Geomechanics and Engineering, *Vol. 31, No. 4 (2022) 385-393*
https://doi.org/10.12989/gae.2022.31.4.385

Numerical study of strength reduction-induced capillary rise effect for unsaturated soil

Bestun J. Shwan*

Department of Geotechnical Engineering, Faculty of Engineering, Koya University, Koya KOY45, Kurdistan Region- F.R. Iraq

(*Received July 4, 2022, Revised November 6, 2022, Accepted November 14, 2022*)

Abstract. Previous studies postulated insignificant capillary rise (h_c) effect above the water table (H_w) for unsaturated soils. In addition, these studies utilised dry unit weight above H_w. This paper, therefore, addresses the effect of these postulations on strength where the influence of h_c using a modified upper bound approach, Discontinuity Layout Optimization (UNSAT-DLO) for a simulated soil was predicted. Two different parametric studies to model passive earth pressure and bearing capacity problems are carried out to provide an insight into the effect of capillary rise on strength. Significant increase in strength, owing to unsaturated conditions, was obtained where the maximum increase was when suction slightly less or greater than the air entry suction. On the other hand, the results showed a negative effect of h_c. For example, up to 8.24% decrease in passive thrust (P_p) was obtained at H_w=0 m when h_c rose 1 m from 0 m. To put this into perspective, this was equivalent to a decrease of about 2° in ϕ at H_w=0 m and h_c =0 m in order to obtain the same result at h_c =1 m. For the bearing capacity problem, the effect was seen to be higher, up to 18.4% decrease in N_γ was obtained when h_c rose from 0 m to 2.5 m at H_w=0 m. In addition, the results revealed a negative influence of assigning dry unit weight above H_w or h_c.. However, considerable increase in strength was obtained when unsaturated unit weight above h_c was assigned.

Keywords: bearing capacity; capillary rise; passive earth pressure; unsaturated soil; upper bound

1. Introduction

The behaviour of unsaturated soils is significantly affected by matric suction, Tarantino (2007), Rahardjo *et al.* (2019) and Lu (2020), which is a consequence of capillarity phenomenon and surface adsorption effects, Murray and Sivakumar (2010) and Lu (2016). Capillarity phenomenon, the ability of water to move up in a small space, e.g., thin tubes, without the resistance of any external forces, is related to the surface tension. Surface tension (T) in a thin tube with a diameter of d balances the capillary rise height (h_c) and can be expressed as

$$h_c d\gamma_w = 4T\cos\theta \quad (1)$$

where γ_w is the unit weight of water (kN/m³) and θ is the contact angle (°) where capillary occurs when θ is less than 90°. In Eq. (1); both T (=72 mN/m for water at 25°C) and θ (=0 for pure water and clean glass) can be constant, therefore, h_c depends only on d. Assumption on the determination of h_c by several scholars based on a gradation parameter (i.e., D_{10} , is the particle size corresponding to 10% passing, equivalent to d) or empirical equations was, therefore, verified. In soils, continuity of voids, however, is not ideal as in the capillary thin tube, d in Eq. (1). This is because of variable cross sections of soils. Hazen (1930) proposed an empirical equation based on D_{10} as

$$h_c = \frac{C}{e \times D_{10}} \quad (2)$$

where e is void ratio, and C is a constant (10 mm² to 50 mm²). On the basis of experimental tests of different types of soil, Lane and Washburn (1946) also proposed an equation based on D_{10} as

$$h_c = -990\ \ln(D_{10}) - 1540 \quad (3)$$

Additionally, based on soil types, h_c was also estimated. For example, Das (2010) listed various values of h_c for different types of soil as shown in Table 1.

It is fairly accepted that h_c depends on unsaturated parameters. Soil water retention curve (SWRC), a fundamental property of unsaturated soils, is influenced by several factors such as particle structures and void size distribution, Fredlund and Rahardjo (1993), Zhai *et al.* (2020) and Qian *et al.* (2022). The SWRC determination was also based on the particle size distribution curve (PSD). This was by matching PSDs (did not have SWRCs) to approximately similar PSDs for soils having SWRCs, e.g., the work of Öberg and Sällfors (1997) and Otalvaro et al. (2016). The attempt led to satisfactory results. This explicitly linked PSD to SWRC. While SWRC is considerably affected by air entry suction (s_o), the relation between PSD and s_o should be taken, therefore, into consideration. That is, the higher s_o, the finer the soil is, under null loading conditions, without hydraulic and mechanical hystereses. As the particle size affects h_c, a direct relationship between unsaturated parameters with h_c

*Corresponding author, Assistant Professor
E-mail: bestun.shwan@koyauniversity.org

http://www.techno-press.org/?journal=gae&subpage=7
ISSN: 2005-307X (Print), 2092-6219 (Online)

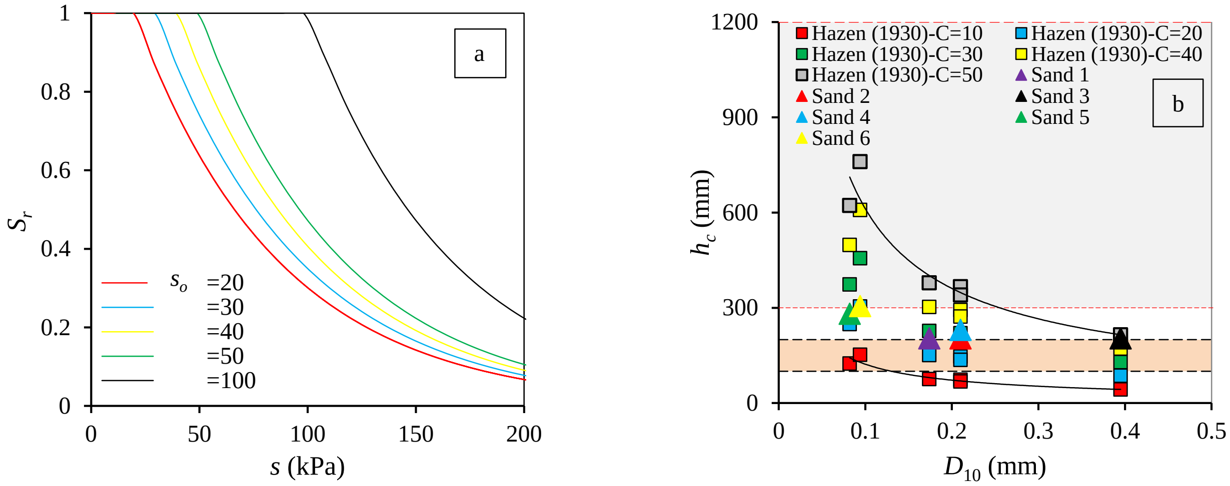

Fig. 1 (a) SWRCs for various s_o values and (b) Estimation of h_c for 6 sands using Eqs. (2)-(4)

Table 1 Typical values of capillary rise height, h_c, for different types of soil, Das (2010).

Soil Type	h_c, (mm)
Coarse sand	100-200
Fine sand	300-1200
Silt	750-7500
Clay	7500-23000

was, then, inevitable. Shwan (2015) proposed, therefore, a relationship between s_o and h_c in a simple form as follows

$$h_c = \frac{s_o}{\gamma_w} \tag{4}$$

Figure 1a plots different SWRCs for various s_o values, using Eq. (7) (discussed later). The SWRCs in Fig. 1(a) exhibited a significant influence of s_o on the capillarity range and therefore on h_c in Eq. (4). The smaller the s_o value, the larger the particle size range and therefore the smaller h_c. Implicitly, Eq. (4) links h_c to the pore sizes. In Fig. 1(a), full saturation was assumed for any suction (s) less than s_o. The assumption of full saturation for $s \leq s_o$ also suggested by Wheeler *et al*. (2003) and Lloret *et al*. (2018).

Eq. (4) was used to determine h_c, based on experimental data, for six fine sands and then validated against Eq. (2). Air entry values for the sands were found from the SWRCs [obtained using the hanging column technique (HCT)]. The validation is shown in Fig. 1(b). The upper and lower bounds presented in Table 1 for fine and coarse sands are also plotted in Fig. 1b as shaded areas. The smaller D_{10} value, the higher h_c obtained. Scatter in obtained h_c values based on Eq. (2) was obvious which attributed to the wide range of the parameter C (from 10 mm^2 to 50 mm^2). However, the obtained h_c values using Eq. (4) were reasonably well defined within the obtained h_c values using Eq. (2) especially at low C (i.e., 10 and 20 mm^2), suggesting a better match of Eq. (4) over Eq. (2) at high values of C.

Classical soil mechanics assumed dry conditions above the water table [Blight (2013)], dismissing the capillary rise effect, or did not take into consideration the effect of unsaturated parameters above the capillary rise, e.g., Eqs. (2) and (3). Only a limited number of studies in the literature consider these issues for unsaturated geotechnical problems. This paper, therefore, investigates the combined effects of capillary rise above the water table and suction above h_c using a modified upper bound approach, Discontinuity Layout Optimization (**UNSAT-DLO**). Parametric studies to model passive earth pressure (**PEP**) and bearing capacity (**BC**-for a surface strip footing) problems are carried out to provide an insight into the effect of capillary rise and suction on strength for both problems.

2. Material and numerical modification

The simulated soil represented sand or silt as it reached its residual suction at low suction less than 200 kPa (Vanapalli *et al*. 1996) see Fig. 2. The simulated soil had an air-entry suction of 20 kPa. The full saturation was achieved for any suction (s) values less than 20 kPa, however, a significant reduction in degree of saturation, S_r, can be seen due to desaturation beyond the air-entry suction.

As stated earlier; a modified upper bound approach, Discontinuity Layout Optimization (**UNSAT-DLO**) was utilised to carry out the two parametric studies. The approach, based on evenly spaced nodes, was first introduced by Smith and Gilbert (2007). The method can determine the failure load for any geometry of stability problems. The modifications involved inclusion the effect of unsaturated conditions on strength [omitted for brevity, see Shwan (2015) and Shwan (2022)] as follows

$$\min \ \lambda \mathbf{f}_L^T \boldsymbol{d} = -\mathbf{f}_D^T \boldsymbol{d} + \mathbf{g}^T p + U_i n_i \tag{5}$$

$$\hat{C} = U \tan\phi = -\int_0^L s S_r \ \tan\phi . dl \tag{6}$$

where λ is the minimum adequacy factor, $\mathbf{f}_L^T$ and $\mathbf{f}_D^T$

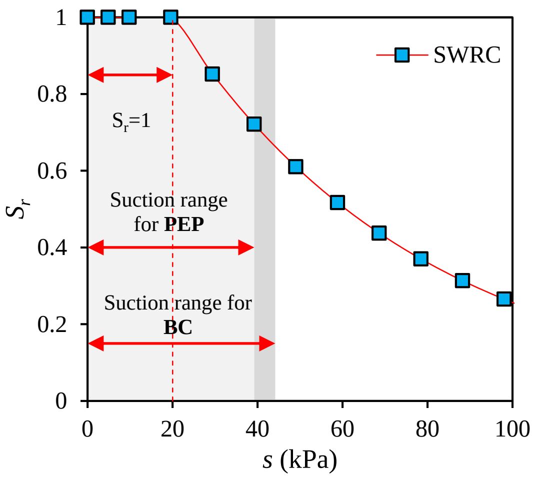

Fig. 2 SWRC for the simulated soil for the **PEP** and **BC** problems

are live and dead loads, respectively, ***d*** is a matrix that contains displacement vectors, $\mathbf{g}^T$ is corresponding dissipation coefficients [cohesion (c)×length of a discontinuity(l)], p is a vector containing plastic multipliers, n_i is the normal displacement accompanying the sliding, $\hat{C}$ is the apparent cohesion (gain in strength due to unsaturated conditions), ϕ is the angle of shearing resistance, S_r is the degree of saturation and L is the length of a discontinuity. It is clear that U accounts for the shear strength increase due to the effect of suction. The adequacy factor (λ) is a factor by which the loads should be increased or the material strength should be decreased in order the problem reaches collapse.

Degree of saturation and suction equations are given by (after Shwan and Smith (2014))

$$S_r = e^{-a(s-s_o)} \tag{7}$$

$$s = \gamma_w (H_w - z) \tag{8}$$

where (a) is a fitting parameter (kPa^{-1}) and H_w is the water table depth (positive downward) (m), z is the vertical coordinate (positive downward) (m).

Despite a lack in experimental data, it was necessary to validate the **UNSAT-DLO** approach against the available results and equations in the literature. Therefore, the approach was validated against the experimental bearing capacity data obtained by Vanapalli and Mohamed (2007) and the Fredlund and Rahardjo (1993) equation for a passive earth pressure problem. Table 2 shows the available parameters by Vanapalli and Mohamed (2007) and the other obtained required parameters in the analysis. The obtained parameters were s_o, a and saturated unit weight (assuming Gs=2.65 for the tested sand). The parameter a was obtained by fitting the experimental SWRC [available by Vanapalli and Mohamed (2007)] data using Eq. (7). The angle of shearing resistance of 39° was used [increased by a factor of 1.1 fold as suggested by Vanapalli and Mohamed (2007)]. This increase in ϕ was due to, as suggested by Vanapalli and Mohamed (2007), dilation and the influence of suction. However, it is fairly accepted that due to aggregation and fabric changes at unsaturated conditions, cohesion also increases, evidence from the literature are those provided by Röhm and Vilar (1996), Kim *et al.* (2011) and Shwan and Smith (2015). Therefore, two scenarios of changing c=5 and 6 kPa at ϕ =39° were assumed in the **UNSAT-DLO** analysis. However, for the fully saturated conditions where the effect of suction eliminates, c=0 kPa and ϕ =39° was used in the analysis. For the **PEP** problem, arbitrarily selected parameters were used in the analysis as shown in Table 2.

Vanapalli and Mohamed (2007) also proposed an equation to determine unsaturated ultimate bearing capacity (q_u) as follows

$$q_u = \begin{bmatrix} c' + (u_a - u_w)_b (1 - S^{\psi}) \tan\phi' \\ + (u_a - u_w)_{AVR} S^{\psi} \tan\phi' \end{bmatrix} \times N_c \; [1.0 + (\frac{N_q}{N_c})(\frac{B}{L})] + 0.5 B \, \gamma \, N_\gamma [1.0 - 0.4(\frac{B}{L})] \tag{9}$$

where c' is the effective cohesion, $(u_a - u_w)_b$ is the air entry suction, S^{ψ} is the degree of saturation and ψ is the bearing capacity fitting parameter, $(u_a - u_w)_{AVR}$ is the average suction directly beneath the footing and to the end of stress bulb, ϕ' is the effective angle of shearing resistance, B and L are footing's dimensions and N_c, N_q and N_γ are the bearing capacity factors.

The Fredlund and Rahardjo (1993) equation for the passive thrust (P_p) is given by

$$P_p = \left[\frac{1}{2} k_p \gamma H^2 + 2c' \sqrt{k_p} H + 2(u_a - u_w) \tan\phi^b \sqrt{k_p} \; H \right] \tag{10}$$

where k_p is the passive earth pressure coefficient, γ is the soil unit weight, H is the height of the wall, $(u_a - u_w)$ is the matric suction and ϕ^b is the angle of friction with respect to suction. The average unit weight above the water table was assumed for both problems. For any $s \le s_o$, the saturated unit weight was used. For the passive thrust problem, ϕ^b =0.25ϕ, ϕ^b =0.5ϕ and ϕ^b =0.75ϕ were used.

Figs. 3(a) and 3(b) the numerical results for both problems. The agreement of the **UNSAT-DLO** approach

Table 2 Used parameters for the **BC** and **PEP** problems

γ_{dry} (kPa)	γ_{sat} (kPa)	γ_{unsat} (kPa)	ϕ (degrees)	c (kPa)	a (kPa^1)	s_o (kPa)	h_c (m)
BC problem							
16.02	19.74	17.88	39	0	0.285	3.5	0
PEP problem							
15	20	17.5	30	0	0.04	10	1.019

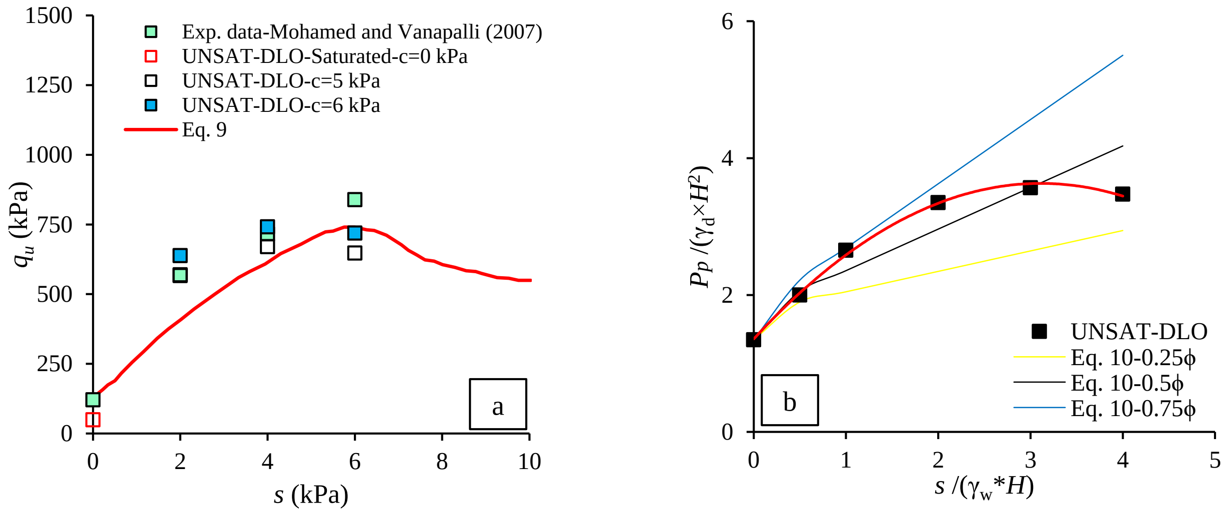

Fig. 3 The **UNSAT-DLO** approach validation against the (a) **BC** and (b) **PEP** problems

Table 3 Soil properties and unsaturated parameters for the simulated soil

γ_{dry} (kPa)	γ_{sat} (kPa)	γ_{unsat} (kPa)	ϕ (degrees)	c (kPa)	a (kPa^{-1})	s_o (kPa)
15	20	17.5	30	0	0.04	20

against the bearing capacity data in Fig. 3(a) was satisfactory. In addition, reasonable agreement was obtained against the Fredlund and Rahardjo (1993) equation. It is clear also that the **UNSAT-DLO** results showed non-linear trend induced desaturation behaviour for both problems. Following the validation, parametric studies were, therefore, carried out, discussed in the next section.

3. Problem geometry

The problem domains for both cases are depicted schematically in Figs. 4(a) and 4(b). The x-boundary was extended to prevent restriction of the slip line mechanism. The analysis was carried out under drained conditions. A frictionless wall of 1m height and a plane strain surface strip footing of 1 m width were used in the simulation. For the wall, the results were normalised over the wall's height, therefore, selecting an arbitrary wall height (H=1 m) was not an issue. Due to the symmetric condition, it was only required to model only half of the footing.

Suction was applied by changing the position of the water table (H_w). Various values of h_c smaller and higher than 2.03 m, obtained using Eq. (4) for s_o =20 kPa, were used. This was to investigate the influence of different h_c values on strength as well as to cover the scatter in Fig. 1(b).

For the **PEP** problem, it was not necessary to model h_c > 1 m as the wall's height was assigned to 1 m. Fig. 4(c) shows the modelled scenarios for h_c and unit weight above h_c or H_w. The modelled suction ranges were 0-39.24 kPa (H_w=1 m-H_w=-3 m) and 0-44.14 kPa (H_w=2.5 m-H_w=-2 m) for the wall and the footing, respectively (see shaded areas in Fig. 2) which included the fully saturated conditions as well as the transition zone of the SWRC. Table 3 shows the used properties and the required unsaturated parameters for the simulated soil in the analysis.

In the analysis; baseline nodal spacings of 0.04m and 0.05 m (high numbers of nodes along the slip line) were assigned for the wall and the footing, respectively. Prior to the modelling, the **UNSAT-DLO** approach was also validated at both fully saturated and fully dry conditions where the exact solutions were obtained for both cases against the original version of the DLO method.

4. Numerical results

Figs. 5(a) and 5(b) show the normalised results. For a surface footing, N_γ was obtained using the equation below

$$N_\gamma = \frac{q_u}{0.5 \times \gamma \times B} \tag{11}$$

where q_u is the ultimate bearing capacity, γ is the soil unit weight, used according to the saturation conditions: 15, 17.5 and 10.19 (buoyant unit weight) kN/m^3 for the dry, unsaturated and saturated conditions, respectively and B is the footing's width.

Generally, for both problems, the influence of unsaturated conditions was significant, see Figs. 5(a) and 5(b). For example, an increase of 3.86 fold in the normalised P_p was obtained when comparing H_w =-2 m with H_w =1 (full saturation) at h_c =0 m. The overall trend, then, decreased towards the full saturation (H_w/H=1 and H_w/B=2.5 in Figs. 5(a) and 5(b), respectively). The maximum P_p and N_γ were obtained when suction was slightly less or greater than s_o (<20 kPa) for the **PEP** and **BC** problems, respectively. In addition, there was no need to model cases of H_w < -3 m and -2 m as the trend decreased

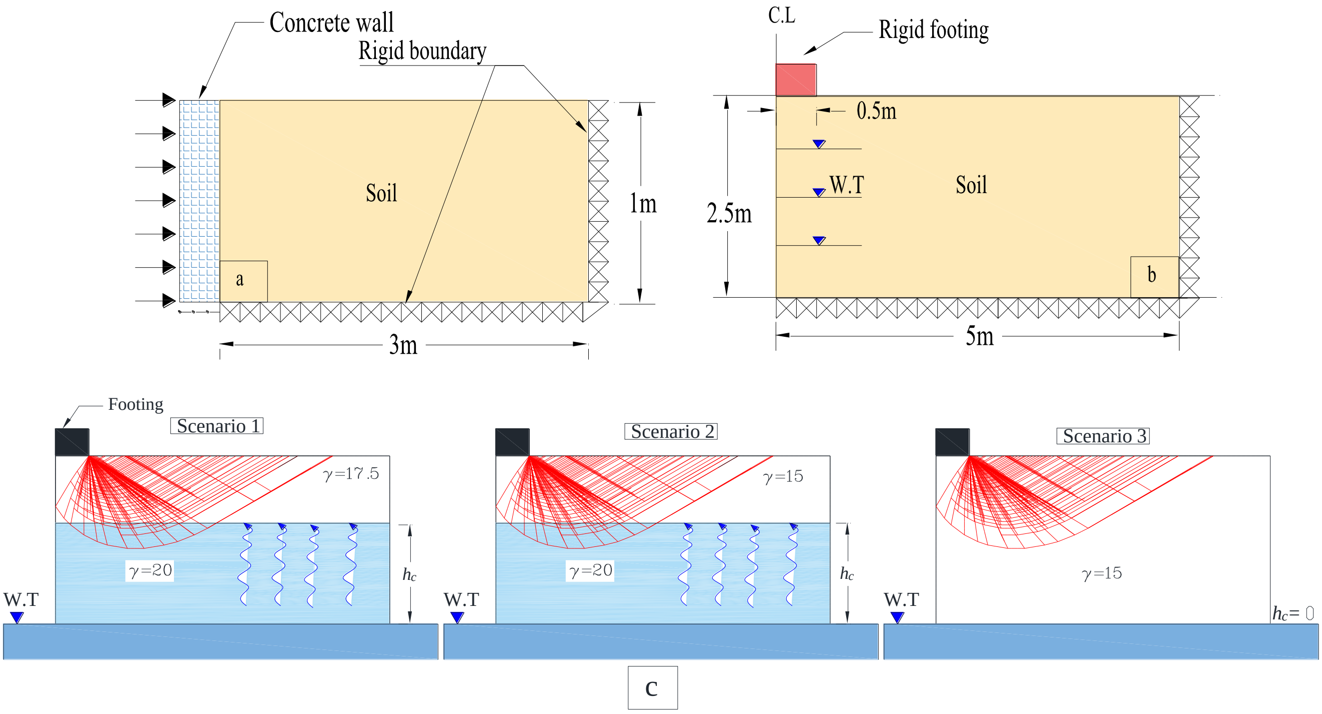

Fig. 4 Problem domain for the (a) **PEP**, and (b) **BC** problem and (c) Modelled scenarios above the water table

beyond these water table heights. The decrease was attributed to desaturation; water cannot transmit suction to the aggregate particles.

On the other hand, the effect of capillary rise above the water table clearly showed a negative influence. The higher the h_c, the lower the P_p and N_γ obtained. Figs. 5(a) and 5(b) also exhibit the maximum obtained difference in strength (in %) between the highest and lowest h_c at H_w=0 m as well as at the fully saturated case. A decrease of about 8.24% in P_p was obtained at H_w=0 m when h_c rose 1 m from h_c = 0 m. To put this into perspective, this was equivalent to a decrease of about 1.92° in ϕ if H_w=0 m was modelled at h_c =0 m. Near or at full saturation, the effect of h_c was negligible, see Fig. 5(a) at H_w/H=0.5. Figs. 5(a) and 5(b) implied the effect of the earlier assumption of dismissing the effect of h_c above H_w which led to strength overestimation.

5. Unit weight effect above capillary height level

The second postulation in previous studies was by assuming dry unit weight above h_c. Owing to the capillary rise phenomenon, soil above h_c is not fully dry. This led to a conservative design of the geotechnical applications. To investigate the influence of this assumption, it was required to model two more scenarios: 2 and 3 in Fig. 4(c). Scenario 2 comprised the effect of h_c but assumed dry unit weight above h_c (γ_{dry} = 15 kN/m^3), while scenario 3 did not take into account neither h_c (=0 m) nor the unsaturated unit weight above h_c (assumed γ_{dry} = 15 kN/m^3).

Figs. 6(a) and 6(b) depict the difference in (%) between scenarios 1 and 2 for various H_w at h_c =0 m. It is clear that the influence of assuming dry unit weight above h_c on strength led to underestimate strength for both problems. For the **PEP** problem, it was required to increase ϕ by 2.52° (from 30°) at h_c =0 m and H_w=0.5 m if γ_{dry}= 15 kN/m^3 was used above h_c in order to obtain the same result as for the scenario 1, γ_{dry} =17.5 kN/m^3 and ϕ=30°.

Tables 4 and 5 show the required additional $\Delta\phi$ (i.e., 30°+ $\Delta\phi$) for all cases. The results in Tables 4 and 5 are in an accuracy of 0.02° (equivalent to about 0.08% difference in P_p at h_c =0 m, H_w=0 m and ϕ=30°). At lower H_w<-1 m, -2 and -3 m, the influence of h_c was eliminated where a constant increase in $\Delta\phi$ was obtained. This was because h_c did not contribute in P_p (h_c was below the developed slip lines). The same finding was obtained for the bearing capacity case as long as h_c did not cross the developed failure lines. The differences in $\Delta\phi$ for the **BC** problem (see Table 4) were less than for the **PEP** problem (Table 5) due to the sensitivity of the **BC** problems on ϕ and the assumed full frictionless of the wall.

Tables 4 and 5 also comprise the results of scenario 3, the bold values. These values represented the maximum obtained difference which indicated how scenario 3 underestimated both P_p and N_γ values. A comparison between scenarios 1 and 3 shown in Figs. 7(a) and 7(b) for both problems further illustrated underestimation of scenario 3, apart from a few cases for scenario 1 where full capillarity was assumed (saturated unit weight was used by the software), e.g., see Fig. 7(a) scenario 1- h_c= 1 m at H_w/H=-1. Comparisons between scenarios 2 and 3 (not in

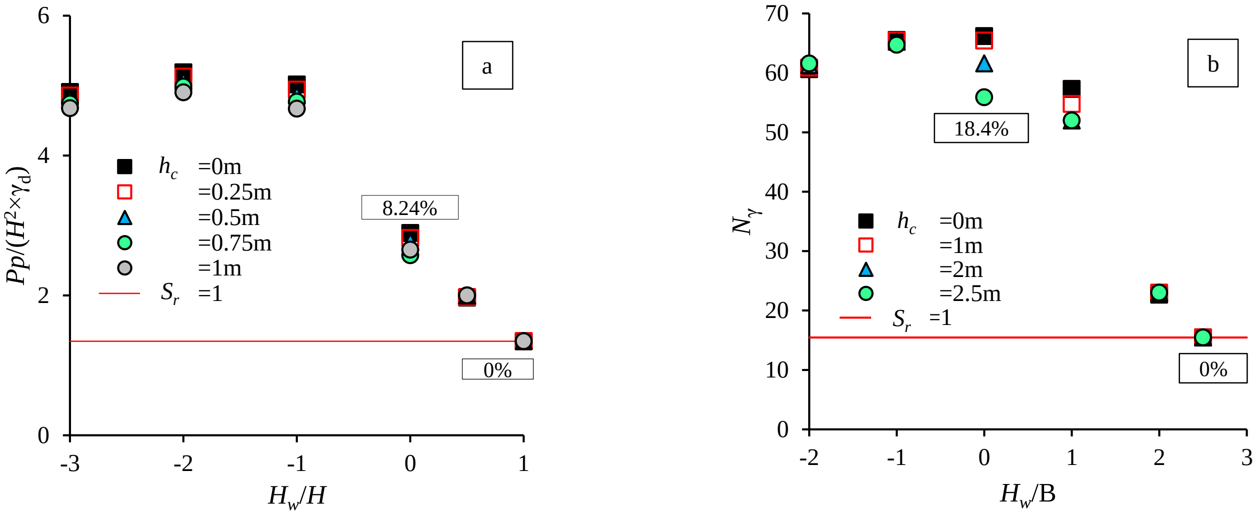

Fig. 5 (a) Normalised P_p against H_w/H at various h_c values and (b) N_γ against H_w/B at various h_c values

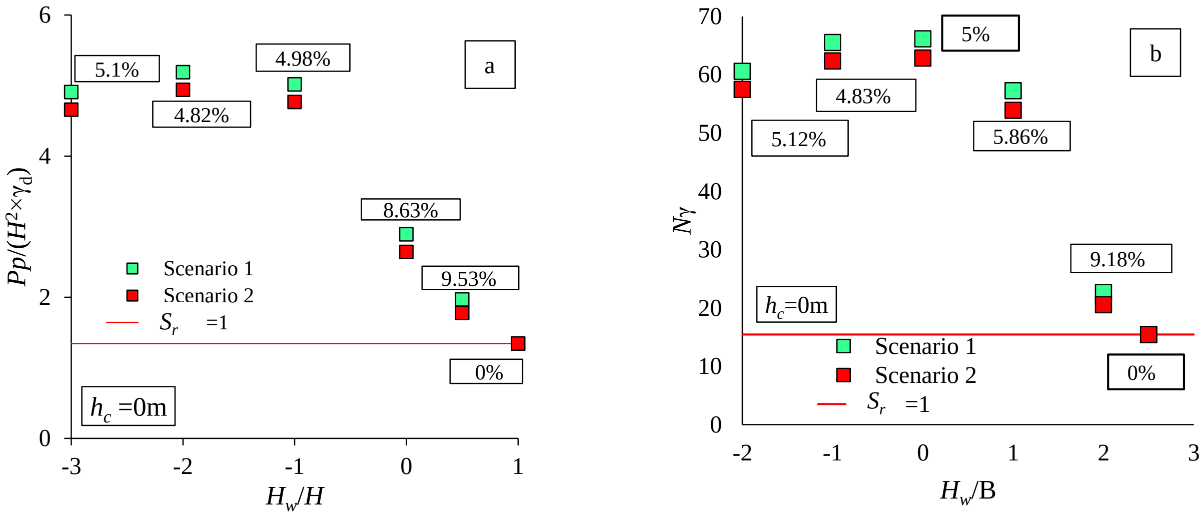

Fig. 6 A comparison between scenario 1 and 2 at h_c =0 m for the (a) normalised P_p, and (b) N_γ

Table 4 The required additional $\Delta\phi$ for scenario 2 to obtain the same results of Scenario 1 for the **PEP** problem

	$\Delta\phi$					
	H_w (m)					
h_c (m)	1	0.5	0	-1	-2	-3
0	**0**	**2.52**	**2.02**	**1.14**	**1.09**	**1.14**
0.25	0	1.42	1.92	1.14	1.09	1.14
0.5	0	0	1.6	1.14	1.09	1.14
0.75	0	0	0.98	1.14	1.09	1.14
1	0	0	0	1.14	1.09	1.14

Table 5 The required additional $\Delta\phi$ for scenario 2 to obtain the same results of Scenario 1 for the **BC** problem

	$\Delta\phi$					
	H_w (m)					
h_c(m)	2.5	2	1	0	-1	-2
0	**0**	**0.68**	**0.5**	**0.42**	**0.38**	**0.38**
1	0	0	0.3	0.42	0.38	0.38
2	0	0	0	0.3	0.38	0.38
2.5	0	0	0	0	0.38	0.38

presented this paper) demonstrated the same findings for both problems.

The implication of Figs. 7(a) and 7(b) was that although the negative influence of h_c on strength, it was balanced by the increase in unit weight due to the unsaturated conditions. For example, although a rise of 1 m of h_c for scenario 1 at H_w=0 m (see Fig. 7(b)), still provided a higher N_γ when compared with scenario 3 at H_w=0 m and h_c =0 m, ϕ =30.42° required for scenario 3 in order to obtain the

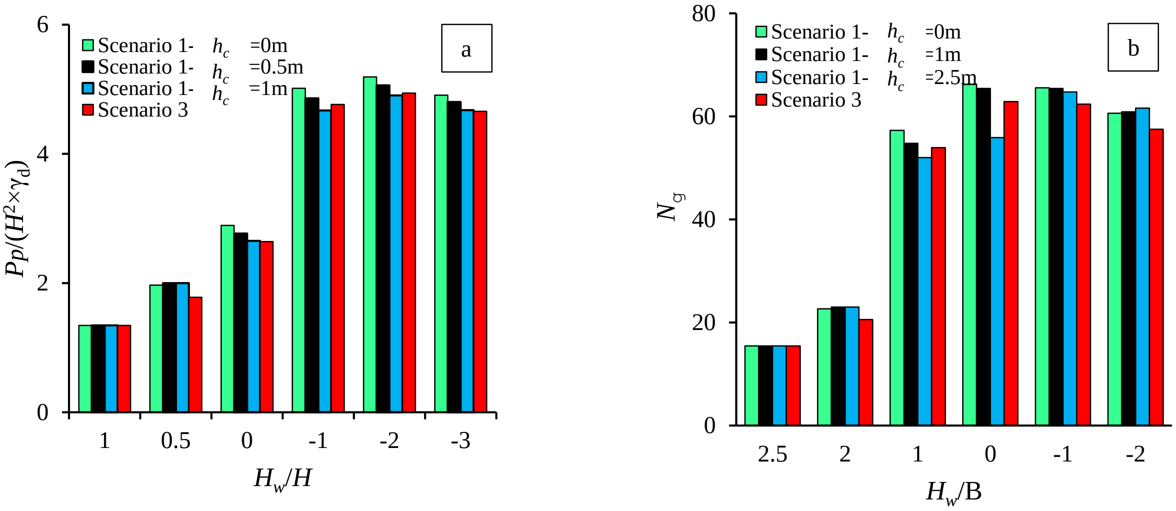

Fig. 7 A comparison between scenario 1 and 3 at various h_c for the (a) normalised P_p, and (b) N_γ

same bearing capacity of scenario 1.

6. Conclusions

The influence of capillary rise height (h_c) on strength has been underestimated in the literature because of 1) ignoring the negative effect of h_c above the water table (H_w) and 2) assuming dry unit weight above H_w or h_c. Consequently, leading to less accurate determination of strength. This paper, therefore, addressed numerically these gaps by carrying out parametric studies on passive earth pressure (**PEP**) and bearing capacity (**BC**) problems. The negative effect of h_c on the mechanical characterization of strength as well as the effect of assigning dry unit weight above h_c for a simulated soil were studied using a modified upper bound approach, Discontinuity Layout Optimization (**UNSAT-DLO**).

The numerical results noticeably showed significant influence of unsaturated conditions for the **PEP** and **BC** problems. The maximum increase in strength, owing to the unsaturated conditions, was obtained when suction was slightly less or greater than the air entry suction.

On the other hand, the numerical results showed a negative influence of capillary rise on strength for both the **PEP** and **BC** problems. Up to 8.24% decrease in P_p at H_w=0 m was obtained when h_c rose 1 m, from 0 m. To put this in context, this was equivalent to a decrease of about 1.92° in ϕ at H_w=0 m and h_c =1 m in order to obtain the same result at h_c =0 m. Similar finding was obtained for the **BC** problem, up to 18.4% decrease in N_γ (equivalent to 1.3° decrease in ϕ when H_w=0 m at h_c =2.5 m was modelled) was obtained when h_c rose from 0m to 2.5 m at H_w=0 m. In addition, the numerical results also showed the negative influence of assigning dry unit weight above h_c. While, assigning an accurate unit weight (unsaturated unit weight) above h_c showed an increase in strength. Due to a deficiency in experimental results, however, it is worth waiting for validations/verifications of the adopted analysis in this study against practical data.

References

Blight, G.E. (2013), "Unsaturated soil mechanics in geotechnical practice", CRC Press.

Das, B.M. (2010), "Principles of geotechnical engineering", CENGAGE Learning. Stamford, Connecticut.

Fredlund, D.G. and Rahardjo, H. (1993), "Soil mechanics for unsaturated soils", John Wiley & Sons.

Hazen, A. (1930), "Water supply. in American Civil Engineering Handbook", Wiley, New York.

Kim, Y.T., Shin, H. and Park, D.K. (2011), "Variation of shear strength of weathered granite soil with water content", Unsaturated soils Theory and Practice, Kasetsart University, ThailandM, 253-258.

Lane, K.S. and Washburn, D.E. (1946), "Capillary tests by capillary meters and by soil filled tubes", *Processdings of the Highway Research Board*, **26**, 460-473, Washington, D.C. December.

Lloret-Cabot, M., Wheeler, S.J., Pineda, J.A., Romero, E. and Sheng, D. (2018), "From saturated to unsaturated conditions and vice versa", *Acta Geotechnica*, **13**(1), 15-37. https://doi.org/10.1007/s11440-017-0577-6.

Lu, N. (2016), "Generalized soil water retention equation for adsorption and capillarity", *J. Geo. Geoenviron. Eng.*, **42**(10). https://doi.org/10.1061/(ASCE)GT.1943-5606.0001524.

Lu, N. (2020), "Unsaturated soil mechanics: Fundamental challenges, breakthroughs, and opportunities", *J. Geotech. Geoenviron. Eng.*, **146**(5). https://doi.org/10.1061/(ASCE)GT.1943-5606.0002233.

Murray, E.J. and Sivakumar, V. (2010), "Unsaturated Soils: A fundamental interpretation of soil behaviour", John Wiley & Sons.

Öberg, A.L. and Sällfors, G. (1997), "Determination of shear strength parameters of unsaturated silts and sands based on the water retention curve", *Geo. testing J.*, **20**(1), 40-48. https://doi: 10.1520/GTJ11419J.

Otalvaro, I.F., Neto, M.P.C., Delage, P. and Caicedo, B. (2016), "Relationship between soil structure and water retention properties in a residual compacted soil", *Eng. Geol*., **205**, 73-80, https://doi.org/10.1016/j.enggeo.2016.02.016.

Qian, J., Lin, Z. and Shi, Z. (2022), "Soil-water retention curve model for fIne-grained soils accounting for void ratio–dependent capillarity", *Can. Geo J*., **59**(4), 498-509. https://doi.org/10.1139/cgj-2021-004,

Rahardjo, H., Kim, Y. and Satyanaga, A. (2019), "Role of unsaturated soil mechanics in geotechnical engineering", *Int. J. of Geo-Eng.*, **1**(10), 1-23. https://doi.org/10.1186/s40703-019-0104-8.

Röhm, S.A. and Vilar, O.M. (1996), "Shear strength of an unsaturated sandy soil", *In Unsaturated Soils*, 189-193, Paris, Septmeber.

Shwan, B.J. (2015), "Experimental and numerical study of the shear strength of unsaturated sand", Doctoral dissertation, University of Sheffield, Sheffield, UK.

Shwan, B.J. (2022), "Upper bound analysis of suction effect on tunnel stability", *Int. J. Geomech.*, **22**(9). https://doi.org/10.1061/(ASCE)GM.19435622.0002504.

Shwan, B.J. and Smith, C.C. (2015), "Investigation of the shear strength of unsaturated sand using a modified direct shear apparatus", *Geo. Eng. for Infrastructure and Development*, 3353-3357, Edinburgh.

Shwan, B.J. and Smith, C.C. (2014), "Application of limit analysis in unsaturated soils: numerical and experimental study of bearing capacity", *In Unsaturated Soils: Research and Applications-Proceedings of the 6th International Conference on Unsaturated Soils.* Sedney, Australia, July.

Smith, C.C. and Gilbert, M. (2007), "Discontinuity layout optimization: A new numerical procedure for upper bound limit analysis", *Proceedings of the IX International Conference on Computational Plasticity*, 170-173.

Tarantino, A. (2007), "A Possible Critical State Framework for Unsaturated Compacted Soils", *Géotechnique*, **57**(4), 385-389. https://doi.org/10.1680/geot.2007.57.4.385.

Vanapalli, S.K., Fredlund, D.G., Pufahl, D.E. and Clifton, A.W. (1996), "Model for the prediction of shear strength with respect to soil suction", *Can. Geo. J.*, **33**(3), 379-392. https://doi.org/10.1139/t96-060.

Vanapalli, S.K. and Mohamed, F.M. (2007), "Bearing capacity of model footings in unsaturated soils", *In Experimental unsaturated soil mechanics. Springer, Berlin, Heidelberg.*, 483-493. https://doi.org/10.1007/3-540-69873-6_48.

Wheeler, S.J., Sharma, R.S. and Buisson, M.S.R. (2003), "Coupling of hydraulic hysteresis and stress–strain behaviour in unsaturated soils", *Géotechnique*, **53**(1), 41-54. https://doi.org/10.1680/geot.2003.53.1.41.

Zhai, Q., Rahardjo, H., Satyanaga, A., Dai, G. and Zhuang, Y. (2020), "Framework to estimate the soil-water characteristic curve for soils with dIfferent void ratios", *Bull. Eng. Geo. Environ.*, **79**(8), 4399-4409. https://doi.org/10.1007/s10064-020-01825-8.

GC

List of abbreviation

T: surface tension
h_c: capillary rise height
γ_w : unit weight of water
θ : contact angle
e: void ratio
D_{10}: particle size corresponding to 10% passing
C: constant
SWRC: soil water retention curve
s_o: air entry suction
s: suction
PEP: passive earth pressure
BC: bearing capacity
UNSAT-DLO: unsaturated discontinuity layout optimization
λ : minimum adequacy factor
$\mathbf{f}_L^T$: live load
$\mathbf{f}_D^T$: dead load
d: matrix contains displacement vectors
$\mathbf{g}^T$: dissipation coefficients
c: cohesion
p : vector containing plastic multipliers
n_i: normal displacement accompanying the sliding
$\hat{C}$: apparent cohesion
L: length of a discontinuity
U: a variable accounts for the shear strength increase due to the effect of suction
a: a fitting parameter
H_w: water table depth
z: vertical coordinate
Gs: specific gravity
ϕ : angle of shearing resistance
S_r : degree of saturation
q_u: unsaturated ultimate bearing capacity
c' : effective cohesion
$(u_a - u_w)_b$: air entry suction
S^{ψ} : degree of saturation
ψ : bearing capacity fitting parameter
$(u_a - u_w)_{AVR}$: average suction directly beneath the footing and to the end of stress bulb
ϕ' : effective angle of shearing resistance
B and *L*: footing's dimensions
N_c, N_q and N_γ : bearing capacity factors
P_p: passive thrust
k_p : passive earth pressure coefficient
γ : soil unit weight
H: height of the wall
$(u_a - u_w)$: matric suction
ϕ^b : angle of friction with respect to suction
γ_{dry} : dry unit weight
γ_{sat} : saturated unit eight
γ_{unsat} : unsaturated unit weight