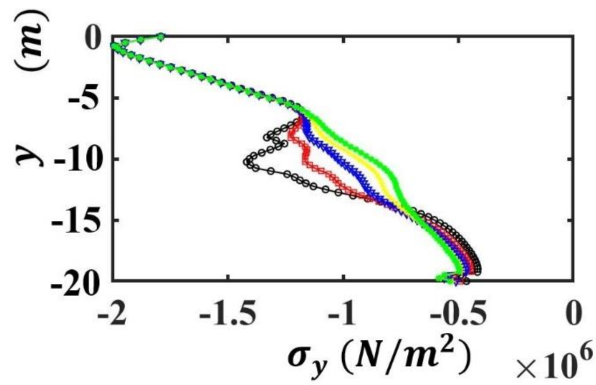


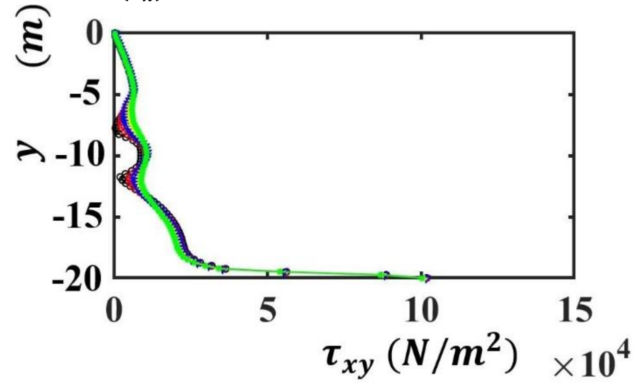
Fig. 13 Effects of the distance between twin tunnels (h_x) on the stress distribution along for soil-depth for section II-II under uniform distributed load



(a) Normal Stress distribution along soil depth for X Direction (σ_x)



(b) Normal Stress distribution along soil depth for Y Direction(σ_y)



(c) Shear Stress distribution along soil depth(τ_{xy})

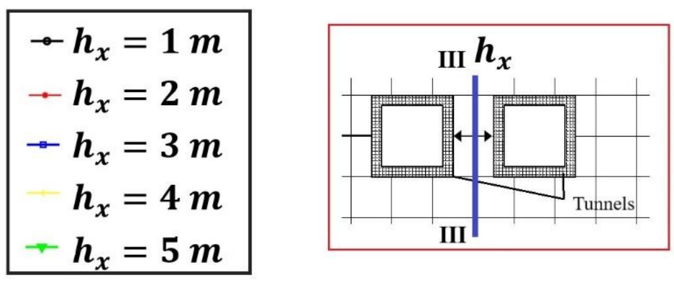


Fig. 14 Effects of the distance between twin tunnels (h_x) on the stress distribution along for soil-depth for section III-III under uniform distributed load

stress distributions do not behave sudden change like section I-I and II-II as seen from Fig. 11. However, the stress concentration occurs in the near of tunnel boundaries.

The position of the twin tunnels goes up to ground surface or first soil layer, the stress values increase considerably. This is because, the effects of load and stress distribution is biggest in the near of the load. So, more stress occurs the shallow tunnels in contrast with deep tunnels under surcharge loads. Also, the biggest stress values occur in I-I section. It shows that the vertical position of the tunnels is very important in the stress distribution behavior.

In Figs. 12-14, effects of the distance between twin tunnels (h_x) on the stress distribution of along soil depth are investigated under uniform distributed load. It is noted that h_x defines the horizontal distance between two tunnels as shown Figs. 12-14. In these figures, the stress distributions are obtained different sections of the soil-depth, for example I-I, II-II and III-III sections are displayed Figs. 12-14, respectively. It is seen from Figs. 12-14 that decreasing the h_x distance, the stress values increase significantly. Especially, σ_y stresses change considerably. The stress distributions for σ_x change very little with the change the hx distance. When the tunnels close up each other, the stress concertation increase considerably. It shows that the distance among tunnels is very important to the obtain minimize stress distribution. Also, as seen from these figures that the biggest stress change occurs the upper region of the soil sections.

4. Conclusions

In this study, stress analysis of two layered semi-infinite soil with twin tunnels is investigated by using the two dimensional (2D) plane solid finite-infinite elements. Plane solid elements are used with sixteen-nodes rectangular finite and eight-nodes infinite shapes. For fast solution of the problem, a finite element with 16 nodes and an infinite element with 8 nodes have been developed. The governing equations of the problem are obtained by using the virtual work principle. The finite-infinite element problem is solved by the 5-point Gauss-Legendre integration method.

It is observed from the obtained results that the position of the tunnels plays an important role on the stress distribution though soil depth. The stress concentration occurs in the near of tunnel boundaries, especially shallow tunnels. It can be concluded that the tunnel structures are subjected to more stress and fracture problems can be occurred frequently in the shallow and close range tunnels. The using finite element method with and infinite elements which defines infinite medium the realistic stress analysis can be obtained. Also, with using the infinite elements in the infinite medium, the computational cost for this complex problem can be reduced. Future work can be devoted to the interpretation of the results in order to possible nonlinear analysis of this problem.

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