

Bearing capacity of strip footings on unsaturated soils under combined loading using LEM

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Abstract. Bearing capacity of shallow foundations is often determined for either dry or saturated soils. In some occasions, foundations may be subjected to external loading which is inclined and/or eccentric. In this study, the ultimate bearing capacity of shallow foundations resting on partially saturated coarse-grained cohesionless and fine-grained cohesive soils subjected to a wide range of combined vertical (V) - horizontal (H) - moment (M) loadings is rigorously evaluated using the well-established limit equilibrium method. The unified effective stress approach as well as the suction stress concept is effectively adopted so as to simulate the behaviour of the underlying unsaturated soil medium. In order to obtain the bearing capacity, four equilibrium equations are solved by adopting Coulomb failure mechanism and Bishop effective stress concept and also considering a linear variation of the induced matric suction beneath the foundation. The general failure loci of the shallow foundations resting on unsaturated soils at different hydraulic conditions are presented in $V - H - M$ spaces. The results indicate that the matric suction has a marked influence on the bearing capacity of shallow foundations. In addition, the effect of induced suction on the ultimate bearing capacity of obliquely-loaded foundations is more pronounced than that of the eccentrically-loaded footings.

Keywords: bearing capacity; combined loading; limit equilibrium method; suction stress; unsaturated soil

1. Introduction

Shallow foundations are commonly used to safely transfer the loads imposed by superstructures to the underlying soil layers. The ultimate induced pressure a shallow foundation can withstand without reaching the failure state is typically denoted as the bearing capacity. The bearing capacity of shallow foundations must be determined carefully so as to decrease the extensive costs of construction in various infrastructure projects. Most of the design methods to estimate the bearing capacity of shallow foundations are based on the conventional equations, with the seminal contributions of Terzaghi (1943), Meyerhof (1953), Hansen (1970) and Vesic (1973). These pioneering conventional methods represent relatively simple procedures in order to evaluate the ultimate bearing capacity of surface footings under the sole vertical concentric loads and might fail to provide a precise stability criterion for foundations under the effect of considerable moment and lateral loads.

Although the vast majority of studies on the bearing capacity of shallow footings have been limited to the estimation of maximum bearing capacity under predominantly vertical loading (Khatri and Kumar 2009,

Dixit and Patil 2013, Uncuoglu 2015, Ukritchon and Keawsawasvong 2017, 2020, Turedi *et al.* 2019, Keawsawasvong 2021, Keawsawasvong and Lai 2021, Yodsomjai *et al.* 2021), few studies have focused on the response of shallow foundations subjected to oblique-eccentric combined loading (Meyerhof 1963, Vesic 1975, Nova and Montrasio 1991, Ukritchon *et al.* 1998, Taiebat and Carter 2000, Okamura *et al.* 2002, Loukidis *et al.* 2008, Krabbenhoft *et al.* 2012, 2014, Cure *et al.* 2014, Tang *et al.* 2015, Yahia-Cherif *et al.* 2017; Conti 2018; Zheng *et al.* 2019, Pham *et al.* 2020, Dastpak *et al.* 2020, Wu *et al.* 2020, Jin *et al.* 2020, Keawsawasvong *et al.* 2021, Fathipour *et al.* 2022a, b, 2023, Payan *et al.* 2022). In a comprehensive study, Taiebat and Carter (2000) performed a three-dimensional finite-element analysis of circular foundations resting on homogeneous, purely cohesive soil subjected to vertical-horizontal-moment combined loading so as to develop their general failure envelopes in the undrained condition. Loukidis *et al.* (2008) adopted the finite element method to study the collapse condition of strip footings resting on purely frictional soils, following the elastic-perfectly plastic Mohr-Coulomb failure criterion, subjected to inclined and eccentric loadings. They demonstrated that inclination factor is directly affected by the value of the internal friction angle; whereas the effective width is literally independent of the soil shear strength parameters. In a more recent study, Pham *et al.* (2020) examined the ultimate bearing capacity of a rigid footing subjected to combined loading on sandy and clayey soils using the rigid plastic finite element method (RPFEM) and showed that the size and shape of the failure envelope depend primarily on the direction of the horizontal load applied on the shallow foundation in sandy soils; whereas

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the corresponding influence was observed to be almost negligible for clayey soils.

Many of the design theories for estimating the bearing capacity of shallow foundations have been developed by considering the underlying soil in either fully dry or entirely saturated states. However, the response of surface footings is majorly influenced by the hydraulic properties of the soil upper layers near the ground surface, which, in turn, are often above the water table and therefore, in a partially saturated condition. Unsaturated soils possess an internal apparent cohesion, manifested in matric suction, which contributes to a considerable increase in the shear strength and stiffness properties of the soil and thus, the ultimate bearing capacity of the overlying foundation (Khalili and Khabbaz 1998, Lu and Likos 2004, 2006, Lu *et al.* 2010). Throughout the literature, several theoretical, experimental and numerical efforts have been made to examine the significant influence of matric suction on the ultimate bearing capacity of shallow foundations resting on partially saturated soils (Oloo *et al.* 1997, Costa *et al.* 2003, Xu 2004, Rojas *et al.* 2007, Oh and Vanapalli 2011, Vanapalli and Mohamed 2013, Vahedifard and Robinson 2016, Tang *et al.* 2017, Vo and Russell 2016, Nouzari *et al.* 2021). In separate studies, Oloo *et al.* (1997), Vanapalli and Mohamed (2013) and Vahedifard and Robinson (2016) proposed different equations to estimate the bearing capacity of foundations resting on unsaturated soils by extending the well-known Terzaghi's equation and including the effect of induced matric suction beneath the surface footing into the formulations. It has been represented that the suction profile in the stress bulb right beneath the footing base could be significantly affected by the depth of groundwater table and the flow condition. Accordingly, Wuttke *et al.* (2013) suggested that the suction profile can be well-predicted by a linear relationship for the given steady state flow condition and a specific depth of water table. In a comprehensive experimental study conducted by Vanapalli and Mohamed (2013), the bearing capacity and settlement behaviour of sandy soils under saturated and unsaturated conditions were carefully examined with several physical modelling tests. It was demonstrated that the bearing capacity of foundations on unsaturated sands increases linearly with matric suction in the saturation zone; the trend of which was reported to be non-linear in the transition zone of the soil-water retention curve. In another study, Vahedifard and Robinson (2016) utilized the well-established suction stress concept, originally proposed by Lu and Likos (2004), to represent a unified method for estimating the ultimate bearing capacity of shallow foundations in partially saturated soils under the steady state flow condition. Vo and Russell (2016) exploited the slip-line theory to determine the bearing capacity of strip footings on unsaturated soils by adopting the unified effective stress concept and the Mohr-Coulomb failure criterion. They approximated the non-uniform suction profile within the soil layers by linearly varying matric suction with depth and reported that both methods yield fairly similar predictions of the ultimate bearing capacity. Tang *et al.* (2017) also proposed alternative equations based on the effective stress principle to evaluate the bearing

capacity of shallow foundations on partially saturated soils by considering both uniform and varying suction profiles.

Apropos of the abovementioned discussion, the individual influences of combined loading and partial saturation of the soil on the bearing capacity of shallow foundations have been investigated in several previous studies in the literature. However, their inclusive effects have not been systematically examined so far. To overcome this shortcoming, in this study, the ultimate bearing capacity of shallow foundations resting on variably saturated soils subjected to eccentric-inclined loadings is estimated by using the well-established limit equilibrium method. The general three-dimensional failure envelopes of the shallow foundations, under combined vertical (V) - horizontal (H) - moment (M) loadings, are also presented for the design purposes.

2. Suction stress concept: Background & implementation

In the two-independent stress states theory, originally proposed by Fredlund *et al.* (1978), the shear strength of unsaturated soils was considered to be an *a priori* function of the mean net stress ($\sigma - p_a$) and the matric suction ($p_a - p_w$)

$$\tau = c' + (\sigma - p_a)\tan\phi' + (p_a - p_w)\tan\phi^b \quad (1)$$

where σ is the total stress, p_a is the pore air pressure, p_w is the pore water pressure, c' is the effective cohesion, ϕ' is the effective internal friction angle corresponding to the change in the mean net stress and ϕ^b is the friction angle with respect to the change in suction for the given net stress. Based on this theory, some experimental and numerical studies have been conducted to develop an equation for the estimation of the bearing capacity of shallow foundations resting on variably saturated soils. In these studies, suction has been considered as an apparent cohesion. Oloo *et al.* (1997) presented a correlation for the estimation of the bearing capacity and showed that there exists a linear relationship between bearing capacity and matric suction, as follows

$$q_u = (c' + s_e \tan\phi' + (s - s_e)\tan\phi^b)N_c + 0.5\gamma BN_\gamma \quad (2)$$

where s_e is the air entry or air expulsion value marking the transition from saturated to unsaturated conditions and vice versa. This linear relationship has been subsequently questioned by a great number of experimental works (Costa *et al.* 2003, Rojas *et al.* 2007, Oh and Vanapalli 2011, Vanapalli and Mohamed 2013).

In later studies, the two independent stress state variables approach was proven to be practically inefficient in the stability analysis of partially saturated soils, as the soil suction angle was reported not to be constant; but dependent on the induced matric suction. This deficiency was appropriately addressed by Khalili and Khabbaz (1998) who extended and validated the unified effective stress concept for unsaturated soils, first developed by Bishop (1959), as follows

$$\sigma' = (\sigma - p_a) + (\chi s) \quad (3)$$

where $s = (p_a - p_w)$ is the matric suction, and χ is the effective stress parameter, equal to 1 for saturated and zero for dry soils. Khalili and Khabbaz (1998) examined the application of the effective stress principle to unsaturated soils using a great number of shear strength and volumetric strain data from literature and proposed a general correlation for the effective stress parameter as a function of air entry pressure and soil inherent characteristics. Based on the effective stress concept, the shear strength of the soil could be expressed as

$$\tau = (\sigma - p_a) \tan \varphi' + (c' + \chi s \tan \varphi') \quad (4)$$

Vahedifard and Robinson (2016) utilized the suction stress-based effective stress approach to present a bearing capacity equation for unsaturated soils using the effective degree of saturation (S_{eff}), as follows

$$q_u = (c' + s_e \tan \varphi' + (s_{AVE} - s_e) S_{eff, AVE} \tan \varphi') N_c \xi_c + q N_q \xi_q + 0.5 \gamma B N_\gamma \xi_\gamma \quad (5)$$

$$S_{eff} = \frac{(S_r - S_{res})}{1 - S_{res}} \quad (6)$$

where S_{res} is the residual degree of saturation and $S_{eff, AVE}$ is the average effective degree of saturation corresponding to the average matric suction values within the stress bulb zone right beneath the foundation. Considering a uniformly distributed matric suction under the footing, Tang *et al.* (2017) presented the following relationship to predict the bearing capacity of shallow foundations resting on variably saturated soils

$$q_u = (c' + \chi s_{AVE} \tan \varphi') N_c d_c + 0.5 B N_\gamma d_\gamma + q N_q d_q \quad (7)$$

Vo and Russell (2016) studied the bearing capacity of strip footings on unsaturated soils using the slip line theory and the effective stress concept. The study was conducted in different hydraulic states, namely evaporation, through which the water is removed from the soil surface, and infiltration, in which the water penetrates through the soil layers most probably due to precipitation. The results showed that the impact of matric suction on the bearing capacity has a meaningful correlation with the depth of groundwater table and the footing width. Vo and Russell (2016) reported that similar bearing capacity and settlement responses of the shallow foundations could be estimated by approximating the non-uniform χs profiles with an appropriate linear variation of χs with depth (Fig. 1)

$$\chi s = \chi s_0 - \beta z \quad (8)$$

where χs_0 is the value of χs at the ground surface, z is the depth below the footing base and β is a constant defining the rate of variation of χs with depth. The linear variation of χs with depth was also confirmed by the results of the study conducted by Lu and Likos (2004), where they showed that suction profile is independent of soil parameters and could be assumed to be linear.

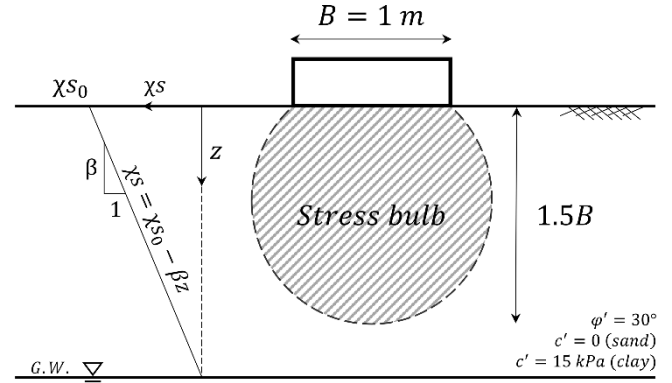


Fig. 1 Schematic illustration of the suction profile in the partially saturated soil beneath the shallow foundation

3. Problem definition & limit equilibrium formulations

In this study, limit equilibrium method is employed to evaluate the ultimate bearing capacity of strip surface footing resting on unsaturated soil while the foundation is subjected to combined vertical (V) – horizontal (H) – moment (M) loading. The shallow foundation is assumed to have a typical width of B (1 m in this study) and the groundwater table is considered to be located at two meters below the foundation which is deemed to be within the influence zone of the footing. A linear variation of the matric suction measure with depth was considered in this study according to the suggestion of Vo and Russell (2016), as described in the previous section. The amount of suction decreases gradually by moving from the soil surface into the depth and it reaches zero at the groundwater table (Fig. 1). Coulomb failure mechanism and half-Prandtl general shear failure mechanism, which are appropriate for the estimation of the bearing capacity of shallow foundations under oblique-eccentric combined loads, are adopted herein so as to develop the limit equilibrium formulations (Fig. 2(a)). According to the half-Prandtl general shear failure theory, there are two active and passive failure wedges which move in the opposite directions along the vertical line. The semi-infinite half-space soil medium beneath the surface footing has the effective soil unit weight of γ , the effective internal friction angle of φ' and the effective cohesion of c' .

According to Figs. 2(b) and 2(c), W_A and W_B are the weights of the active and passive failure wedges, respectively, which are applied at their centre of gravity; R_A and R_B are the reaction forces of the active and passive wedges, respectively; ρ_A and ρ_B are the angles of the active and passive wedges with horizontal axis, respectively; and P_L is the bearing capacity pressure applied on the shallow foundation with an inclination angle of " α " and an eccentricity of " e ". By applying the external load on the foundation, suction stress and cohesion are both mobilized so as to react against the movement along the boundaries. According to the equilibrium condition, effective stress concept for unsaturated soils and also the linear variation of suction measure with depth, the values of suction stress and cohesion between the wedges could be expressed as follow

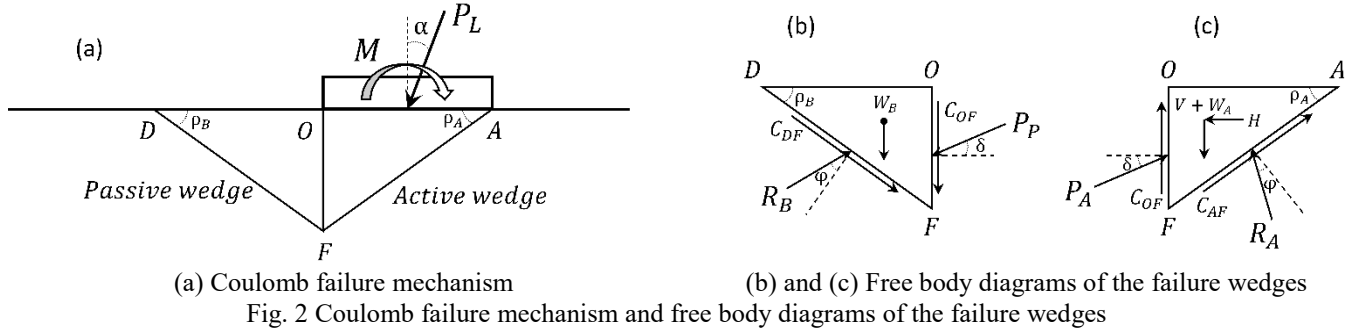


Fig. 2 Coulomb failure mechanism and free body diagrams of the failure wedges

$$C_{OF} = B \tan \rho_A (c' + \chi s_0 \tan \varphi' - \frac{B \tan \rho_A}{2} \beta \tan \varphi') \quad (9)$$

$$C_{DF} = \frac{B \tan \rho_A}{\sin \rho_B} (c' + \chi s_0 \tan \varphi' - \frac{B \tan \rho_A}{2} \beta \tan \varphi') \quad (10)$$

$$C_{AF} = \frac{B \tan \rho_A}{\sin \rho_A} (c' + \chi s_0 \tan \varphi' - \frac{B \tan \rho_A}{2} \beta \tan \varphi') \quad (11)$$

where C_{OF} , C_{DF} and C_{AF} are the combinations of mobilized suction stress and cohesion along the respective boundaries. It should be noted that the matric suction is zero below the groundwater table which is assumed to be located two meters below the foundation. Thus, for the values of $(B \tan \rho_A)$ larger than two meters, while the cohesion is fully mobilized at the corresponding alignment, the matric suction is mobilized only within the two meters below the foundation up to the groundwater table. Therefore, the abovementioned equations are to be modified for these conditions, as follow

$$C_{OF} = B \tan \rho_A c' + 2 \left(\frac{\chi s_0 \tan \varphi'}{2} \right) \quad (12)$$

$$C_{DF} = \frac{B \tan \rho_A c'}{\sin \rho_B} + 2 \left(\frac{\chi s_0 \tan \varphi'}{2 \sin \rho_B} \right) \quad (13)$$

$$C_{AF} = \frac{B \tan \rho_A c'}{\sin \rho_A} + 2 \left(\frac{\chi s_0 \tan \varphi'}{2 \sin \rho_A} \right) \quad (14)$$

Another noticeable point is that the bearing capacity can be affected by the different values of χs_0 and β which can vary in different flow conditions, such as evaporation and infiltration. The equilibrium equations between the body forces related to the passive wedge have been illustrated in Eqs. (15) and (16) and those corresponding to the active wedge have been presented in Eqs. (17) and (18).

$$\rightarrow \sum F_x = 0 \quad -P_P \cos \delta + C_{DF} \cos \rho_B + R_B \sin(\rho_B + \varphi') = 0 \quad (15)$$

$$\uparrow \sum F_y = 0 \quad -P_P \sin \delta - C_{OF} - C_{DF} \sin \rho_B + R_B \cos(\rho_B + \varphi') - W_B = 0 \quad (16)$$

$$\rightarrow \sum F_x = 0 \quad P_A \cos \delta + C_{AF} \cos \rho_A - R_A \sin(\rho_A - \varphi') - H = 0 \quad (17)$$

$$\uparrow \sum F_y = 0 \quad P_A \sin \delta + C_{OF} + C_{AF} \sin \rho_A + R_A \cos(\rho_A - \varphi') - V - W_A = 0 \quad (18)$$

$$\frac{H}{V} = \tan \alpha \quad (19)$$

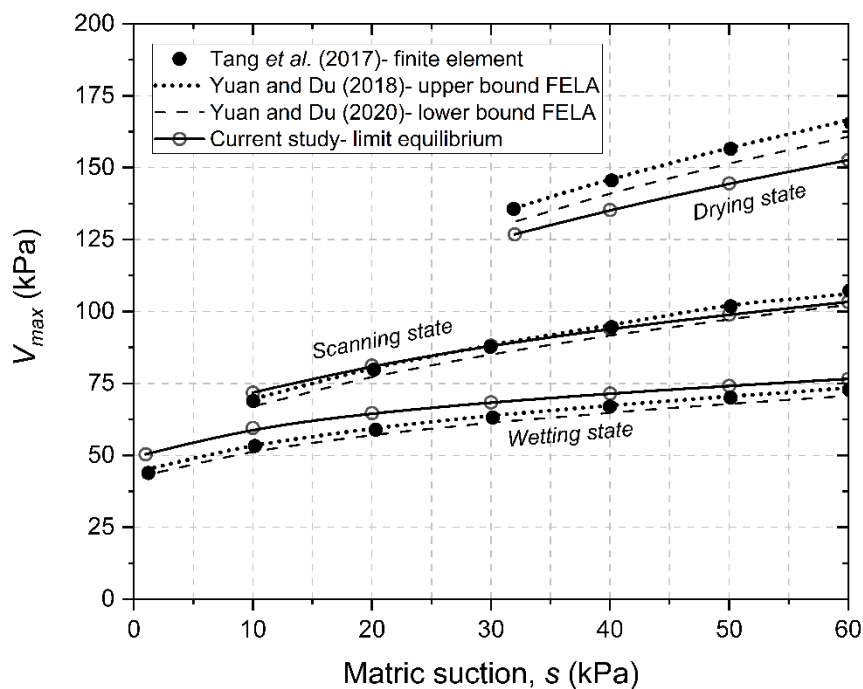
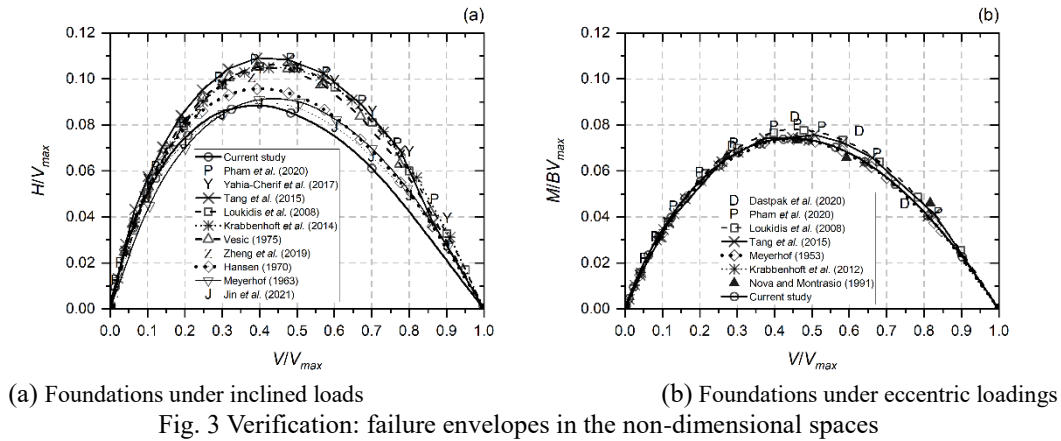
$$P_L = \sqrt{(H^2 + V^2)} \quad (20)$$

It is worth noting that the inclination of α should be considered towards the passive wedge to satisfy the so-called Prandtl's theory. According to the method proposed by Meyerhof (1963), the width of the shallow foundation is modified to $(B - 2e)$ in order to consider the effect of eccentric loading in the limit equilibrium analysis. A wide range of values for ρ_A and ρ_B are considered and the corresponding values of P_L are acquired through an optimization procedure adopted in MATLAB.

4. Validation of limit equilibrium approach

The limit equilibrium method adopted in the course of this study was rigorously validated against several high-quality data from literature. In spite of idealization related to considering Prandtl mechanism, the adopted LE method can estimate the bearing capacity of shallow foundations under combined loads appropriately as the verification results match well with the results of the studies in the literature.

Two series of data for the dry and saturated states of the underlying soil were exploited herein for the validation purpose. In the first series, a great number of experimental and theoretical/numerical studies of shallow foundations resting on fully dry soil under combined loading (either inclined or eccentric) were considered. In order to compare the results of current limit equilibrium simulations and those of the previous studies, the values of effective internal friction angle (φ') and soil unit weight (γ) were assumed to be 35° and 20 kN/m^3 , respectively. The value of interface friction angle (δ) was also considered to be $0.5\varphi'$, so as to render a satisfactorily good agreement with the previous studies. The ultimate vertical load capacity of the foundation (V_{max}) was obtained when α or e is zero, so that the load is applied vertically at the centre of the surface footing. Fig. 3(a) shows the comparison between the failure envelopes for obliquely-loaded foundations in the



normalized $V-H$ space obtained from the limit equilibrium approach in this study and those presented in the literature by Meyerhof (1963), Vesic (1975), Hansen (1970), Loukidis *et al.* (2008), Krabbenhoft *et al.* (2013), Tang *et al.* (2015), Yahia-Cherif *et al.* (2017), Zheng *et al.* (2019), Pham *et al.* (2020) and Jin *et al.* (2021). It can be observed that all failure envelopes follow quite similar patterns and the locus obtained from the limit equilibrium method in the current study closely matches the results of the majority of studies in the literature, particularly Meyerhof (1963) and Jin *et al.* (2021). Fig. 3(b) illustrates the comparison of the results for eccentrically-loaded foundations in the normalized $M-V$ failure envelope between the current study and the predictions of Meyerhof (1953), Nova and Montrasio (1991), Loukidis *et al.* (2008), Krabbenhoft *et al.* (2012), Tang *et al.* (2015), Pham *et al.* (2020) and Dastpak *et al.* (2020). Although the results of this study are generally in satisfactorily good agreement with all studies in the

literature, it can be seen that there is a remarkable match between the failure locus of the current study and that of Nova and Montrasio (1991). In addition, as $(B' = B - 2e)$ approach is used to account for the moment load, the results also agree very well with the pioneering study of Meyerhof (1953).

In the second series of verification data, three major experimental and numerical studies in the literature on the bearing capacity of shallow foundations resting on partially saturated soils subjected to vertical concentric loading were considered. The bearing capacity of a rigid smooth strip footing placed on unsaturated soil was first compared with the results presented by Yuan and Du (2020), whose saturation parameters are summarized in Table 1. As presented by Yuan and Du (2020), suction stress was assumed to be uniform through the soil layer beneath the foundation. The foundation width, total unit weight, effective cohesion and internal friction angle were

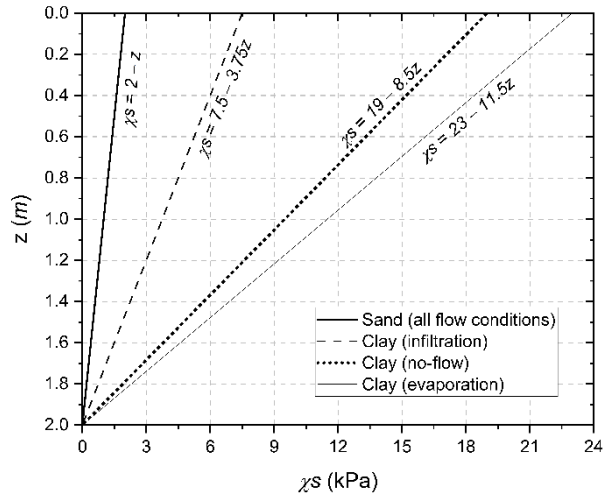


Fig. 5 Suction profiles for partially saturated sand and clay soils subjected to various hydraulic conditions

Table 1 Saturation parameters at different hydraulic states (data from Yuan and Du 2020)

Matric suction, <i>s</i> (kPa)	χs			Saturation		
	Drying	Scanning	Wetting	Drying	Scanning	Wetting
1	-	-	1	-	1	-
10	-	5.31	2.82	-	0.355	0.596
20	-	7.26	3.86	-	0.260	0.436
30	-	8.7	4.62	-	0.216	0.363
32	16.864	8	4.768	0.592	0.210	0.353
40	18.68	9.92	5.24	0.536	0.190	0.319
50	20.65	10.95	5.8	0.485	0.172	0.289
60	22.38	11.88	6.3	0.447	0.158	0.266

considered to be 1 m , 20 kN/m^3 , 1 kPa and 20° , respectively. Fig. 4 shows the comparison of the ultimate vertical bearing capacity between the results of this study using the limit equilibrium technique with those presented by Tang *et al.* (2017) using theoretical/numerical analysis, Yuan and Du (2018) using the upper-bound finite element limit analysis (FELA) method and Yuan and Du (2020) adopting the lower-bound finite element limit analysis (FELA) approach. According to this Figure, there exists a very good agreement between the vertical bearing capacity values obtained in the current study and those reported in the aforementioned works with the maximum error of around 5%, which is quite acceptable considering different approaches adopted in these studies.

5. Results & discussion

5.1 General outline & suction profile

In this study, the significant influence of matric suction induced beneath the shallow foundations on their ultimate bearing capacity was evaluated for typical sandy and clayey underlying soils and consequently, the general failure envelopes in the normalized $V-H$, $V-M$ and $V-H-M$ planes were derived. Generally, the maximum ultimate

vertical load for foundations resting on unsaturated soils can be practically estimated using the aforementioned equations presented by Oloo *et al.* (1997), Vahedifard and Robinson (2016) and Tang *et al.* (2017). However, in this study, numerical analyses have been conducted to obtain the maximum vertical load for dry or unsaturated soils.

To this end, the typical value of effective internal friction angle for sand and clay was assumed to be 30° (Fathipour *et al.* 2021a, b, c, 2022c, Safardoost *et al.* 2022, Bahmani Tajani *et al.* 2022) and the typical cohesion for the clay deposit was considered as 15 kPa (Di Matteo *et al.* 2013, Wang and Akeju 2016). Furthermore, a linear variation of the matric suction measure (χs) with depth was considered according to the suggestion of Vo and Russell (2016). Considering the water table to be located at the two meters below the ground surface, the corresponding suction profiles are plotted and shown in Fig. 5 for sand and clay deposits at different flow conditions (infiltration, no-flow, evaporation). It should be noted that the values of matric suction measure at the ground surface (χs_0) are the typical values suggested by Lu *et al.* (2010) and Fathipour *et al.* (2020).

5.2 Effect of load inclination

Fig. 6 shows the influence of load inclination on the

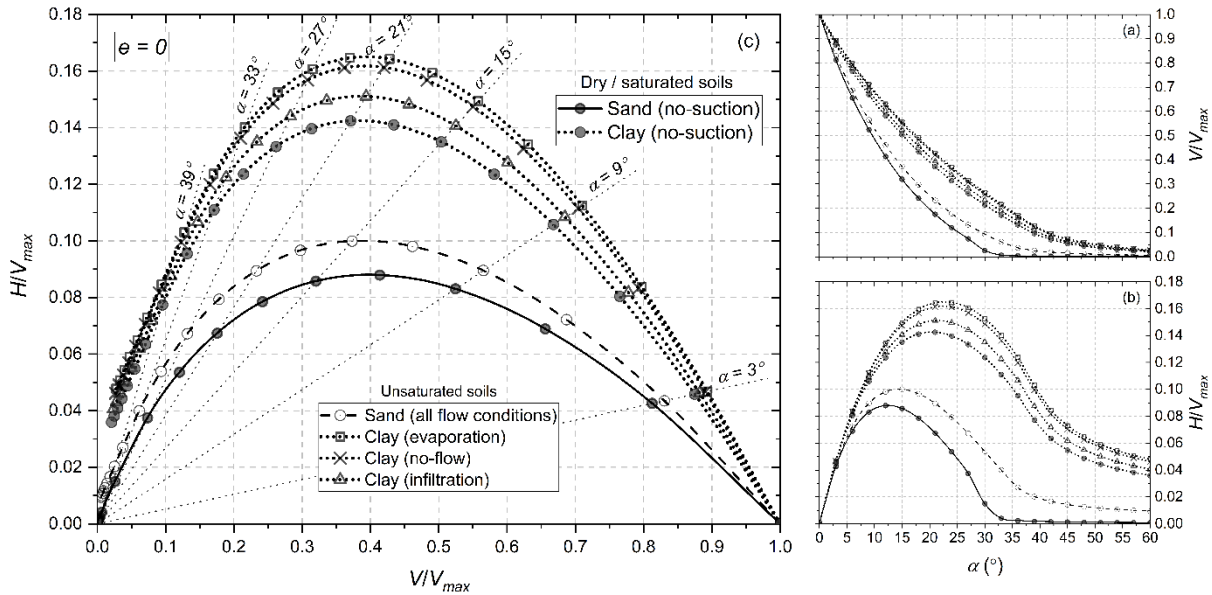


Fig. 6 Failure envelopes for shallow foundations resting on unsaturated sand and clay deposits subjected to inclined loading

failure envelopes in the normalized $V - H$ plane for the obliquely-loaded foundations resting on dry/saturated and partially saturated sand and clay deposits subjected to various flow conditions. As can be observed in Fig. 6(a), the value of V/V_{max} decreases with the increase in the inclination angle and turns out to be quite negligible beyond the values of around $\alpha = 30^\circ$ for sandy soils and $\alpha = 40^\circ$ for clayey soils. To clarify this phenomenon, it should be noted that the bearing capacity of the foundations subjected to inclined loading is related to both the size of the failure wedges and the angle of load inclination, where the former is indeed a function of the latter. In fact, the value of ρ_A decreases with the increase in the inclination angle; thus, yielding a shallower active failure wedge. This trend continues until the value of ρ_A reaches ϕ' , where it remains constant thereafter; so that the size of the failure wedges would then remain fairly constant and the bearing capacity would not change considerably.

The variations of normalized H with inclination angle for different soils and hydraulic conditions are represented in Fig. 6(b). According to this Figure, the H/V_{max} values for all conditions increase with the inclination angle by a sharp rise until reaching their maximum level, beyond which they decrease continuously. The inclination angle at which the maximum value of H/V_{max} (ultimate horizontal load) occurs is generally greater in clay deposits than sand layers and increases with the introduction of matric suction into the porous structure of soil medium. This trend of variation can be attributed to the very fact that introducing any cohesion, in either true or apparent form, would impart augmented shear strength to the failure wedges underneath, which is, in turn, translated into increased horizontal ultimate load. Accordingly, the H/V_{max} reaches its highest value at the inclinations of $\alpha = 12.5^\circ$, $\alpha = 15^\circ$, $\alpha = 21^\circ$ and $\alpha = 25^\circ$ for dry/saturated sand, unsaturated sand (all flow conditions), dry/saturated clay and unsaturated clay (in the evaporation state), respectively. The shallow foundation

resting on cohesionless granular soils in the no-suction state could be observed to have practically no resistance to shearing for the load inclination angles over around 30° to 35° . However, there exists a residual amount of lateral load resistance for similar foundation overlying the partially saturated sand, due primarily to the marked influence of matric suction in different sliding surfaces underneath; thus producing a kind of barrier against the shear sliding of the shallow foundation. Such effects are much further pronounced in the clay deposits, where the corresponding positive effect could be the result of both soil cohesion and imposed suction stress.

Fig. 6(c) shows the general failure envelopes of obliquely-loaded shallow foundation resting on cohesive (clay) and cohesionless (sand) soils at various hydraulic conditions in the $V/V_{max} - H/V_{max}$ plane. Superimposed on this Figure are the contours of the various inclination angles of the applied external load. As can be seen, suction stress bears a great effect on the expansion of the failure envelope and consequently, on the ultimate bearing capacity of the shallow foundation against inclined loading. Once again, the H/V_{max} ratio for all conditions increases with the inclination angle by a sharp rise until a maximum value, occurring at approximately $V/V_{max} = 0.4$, where it decreases thereafter. For the cases of clay deposits in the no-suction state and also unsaturated sand and clay underlying soils, the values of H/V_{max} would not reach zero, which is once again due to the very fact that the cohesion and/or suction stress impart additional shear resistance to the failure wedges underneath, thus providing a residual shearing stability against sliding for the surface footing. The maximum horizontal load, H_{max} , varies in the range of $0.09 V_{max}$ to $0.16 V_{max}$. Furthermore, as the inclination angle increases, points on the failure envelopes get much closer to each other, demonstrating that the load inclination poses a quite limited effect on the bearing capacity of shallow foundations at high α values.

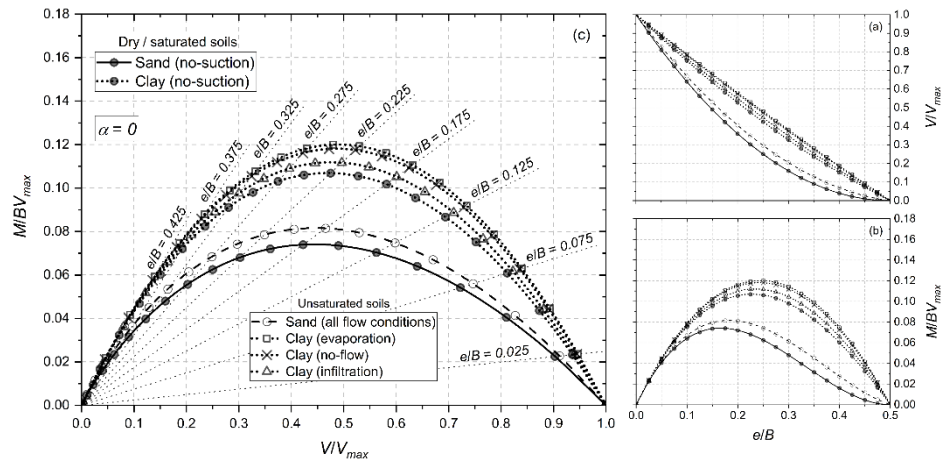


Fig. 7 Failure envelopes for shallow foundations resting on unsaturated sand and clay deposits subjected to eccentric loading

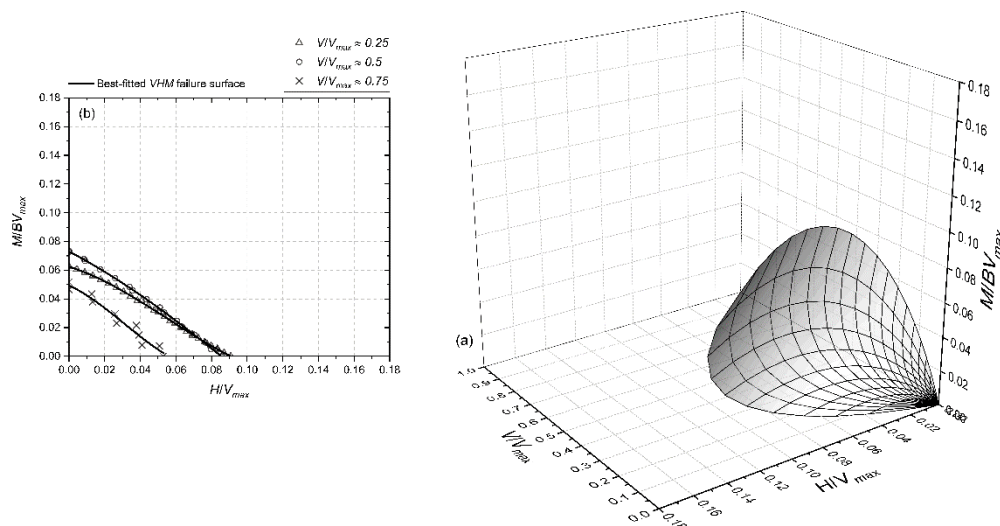


Fig. 8 Three-dimensional failure locus for shallow foundations resting on the sand layer (no-suction state) subjected to combined inclined

5.3 Effect of load eccentricity

Fig. 7 shows the effect of load eccentricity on the failure loci in the normalized $V - M$ plane for the eccentrically-loaded foundations resting on dry/saturated and partially saturated sand and clay deposits subjected to various flow conditions. Fig. 7(a) demonstrates that as eccentricity increases, the value of V/V_{max} decreases, which is deemed to be more linear in clay deposits. The variation of the normalized M value against the load eccentricity is shown in Fig. 7(b). According to this Figure, M/BV_{max} increases with load eccentricity and reaches its highest value at $e/B = 0.18$, $e/B = 0.2$, $e/B = 0.22$ and $e/B = 0.25$ for dry sand, unsaturated sand (all flow conditions), dry clay and unsaturated clay (in the evaporation state), respectively. After the peak, the normalized M values all decrease and reach zero at considerably high load eccentricities ($e/B = 0.5$). Moreover, as the induced suction in the underlying soil layer rises, the value of eccentricity, at which the maximum ultimate moment load is obtained, also increases.

The failure envelopes of the eccentrically-loaded shallow foundations in the normalized $V - M$ plane are shown in Fig. 7(c). Superimposed on this Figure are the contours of various eccentricities of the applied external load. As can be observed, the induced suction causes all the failure envelopes to markedly expand, thereby rendering significant increase in the ultimate moment load the foundation can withstand. According to this Figure, the M/BV_{max} ratios for all hydraulic conditions rose with V/V_{max} up to a maximum value, occurring at $V/V_{max} = 0.45$ to 0.5 , and decreased thereafter until reaching zero. The maximum moment load, M_{max} , varies in the range of $0.07 BV_{max}$ to $0.12 BV_{max}$ for different types of underlying soils and variable hydraulic states. Quite similar to the case for the inclined loading, as the load eccentricity increases, points on different failure surfaces get closer to each other, highlighting the fairly insignificant influence of load eccentricity on the ultimate bearing capacity as the load eccentricity grows considerably large. A comparison between the failure envelopes shown in Figs. 6 and 7

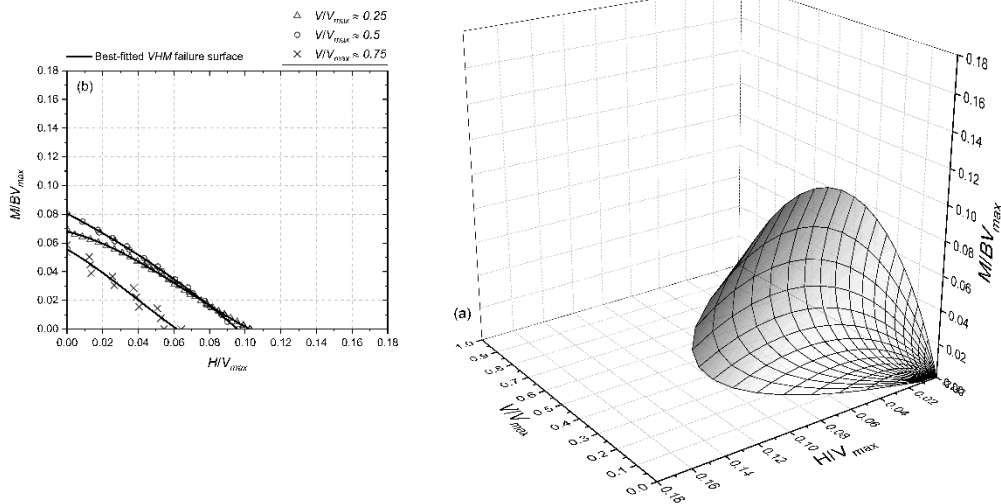


Fig. 9 Three-dimensional failure locus for shallow foundations resting on partially saturated sand layer (all flow conditions) subjected to combined inclined and eccentric loadings

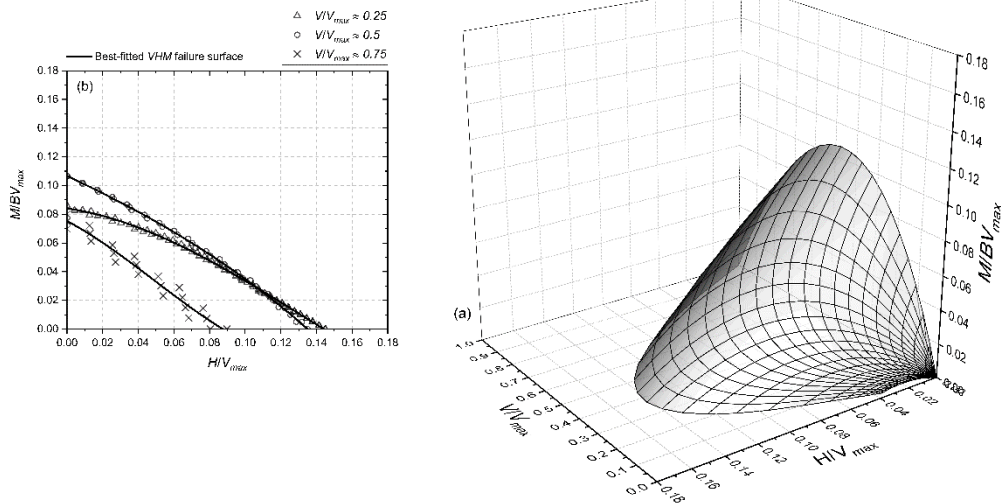


Fig. 10 Three-dimensional failure locus for shallow foundations resting on the clay layer (no-suction state) subjected to combined inclined and eccentric loadings

reveals that the effect of induced matric suction on the ultimate horizontal load (in obliquely-loaded foundations) is more significant than its influence on the ultimate moment load (in eccentrically-loaded foundations).

5.4 General failure loci for ultimate bearing capacity in the normalized $V-H-M$ space

Figs. 8(a) to 13(a) show the general failure envelopes of shallow foundations resting on dry/saturated and partially saturated soils subjected to inclined and eccentric combined loading in the three-dimensional normalized $V-H-M$ space. As the Prandtl’s failure mechanism was adopted in this study, the analyses for positive inclination angles and eccentricities were conducted; thus, failure envelopes in the $V-H-M$ space represent a quarter of the predicted ellipsoids. These Figures demonstrate that the size of the failure envelopes is significantly dependent on the soil

cohesion and the value of the suction stress imposed within the soil medium. Accordingly, the failure envelopes for the unsaturated clay in the evaporation state and the dry/saturated sand are the largest and smallest loci, respectively. As mentioned earlier, in the clayey soil at various hydraulic conditions, the amounts of H/V_{max} would not reach zero due to the effect of cohesion and induced suction stress. On the other hand, regardless of the type of the underlying soil, the amount of M/BV_{max} reaches zero, as the Meyerhof (1963) approach was adopted. The best-fitted two-dimensional $H-M$ failure surfaces have also been plotted at different given values of V/V_{max} (Figs. 8b to 13b). It can be seen that the sizes and shapes of the failure surfaces depend on the level of normalized vertical load and, in all Figures, the maximum size of the failure surface is related to $V/V_{max} = 0.5$. Fig. 14 demonstrates the best-fitted two-dimensional $H-M$ failure surfaces at different

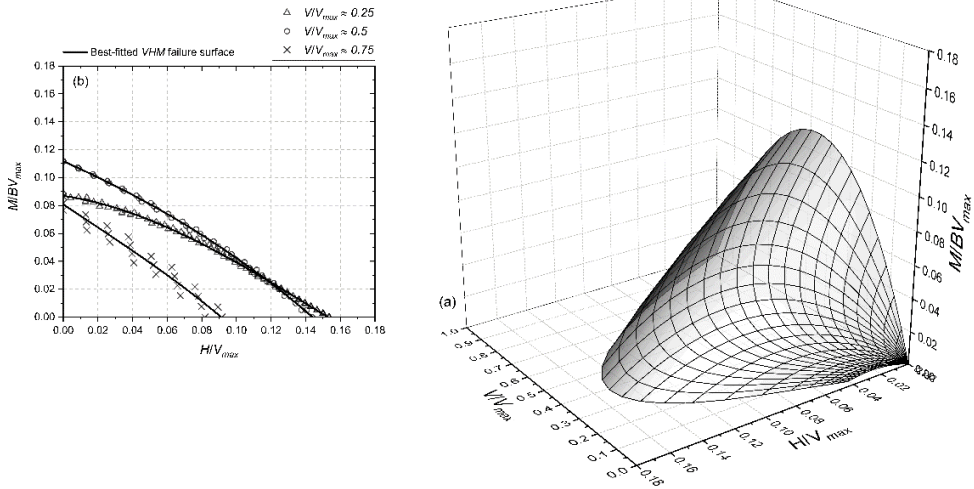


Fig. 11 Three-dimensional failure locus for shallow foundations resting on partially saturated clay layer (infiltration condition) subjected to combined inclined and eccentric loadings

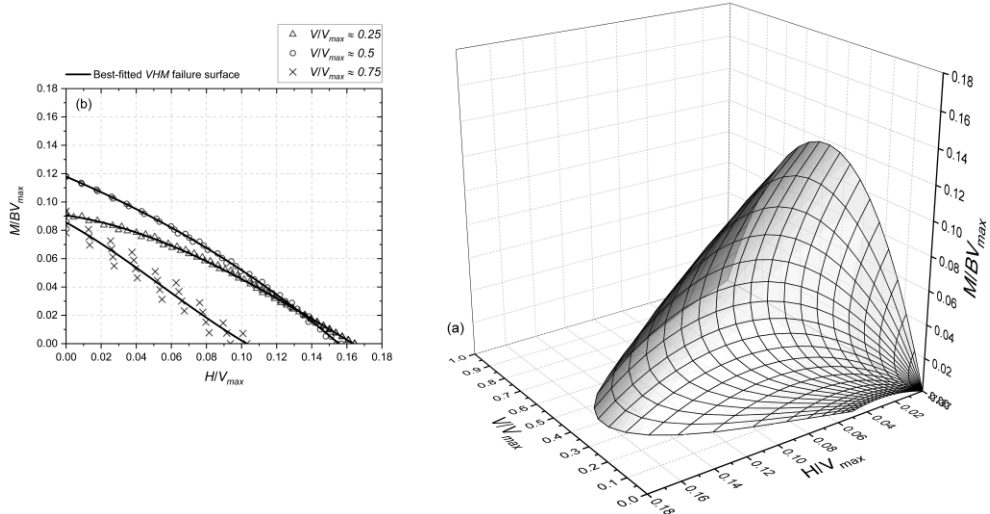


Fig. 12 Three-dimensional failure locus for shallow foundations resting on partially saturated clay layer (no flow condition) subjected to combined inclined and eccentric loadings

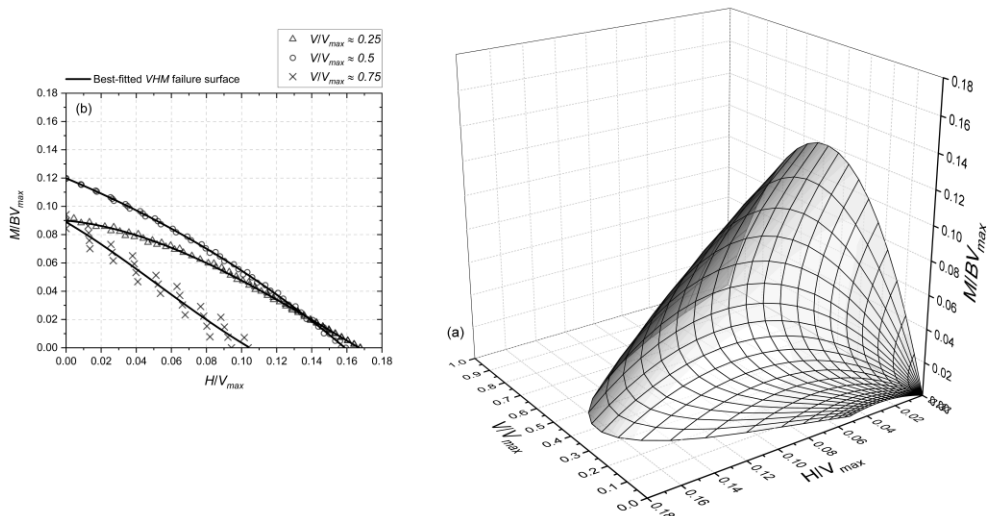


Fig. 13 Three-dimensional failure locus for shallow foundations resting on partially saturated clay layer (evaporation condition) subjected to combined inclined and eccentric loadings

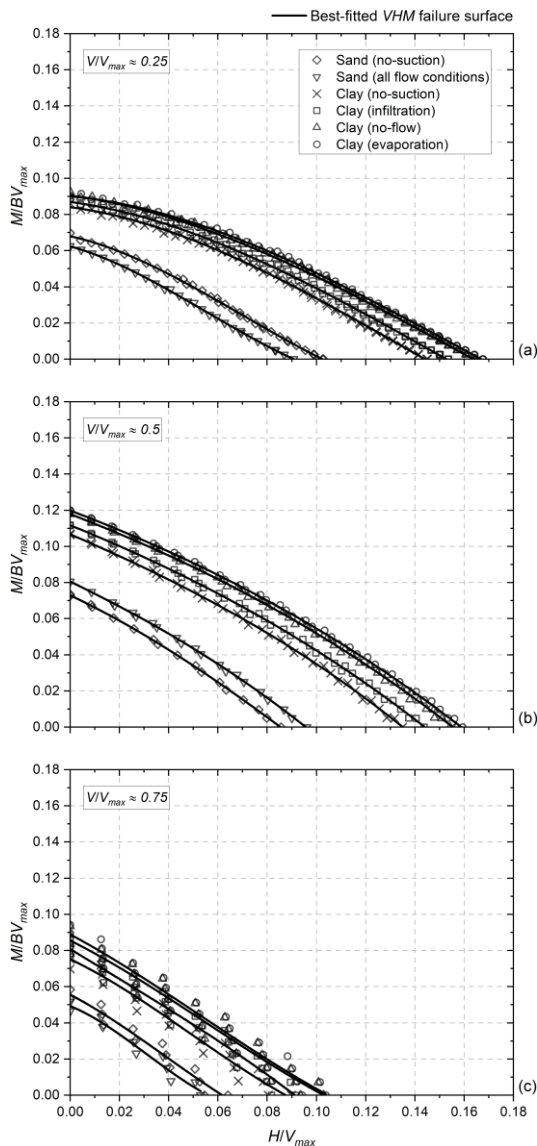


Fig. 14 Failure surfaces of shallow foundations resting on sand and clay deposits with variable hydraulic conditions in $H-M$ plane for different values of normalized vertical load

values of V/V_{max} for all the cases under study. As highlighted before, it can be observed that the sizes of failure surfaces increase with the induced matric suction and are generally larger in clayey soils as compared to sand deposits. Fig. 14 also shows that the contribution of matric suction to the size of failure surfaces in clayey soils turns out to be more pronounced when $V/V_{max} = 0.5$, as compared to the other cases.

6. Conclusions

In this study, the well-established limit equilibrium method was deployed to obtain the ultimate bearing capacity of shallow foundations resting on partially saturated cohesionless (sand) and cohesive (clay) soils subjected to eccentric and inclined loads. The limit equilibrium formulation adopted in this study was

rigorously validated against a large number of experimental and theoretical/numerical data throughout the literature. The significant influence of matric suction on the general failure envelopes in the normalized $V-H$, $V-M$ and $V-H-M$ planes were demonstrated and thoroughly discussed. Based on the results presented in this study, the following conclusions were drawn:

- The inclination angle at which the maximum value of H/V_{max} (ultimate horizontal load) occurs is generally greater in clay deposits than sand layers and increases with the introduction of matric suction into the porous structure of soil medium.
- Suction stress has a great impact on the expansion of the failure envelope and consequently, on the ultimate bearing capacity of the shallow foundation against inclined loading.
- As either inclination angle or load eccentricity increases, points on the failure envelopes get much closer to each other, demonstrating that the load inclination and eccentricity pose a quite limited effect on the bearing capacity of shallow foundations at high α and e/B values.
- As the induced suction in the underlying soil layer rises, the value of eccentricity at which the maximum ultimate moment load is obtained also increases.
- Suction stress causes all the general failure loci in the normalized $V-H-M$ plane to markedly enlarge, thereby rendering significant increases in the ultimate shear and moment loads the foundation can sustain.
- The effect of induced matric suction on the ultimate horizontal load (in obliquely-loaded foundations) is more significant than its influence on the ultimate moment load (in eccentrically-loaded foundations).

References

- Bahmani Tajani, Sh., Fathipour, H., Payan, M., Jamshidi Chenari, R. and Senetakis, K. (2022), "Temperature-dependent lateral earth pressures in partially saturated backfills" *Eur. J. Environ. Civil Eng.*, <https://doi.org/10.1080/19648189.2022.2125911>.
- Bishop, A. (1959), *The Principal of Effective Stress*. Teknisk Ukeblad.
- Conti, R. (2018), "Simplified formulas for the seismic bearing capacity of shallow strip foundations", *Soil Dyn. Earthq. Eng.*, **104**, 64-74. <https://doi.org/10.1016/j.soildyn.2017.09.027>.
- Costa, Y.D., Cintra, J.C. and Zornberg, J.G. (2003), "Influence of matric suction on the results of plate load tests performed on a lateritic soil deposit", *Geotech. Test. J.*, **26**(2), 219-227. <https://doi.org/10.1520/GTJ11326J>.
- Cure, E., Sadoglu, E., Turker, E. and Uzuner, B.A. (2014), "Decrease trends of ultimate loads of eccentrically loaded model strip footings close to a slope", *Geomech. Eng.*, **6**(5), 469-485. <https://doi.org/10.12989/gae.2014.6.5.469>.
- Dastpak, P., Abrishami, S., Sharifi, S. and Tabaroei, A. (2020), "Experimental study on the behavior of eccentrically loaded circular footing model resting on reinforced sand", *Geotext. Geomembranes*, **48**(5), 647-654. <https://doi.org/10.1016/j.geotextmem.2020.03.009>.
- Di Matteo, L., Valigi, D. and Ricco, R. (2013), "Laboratory shear strength parameters of cohesive soils: variability and potential effects on slope stability", *Bull. Eng. Geol. Environ.*, **72**(1), 101-

106. <https://doi.org/10.1007/s10064-013-0459-6>.
- Dixit, M.S. and Patil, K.A. (2013), "Experimental estimate of N_y values and corresponding settlements for square footings on finite layer of sand", *Geomech. Eng.*, **5**(4), 363-377. <https://doi.org/10.12989/gae.2013.5.4.363>.
- Fathipour, H., Payan, M. and Chenari, R.J. (2021a), "Limit analysis of lateral earth pressure on geosynthetic-reinforced retaining structures using finite element and second-order cone programming", *Comput. Geotech.*, **134**, 104119. <https://doi.org/10.1016/j.compgeo.2021.104119>.
- Fathipour, H., Payan, M., Chenari, R.J. and Fatahi, B. (2022a), "General failure envelope of eccentrically and obliquely loaded strip footings resting on an inherently anisotropic granular medium", *Comput. Geotech.*, **146**, 104734.
- Fathipour, H., Payan, M., Safardoost Siahmazgi, A., Jamshidi Chenari, R. and Senetakis, K. (2022b), "Numerical study on the bearing capacity of strip footing resting on partially saturated soil subjected to combined vertical-horizontal-moment loading", *Eur. J. Environ. Civil Eng.*, 1-34.
- Fathipour, H., Payan, M., Jamshidi Chenari, R. and Senetakis, K. (2021b), "Lower bound analysis of modified pseudo-dynamic lateral earth pressures for retaining wall-backfill system with depth-varying damping using FEM-Second order cone programming", *Int. J. Numer. Anal. Method. Geomech.*, **45**(16), 2371-2387.
- Fathipour, H., Siahmazgi, A.S., Payan, M. and Chenari, R.J. (2020), "Evaluation of the lateral earth pressure in unsaturated soils with finite element limit analysis using second-order cone programming", *Comput. Geotech.*, **125**, 103587. <https://doi.org/10.1016/j.compgeo.2020.103587>
- Fathipour, H., Siahmazgi, A. S., Payan, M., Veiskarami, M. and Jamshidi Chenari, R. (2021c), "Limit analysis of modified pseudodynamic lateral earth pressure in anisotropic frictional medium using finite-element and second-order cone programming", *Int. J. Geomech.*, **21**(2), 04020258. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0001924](https://doi.org/10.1061/(ASCE)GM.1943-5622.0001924).
- Fathipour, H., Tajani, S.B., Payan, M., Chenari, R.J. and Senetakis, K. (2022c), "Influence of transient flow during infiltration and isotropic/anisotropic matric suction on the passive/active lateral earth pressures of partially saturated soils", *Eng. Geol.*, 106883.
- Fathipour, H., Tajani, S.B., Payan, M., Chenari, R.J. and Senetakis, K. (2023), "Impact of transient infiltration on the ultimate bearing capacity of obliquely-eccentrically loaded strip footings on partially saturated soils", *Int. J. Geomech.*, <https://doi.org/10.1061/IJGNAL/GMENG-7463>.
- Fredlund, D.G., Morgenstern, N.R. and Widger, R.A. (1978), "The shear strength of unsaturated soils", *Can. Geotech. J.*, **15**(3), 313-321. <https://doi.org/10.1139/t78-029>.
- Hansen, J.B. (1970), "A revised and extended formula for bearing capacity", Danish Geotech. Instit., Bull. No. **28**, 5-11
- Jin, L., Zhang, H. and Feng, Q. (2021), "Ultimate bearing capacity of strip footing on sands under inclined loading based on improved radial movement optimization", *Eng. Optim.*, **53**(2), 277-299. <https://doi.org/10.1080/0305215X.2020.1717483>.
- Keawsawasvong, S. (2021), "Bearing capacity of conical footings on clays considering combined effects of anisotropy and non-homogeneity", *Ship. Offshore Struct.*, 1-12.
- Keawsawasvong, S. and Lai, V.Q. (2021), "End bearing capacity factor for annular foundations embedded in clay considering the effect of the adhesion factor", *Int. J. Geosynthetic Ground Eng.*, **7**(1), 1-10.
- Keawsawasvong, S., Thongchom, C. and Likitlersuang, S. (2021), "Bearing capacity of strip footing on Hoek-Brown rock mass subjected to eccentric and inclined loading", *T. Infrastruct. Geotech.*, **8**(2), 189-202.
- Khalili, N. and Khabbaz, M.H. (1998), "A unique relationship for χ for the determination of the shear strength of unsaturated soils", *Geotechnique*, **48**(5), 681-687. <https://doi.org/10.1680/geot.1998.48.5.681>.
- Khatri, V.N. and Kumar, J. (2009), "Bearing capacity factor N for a rough conical footing", *Geomech. Eng.*, **1**(3), 205-218. <https://doi.org/10.12989/gae.2009.1.3.205>.
- Krabbenhoft, S., Damkilde, L. and Krabbenhoft, K. (2012), "Lower-bound calculations of the bearing capacity of eccentrically loaded footings in cohesionless soil", *Can. Geotech. J.*, **49**(3), 298-310. <https://doi.org/10.1139/t11-103>.
- Krabbenhoft, S., Damkilde, L. and Krabbenhoft, K. (2014), "Bearing capacity of strip footings in cohesionless soil subject to eccentric and inclined loads", *Int. J. Geomech.*, **14**(3), 04014003. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000332](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000332).
- Loukidis, D., Chakraborty, T. and Salgado, R. (2008), "Bearing capacity of strip footings on purely frictional soil under eccentric and inclined loads", *Can. Geotech. J.*, **45**(6), 768-787. <https://doi.org/10.1139/T08-015>.
- Lu, N. and Likos, W.J. (2004), *Unsaturated Soil Mechanics*, John Wiley&Sons. Inc., Hoboken, USA.
- Lu, N. and Likos, W.J. (2006), "Suction stress characteristic curve for unsaturated soil", *J. Geotech. Geoenviron. Eng.*, **132**(2), 131-142. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2006\)132:2\(131\)](https://doi.org/10.1061/(ASCE)1090-0241(2006)132:2(131)).
- Lu, N., Godt, J.W. and Wu, D.T. (2010), "A closed-form equation for effective stress in unsaturated soil", *Water Resour. Res.*, **46**(5). <https://doi.org/10.1029/2009WR008646>.
- Meyerhof, G. (1953), "The bearing capacity of foundations under eccentric and inclined loads", *Proceedings of the 3rd Int. Conf. on SMFE*.
- Meyerhof, G.G. (1963), "Some recent research on the bearing capacity of foundations", *Can. Geotech. J.*, **1**(1), 16-26. <https://doi.org/10.1139/t63-003>.
- Nouzari, M.A., Chenari, R.J., Payan, M. and Pishgar, F. (2021), "Pseudo-static seismic bearing capacity of shallow foundations in unsaturated soils employing limit equilibrium method", *Geotech. Geol. Eng.*, **39**(2), 943-956. <https://doi.org/10.1007/s10706-020-01535-8>.
- Nova, R. and Montrasio, L. (1991), "Settlements of shallow foundations on sand", *Géotechnique*, **41**(2), 243-256. <https://doi.org/10.1680/geot.1991.41.2.243>.
- Oh, W.T. and Vanapalli, S.K. (2011), "Modelling the applied vertical stress and settlement relationship of shallow foundations in saturated and unsaturated sands", *Can. Geotech. J.*, **48**(3), 425-438. <https://doi.org/10.1139/T10-079>.
- Okamura, M., Mihara, A., Takemura, J. and Kuwano, J. (2002), "Effects of footing size and aspect ratio on the bearing capacity of sand subjected to eccentric loading", *Soils Found.*, **42**(4), 43-56. https://doi.org/10.3208/sandf.42.4_43.
- Oloo, S.Y., Fredlund, D.G. and Gan, J.K. (1997), "Bearing capacity of unpaved roads", *Can. Geotech. J.*, **34**(3), 398-407. <https://doi.org/10.1139/t96-084>.
- Payan, M., Fathipour, H., Hosseini, M., Chenari, R.J. and Shiau, J. S. (2022), "Lower bound finite element limit analysis of geostructures with non-associated flow rule", *Comput. Geotech.*, **147**, 104803.
- Pham, Q.N., Ohtsuka, S., Isobe, K. and Fukumoto, Y. (2020), "Limit load space of rigid footing under eccentrically inclined load", *Soils Found.*, **60**(4), 811-824. <https://doi.org/10.1016/j.sandf.2020.05.004>.
- Rojas, J.C., Salinas, L.M. and Sejas, C. (2007), "Plate-load tests on an unsaturated lean clay", *Experimental unsaturated soil mechanics* (pp. 445-452). Springer, Berlin, Heidelberg. https://doi.org/10.1007/3-540-69873-6_44.
- Safardoost Siahmazgi, A., Fathipour, H., Jamshidi Chenari, R., Veiskarami, M. and Payan, M. (2022), "Evaluation of the pseudo-dynamic bearing capacity of surface footings on

- cohesionless soils using finite element lower bound limit analysis”, *Geomech. Geoeng.*, **17**(3), 765-777.
- Taiebat, H.A. and Carter, J.P. (2000), “Numerical studies of the bearing capacity of shallow foundations on cohesive soil subjected to combined loading”, *Géotechnique*, **50**(4), 409-418. <https://doi.org/10.1680/geot.2000.50.4.409>
- Tang, C., Phoon, K.K. and Toh, K.C. (2015), “Effect of footing width on N_y and failure envelope of eccentrically and obliquely loaded strip footings on sand”, *Can. Geotech. J.*, **52**(6), 694-707. <https://doi.org/10.1139/cgj-2013-0378>.
- Tang, Y., Taiebat, H.A. and Senetakis, K. (2017), “Effective stress based bearing capacity equations for shallow foundations on unsaturated soils”, *J. Geoeng.*, **12**(2), 59-64. [https://doi.org/10.6310/jog.2017.12\(2\).2](https://doi.org/10.6310/jog.2017.12(2).2).
- Tang, Y., Taiebat, H.A. and Russell, A.R. (2017), “Bearing capacity of shallow foundations in unsaturated soil considering hydraulic hysteresis and three drainage conditions”, *Int. J. Geomech.*, **17**(6), 04016142. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000845](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000845).
- Terzaghi, K. (1943), *Theoretical Soil Mechanics*, John Wiley and Sons, New York, NY, USA.
- Turedi, Y., Emirler, B., Ornek, M. and Yildiz, A. (2019), “Determination of the bearing capacity of model ring footings: Experimental and numerical investigations”, *Geomech. Eng.*, **18**(1), 29-39. <https://doi.org/10.12989/gae.2019.18.1.029>.
- Ukritchon, B., Whittle, A.J. and Sloan, S.W. (1998), “Undrained limit analyses for combined loading of strip footings on clay”, *J. Geotech. Geoenviron. Eng.*, **124**(3), 265-276. [https://doi.org/10.1061/\(ASCE\)1090-0241\(1998\)124:3\(265\)](https://doi.org/10.1061/(ASCE)1090-0241(1998)124:3(265)).
- Ukritchon, B. and Keawsawasvong, S. (2017), “Unsafe error in conventional shape factor for shallow circular foundations in normally consolidated clays”, *J. Geotech. Geoenviron. Eng.*, **143**(6), 02817001.
- Ukritchon, B. and Keawsawasvong, S. (2020), “Undrained lower bound solutions for end bearing capacity of shallow circular piles in non-homogeneous and anisotropic clays”, *Int. J. Numer. Anal. Method. Geomech.*, **44**(5), 596-632.
- Uncuoglu, E. (2015), “The bearing capacity of square footings on a sand layer overlying clay”, *Geomech. Eng.*, **9**(3), 287-311. <https://doi.org/10.12989/gae.2015.9.3.287>.
- Vahedifard, F. and Robinson, J.D. (2016), “Unified method for estimating the ultimate bearing capacity of shallow foundations in variably saturated soils under steady flow”, *J. Geotech. Geoenviron. Eng.*, **142**(4), 04015095. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001445](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001445).
- Vanapalli, S.K. and Mohamed, F.M. (2013), “Bearing capacity and settlement of footings in unsaturated sands”, *Int. J. Geomate*, **5**(1), 595-604.
- Vesić, A.S. (1973), “Analysis of ultimate loads of shallow foundations”, *J. Soil Mech. Found. Division*, **99**(1), 45-73. <https://doi.org/10.1061/JSFEAQ.0001846>.
- Vesic, A.S. (1975), *Bearing Capacity of Shallow Foundations*, Van Nostrand Reinhold, New York, USA
- Vo, T. and Russell, A.R. (2016), “Bearing capacity of strip footings on unsaturated soils by the slip line theory”, *Comput. Geotech.*, **74**, 122-131. <https://doi.org/10.1016/j.compgeo.2015.12.016>.
- Wang, Y. and Akeju, O.V. (2016), “Quantifying the cross-correlation between effective cohesion and friction angle of soil from limited site-specific data”, *Soils Found.*, **56**(6), 1055-1070. <https://doi.org/10.1016/j.sandf.2016.11.009>.
- Wu, G., Zhang, R., Zhao, M. and Zhou, S. (2020), “Undrained stability analysis of eccentrically loaded strip footing lying on layered slope by finite element limit analysis”, *Comput. Geotech.*, **123**, 103600. <https://doi.org/10.1016/j.compgeo.2020.103600>.
- Wuttke, F., Kafle, B., Lins, Y. and Schanz, T. (2013), “Macroelement for statically loaded shallow strip foundation resting on unsaturated soil”, *Int. J. Geomech.*, **13**(5), 557-564. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0000254](https://doi.org/10.1061/(ASCE)GM.1943-5622.0000254).
- Xu, Y. (2004), “Bearing capacity of unsaturated expansive soils”, *Geotech. Geol. Eng.*, **22**(4), 611-625. <https://doi.org/10.1023/B:GEGE.0000047043.29898.17>.
- Yahia-Cherif, H., Mabrouki, A., Benmeddour, D. and Mellas, M. (2017), “Bearing capacity of embedded strip footings on cohesionless soil under vertical and horizontal loads”, *Geotech. Geol. Eng.*, **35**(2), 547-558. <http://doi.org/10.1007%2Fs10706-016-0124-5>.
- Yodsomjai, W., Keawsawasvong, S. and Lai, V.Q. (2021), “Limit analysis solutions for bearing capacity of ring foundations on rocks using Hoek–Brown failure criterion”, *Int. J. Geosynthetic Ground Eng.*, **7**(2), 1-10.
- Yuan, S. and Du, J. (2018), “Effective stress-based upper bound limit analysis of unsaturated soils using the weak form quadrature element method”, *Comput. Geotech.*, **98**, 172-180. <https://doi.org/10.1016/j.compgeo.2018.02.008>.
- Yuan, S. and Du, J. (2020), “A lower-bound formulation for unsaturated soils”, *Géotechnique*, **70**(2), 123-137. <https://doi.org/10.1680/jgeot.18.P.103>.
- Zheng, G., Zhao, J., Zhou, H. and Zhang, T. (2019), “Ultimate bearing capacity of strip footings on sand overlying clay under inclined loading”, *Comput. Geotech.*, **106**, 266-273. <https://doi.org/10.1016/j.compgeo.2018.11.003>.

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