Geomechanics and Engineering, *Vol. 32, No. 1 (2023) 49-68*
https://doi.org/10.12989/gae.2023.32.1.049

A new extended Mohr-Coulomb criterion in the space of three-dimensional stresses on the in-situ rock

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(*Received June 11, 2022, Revised December 13, 2022, Accepted December 23, 2022*)

Abstract. The three-dimensional failure criterion is essential for maintaining wellbore stability and sand production problem. The convenient factor for a stable wellbore is mud weight and borehole orientation, i.e., mud window design and selection of borehole trajectory. This study proposes a new three-dimensional failure criterion with linear relation of three in-situ principal stresses. The number of failure criteria executed to understand the phenomenon of rock failure under in-situ stresses is the Mohr-Coulomb criterion, Hoek-Brown criterion, Mogi-Coulomb criterion, and many more. A new failure criterion is the extended Mohr-Coulomb failure criterion with the influence of intermediate principal stress (σ_2). The influence of intermediate principal stress is considered as a weighting of (σ_2) on the mean effective stress. The triaxial compression test data for eleven rock types are taken from the literature for calibration of material constant and validation of failure prediction. The predictions on rock samples using new criteria are the best fit with the triaxial compression test data points. Here, Drucker-Prager and the Mogi-Coulomb criterion are also implemented to predict the failure for eleven different rock types. It has been observed that the Drucker-Prager criterion gave over prediction of rock failure. On the contrary, the Mogi-Coulomb criterion gave an equally good prediction of rock failure as our proposed new 3D failure criterion. Based on the yield surface of a new 3D linear criterion it gave the safest prediction for the failure of the rock. A new linear failure criterion is recommended for the unique solution as a linear relation of the principal stresses rather than the dual solution by the Mogi-Coulomb criterion.

Keywords: 3D failure criterion; extended Mohr-Coulomb; geomechanics; in-situ stresses; triaxial principal stresses

1. Introduction

Over the last three decades, the petroleum industry's problem with wellbore stability increased with the exploration of new oil and gas fields onshore or offshore. New explored reserves of oil and gas required new technology for drilling and production operations. If the reservoir is more profound than 4 km, then the uncertainty of subsurface formation pressure, tectonic effect, and temperature gradient play roles. Uncertainty about the drilling well is solved by modeling a well with maximum wellbore stability. Modeling well is done with the help of a numerical solution to a problem with considering the role of geomechanical stress (Zhang 2019, Aslannezhad *et al.* 2020).

The primary wellbore instability is due to the brack-out of rock by compression or rock fracture by tension. Many wells are missed due to the instability of the borehole by rock failure. Prediction of fracture can be made with the help of the failure criterion. Three perpendicular stresses produced on the in-situ rock are identified as vertical stress (overburden stress), minimum horizontal stress, and maximum horizontal stress (Das and Chatterjee 2017).

The Mohr-coulomb failure criterion is the best-known failure criterion for rock failure in geotechnical engineering (Ulusay, R (Ed.), 2015). Mohr-Coulomb is a linear relation of shear stress and normal stress with neglecting the effect of intermediate principal stress (σ_2) on the strength of failure of the material. Hoek-Brown failure criterion is the empirically derived relation between two principal stress $\sigma_1 and\ \sigma_3$ (Hoek and brown 1997, Hoek *et al.* 2002, Si *et al.* 2019). Hoek-Brown is a nonlinear-parabolic failure criterion which is also neglecting the effect of intermediate principal stress at the failure. Experimental tests suggest that intermediate principal stress has a substantial effect on the strength of rock material. (Rahimi and Nygaard 2018, He *et al.* 2022).

The numbers of three-dimensional (3D) failure criteria have been proposed by considering the effect of intermediate principal stress based on the extension of Mohr-Couloum failure criterion in 3D principal stress are; Drucker and Prager (1952), Lade criterion (Kim and Lade 1984), Krabbenhøft *et al.* (2007), Lee *et al.* (2012), Meyer and Labuz (2013), Jiang (2015), Jiang (2018), Barsanescu *et al.* (2018), Comanici and Barsanescu (2018), Yu and Wang (2019), Staat (2021) and many more. This 3D failure criterion is derived as the comprehensive failure criterion in terms of the octahedral shear stress (τ_{oct}) and octahedral normal stress (σ_{oct}). The rock strength value for triaxial

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ISSN: 2005-307X (Print), 2092-6219 (Online)

compression comprehensive failure criterion on the deviatoric stress plane may be underestimated or overestimated. Mogi (1971) proposed a failure criterion based on the triaxial compression data as a function of power low. The parameter from the Mogi criterion does not concern the standard parameters of the Mohr-coulomb criterion. Al-Ajmi and Zimmerman (2005) proposed a new failure criterion with a combination of the Mohr-Coulomb and Mogi criteria, called Mogi-Coulomb. Al-Ajmi and Zimmerman (2006) used the Mogi-Coulomb criterion for borehole stability analysis. It is recommendable that before applying any of the 3D failure criteria, further research and triaxial rock testing are necessary. Application of these failure criteria on the analytical solution to wellbore stability given and damage analysis of tunnels is given by (Das and Chatterjee 2017, Zhang *et al.* 2010, Singh *et al.* 2019, Chinaei *et al*. 2021, Noohnejad *et al.* 2021). All these failure criteria are also considered for the application of hydrofracturing for enhanced oil recovery. (Yan *et al*. 2019)

This study aims to propose a new 3D failure criterion based on the Mohr-Coulomb failure criterion by considering the effect of σ_2 on the rock strength. A new 3D failure criterion develops the plane surface as a failure envelope on the 3D space of $(\sigma_1, \sigma_2, \sigma_3)$, which is easy to derive and interpret. Available triaxial data on the eleven rock types are used to calibrate the proposed new 3D failure criterion on principal stress space $(\sigma_1, \sigma_2, \sigma_3)$. The proposed 3D failure criterion is validated by comparing predicted results with the Drucker-Prager and Mogi-Coulomb criterion on the eleven rock types. A new 3D failure criterion considers the weighting of σ_2 on mean stress instead of the average of principal stresses.

2. Convention and definition

The failure criterion is visualized and expressed by principal stress space $(\sigma_1, \sigma_2, \sigma_3)$ or a version of 3D Haigh-Westergaard coordinates stress space (ξ, ρ, θ) which is a direct physical interpretation (Ottosen 1977). The Geometric representation of a stress state in the space of Haigh–Westergaard and principal stresses space is given in Fig. 1. Haigh-Westergaard coordinates are widely used invariants in (ξ, ρ, θ) and define the principal stresses in the order ($\sigma_1 \geq \sigma_2 \geq \sigma_3$). The Haigh- Westergaard coordinates are given as

$$\xi = \sqrt{3}\,\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}} \tag{1}$$

$$\rho = \sqrt{3}\tau_{oct} = \frac{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}}{\sqrt{3}} \tag{2}$$

$$\theta = ar\cos\frac{2\sigma_1 - \sigma_2 - \sigma_3}{2\sqrt{3}\sqrt{J_2}} \tag{3}$$

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \tag{4}$$

Where the $\xi - \rho$ plane is also called the rendulic plane.

Principal stress $\sigma_1, \sigma_2\ and\ \sigma_3$ in the terms of the (ξ, ρ, θ) are given by Brannon *et al.* 2009.

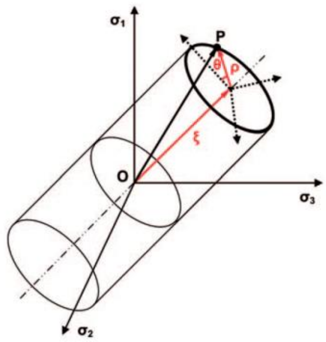


Fig. 1 Geometric representation of a stress state in Haigh–Westergaard and principal stresses space. (Lian *et al.* 2013)

$$\sigma_1 = \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}(\cos\theta - \frac{\sin\theta}{\sqrt{3}}) \tag{5}$$

$$\sigma_2 = \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}\left(\frac{2\ \sin\theta}{\sqrt{3}}\right) \tag{6}$$

$$\sigma_3 = \frac{1}{\sqrt{3}}\xi + \frac{\rho}{\sqrt{2}}\left(-\frac{\sin\theta}{\sqrt{3}} - \cos\theta\right) \tag{7}$$

Angle θ is called the stress angle in the range of 0 to $\frac{\pi}{3}$ and the cos (3θ) is also called the Lode parameter.

3. New three-dimensional failure criterion

Mohr-coulomb failure criterion gives the linear relationship between the shear strength of material versus normal stress. This relation is given by the following equation (Labuz and Zang 2012)

$$\tau = \sigma\tan(\phi) + c \tag{8}$$

Here, τ is the shear strength, σ is the applied normal stress, c is the cohesion of material and ϕ is the internal friction angle.

Now from Mohr's circle

$$\sigma = \sigma_m - \tau_m \sin\phi \ \text{ and } \ \tau = \tau_m\cos\phi \tag{9}$$

Here σ_m(mean stress) and τ_m (maximum shear stress) are expressed in the form of principal stresses as

$$\sigma_m = \frac{\sigma_1+\sigma_3}{2} \text{ and } \tau_m = \frac{\sigma_1-\sigma_3}{2}\ ;\ [\text{ Here } (\sigma_1 \geq \sigma_2 \geq \sigma_3)] \tag{10}$$

Mohr-Coulomb can be written in the terms of principal stress as.

$$\sigma_1 = q.\sigma_3 + UCS \tag{11}$$

Where $q = \frac{(1+\sin\phi)}{(1-\sin\phi)}$ and UCS is the uniaxial compressive strength $UCS = \frac{2c\cos\phi}{(1-\sin\phi)}$. UCS is also estimated with a different method based on the formation property, surrounding stress profile, and application of methods. (Qi *et al.* 2020, Lee *et al.* 2019, Chai *et al.* 2019)

Rock failure due to the compressive stress is without elastic and plastic deformation just acts like a brittle material. Brittle material has higher compressive strength than tensile strength. The Mohr-Coulomb criterion is suitable for brittle material with higher prediction efficiency. In the Mohr-Coulomb failure criterion, rock failure is related to the magnitude of maximum and minimum principal stresses acting on the material. Based on the experimental result of the triaxial compression failure test on the rock, there is an indication that intermediate principal (σ_2) stress impact on the failure of the rock. Rock strength decrease with the increment in the value of σ_2. Here positive values of the stresses indicated tension, and the negative value indicated compression stress on the rock.

The failure criterion for the in-situ condition must include all the stress on the rock. There are three perpendicular forces applied in the minor component of the rock, vertical stress, minimum horizontal stress, and maximum horizontal stress. Stress magnitude and direction depend on the well's inclination and direction. Now mean effective stress and maximum shear stress are given by the following equation, where the order of the principal stresses is given as ($\sigma_1 \geq \sigma_2 \geq \sigma_3$)

$$\sigma_m = \frac{(\sigma_1+\sigma_2+\sigma_3)}{3} \text{ and } \tau_m = \frac{\sigma_1-\sigma_3}{2} \tag{12}$$

From the Coulomb correlation in Eq. (8), and the result of the Mohr circle in Eq. (9), the theoretical relation between mean effective stress and maximum shear stress is given as

$$\tau_m(1+\tan^2\phi) = \sigma_m \sec\phi \tan\phi + c \sec\phi \tag{13}$$

Implementing the value of the mean effective stress and maximum shear stress given in the above equation. Now friction angle (ϕ) is considered as friction angle in triaxial compression (ϕ_m).

$$\frac{\sigma_1-\sigma_3}{2}(1+\tan^2\phi_m) = \frac{(\sigma_1+\sigma_2+\sigma_3)}{3}\sec\phi\tan\phi + c\sec\phi \tag{14}$$

we get a new equation for 3D stress, as

$$\sigma_1 = \frac{2\sin\phi_m}{(3-2\sin\phi_m)}\sigma_2 + \frac{(3+2\sin\phi_m)}{(3-2\sin\phi_m)}\sigma_3 + \frac{6\,c\cos\phi_m}{(3-2\sin\phi_m)} \tag{15}$$

Here

σ_1, σ_2 *and* σ_3 three principal stress and $\sigma_1 \geq \sigma_2 \geq \sigma_3$

if $q_1 = \frac{2\sin\phi_m}{(3-2\sin\phi_m)}$, $q_2 = \frac{(3+2\sin\phi_m)}{(3-2\sin\phi_m)}$ and $CS = \frac{6\,c\cos\phi_m}{(3-2\sin\phi_m)}$ then Eq. (15), is rewiring as a

$$\sigma_1 = q_1\sigma_2 + q_2\sigma_3 + CS \tag{16}$$

ϕ_m is the angle of internal friction of rock in 3D space.

In Eq. (15), the mean effective stress is the average of three principal stresses at in-situ conditions. Principal stresses are not equally responsible for the failure of rock in in-situ conditions. It is advisable to consider the weighting average of stresses for mean effective stress

$$\sigma = \frac{(P_1\sigma_1 + P_2\sigma_2 + P_3\sigma_3)}{P_1+P_2+P_3} \tag{17}$$

Here P_1, P_2 *and* P_3 is the weighting of σ_1, σ_2 *and* σ_3 respectively. In the Mohr-Coulomb failure criterion, mean stress is equal to the average of the maximum principal stress and the minimum principal stress. It signifies that the impact of σ_1 *and* σ_3 for rock, failure is equal to unity. From the experimental data, there are some impacts of σ_2 on rock, failure is observed. So, some assumptions are taken for simplification of the equation.

The impact of maximum and minimum principal stress for the rock failure is equal to unity, since the value of σ_1 *and* σ_3 are equally responsible for the failure of rock. The weighting of σ_1 *and* σ_3 are taken as unity as its direct impact on rock strength as developed by the Mohr-Coulomb criterion. So, $P_1 = P_3 = 1$

The Impact of intermediate principal stress is given by $P_2 = P$

Mohr-Coulomb criterion is the function of maximum principal stress (σ_1) and minimum principal stress (σ_3), as shown in Eq. (11). The value of maximum principal stress and the value of minimum principal stress is directly responsible for the failure of the rock. The influence of intermediated principal stress (σ_2) on the rock strength is considered as a weighting of σ_2. During the derivation of A new 3D failure criterion, the influences of σ_1 and σ_3 are equal to unity ($P_1 = P_3 = 1$), same as the Mohr-Coulomb criterion, but the influence of σ_2 on the strength of rock is considered a variable $P_2 = P$. The value of P may change with the rock material property.

Now by considering the effect of intermediate principal stress on the failure of rock where the order of principal stresses is ($\sigma_1 \geq \sigma_2 \geq \sigma_3$). The following equation gives the value of mean effective stress and maximum shear stress.

$$\sigma_m = \frac{(\sigma_1 + P*\sigma_2 + \sigma_3)}{P+2} \qquad \tau_m = \frac{\sigma_1-\sigma_3}{2} \tag{18}$$

Here P is the weighting of the intermediate principal stress. The value of P is contingent on rock property.

The value of the σ_m and τ_m are put in Eq. (9), to get the value of σ and τ, which is given by

$$\sigma = \frac{(\sigma_1 + P*\sigma_2 + \sigma_3)}{P+2} - \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin\phi$$

and $\tau = \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos\phi$

Now put the value of σ and τ in Eq. (8) and replace ϕ with friction angle in triaxial compression (ϕ_m) for A new failure criterion.

$$\left\{\left(\frac{\sigma_1-\sigma_3}{2}\right)\cos\phi_m\right\} = \left\{\frac{(\sigma_1 + P*\sigma_2 + \sigma_3)}{P+2} - \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin\phi_m\right\}\tan(\phi_m) + c$$

By Simplifying the above equation, we get the following A new 3D failure criterion with a significant impact of σ_2.

$$\sigma_1 = \frac{2P\sin\phi_m}{(2-2\sin\phi_m+P)}\sigma_2 + \frac{2+2\sin\phi_m+P}{(2-2\sin\phi_m+P)}\sigma_3 + \frac{2(2+P)*c*\cos\phi_m}{(2-2\sin\phi_m+P)} \tag{19}$$

Or

$$\sigma_1 = q_1\sigma_2 + q_2\sigma_3 + CS \tag{20}$$

Here

$$q_1 = \frac{2P\sin\phi_m}{(2 - 2\sin\phi_m + P)}$$

$$q_2 = \frac{2 + 2\sin\phi_m + P}{(2 - 2\sin\phi_m + P)}$$

$$CS = \frac{2(2+P) * c * \cos\phi_m}{(2 - 2\sin\phi_m + P)}$$

Eqs. (19) and (20), is the new 3D failure criterion directly derived from the Mohr-Coulomb failure criterion by considering the effect of σ_2 on rock strength. Here P, c, and ϕ_m are the additional material parameter of the new 3D failure criterion. P is the coefficient of the σ_2 and its change with the impact on σ_2 on the strength of rock. Eq. (19) produces a plane surface on the principal stress space $(\sigma_1, \sigma_2, \sigma_3)$. the value of c is given by the cohesion of the rock, ϕ_m is the new friction angle on the 3D principal stress space. There is no influence of P on the value of c and ϕ_m. The value of P is a function of c and ϕ_m. On the other hand, the values of C and ϕ_m influence the P's value. The value of P may vary for each rock type as P is the material parameter.

If P = 0, then the effect of intermediate principal stress on the rock strength is no more so, $q_1 = 0$, $q_2 = \frac{2+2\sin\phi_m}{(2-2\sin\phi_m)}$ and $CS = \frac{2c\cos\phi_m}{(1-\sin\phi_m)}$ and the new 3D linear criterion reduces to the Mohr-Coulomb criteria. as in Eq. (11).

$$\sigma_1 = \frac{1+\sin\phi_m}{(1-\sin\phi_m)}\sigma_3 + \frac{2c\cos\phi_m}{(1-\sin\phi_m)}$$

Eq. (19), reduced to the Mohr-Coulomb failure criterion if the value of P is zero. It has been shown that the Mohr-coulomb criterion fulfills the new failure criterion if we neglect the effect of intermediate principal stress.

Eq (19), can be written in the Haigh- Westergaard coordinates as

$$\rho = \frac{\sqrt{2}(q_1 + q_2 - 1)\xi + \sqrt{3}\,CS}{(\sin\theta + \sqrt{3}\cos\theta)q_2 - (2\sin\theta)q_1 - \sin\theta + \sqrt{3}\cos\theta} \tag{21}$$

When θ is equal to zero it's called the compressive meridian and for θ is equal to 60° it's called the tensile meridian.

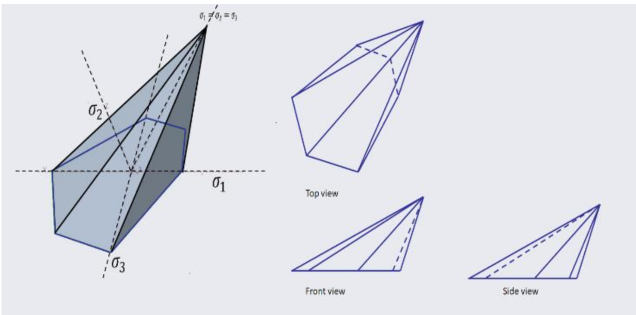
Eqs. (19) and (21), represent failure criterion in the coordinate of $(\sigma_1, \sigma_2, \sigma_3)$ and (ξ, ρ, θ) respectively. Both equations are newly developed and never mentioned before in literature as a failure criterion. The material parameter from both equations is calibrated with the triaxial data.

Here P is the weighting of the intermediate principal stress. The value of P is contingent on rock property and calculated with the help of Mohr-Coulomb criteria to find out UCS at $\phi = \phi_m$ given as

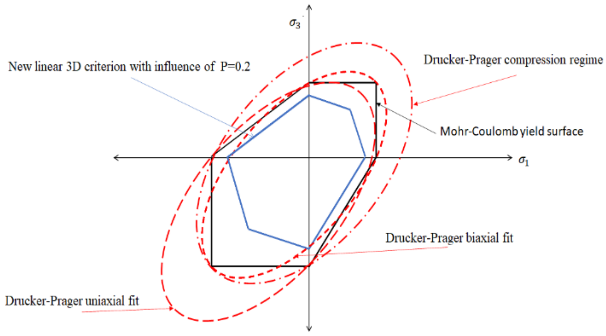
$$P = \frac{(2 - 2\sin\phi)\,UCS - 4\,c\cos\phi}{(2\,c\cos\phi - UCS)} \tag{22}$$

Shear failure surface in 3D Stress Space.

The stress profile of the new 3D linear failure criterion on the stress space is given in Fig. 2(a). Volume cover in this space is considered a safe zone when at the surface and outer space is considered a failure zone. Fig. 2(b), is the yield surface in 2D space of principal stress $\sigma_3 = 0$ for the new 3D linear, the Mohr-Coulomb, and the Drucker-Prager failure criterion (Drucker and Prager 1952). The yield stresses for the uniaxial compression and tension are given as S_{yc} and S_{yt}, respectively. The blue shaded area given by the new 3D linear failure criterion behave differently in the 1st and 2nd quadrant with the Mohr-Coulomb criterion. The slope for the new 3D linear criterion is due to the weighting of stresses on the failure of rock. the red dotted curve is indicated the Drucker-Prager failure criterion for uniaxial, biaxial and compression regime fit with the Mohr-Coulomb criterion. Fig. 2(b), develops for $P = 0.2, \theta = 30°$ and K=0.2 than value of $q_1 = 0.1667, q_2 = 2.667$ and $CS = 2.886\,c$. strength of the rock based on the Mohr-Coulomb as $CS = 3.4641\,c$ with $q_1 = 0, q_2 = 3$. intercept on the failure surface in the x-axis is given by $\sigma_1 = 0.833\,S_{yc}$, y-axis intercepts are given $\sigma_3 = 0.833\,S_{yc}$ in the compressive failure and x axis is given by $\sigma_1 = 0.833\,S_{yt}$, y-axis intercepts are given $\sigma_3 = 0.833\,S_{yt}$ in the tension failure. With the increment in the value of σ_3 strength value decrease as a function of P based on Eq. (20). The apex of the pyramid given for $\sigma_1 = \sigma_2 = \sigma_3 = 1.574\,c$ and Apex for the Mohr-Coulomb is given as $\sigma_1 = \sigma_2 = \sigma_3 = 1.7320\,c$. The volume covered by the pyramid of the new criterion is lesser than the pyramid of the Mohr-Coulomb criterion hence the new 3D linear failure criterion is safe compared to the Mohr-Coulomb criterion and the Drucker-Prager failure criteria.



(a)



(b)

Fig. 2 hear failure surface in 3D Stress Space. (a) The new 3D linear failure creation on σ_1, σ_2 and σ_3 plane. ($P = 0.2, \theta = 30$ and $K = 0.8$) and (b) Yield surface in 2D space of principal stresses ($\sigma_3 = 0,\ P = 0.2, \theta = 30$)

Table 1 Calibration result using the triaxial compression test for eleven rock types

Rock type	P	ϕ_m	c (MPa)	RMSE of σ_1 New 3D Criterion. (MPa)
KTB amphibolite	0.21	52°	60.03	55.028
Dunham dolomite	0.52	38°	119.21	35.345
Solnhofen limestone	0.286	37°	92.45	30.397
Shirahama sandstone	0.14	40°	23.35	7.903
Yuubari shale	0.37	28°	37.61	9.827
Yamaguchi marble	0.357	45°	24.04	22.273
Mizuho trachyte	0.46	26°	77.47	15.213
Manazuru andesite	0.18	51°	64.64	35.616
Inada granite	0.35	48°	123.58	46.088
Orikabe monzonite	0.74	43°	157.93	53.597
Coconino sandstone	≈ 0	45°	29.76	13.596

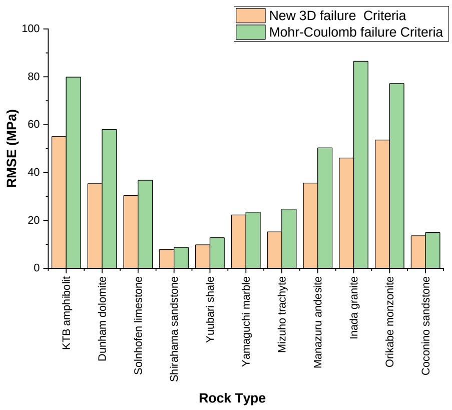


Fig. 3 Comparisons of RMSE of σ_1 in triaxial compression test data for calibration of the new 3D criterion.

4. Calibration and validation

4.1 Calibration of new 3D failure criterion

Typically, in in-situ conditions, rock break-out done by the compression and rock fracture is due to the combination of compression and tension. Experimental data on the compressive strength of rock with three perpendicular forces are available in the literature review. The triaxial and biaxial compression tests are the methods to determine compressive strength. For calibration of the constant in the new equation, the triaxial compression test data is used for the best value of P and ϕ_m.

Eleven types of rock that have both true-triaxial and triaxial compression strength data are selected for further calibration and validation of the new failure criterion. Testing data on the German Continental Deep Drilling Program (KTB) amphibolite was taken from Colmenares and Zoback (2002), which was presented and provided by Haimson and Chang (2000). Triaxial compression test data for Shirahama sandstone and Yuubari shale were also taken from Colmenares and Zoback (2002), which was the digitized form of the plot on Takahashi and Koide (1989). Data for the Inada granite, Orikabe monzonite, Dunham dolomite, Solenhofen limestone, Mizhuho trachyte, Yamaguchi marble, and Manazuru andesite were taken from Mogi (2006), while the data of Coconino sandstone were acquired from the Xiaodong and Haimson (2016). Comparisons for the RMSE of σ_1 in with the Mohr-Coulomb and New 3D linear criterion is given in Fig. 3.

Root means square error (RMSE) was used to find the misfits between the triaxial test data and predicted strength by the failure criterion.

RMSE is defined by the standard deviation of the residuals.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} r_i^2} \quad , \text{i=1,2,…,n} \qquad (23)$$

Here,

n is the number of the test data point of a specific rock.

r_i is the Residuals or prediction errors defined as

$$r_i = {\sigma_i}^{calc} - {\sigma_i}^{test} \qquad (24)$$

${\sigma_i}^{calc}$ is i^{th} calculated value by the failure criterion and ${\sigma_i}^{test}$ is the i^{th} tested data. For the best fitting RMSE value is the minimum. For the best-fit prediction of test data, the value of residuals is always equal to zero.

Fig. 4, indicates the calibration result on rock strength predicted by the new 3D failure criterion on eleven rock types with a triaxial compression test. Scatter data indicate triaxial compression test data, and solid lines represent the prediction of data by Eq. (16).

From the new failure criterion, rock strength decreased with the increment of σ_2, shown in Fig. 4. For the best fit, RMSE for σ_1 must be minimum for all data points.

Comparison of the RMSE values for σ_1 considering the influence of σ_2 (The New 3D failure Criterion) and without considering the influence of σ_2 (The Mohr-Coulomb failure criterion) is shown in Fig. 3. The values of the material parameter from the calibration and the RMSE value of σ_1 by the new 3D failure criteria are given in Table 1. The

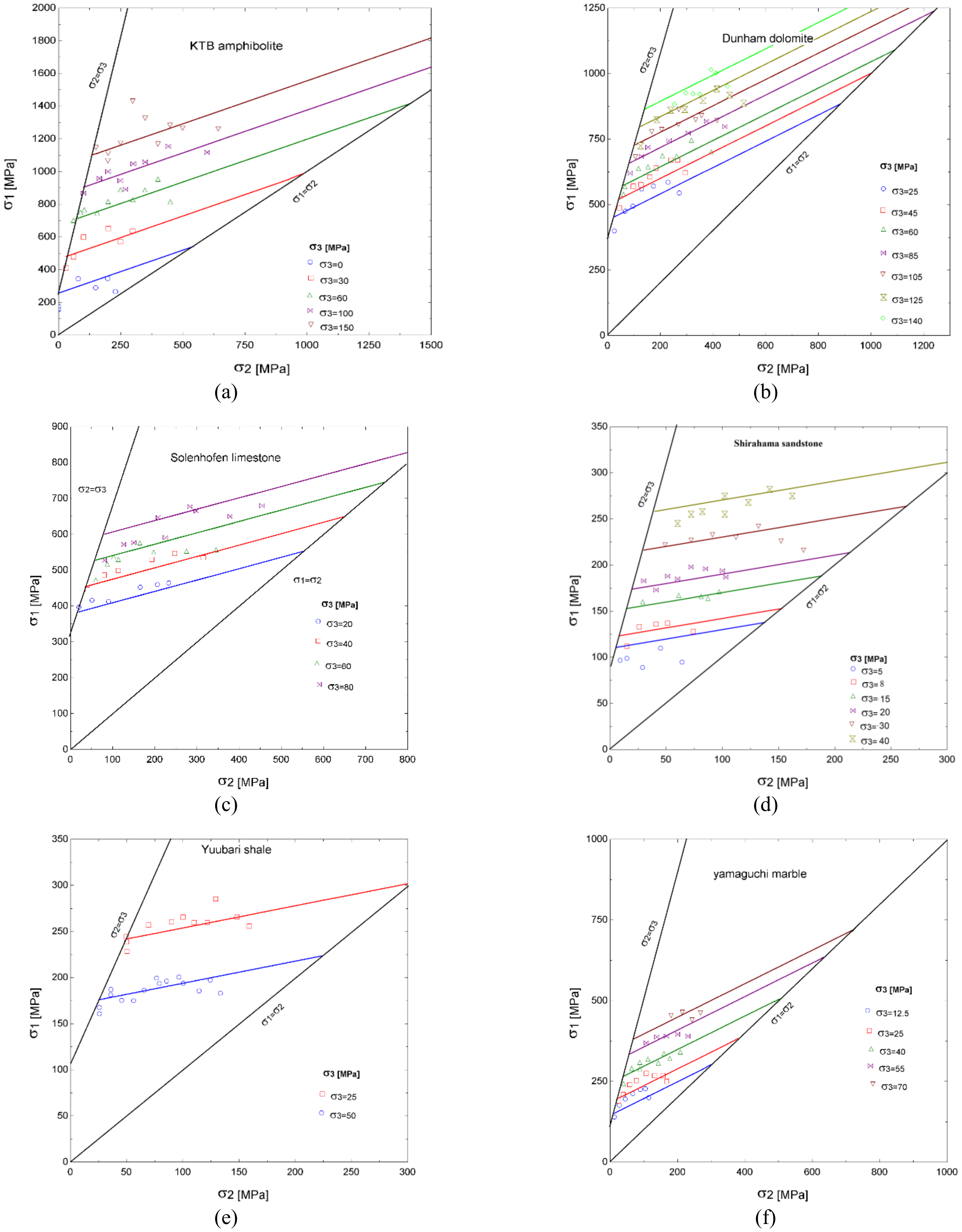


Fig. 4 Calibrations of the new 3D Failure Criterion in $\sigma_2 - \sigma_1$ space for eleven rock type

minimum RMSE of σ_1 is 7.903 MPa for Shirahama sandstone and the maximum RMSE of σ_1 is 55.02 MPa for the KTB Amphibolite for the best-fitting triaxial data point. RMSE value of σ_1 are between 7.903 MPa to 55.02 MPa for the rest of all rock types shown in Table 1. From calibration, values of material parameters are calculated and further used for the prediction of rock strength.

4.2 Validation of the new 3D failure criterion

To validate the proposed new failure criterion, we compare the predicted value with experimental triaxial test data of eleven rock types. The values of the q_1, q_2 and CS is calculated from calibration with the help of Eq. (16) along with linear regression of the triaxial test data in the plot of

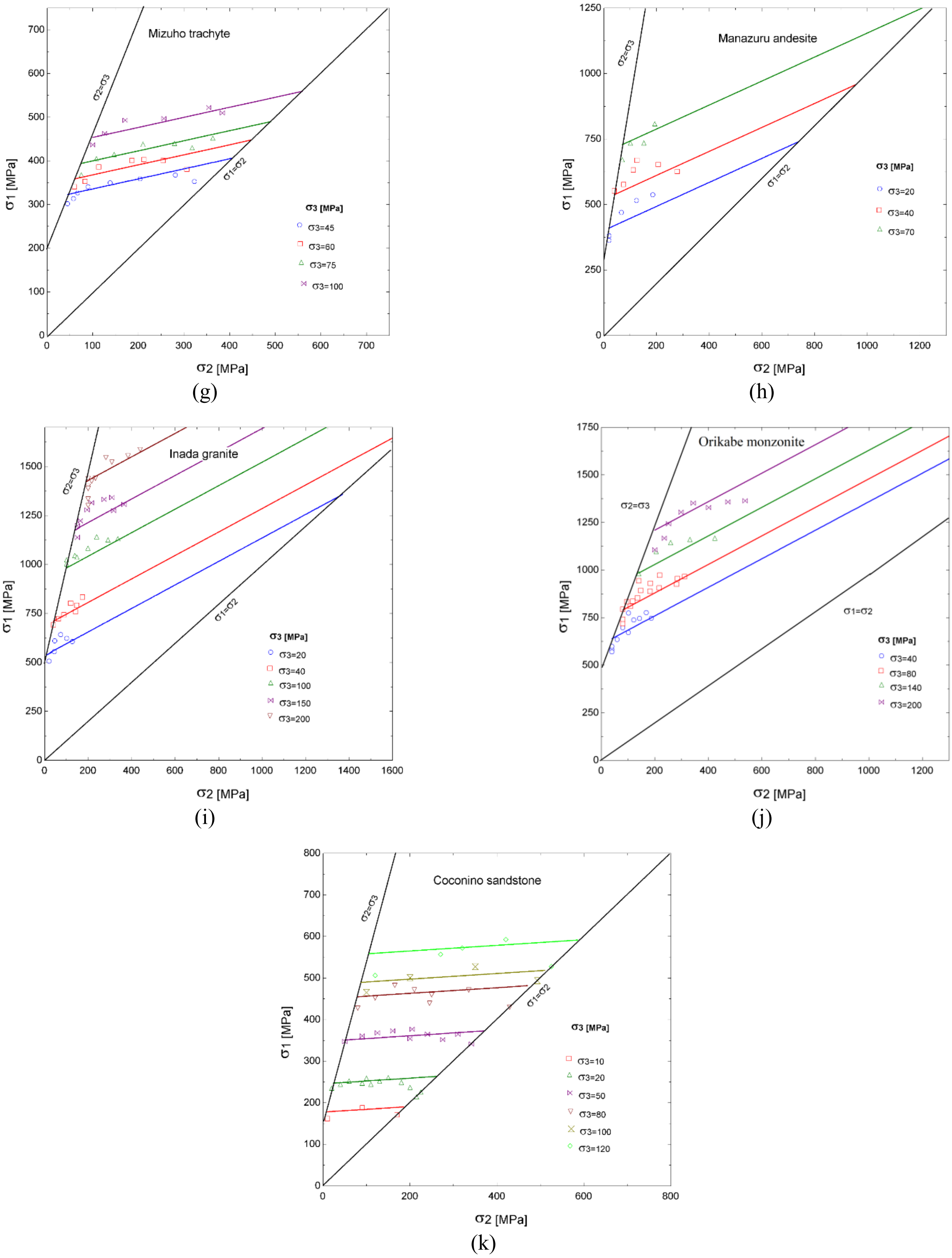


Fig. 4 Continued-

the σ_1 vs σ_2. The values of P, ϕ_m and cohesion C (MPa) are also calculated from Eq. (20), as shown in Table 1.

Haigh-Westergaard coordinates system (ξ, ρ, θ) is used to further validate failure criterion as shown in Fig. 1, Here ρ is the dependent variable, ξ is the independent variable, and θ depends on rock type. The comparison of the proposed 3D failure criterion with triaxial.

Compression test data is shown in Fig. 5, Scatter data on the plot indicate the triaxial test data, where the solid red line indicates the failure criterion by the Eq. (21). Validation of the new 3D failure criterion is done with the help of the coefficient of determination (DC) and RMSE of the ρ shown in Table 2. DC is a scalar indicator used to assess the reliability prediction of ρ by the failure criterion.

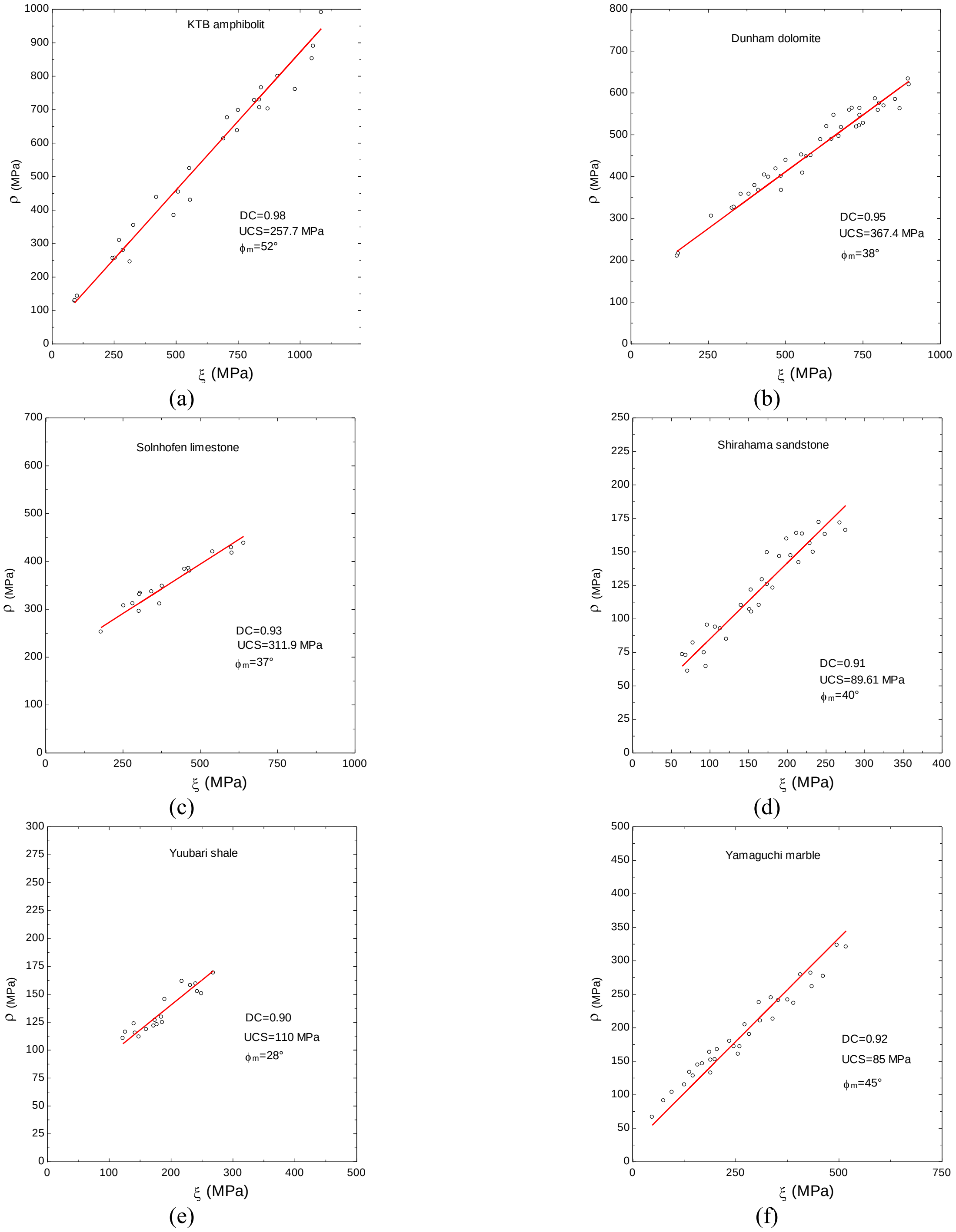


Fig. 5 Validation of the new 3D failure criterion for the eleven rock types

$$DC = 1 - \frac{\sum_{i=1}^{n}(\rho_i^{calc} - \rho_i{}^{test})^2}{\sum_{i=1}^{n}(\rho_i^{test} - \bar{\rho}_{\,test})^2} \tag{25}$$

Where ρ_i^{calc} i^{th} calculated value, ρ_i^{test} is i^{th} tested data on the triaxial test. $\bar{\rho}_{\,test}$ is the mean value of tested data by triaxial test. DC=1 means it's an ideal case where test data agree with the zero misfits. The desirable value of the DC must be higher, mostly near one. zero value of the DC indicates the baseline model and a negative value is a worse prediction than the baseline model.

Table 2, represents the value of DC of ρ for the predicted strength by the new failure criterion on the eleven rock types. Desirable values of DC are for Inada granite (DC=0.98), KTB amphibolite (DC=0.98), Orikabe monzonite (DC=0.95), and Dunham dolomite (DC=0.95). DC for the Manazuru andesite (DC=0.87) is the minimum among all the rock types. DC for the rest of the rock types is also good. i.e., Yamaguchi marble (DC=0.92), Solnhofen limestone (DC=0.93), Shirahama sandstone (DC=0.91), Yuubari shale (DC=0.90), Manazuru andesite (DC=0.92), and Coconino sandstone (DC=0.92).

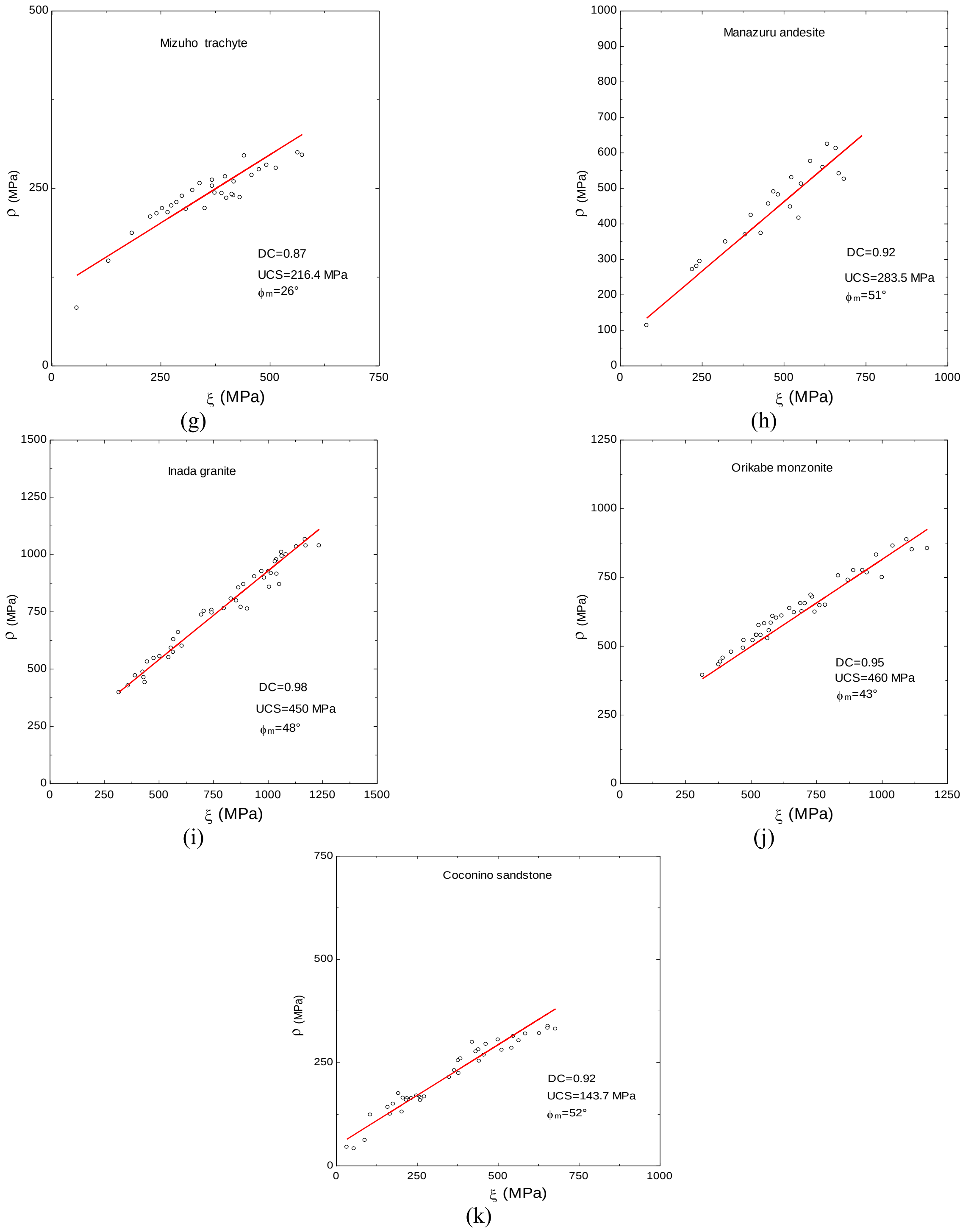


Fig. 5 Continued-

Fig. 6, is the distribution of the percentage error for predicted ρ for all the rock types. Percentage error is defined as

$$\varepsilon_i = \frac{clcuated\ data - tested\ data}{tested\ data} * 100\% \quad (26)$$

Or in the terms of the ρ

$$\varepsilon_i = \frac{\rho_i^{calc} - \rho_i^{test}}{\rho_i^{test}} * 100\% \quad (27)$$

Where ρ_i^{calc} is the i^{th} the predicted value of ρ from the triaxial data point, and ρ_i^{test} is the tested value of ρ on the triaxial data.

The percentage errors are calculated by considering triaxial compression test data for all the rock types, as shown in Eq. (26). The histogram showed the range of outcomes into columns along the x-axis and the number of triaxial data points for that range on the Y-axis.

The percentage error distribution for all eleven rock types is in the range of -10 % to 10%, with the maximum

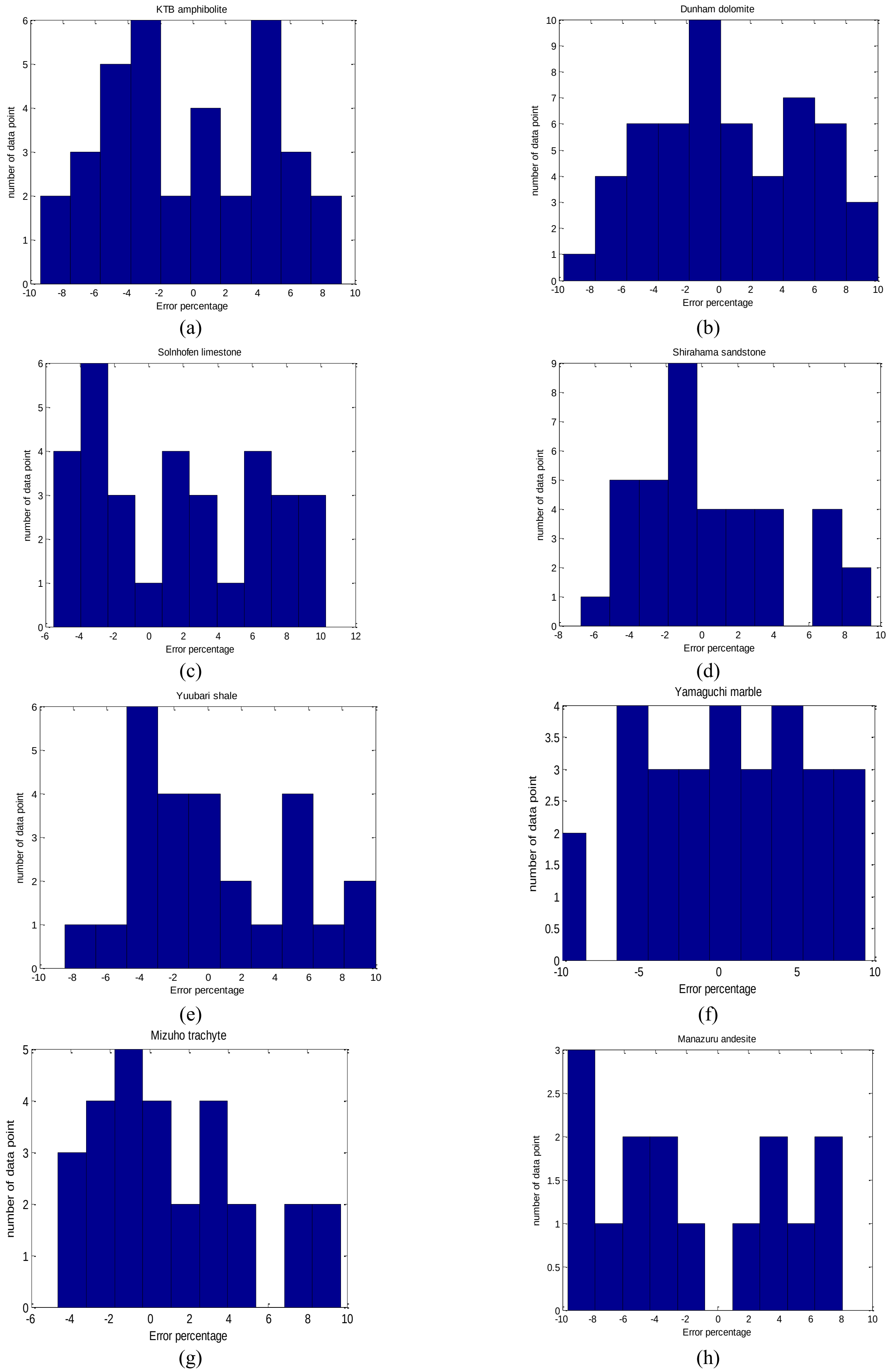
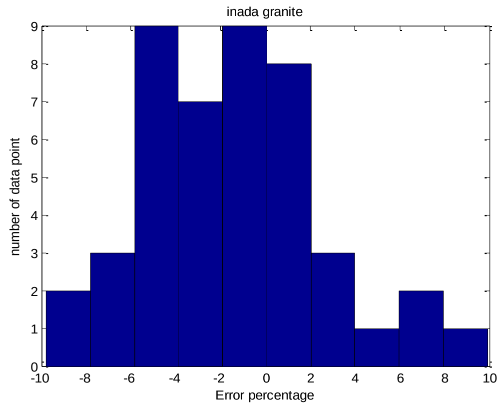
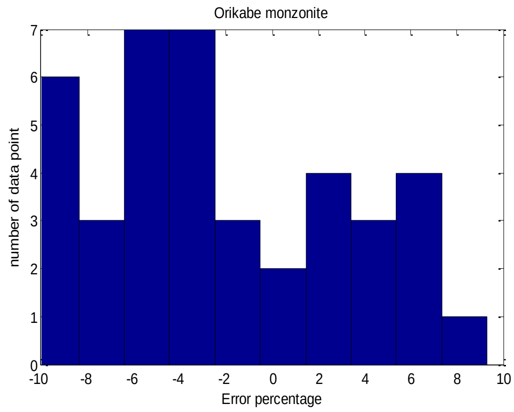


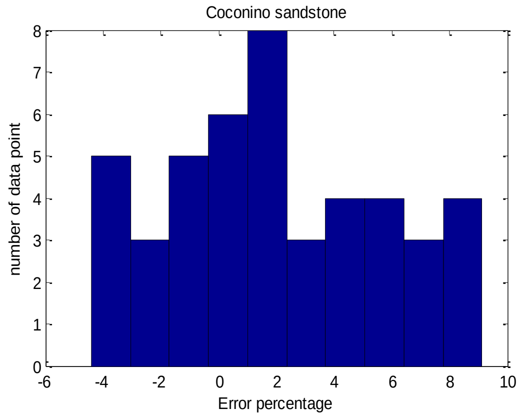
Fig. 6 Distribution of the percentage Error from triaxial compression test for eleven rock types



(i)



(j)



(k)

Fig. 6 Continued-

Table 2 Comparisons of new 3D failure criterion with existing criteria for $\boldsymbol{\rho}$ (MPa)

Rock Type	Number of Data Points	DC New 3D failure Criterion	DC Drucker-Prager criterion	DC Mogi-Coulomb criterion	RMSE The new 3D failure criterion	RMSE Drucker-Prager criterion	RMSE Mogi-Coulomb criterion	RMSE 3D M-C criterion A	RMSE 3D M-C criterion B
KTB amphibolite	42	0.98	0.889	0.97	39.96	73.63	40.64	136.45	121.5
Dunham dolomite	54	0.95	0.7148	0.96	24.34	47.88	16.22	100.03	104.36
Solnhofen limestone	26	0.93	0.6128	0.93	15.27	53.93	14.92	53.92	58.65
Shirahama sandstone	38	0.91	0.7825	0.87	10.76	14.89	11.06	23.93	24.38
Yuubari shale	18	0.90	0.6889	0.92	6.01	10.98	4.72	16.82	18.03
Yamaguchi marble	38	0.92	0.9014	0.95	14.50	18.86	8.36	-	-
Mizuho trachyte	31	0.87	0.6204	0.94	18.04	31.84	9.74	40.04	40.25
Manazuru andesite	23	0.92	0.7961	0.93	30.07	55.10	29.98	91.35	86.18
Inada granite	46	0.98	0.9285	0.97	34.59	61.04	36.16	-	-
Orikabe monzonite	44	0.95	0.8192	0.88	38.23	69.94	55.15	-	-
Coconino sandstone	48	0.92	0.71	0.95	23.04	44.50	17.51	-	-

number of data points distributed near zero percentage error shown in Fig. 6. The distribution of percentage error follows the Gaussian distribution. It signifies that the predicted values are reliable. The distribution of data points for Inada granite is excellent, where eight triaxial data points are between 0% to 2%, nine data points are between 0 % to-2%, three data points are between 2% to 4%, seven data points are -2% to -4%, eight data points are between -4% to -6%, four data points are between 4% to 10% and five data points are -6% to -10%.

5. Comparative study with some existing failure Criteria

5.1 Existing 3D criterion

5.1.1 Drucker- prager criterion

The Drucker-Prager failure criterion is a curvilinear 3D generalized form of the Mohr-Coulomb criterion for soils. Drucker-Prager criterion is the relation between octahedral stress σ_{oct} and octahedral shear stress τ_{oct} through material constant.

Drucker-Prager Criterion (Drucker and Prager 1952) is given by

$$\tau_{oct} = \sqrt{\frac{2}{3}}(3\lambda\sigma_{oct} + k) \tag{28}$$

λ *and* k are material constants. Drucker-Prager Criterion in Haigh-Westergaard coordinates. (ξ, ρ, θ) is given as

$$\rho = \sqrt{2}\left(\sqrt{3}\lambda\xi + k\right) \tag{29}$$

λ *and* k can be expressed in the terms of cohesion intercept and internal friction angle (Yi *et al.* 2005)

$$\lambda = \frac{2\sin\phi}{\sqrt{3}\,(3-\sin\phi)} \qquad k = \frac{6c\cos\phi}{\sqrt{3}\,(3-\sin\phi)} \tag{30}$$

5.1.2 Mogi-Coulomb criterion

Al- Ajami and Zimmerman (2005) proposed a failure criterion by considering the Mogi criterion (1971) and linear Mohr-coulomb failure criterion called the Mogi-Coulomb criterion. The Mogi-Coulomb criterion is the linear relation between octahedral stress (τ_{oct}) and mean effective stress (σ_{m2}).

$$\tau_{oct} = a + b * \sigma_{m2} \tag{31}$$

Here,

$\tau_{oct} = \frac{\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}}{3}$, $\sigma_{m2} = \frac{\sigma_1+\sigma_3}{2}$

Material constant a *and* b is simply related to the angle of internal friction (∅) and cohesion (c) is given by

$a = \frac{2\sqrt{2}}{3}\, c\cos\emptyset$

$b = \frac{2\sqrt{2}}{3}\sin\emptyset$.

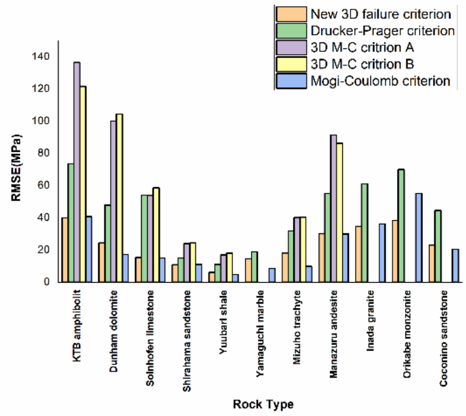
Mogi-coulomb criterion in Haigh-Westergaard coordinates. (ξ, ρ, θ) is given as

$$\rho = \sqrt{3}\,a + \frac{b}{2}(3\,\xi - \sqrt{3}\,b) \tag{32}$$

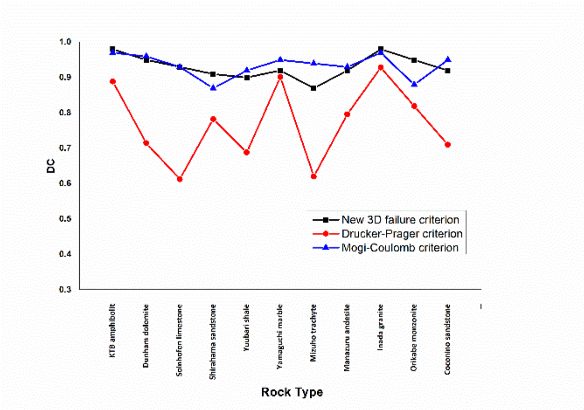
5.1.3 3D M-C criteria A, 3D M-C criteria B

Two 3D M-C criteria formulated by Jiang (2018) in terms of the principal stresses as extended Mohr-Coulomb criterion with the effect of σ2. that criteria established a linear relationship between the octahedral stress τ_{oct} and major principal stress σ_1 mathematical derivation of both criteria is developed in

$$\frac{3}{2}\tau_{oct} = \sigma_t + \left(1 - \frac{\sigma_t}{\sigma_c}\right)\sigma_1 \tag{33}$$



(a)



(b)

Fig. 7 (a) Comparisons of RMSE (MPa) for ρ of 3D failure Criteria (b) Comparisons of DC of the 3D failure criterion with existing criteria for ρ

and

3D M-C criteria A

$$\frac{3}{2}\tau_{oct} = \sigma_c + \left(\frac{\sigma_c}{\sigma_t} - 1\right)\sigma_1 \tag{34}$$

maximum horizontal stress based on the above criteria at the failure plane is given as

$\sigma_1 = \frac{a_1+\sigma_3}{1-b_1}$ by 3D M-C criteria A and, $\sigma_1 = a_2 + (b_2+1)\,\sigma_3$ for 3D M-C criteria B. value for the variable a_1, b_1, a_2, b_2 is taken from Jiang (2018) for the best fit on the triaxial data.

5.2 Comparison between existing and proposed 3D Criteria

The comparison is based on DC and RMSE values of ρ (MPa) for eleven rock types triaxial data points. The exact predicted value of ρ (MPa) is also done by Drucker-Prager Criterion and Mogi-Coulomb criterion. Fig. 7(a) compares the RMSE of the new failure criterion with the Drucker-Prager Criterion, 3D M-C criteria-A, 3D M-C criteria-B, and Mogi-Coulomb criterion. The value for DC and RMSE is listed in Table 2, in Haigh - Westergaard coordinates (ξ, ρ, θ). The values of ρ are estimated using Eqs. (25) and

(28). The RMSE values are higher in the case of the 3D M-C criteria-A and 3D M-C criteria-B as compared to the other three criteria. It is also observed that the RMSE of the proposed 3D linear and Mogi-Coulomb criterion are comparatively better than Drucker-Prager Criterion. Our proposed 3D failure criterion shows equivalent RMSE with the Mogi-Coulomb criterion. The maximum value of RMSE is 39.96 MPa for a KTB, and the minimum value of RMSE is 6.07 MPa for a Yuubari shale by the new 3D criterion. For Drucker-Prager Criterion and Mogi-Coulomb criterion, the maximum RMSE is 73.63 MPa (KTB amphibolite) and 55.15 MPa (Orikabe monzonite), and the minimum RMSE is 10.98 MPa (Yuubari shale) and 4.42 MPa (Yuubari shale) respectively. RMSE value in MPa for all rock types is listed in Table 2, for all five criteria. It signifies that our proposed 3D failure criterion gives equivalent results to the Mogi-Coulomb criterion and is better than the remaining criteria.

The range of DC is between 0 to 1, and the preferable value of DC is near one. DC value for an Inada granite is the maximum among all rock types in all three failure criteria shown in fig 7 (B). DC value for an Inada granite is 0.98, 0.9285, and 0.97 in the case of the new 3D criterion, Drucker-Prager Criterion, and Mogi-Coulomb criterion, respectively. DC value for the KTB amphibolite is also suitable as 0.98, 0.889, and 0.97 in the case of the new 3D criterion, Drucker-Prager Criterion, and Mogi-Coulomb criterion, respectively. The new 3D criterion has shown better DC than the Mogi-Coulomb criterion for KTB amphibolite, Shirahama sandstone, Inada granite, and Orikabe monzonite. Similarly, our proposed criterion also shows equivalent DC values with Mogi-Coulomb for Yuubari shale, Yamaguchi marble, Mizuho trachyte, Coconino sandstone, Dunham dolomite, Solnhofen limestone, and Manazuru andesite.

The comparison of the new 3D linear criterion with existing criteria gives better visualization in the deviatoric plane as given in Fig. 8. The Drucker-Prager failure criterion in the deviatoric plane is given as a black circle. the Mogi-Coulomb criterion is given by a blue dashed line and the Mohr-Coulomb criteria is given as a maroon solid line. The new 3D criteria are developed based on the effect of the (P=0.2) on the intermediated principal stress on rock strength given by the dark blue solid line. Based on the theoretical derivation of the new criteria develop the effect of p on rock strength is directly visible. The shear surface for the new 3D criteria covers the smallest volume which indicates the new 3D criteria is the safest among these criteria for the prediction of the failure in three-dimensional principal. Extended Mohr-Coulomb criteria for rocks based on smooth approximations failure functions by Lee *et al.* (2012) varying in the deviatoric plane from the Mohr-Coulomb criterion to the Drucker-Prager criterion based on the effect of smoothness.

From the new failure criterion and the Mogi-Coulomb failure criterion, it has been observed that a new failure criterion is easy to implement for predicting failure stress if another two stresses are known. However, in the Mogi-Coulomb criterion, the prediction of failure stress is not as simple as a new failure criterion because of complicated relationships among the principal stress.

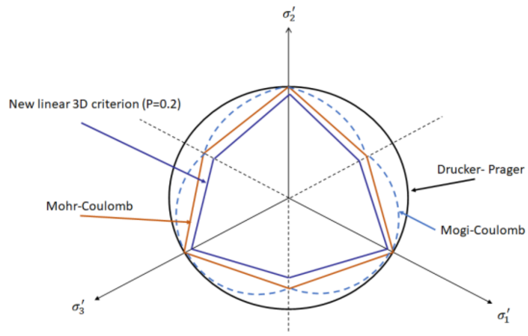


Fig. 8 Comparison of the new 3D linear criterion with existing criteria on the deviatoric plane

6. Conclusions

This study proposes a new 3D failure criterion by considering the role of intermediate principal stress (σ_2). A new 3D failure criterion is developed by considering the experimental evidence of the effect of the intermediated principal stress on the failure of the rock as an extended Mohr-Coulomb criterion that is also easy to implement on the failure of in-situ rock formation.

The proposed new 3D failure criterion stimulates the Mohr-Coulomb failure criterion on 3D principal stress space ($\sigma_1, \sigma_2, \sigma_3$). The effect of intermediate principal stress on the rock strength is considered as a weighting (P) of σ_2 on the mean effective stress. If the influence of σ_2 is not considered (p=0), a new 3D failure criterion reduces to the Mohr-Coulomb failure criterion. The yield surface for the 2D stresses suggests that a new criterion is the safest among the existing criteria. The triaxial compression test data for eleven rock types are taken from the literature for calibration of material constant and validation of failure prediction. The accuracy of a new failure criterion depends on the quality of the triaxial compression test data. The new 3D criteria on the deviatoric cover the smallest area that indicates the safest criterion for prevention failure. The shear surface for a new 3D criterion covers the smallest volume compared to the existing criteria.

The Drucker- Prager, the Mogi-coulomb failure criteria, 3D M-C criteria-A, and 3D M-C criteria -B were also performed on the triaxial compression test dataset to understand the failure phenomenon. This study found that the Mogi-Coulomb and the new failure criteria show a good value of DC and RMSE for all rock types. The significance of the new failure criterion is easy to implement for predicting the failure stress if another two stresses are known. However, the prediction of the failure stress from the Mogi-Coulomb failure criterion is difficult as the complicated correlation among the principal stresses. The solution of the Mogi-Coulomb failure criterion gave two results in solving the complex equation. The selection of one of the results was difficult for wellbore stability and sand production problems. The solution of the proposed 3D failure criterion always gives unique results and can quickly be implemented for wellbore stability and sand production problems. Although the Mogi-coulomb and proposed 3D criterion show equivalent results, we recommend our proposed 3D failure criterion as it is easier to implement. This extended Mohr-Coulomb criterion is developed as linear relation of all three principles stresses, and the variable of this criteria is also derived from the internal friction angle and cohesion (c). Other criteria will be developed as a nonlinear criterion, whose parameter is also derived easily with the best fitting with the experimental result and easy to be implied for the failure of in-situ rock formation.

Funding sources

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Author contribution

Mohatsim mehtaji: Conceptualization, Writing - original Draft, Methodology, Software, Validation and Visualization. **Jwngsar Brahma:** Formal analysis, Writing - Review & Editing, Supervision, **Rakesh Kumar Vij:** Supervision.

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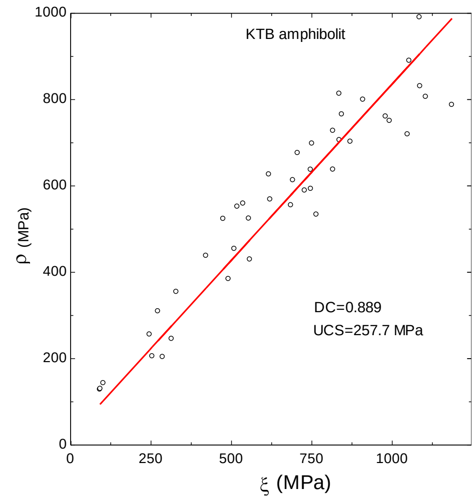
IC

APPENDIX I

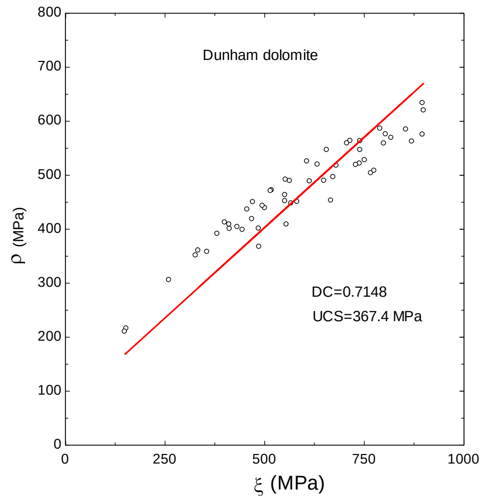
Failure prediction for eleven rock type by Drucker-Prager Criterion

Rad solid line = Drucker-Prager Criterion

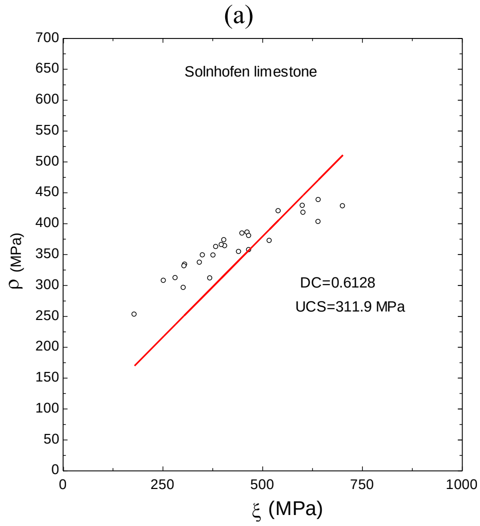
Black scatter data = Triaxial Compression test data



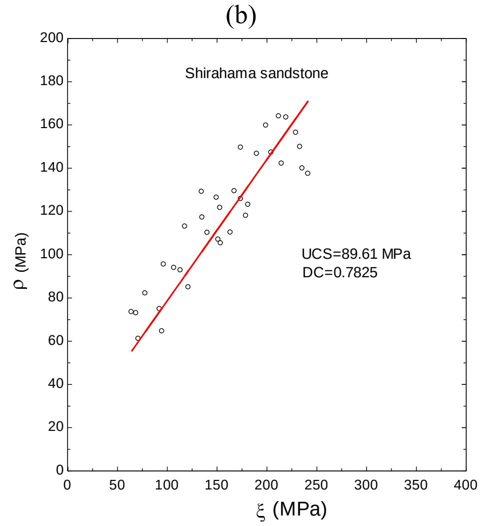
(a)



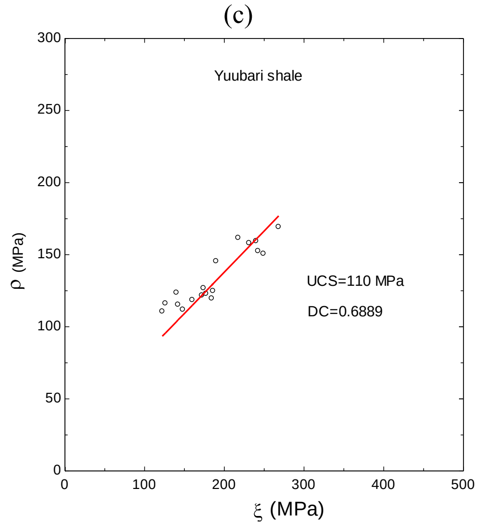
(b)



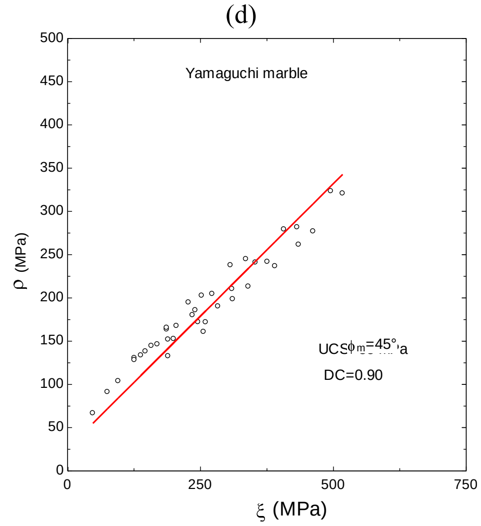
(c)



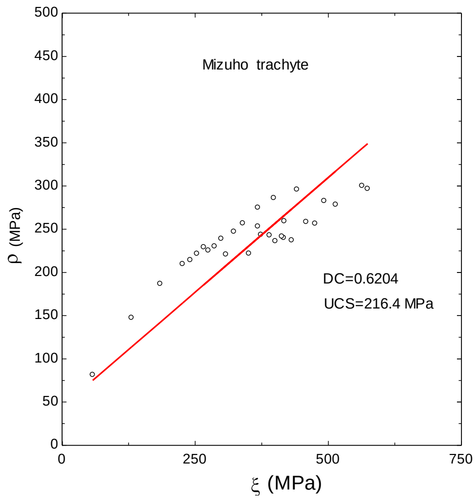
(d)



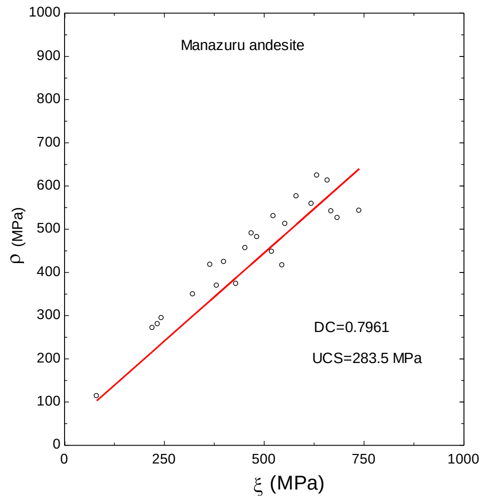
(e)



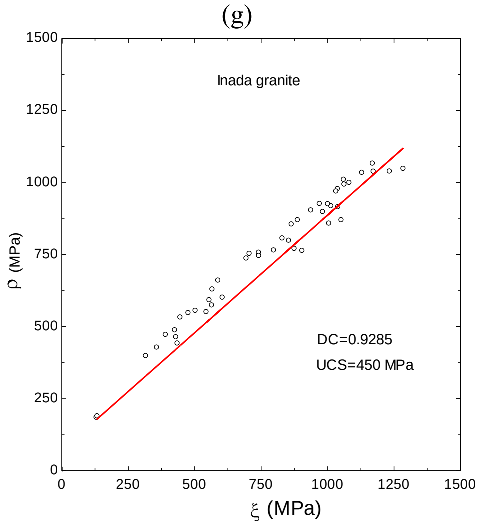
(f)


Mizuho trachyte
DC=0.6204
UCS=216.4 MPa
ρ (MPa)
ξ (MPa)
0
50
100
150
200
250
300
350
400
450
500
0
250
500
750

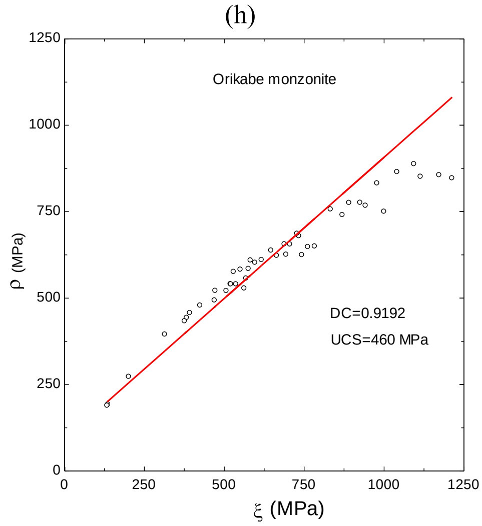
(g)


Manazuru andesite
DC=0.7961
UCS=283.5 MPa
ρ (MPa)
ξ (MPa)
0
100
200
300
400
500
600
700
800
900
1000
0
250
500
750
1000

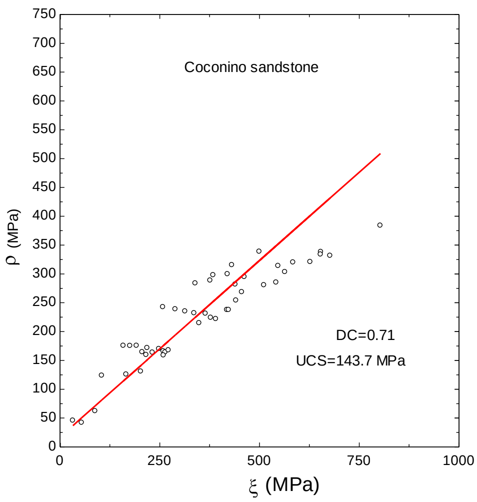
(h)


Inada granite
DC=0.9285
UCS=450 MPa
ρ (MPa)
ξ (MPa)
0
250
500
750
1000
1250
1500
0
250
500
750
1000
1250
1500

(i)


Orikabe monzonite
DC=0.9192
UCS=460 MPa
ρ (MPa)
ξ (MPa)
0
250
500
750
1000
1250
0
250
500
750
1000
1250

(j)


Coconino sandstone
DC=0.71
UCS=143.7 MPa
ρ (MPa)
ξ (MPa)
0
50
100
150
200
250
300
350
400
450
500
550
600
650
700
750
0
250
500
750
1000

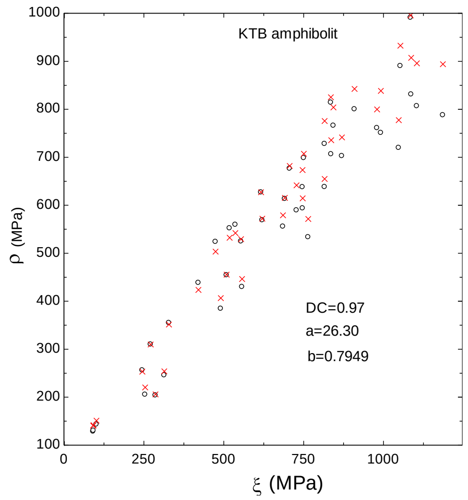
(k)

APPENDIX II

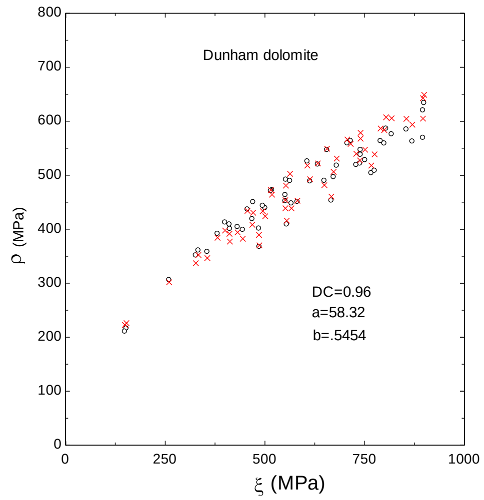
Failure prediction for eleven rock types by Mogi-Coulomb criterion.

Rad scatter data = Mogi-coulomb Criterion

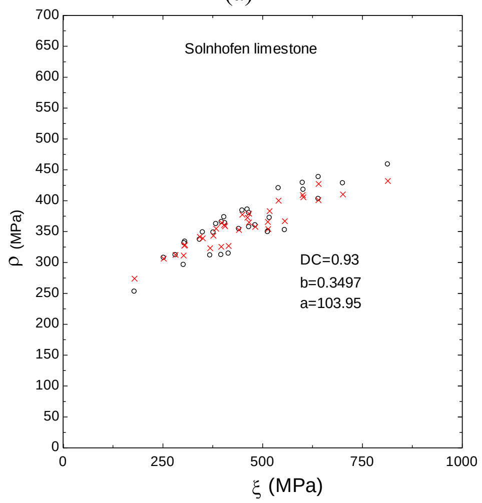
Black scatter data = Triaxial Compression test data



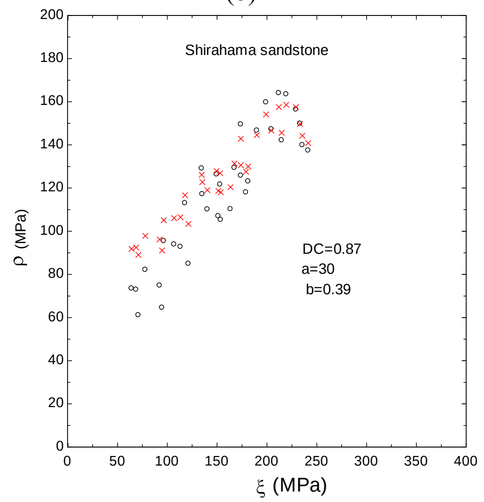
(a)



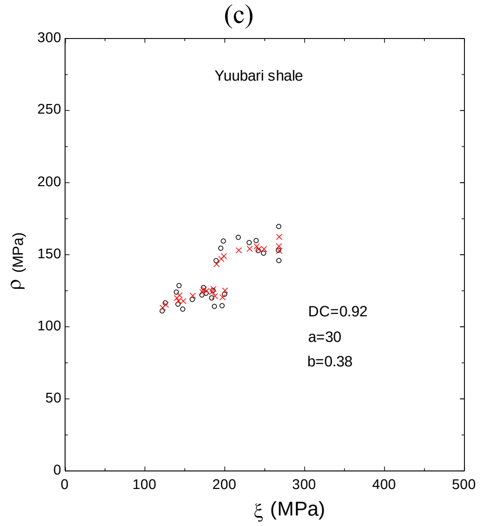
(b)



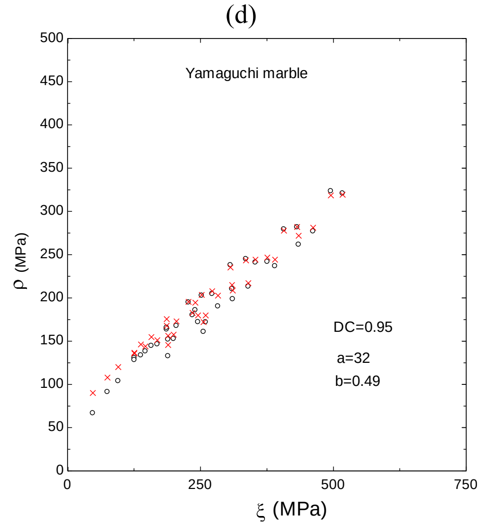
(c)



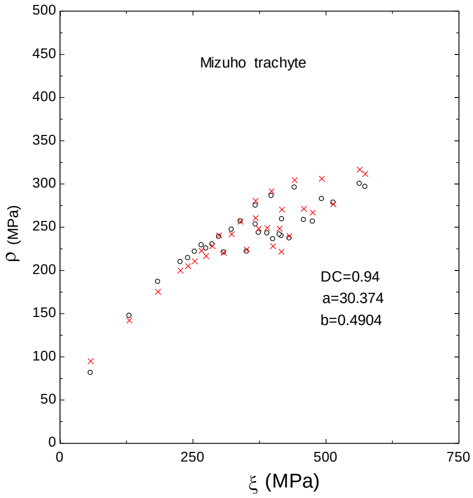
(d)



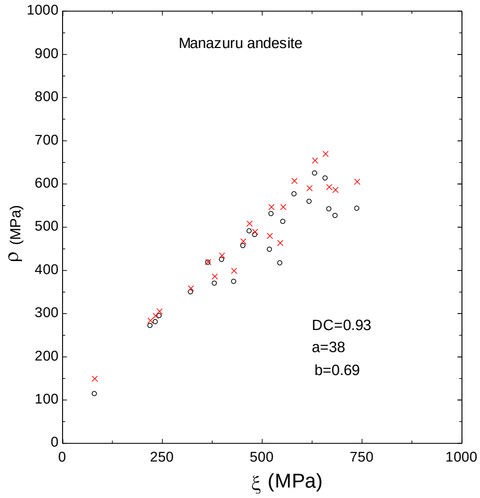
(e)



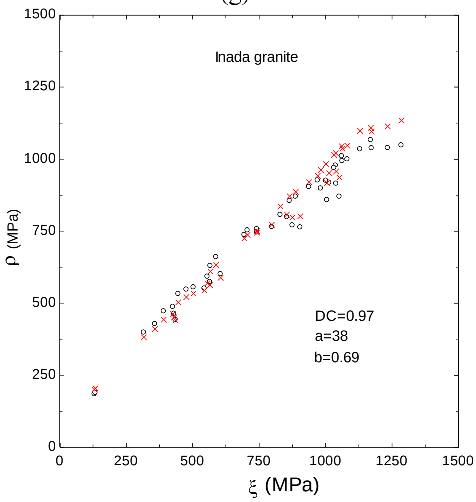
(f)


Mizuho trachyte
DC=0.94
a=30.374
b=0.4904
ρ (MPa)
ξ (MPa)

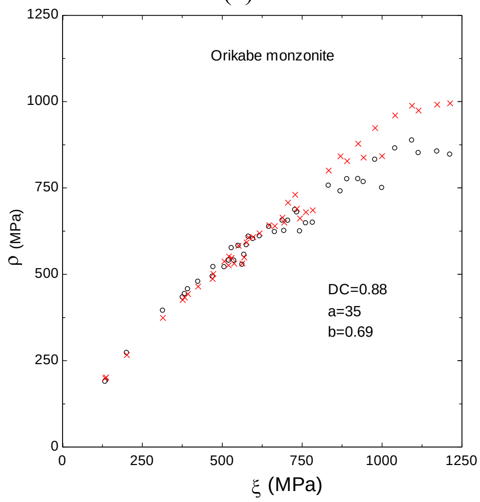
(g)


Manazuru andesite
DC=0.93
a=38
b=0.69
ρ (MPa)
ξ (MPa)

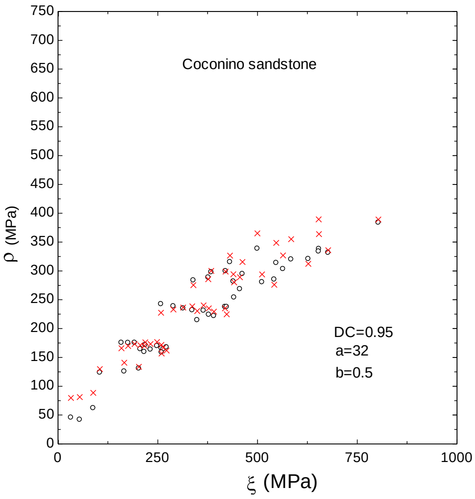
(h)


Inada granite
DC=0.97
a=38
b=0.69
ρ (MPa)
ξ (MPa)

(i)


Orikabe monzonite
DC=0.88
a=35
b=0.69
ρ (MPa)
ξ (MPa)

(j)


Coconino sandstone
DC=0.95
a=32
b=0.5
ρ (MPa)
ξ (MPa)

(k)

List of symbols

σ_1, σ_2 σ_3	Principal stresses
ξ, ρ, θ	Haigh-Westergaard coordinates.
J_2	Second invariants of the deviatoric stress tensor.
τ_{oct}	The octahedral shear stress.
σ_{oct}	The octahedral normal stress.
UCS	uniaxial compressive strength
CS	Compressive strength for a new criterion
c	The cohesion of the rock material
ϕ	The angle of internal friction
P	The weighting σ_2
RMSE	The root-mean-square-error
q_1, q_2	The material constant of intact rock
r_i	Prediction error residuals of i^{th} test
DC	Coefficient of determination.
ε_i	Error percentage
σ_{m2}	mean effective stress
λ, k	Material constant for Drucker-Prager Criterion
K	The ratio of the tensile to the compressive meridian radius
a, b	Material constant for Mogi-coulomb criterion