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Waves dispersion in an imperfect functionally graded beam resting on visco-Pasternak foundation

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Abstract. This article investigates the effect of viscoelastic foundations on the waves' dispersion in a beam made of ceramic-metal functionally graded material (FGM) with microstructural defects. The beam is considered to be shear deformable, and a simple three-unknown sinusoidal integral higher-order shear deformation beam theory is applied to represent the beam's displacement field. Novel to this study is the investigation of the impact of viscosity damping on imperfect FG beams, utilizing a few-unknowns theory. The stresses and strains are obtained using the two-dimensional elasticity relations of FGM, neglecting the normal strain in the beam's depth direction. The variational operation is employed to define the dispersion relations of the FGM beam. The influences of the material gradation exponent, the beam's thickness, the porosity, and visco-Pasternak foundation parameters are represented. Results showed that phase velocity was inversely proportional to the damping and porosity of the beams. Additionally, the foundation viscous damping had a stronger influence on wave velocity when porosity volume fractions were low.

Keywords: FGM beam; porosity; visco-Pasternak foundation; wave dispersion

1. Introduction

The extraordinary characteristics of functionally graded materials make them attractive options for structural elements under extreme loading and thermal conditions. FGMs applications in marine, aircraft, aerospace structural parts, and military equipment sectors have proven their superior effective stiffness, strength, and hygrothermal resistance. This type of composite material is distinguished for having a smooth stress distribution and delamination-preventing composition (Reddy 2000, Yaylacı 2016, Cuong-Le *et al.* 2022a, b). FGM design parameters can be tailored to maintain the structural stiffness and integrity required for a specific application.

FGM beams are simple elements commonly used for different structural purposes. Many research articles investigated the behaviors of these elements in the literature. For instance, a finite element method (FEM) based solution was presented for the static behavior of FGM beams using the 1D Carrera Unified Formulation (CUF), which can generate different displacement field functions automatically (Filippi *et al*. 2015). The static and vibration response of nonlinear FGM viscoelastic sandwich beams was investigated via a numerical FEM approach adopting the zigzag theory. The investigation showed that the loss factor significantly influences the FGM beam behavior (Koutoati *et al.* 2021). A special mixed finite element approach was adopted to analyze the bending of uniformly loaded FGM beams (Benmalek *et al*. 2021). Frequency analysis was presented for bi-directional FGM beams under thermal effect using FEM software (Sharma and Khinchi 2021). Zhang and Zheng (2021) studied the elastoplastic buckling of axially compressed FGM beams with thermal-dependent properties. More studies that utilized the FEM method include (Yaylaci 2022, Yaylacı *et al.* 2021).

FG structures are studied for various shapes, behavior, and loadings. An Investigation of waves in sigmoid curved beams was performed by (Zhou *et al.* 2022). The authors used the Euler-Bernoulli curved beam theory and described various materials for the beam gradation. An analytical model for the free vibration and buckling behavior was presented for FGM graphene platelet-reinforced beams with variable axial loads (Priyanka *et al.* 2021). It was found that the pattern of porosity distribution in the FGM beams has more influence on the buckling than on the free vibration behaviors. Shafiei and Kazemi (2017) used the generalized differential quadrature method (GDQM) to investigate the nonlinear buckling of nano and micro-scaled FGM porous beams. The GDQM was also implemented for the nonlinear Von Kármán strains for vibration analysis of porous FGM nanobeams (Mirjavadi *et al.* 2018). The free vibration and buckling studies were also conducted for tapered

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microbeams with imperfection porosities using similar nonlinear models (Shafiei and Kazemi 2017a, Shafiei *et al.* 2016). Many other studies investigated the influences of porosities on the behaviors of FGM structures (Dastjerdi *et al.* 2021, Kaddari *et al.* 2020, Kiarasi *et al.* 2021, Malikan *et al.* 2018, Öner *et al.* 2014, Shahgholian *et al.* 2020, Sharifan and Jabbari 2020, Cuong-Le *et al.* 2020a, b, Ton-That *et al.* 2021, Tran and Cuong-Le 2022).

Various methods are used in the analysis of FG structures. Rational shear stress distribution-based HSDT was used to analyze the vibration response of FGM beams, and the significant influence of the bending moment on the FGM beams' vibration frequencies was shown in a study by (Chen *et al.* 2021). The nonlinear bending of FGM sandwich beams with various loadings was presented using Reddy's third-order shear deformation theory (TSDT) and von Kármán's nonlinear strain–displacement relations (Srikarun *et al.* 2021). Wave propagation in circular plates was studied using the concept of the physical neural surface by (She *et al.* 2022). This concept refers to a surface that is free of strain or stress as compared to the middle surface which is only free of stress in bending. The governing equations were solved using the Laplace transformation and the solution was used to describe the effect of material exponent and foundation parameters on the plate behavior.

The Fourier series method was used by (Civalek *et al.* 2022) to obtain the solution of a nano FG road subjected to torsion. The static and dynamic responses were obtained, and the study concluded -that how functionally graded nanorods twist, depends on their shape, size, edges, and material. (Ghannadpour and Khajeh 2022) utilized the Ritz method and the strain gradient theory to investigate the buckling of small-scaled FG plates.

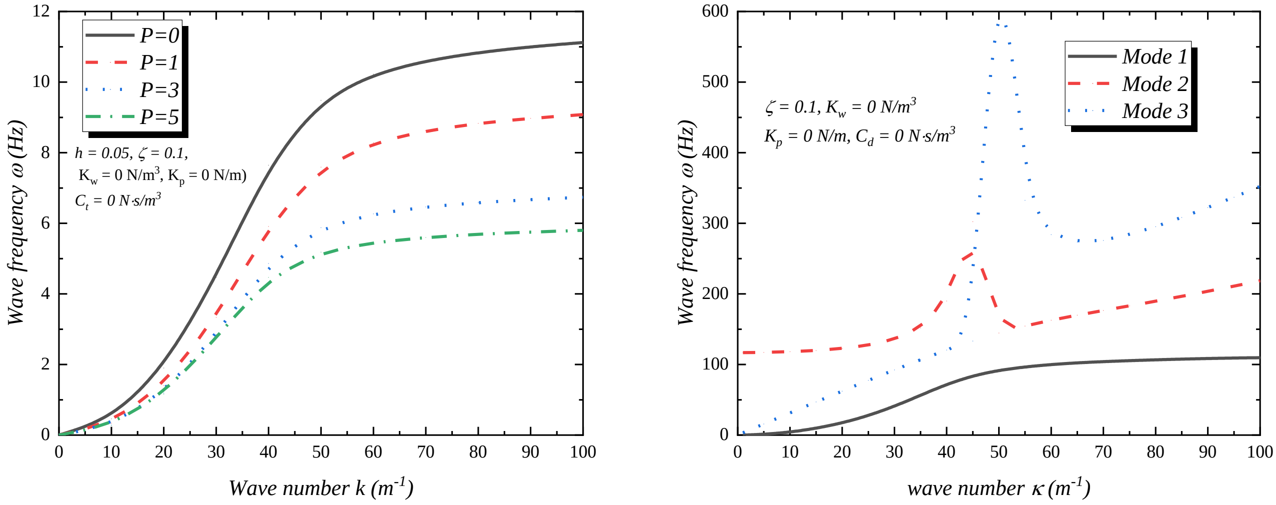
The wave propagation analysis has attracted significant research efforts, especially for the FGM structures (Aminipour and Janghorban 2017, Dai *et al.* 2022, Gao *et al.* 2020, Sepehri *et al.* 2022, Shahsavari *et al.* 2021, Shan *et al.* 2020, She *et al.* 2019, Zeighampour *et al.* 2018).

Arani *et al.* (2019) studied the surface and flexoelectric effects on FGM nanobeam wave propagation on the Winkler-Pasternak foundation. She *et al.* (2018) analyzed the wave propagation of porous FGM nanobeams using nonlocal strain gradient beam theory. (Yaylaci *et al.* 2020) explored continuous/discontinuous contact of a FG layer on a hard base with a distributed load on top. The layer's shear modulus and density changed exponentially with depth, but its Poisson ratio was constant. Fourier integral transform was used to get the stress and displacement components for the FG layer. A general nonlocal theory (GNT) was presented to analyze wave propagation in a rotating 2-D FGM nanobeam with porosities (Faroughi *et al.* 2020). The magneto-thermal influences on wave propagation in simply supported FGM beams resting on an elastic foundation were investigated using a refined higher-order beam theory (Ebrahimi *et al.* 2021). Integral formulations of nonlocal elasticity were presented for the wave propagation analysis of nanobeams and nanotubes (Norouzzadeh *et al.* 2020).

Yao *et al.* (2020) presented the nonlocal theory and the Timoshenko beam model for the wave propagation and free vibration analysis of axially moving FGM microbeam.

The wave propagation of graphene nanoplatelets FG composite nanoplates was investigated by She (2020) using the second-order shear deformation theory and the theory of nonlocal strain gradient that incorporates a stiffening and a softening mechanism. The author derived the equations of motion, which were solved with the help of the Trial function method, by using the Halpin-Tsai model as the distribution of material properties. By using ABAQUS and Artificial Neural Network (ANN) models, Madenci and Ozkilic (2021) investigated the free vibration of a porous FG beam with even and uneven porosity distribution. Using the same method, (Yaylacı *et al.* 2020) investigate the receding contact problem in terms of maximum pressure and contact area. In another study by the authors (Yaylacı *et al.* (2021), the problem was investigated in FG materials with various approaches. A mixed finite element model with 10 degrees of freedom was used to analyze the static and free vibrational properties of FG beams reinforced with spherical particles (Madenci 2021). Zhang and She (2022) studied wave dispersion in fluid-transporting pipes. The classical beam theory was used to derive the equations of motions and the results showed that the wave speeds go down with lower temperatures and gradients, but up with the faster liquid flow. The vibration frequency also goes down with all these factors.

It is important to note that waves' propagation is of interest in many other fields and materials for example Lata and Himanshi (2022) looked at how Rayleigh waves travel in a rotating material with heat, magnetism, and three delays using fractional theory with rotation and hall effect. Of the problems that emerged when using FGMs is the contact problem. It is relatively straightforward to solve the contact mechanics of systems composed of traditional materials that have a homogeneous microstructure and mechanical distribution, but it may be more challenging to solve the contact problem of new-generation materials that exhibit a non-homogeneous distribution (Yaylacı *et al.* 2022). Many studies have been conducted in this regard for example, (Oner *et al.* 2015) compared an analytical method and a finite element method (FEM) for analyzing two elastic layers pressed by a rigid circular punch and rest on a semi-infinite plane. The authors first used elasticity theory and integral transforms to get an analytical solution, then used ANSYS software to create a finite element model and perform a two-dimensional analysis. The results show that the contact areas from FEM are also close to those from analytical method; they differ by 0.03-1.61% providing good agreement in the two methods. Using analytical and FEM methods, (Yaylaci *et al.* 2021) investigated how a non-homogeneous FG layer with exponential properties and body force behaves when pressed by a uniform load on a frictionless homogeneous half plane. (Adıyaman *et al.* (2016) investigated how a functionally graded layer behaves when it contacts two rigid quarter planes and moves away from them. The layer is pushed by some forces on a portion of its top surface. The layer's stiffness varies with depth following an exponential function, but its Poisson ratio is constant. Other studies on this problem could be found in (Birinci *et al.* 2015, Öner *et al.* 2022, Yaylaci *et al.* 2022, Yaylacı and Birinci 2013).



(a) Various material exponents (b) Porosity models

Fig. 1 The dispersion relation between the principal frequency and the wavenumber of imperfect FG beams

Table 1 Ceramic and metal material properties

Materials	E (GPa)	ρ $\left(\frac{\mathrm{kg}}{\mathrm{m}^3}\right)$	α (1/°C)	β $\left(\frac{\mathrm{wt\%}}{\mathrm{H_2O}}\right)^{-1}$	ν
Alumina (ceramic)	380	3800	7×10^{-6}	0.001	0.3
Aluminum (metal)	70	2707	23×10^{-6}	0.44	0.3

From the above synthesis, it is clear that a gap exists for a wave propagating in FG imperfect beams. Therefore, the present study investigates the influences of the material gradation exponent, the beam's thickness, the porosity, and visco-Pasternak foundation parameters on the wave propagation in FGM porous beams. A simple three-unknown integral higher-order shear deformation beam theory is presented in the study. Unlike the previous works in this area, the present model requires less computational cost due to the use of fewer variables. New literature-enriching results were obtained, illustrated, and discussed in this article.

3. Results and discussion

The previous section conducted an analytical analysis of wave dispersion on a beam resting upon a visco-Pasternak foundation. This section presents numerical results obtained using the mathematical model proposed for FGM sandwich ceramics-metals beams with the properties shown in Table 1.

Fig. 1 shows the relationship between the wavenumber and the wave frequency for various power-law indices. The frequency of the wave is higher when the beam is made up entirely of metal $(p = 0)$ than for all the other cases of mixed materials. This means that for the same wavenumber, increasing the power-law index causes a reduction in the frequency of the waves. The lowest wave frequency is found in mode 1 of the three modes shown in Fig. 1. With high wavenumbers (> 50), mode 3 shows the greatest angular wave frequency, whereas, with low wavenumbers (< 50), mode 2 showed the highest.

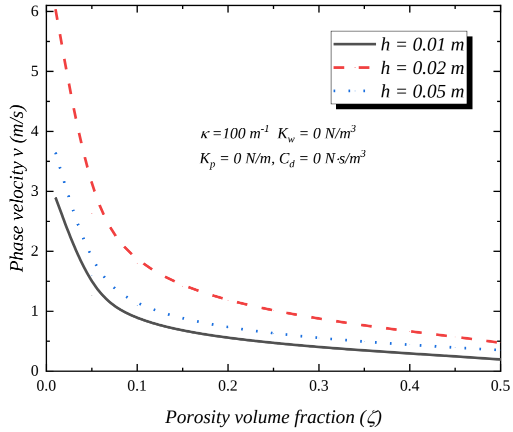
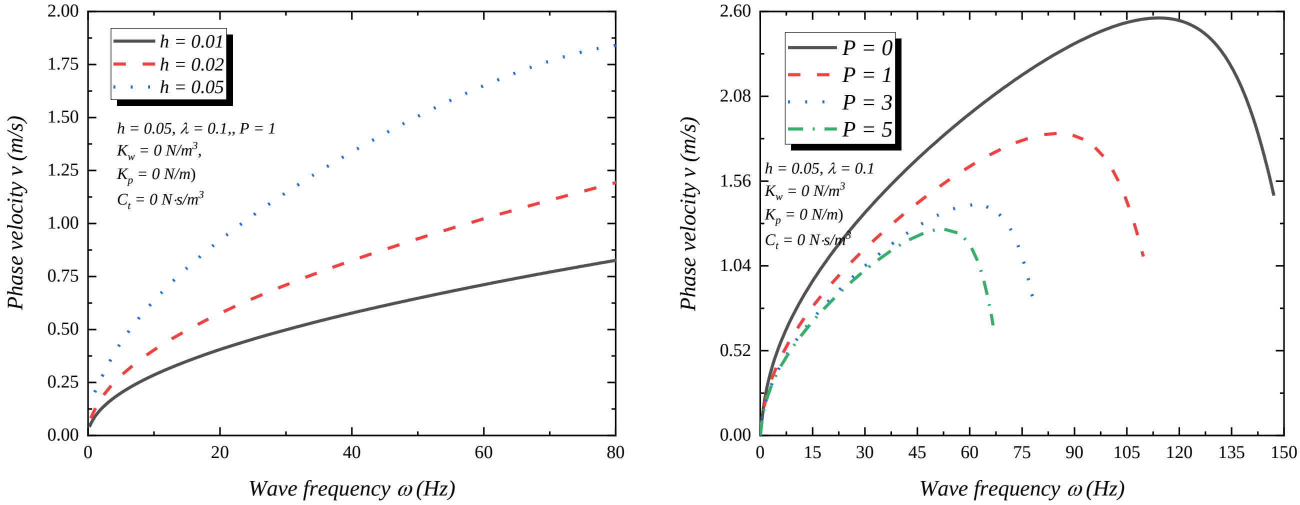


Fig. 3 The effect of Porosity volume fraction on the waves' dispersion in imperfect FG beams

Based on various thicknesses of the beam, Fig. 2 shows the relationship between material power-law exponent and phase velocity in meters per second. Low power-law exponent values are associated with the highest speeds. It is noteworthy that waves travel faster in beams made entirely of ceramic $(p = 0)$, and when the beam's composition is a mix of ceramic and metal, these velocities tend to decrease. When the three thicknesses of the beam in Fig. 2 were considered, the middle thickness $(h = 2cm)$ resulted in the highest wave velocity values for all material indices.

Fig. 3 shows how imperfections in the beam affect the phase velocity for various beam thicknesses with wave number $\kappa = 100\ m^{-1}$. In general, phase velocity decreases with increased porosity volume fraction, regardless of the beam thickness. As waves disperse in a medium, a decrease in velocity is expected because voids reduce the medium of travel resulting in a slower wave speed. The phase velocity of medium-thickness porous beams is higher than that of thinner or thicker beams.



(a) Various thicknesses (b) Various material exponents

Fig. 4 The dispersion relation between the phase velocity and the wave frequency of imperfect FG beams

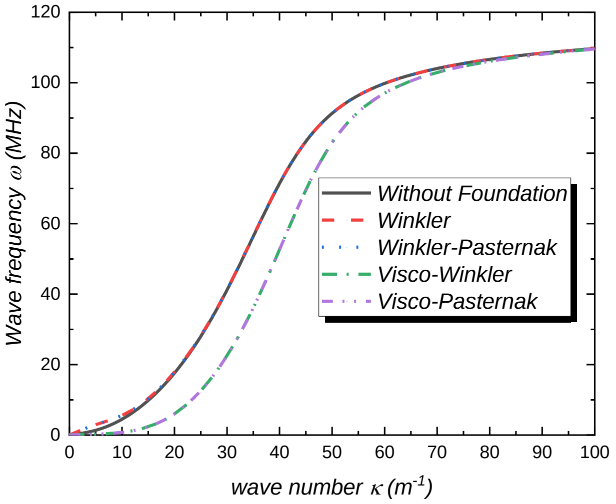


Fig. 5 The effects of various foundation models on the dispersion relation between the phase velocity and the wave frequency of imperfect FG beams

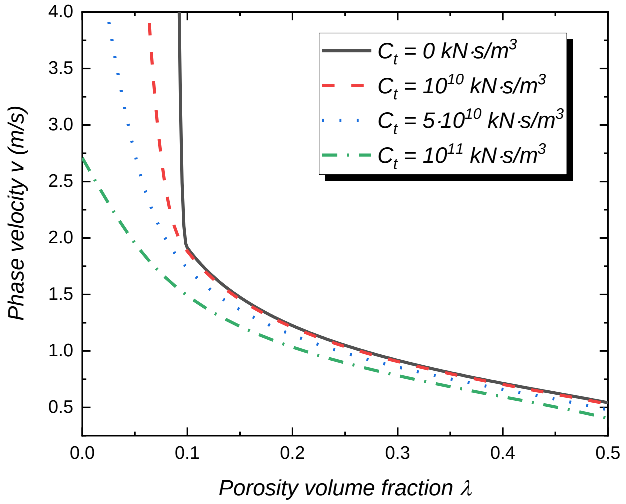


Fig. 6 The dispersion relation between the phase velocity and porosity volume fraction for various damping coefficients in imperfect FG beams

Fig. 5 illustrates how beam foundation models affect wave frequency. In contrast to the absence of foundation, a small effect can be seen when using the Winkler or Winkler-Pasternak models. There is a remarkable drop in wave frequency when the viscoelastic foundation is included, especially at low wavenumbers. In theory, this makes sense because viscosity is a parameter in the Visco-Winkler and Visco-Pasternak foundations that results in more absorption of waves, which causes the waves' frequencies to drop. Because of this, the viscosity effect is a crucial factor when modeling and designing FG beams.

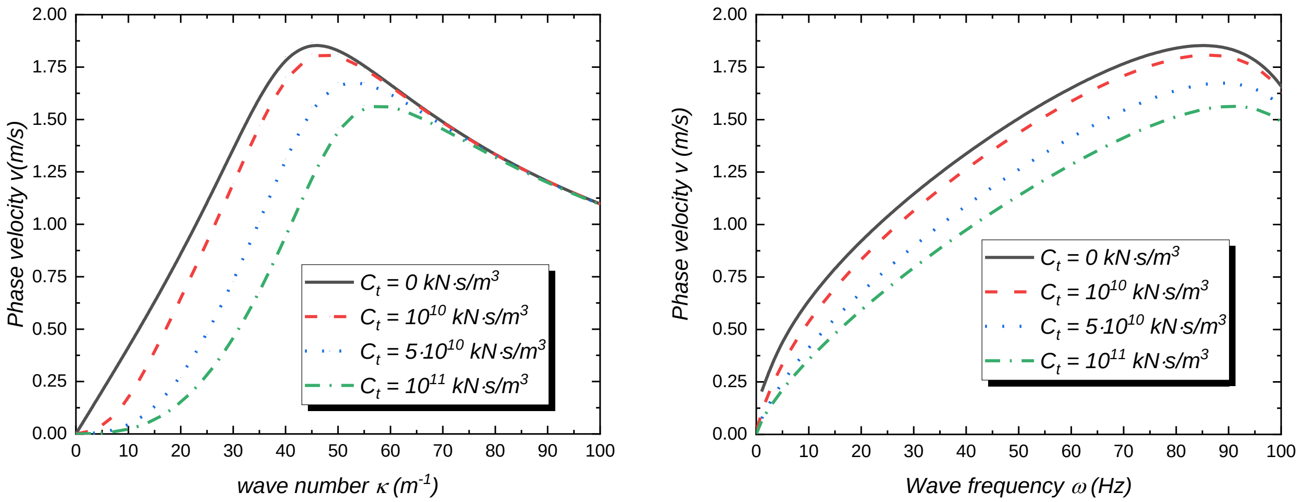
Considering the beam porosity volume fractions and the foundation damping, Fig. 6 plots their combined effects on the waves' phase velocities. Beams having the highest damping and porosity exhibited the lowest velocity and vice versa. In low porosity volume fractions, damping has a greater impact on wave velocity. This leads to the observation that the more the presence of porosity in the FG beam, the less the damping effect on the phase velocity.

The effect of increasing the damping parameter on the phase velocity of the porous FG beam is shown in Fig. 7.

When the viscosity is increased for the same wavenumber, the phase velocity decreases because the viscosity absorbs the vibration. There is no such effect for waves with high wave numbers $(> 65\, m^{-1})$, where the same phase velocity is observed for the various damping parameters. Furthermore, as a result of adding more damping to the foundation, the phase velocity becomes lower (Fig. 7(b)) for the same frequency, and this is the case for the entire frequency range.

4. Conclusions

The wave propagation in a sandwich FGM sandwich beam was explored by employing a three-unknown displacement theory. Hamilton's principle has been applied in the analytical solution incorporating the viscous



(a) Wave number (b) Wave frequency

Fig. 7 The effects of the damping coefficient on the phase velocity dispersion relations of phase velocity in imperfect FG beams

parameter effects. Numerical examples illustrating the effects of the various parameters highlight the following main findings, which shall enhance the design aspects of FGM porous beams:

- By increasing the power-law index, the frequency of waves decreases for the same wavenumber.
- When beams consist entirely of ceramic ($p = 0$), waves travel faster, and when the beam is made up of ceramic and metal (FGM), these velocities tend to decrease.
- No matter how thick the beam is, phase velocity decreases with increased porosity volume fraction.
- As the beam thickness increases, the phase velocity of the beam becomes higher for the same wave frequency.
- With the inclusion of the viscoelastic foundation, wave frequency falls dramatically, especially at low wavenumbers.
- In a given wavenumber, the phase velocity decreases with increasing viscosity.
- A larger porosity in the FG beam reduces the impact of damping on phase velocity.

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