

Metaheuristic models for the prediction of bearing capacity of pile foundation

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Abstract. The properties of soil are naturally highly variable and thus, to ensure proper safety and reliability, we need to test a large number of samples across the length and depth. In pile foundations, conducting field tests are highly expensive and the traditional empirical relations too have been proven to be poor in performance. The study proposes a state-of-art Particle Swarm Optimization (PSO) hybridized Artificial Neural Network (ANN), Extreme Learning Machine (ELM) and Adaptive Neuro Fuzzy Inference System (ANFIS); and comparative analysis of metaheuristic models (ANN-PSO, ELM-PSO, ANFIS-PSO) for prediction of bearing capacity of pile foundation trained and tested on dataset of nearly 300 dynamic pile tests from the literature. A novel ensemble model of three hybrid models is constructed to combine and enhance the predictions of the individual models effectively. The authenticity of the dataset is confirmed using descriptive statistics, correlation matrix and sensitivity analysis. Ram weight and diameter of pile are found to be most influential input parameter. The comparative analysis reveals that ANFIS-PSO is the best performing model in testing phase ($R^2 = 0.85$, RMSE = 0.01) while ELM-PSO performs best in training phase ($R^2 = 0.88$, RMSE = 0.08); while the ensemble provided overall best performance based on the rank score. The performance of ANN-PSO is least satisfactory compared to the other two models. The findings were confirmed using Taylor diagram, error matrix and uncertainty analysis. Based on the results ELM-PSO and ANFIS-PSO is proposed to be used for the prediction of bearing capacity of piles and ensemble learning method of joining the outputs of individual models should be encouraged. The study possesses the potential to assist geotechnical engineers in the design phase of civil engineering projects.

Keywords: dynamic pile load tests; meta-heuristic optimization; pile foundations; rank analysis; reliability analysis

1. Introduction

There has been a growing tendency for multistoried buildings regardless of the strength of the soil due to acute shortage of land and growing demand and the focus is on pile foundations which are economical as well as safe. Constructing sites having soft soils have witnessed high settlements and bearing capacity problems (Charlie *et al.* 2009, Gabriellaitis *et al.* 2013, Wang *et al.* 2016, 2022). Soil is highly variable in nature. Geotechnical uncertainties arising from three categories of sources: inherent variability, transformation uncertainty and measurement error. (Kulhawy 1993, Liu *et al.* 2021). Thus, to ensure reliability, large number tests are required to be performed and also, a balance needs to be established between economy and risk. Studies suggest that more pile tests need to be done than what suggested by IS codes (Sundaram and Gupta 2016). Pile load tests require highly skilled labor and considerable amount of testing time and due to large cost associated with the foundations, contractors often tend to accept the higher values of bearing capacity. Thus, ensuring safety has been challenging in pile foundations. As per the study of Rybak

and Krol (2018), limit state is rarely achieved and its often impossible to estimate ultimate load. The determination of an acceptable safe pile capacity value becomes a difficult and contentious task (Sundaram and Gupta 2016). Empirical formulas too are least reliable. Much earlier, Terzaghi (1929) warned against, "blindly trusting in purely statistical relations". Studies reveal that predictions of empirical relations result in negative values of coefficient of correlation and very high values of RMSE which signifies poor fitting of data and significant disagreement between input and predictor variable (Al-atrroush *et al.* 2020, Chaallal *et al.* 2015, Fishman *et al.* 2003, Guo *et al.* 2022, Kalinli *et al.* 2011, Park and Rilett 1999, Shan *et al.* 2022, Wu *et al.* 2022). Despite the fact that most natural soil deposits have different shear strength distributions even within a given layer, most previous formulae and design charts for estimating bearing capacity have dealt with homogeneous or layered clay whose shear strength is consistent in each layer (Park *et al.* 2010). Sieffert and Bay-Gress (2000) concluded that new parametric and numerical analyses are needed to better understand the bearing capacity.

Artificial Intelligence (AI) based regression models enable computers to learn from correlation between input and predictor data without needing to be explicitly programmed. Instead of obtaining civil engineering design parameters by conducting rigorous field or lab tests, they

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are efficiently simulated by developing regression fitting to datasets in nonlinear multi-dimensional ML based simulation modelling techniques. Artificial neural network (ANN) has been successfully applied in literature to predict the bearing capacity of pile foundation (Fatehnia and Amirinia 2018, Wu *et al.* 2022). However, ANN is observed to have serious drawbacks like significant training time, overfitting and difficulty in fine-tuning the architectures which is generally done by trial and error and varies significantly from one problem to another (Tu 1996). Scientists have innovated various approaches to improve over ANN. Kiefa (1998) developed general regression neural network (GRNN) based models to predict bearing capacity of piles in cohesionless soil. Wei *et al.* (2019) applied RBF neural network for predicting the bearing capacity of composite foundation and concluded to outperform traditional ANN. (Armaghani *et al.* 2020) showed the improved performance of neuro-swarm models compared to ANN for estimating pile settlement. (Zeng *et al.* 2021) successfully improvised ANN using bagging and boosting ensemble techniques. (Momeni *et al.* 2020) proposed Gaussian process regression (GPR) based simulation model for dynamic pile tests. Although the ANN and Fuzzy Logic (FL) have a range of merits, they also have certain flaws. The benefits of both ANN and FL are achieved by Neuro-Fuzzy models. Adaptive Neuro Fuzzy Inference System (ANFIS), an integration of neural network (NN) and AI based fuzzy logic, behaves like NN in learning phase and as FL in execution phase and have been proved to improve over ANN and FL individually (Atsalakis *et al.* 2018, Pezeshki and Mazinani 2019). (Kumar *et al.* 2021, Bai *et al.* 2021, Chen *et al.* 2021, Huang *et al.* 2021, Kumar *et al.* 2021, Xie *et al.* 2021, Xie *et al.* 2021) concluded that ANFIS is more reliable than GP, RVM and GRNN for prediction of bearing capacity of pile foundation. Extreme learning machine (ELM) is another development in neural networks having feed-forward mechanism and single layer or multiple layers of hidden nodes. Kumar and Samui (2019) applied ELM and MARS to reliability analysis of pile foundation and concluded that ELM is more robust than MARS.

The paper proposes novel PSO-ANN (hybrid model of ANN and particle swarm optimization), PSO-ANFIS (hybrid model of ANFIS and particle swarm optimization), and PSO-ELM (hybrid model of ELM and particle swarm optimization) based prediction models for bearing capacity of pile foundation. There is a rising tendency to combine ANN with metaheuristic optimization techniques to increase ANN simulation performance. To increase the performance of the neural network, optimization techniques are utilized to optimize various parameters such as weight and bias. Particle swarm optimization (PSO) based ANN is concluded to give satisfactory results and outperform traditional ANN based model (Murlidhar *et al.* 2020). PSO-ANFIS has limited but encouraging application in foundation engineering (Moayedi *et al.* 2020, Moayedi and Rezaei 2021, Ray *et al.* 2021, Xu *et al.* 2022, Yuan *et al.* 2022), however, it has not been applied in bearing capacity of pile so far. Hybrid ELM models have very limited applications in geotechnical engineering so far (Chen *et al.*

2021, Kardani *et al.* 2021, Yu *et al.* 2021). ELM-PSO has not been applied for the pile foundation problems however its application in other domains have very encouraging (Jing 2019, Li *et al.* 2020, Zeng *et al.* 2021, Zhang and Phoon 2022). In the present paper, the performances of ANN-PSO, ANFIS-PSO and ELM-PSO are compared to each other and a novel hybrid ensemble model is proposed. Fitting and assessing models on a dataset is a common part of applied machine learning. While we can't predict which model will perform best on the dataset advance, we may have to do a lot of trial and error until we find a model that works well or best for our project. Another alternative is to make several separate models and then integrate their results. This is known as an ensemble machine learning model, or simply an ensemble. Ensemble have not so far been applied in foundation engineering, however it has been successfully applied in other streams of geotechnical engineering and found robust (Bardhan *et al.* 2021, Bharti *et al.* 2021, Zhang *et al.* 2021).

The models are trained and tested on field data of dynamic pile tests. Dynamic testing of pile (PDA test) is an innovative method to determine the load capacity of piles (Fellenius 1999, Rausche *et al.* 1985, Smith 2002). The details of the available in detail in literature and need not be discussed here. Dynamic tests are being encouraged due to being cheaper and requiring less time and effort and at the same time, giving the results analogous to static tests (Basarkar 2011, Bradshaw and Baxter 2006, Lin *et al.* 2022, Liu *et al.* 2020, Long 2007, Nayak *et al.* 2000, Rausche *et al.* 2004, Sakr 2013, Shi *et al.* 2022, Wei *et al.* 2021, Wengang Zhang *et al.* 2022, Wei Zhang *et al.* 2022). If the soil is highly variable, the results of static tests become highly unreliable. The dataset of 257 dynamic pile tests is taken from the literature (Armaghani *et al.* 2021) for training and testing the models. The details of the dataset are presented in the section 3.1.

2. Research significance

Computers have revolutionized practically every sector of the economy during the last several decades. Machine learning is one of the most fascinating tools to enter the engineering toolkit in recent years. Because of recent developments in machine learning, we are now at the start of an even larger and more rapid shift. It has already demonstrated its ability to significantly accelerate both fundamental and applied research. Machine learning (ML) introduces numerous new and unique techniques to engineering, boosting system efficiency, flexibility, and quality. Machine Learning focuses on training computers using historical data so that they can make human-like judgements. A model is the algorithm (series of instructions) that is used to train a computer. Traditionally, experiments were the most important part of solving engineering difficulties. Experimental research must be carried out over a long period of time due to the high resource and equipment needs. Pile tests are highly expensive and thus the number of tests conducted are practically very few in number which is often insufficient to

take into account highly variable nature of soil-pile interaction problems. The empirical relations proposed in the various studies are unable to simulate strongly non-linear correlations in the pile bearing capacity problems. That's why, at the moment, there is an explosion of work developing and applying machine learning models to estimate the bearing capacity of piles. Since machine learning methods have only recently been introduced to materials science, many published applications are extremely simple in nature and complexity.

The study proposes state-of-art hybrid neural network model, hybrid neuro-fuzzy model and hybrid ELM model along with state-of-art ensemble technique to enhance the quality of predictions. The study along with fellow researches in the domain can revolutionize the methodology of estimating the bearing capacity of pile foundation. The study will aid in smooth transition from faulty traditional wisdom of designing engineering constructions based on limited field tests to ML based smart and reliable methodology. The significance of this study resides in the fact that developing an intelligent model takes less time than other methods for calculating pile carrying capacity. From an economic standpoint, the proposed intelligent model is advantageous and can be used for early bearing capacity assessment, at least in exploratory investigations. The study can be extended to other civil engineering problems once a reliable database is created. The only drawback of the ML models is that biased and non-inclusive data gives misleading results. In the study, the authenticity of database is established in section 4, however, the models need to be tested on more variety of datasets of different origins to attain a reliable ML based alternative to traditional methods.

3. Methodology and theoretical background

3.1 Details of models and meta-heuristic optimization algorithms

3.1.1 Artificial neural network

ANN is a common approximation tool for simulating and predicting output that was created by simulating the human body's neural system. The input layer, hidden layer, and output layer are three parallel layers coupled by weights and biases (Mohamed *et al.* (2013)). In feedforward ANN models, backpropagation (BP) is the most often used learning method. It employs the gradient descent optimization method. The ANN can manage complicated and non-linear association between input and output variables known for its high neural interconnectivity. Users can change the number of neurons in the hidden layer to get the best results. ANNs have been used as an effective soft computing technology for many applications such as function approximation and pattern recognition in several engineering fields by utilizing such a structure.

3.1.2 Adaptive neuro-fuzzy inference system

Intelligent systems are built using soft computing techniques such as neural networks and fuzzy set theory

(Mohabbi *et al.* 2017). For the sake of simplicity, the fuzzy inference system under investigation is assumed to contain two inputs and one output. The fuzzy inference system, which includes fuzzy analysis and the most common fuzzy structure, always includes fuzzy rules. The rules are made up of fuzzy propositions and linguistic variables, and they are summarized as follows

$$\text{If } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z \text{ is } f(x, y) \quad (1)$$

If a rule is invalid, it should be omitted from the computation in a fuzzy system; otherwise, it should be included. A and B are fuzzy sets in the antecedents, while $z = f(x, y)$ is a crisp function in the following. $f(x, y)$ is generally a polynomial function for the input variables x and y . It might, however, be any other function that can approximately characterise the system's output inside the antecedent's fuzzy region. When $f(x, y)$ is constant, a zero order Sugeno fuzzy model develops, which is a special case of the Mamdani fuzzy inference system, in which each rule consequence is characterized by a fuzzy singleton. If $f(x, y)$ is a first order polynomial, a first order Sugeno fuzzy model is created. The following are the two rules of a first-order Sugeno fuzzy inference system

$$\begin{aligned} \text{Rule 1: If } x \text{ is } A_1 \text{ and } y \text{ is } B_1 \text{ then } f_1 &= p_1 + q_1 + r_1 \\ \text{Rule 2: If } x \text{ is } A_2 \text{ and } y \text{ is } B_2 \text{ then } f_2 &= p_2 + q_2 + r_2 \end{aligned} \quad (2)$$

A type-3 fuzzy inference system was presented by Takagi and Sugeno and it is used here. In this inference system, the output of each rule is a linear combination of the input variables plus a constant term. The final result is a weighted average of each rule's output.

3.1.3 Extreme learning machine

One of the major drawbacks of the feed-forward neural networks is sluggish learning speed. ELM is a single layer feed-forward NN (Ghani *et al.* 2021). Hidden nodes are assigned randomly using Moore-Penrose generalized inverse in ELM, unlike gradient-based learning methods where the network's parameters are tuned iteratively. Thus, the learning becomes extremely fast, thus the name 'Extreme Learning machine'. Since input weight and biases are set randomly and remains frozen, the output weight is linear and doesn't require iterations. The output of ELM is given by equation

$$y_i = \sum_{j=1}^h \beta_j g(W_j X_j + b_j) \quad j=1, \dots, N \quad (3)$$

Here,

W_j and b_j are input weight and biases respectively for i^{th} hidden neuron, h is the total number of hidden neurons, N is total number of neurons, X_j is the input vector for j^{th} sample. β_j is weight matrix of the output layer corresponding to i^{th} hidden neuron and g denotes neuron activation function.

3.1.4 Particle swarm optimization (PSO)

PSO is a widely used optimization technique which belongs to swarm intelligence family, proposed by Kennedy and Eberhart in 1995. The metaheuristic algorithm imitates the behaviors of swarm of birds. They share information among themselves, lands at a particular place function to

various factors like food availability the danger of presence of slayers etc. Once proper place is searched after long search for the same, entire flock lands. The birds are the 'particles' in PSO (potential solutions) and possess some fitness value. The particle owns some velocity and flies through the population in search of the best solutions. The Swarm population P is represented by

$$P = (p_1, p_2, p_3 \dots p_n) \quad (4)$$

The velocities of the individual particles u are denoted by

$$u = (u_1, u_2, u_3 \dots u_n) \quad (5)$$

Previously visited best location (l best) is shown as

$$l = (l_1, l_2, l_3 \dots l_n) \quad (6)$$

The swarm is updated as follows {for ($i=1, 2, \dots, n$) and k being current iteration}

$$u_i^{k+1} = w^k u_i^k + d_1 k_1^k (l_i^k - w_i^k) + d_2 k_2^k (l_g^k - w_i^k) \quad (7)$$

$$w_i^{k+1} = w_i^k + v_i^{k+1} \quad (8)$$

where n is total dimension, l_g denotes the best particle and superscripts depict the number of iterations. w is weight and d_1, d_2 are two learning factors called cognitive and social parameters respectively (position constants). Best performance of the model requires proper tuning of the two position constants. k_1 and k_2 are uniformly distributed random numbers in range 0 to 1.

Unlike evolutionary algorithms, PSO doesn't use Darwinian principles of 'survival of fittest' or genetic operators. In PSO, sociometric principle of exchange of information between the experience of the individual swarm and best performer is the working principle (Gaitonde and Karnik 2012).

3.1.5 Hybridization of Neural Network models (ANN, ANFIS, ELM) and metaheuristic algorithms

Since Back Propagation (BP) fails to identify an accurate global minimum, ANN models may produce undesirable results. Furthermore, ANN models are more prone to becoming trapped in local minima. In PSO-ANN, PSO assigns weights and biases to solve the ANN problems (Armaghani *et al.* 2020). Selecting the optimum number of particles is very critical in PSO. The swarms tend to fall in local optima if the number of particles is low and high number of particles shoots up computational time. The individual particles are treated as a neural network and its fitness can be determined using cost functions. The position and velocity of the individual particles are updated in each iteration till it attains optimum fitness. The optimum global best weight and biases are fed into the neural network and it produces optimum output. The output weight in ELM is determined by random initialization and pseudo-inverse matrix, however, it's performance can be further enhanced using optimization techniques like PSO. It's worth noting that weights and biases in initialization may have non-optimal values which results in unsatisfactory performance. In order to get an expected outcome, ELM requires a large number of hidden layer nodes, which could lead to

overfitting. This work uses PSO to optimize the ELM parameters. Weight and biases in the hidden layer are the potential solution in PSO-ELM and the root mean square error (RMSE) is the fitness function. In PSO-ANFIS the two parameters of ANFIS are updated using PSO: (a) consequent parameters and (b) premise parameters. The membership functions and the member parameters are fine-tuned as per fitness function i.e., lower RMSE.

3.1.6 Ensembling unit:

In this section by merging different model outputs, assembling strategies as post-process approaches have demonstrated the ability to improve model prediction (Kardani *et al.* 2021). It has been proven that using a collection of relatively simple models is less dangerous than using a single model, which is more complicated and expensive method. The ensembling technique were used to combine the output of the ANN-PSO, ANFIS-PSO, and ELM-PSO models in this study and the flowchart is given in Fig. 1. The Eq. (9) was used to implement the proposed linear ensembling procedures, which included weight average.

$$\bar{X}_u(t) = \sum_{i=1}^n w_i X_{u_i}(t) \quad (9)$$

Where $\bar{X}_{u_i}(t)$ is the i th individual model output, in this study output of ANN-PSO, ANFIS-PSO, and ELM-PSO output, $\bar{X}_u(t)$ is the output of the linear ensemble model and n is the total number of single models. In this study value of n is taken as three. w_i is denoted as the applied weight on the i th model represented as

$$w_i = \frac{DX}{\sum_{i=1}^n DX_i} \quad (10)$$

Where DX_i is represented as determination coefficient of the i^{th} individual model.

4. Data processing and analysis

4.1 Data processing and computation of models

It is fundamental to normalize the inputs and output variables with a prescribed range in machine learning methods to enhance model accuracy. The objective of normalization is to transform numeric data values to a uniform scale with no ambiguous value range variances (Biswas *et al.* 2020). The procedure isn't needed for all machine learning datasets, but it is necessary when the parameters have varying ranges like in the present study. The parameters have been normalized in the range 0 to 1 using Eq. (11).

$$x_{\text{Normalized}} = \frac{(x_{\text{Actual}} - x_{\text{min}})}{(x_{\text{max}} - x_{\text{min}})} \quad (11)$$

4.2 Descriptive statistics and statistical visualization:

The 257 datasets of five input parameters and one output

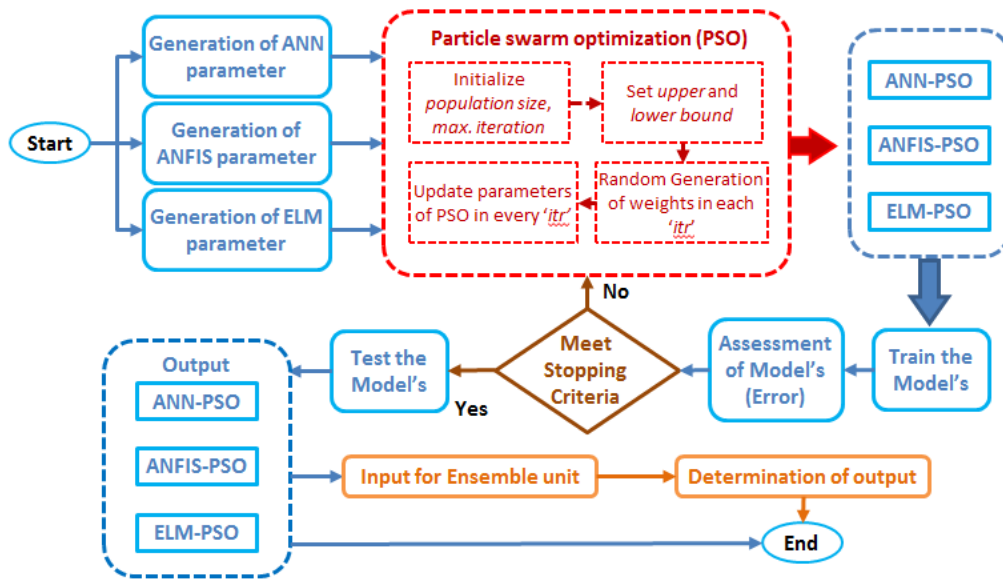


Fig. 1 Flowchart of proposed model with metaheuristic algorithm

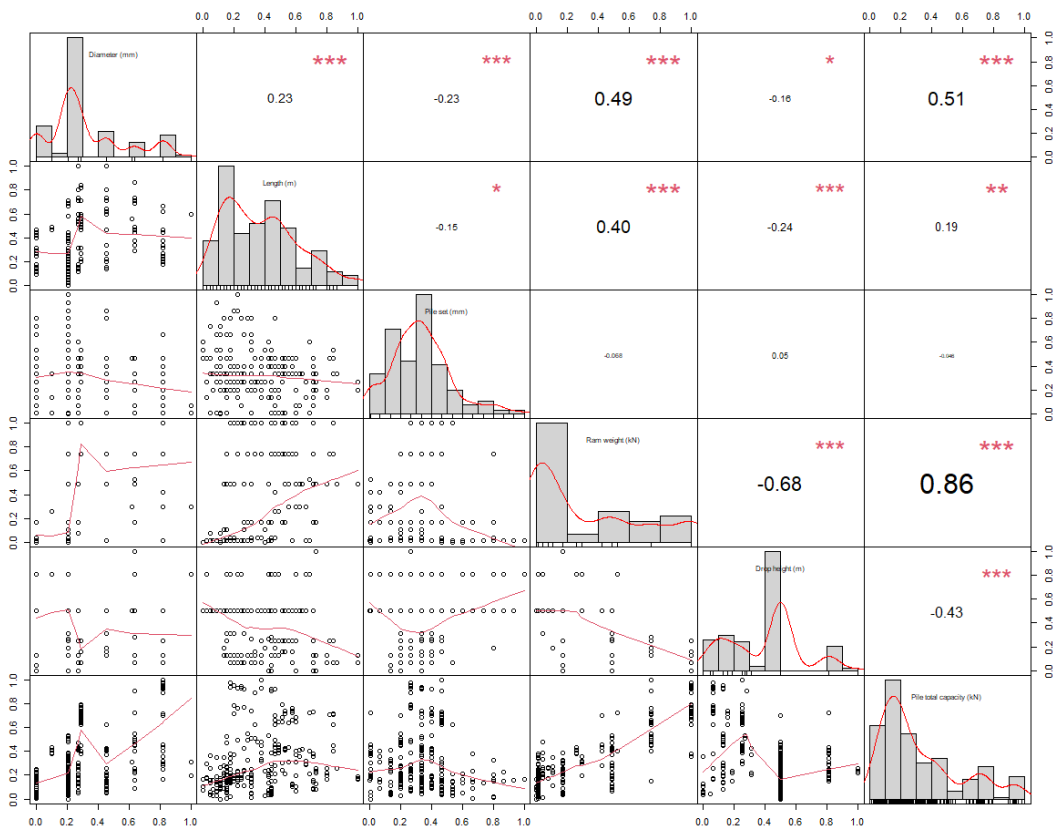


Fig. 2 Correlation matrix of data

parameter are taken from literature. The input parameters are diameter (mm), drop height (m), Ram weight (kN) and length (m) while Ultimate pile capacity (kN) is the output parameter. The descriptive statistics of the dataset is presented in Table 1. As it can be inferred from the table, the sample variances (scattered in the range of 8.02 to 4358)

indicates wide range of input parameters. The variance for the output parameter is also very high. The values of standard error are also scattered in wide range from 0.18 to 53.95 and thus confirms the credibility of the dataset. The frequency histogram of the dataset is given in Fig. 2.

Table 1 Descriptive statistics of the dataset

	Diameter (mm)	Length (m)	Pile set (mm)	Ram weight (kN)	Drop height (m)	Pile total capacity (kN)
Minimum	226.00	3.00	0.00	12.00	0.20	291.00
Mean	312.51	19.64	4.91	36.56	0.83	1367.11
Standard Error	4.12	0.66	0.18	1.73	0.02	53.95
Median	300.00	17.00	5.00	25.00	1.00	1040.00
Mode	282.00	10.00	5.00	13.00	1.00	2790.00
Maximum	500.00	48.00	15.00	90.00	1.80	3680.00
Standard Deviation	66.02	10.51	2.83	27.81	0.38	864.94
Sample Variance	4358.63	110.39	8.02	773.50	0.14	748129.84
Kurtosis	0.17	-0.33	0.96	-0.78	-0.55	0.13
Skewness	0.92	0.60	0.67	0.83	0.15	1.08
Range	274.00	45.00	15.00	78.00	1.60	3389.00

4.3 Sensitivity analysis

In general, sensitivity analysis (SA) is a technique that is used to determine how changes in input parameters affect the response of the proposed models. This will assist us in identifying the input parameters based on their influence on the result. The Cosine Amplitude Method (Biswas *et al.* 2021) is used in this work to calculate the amount of influence of the inputs on the response, i.e., the bearing capacity of pile foundation. The data pairings in this study are represented in a data array, X , as follows

$$X = \{x_1, x_2, x_3, \dots, x_i, \dots, x_n\} \quad (12)$$

and variable x_i in X , is a length vector of m as

$$x_i = \{x_{i1}, x_{i2}, x_{i3}, \dots, x_{im}\} \quad (13)$$

The correlation between the strength of the relation (R_{ij}) and datasets of x_i and x_j is provided by

$$R_{ij} = \frac{\sum_{k=1}^m x_{ik}x_{jk}}{\sqrt{\sum_{k=1}^m x_{ik}^2 \sum_{k=1}^m x_{jk}^2}} \quad (14)$$

The graphical representation of R_{ij} shows the relation between the bearing capacity of soil and the input parameters as shown in Fig. 3. SA reveals that, the ram weight (W) has the greatest influence on pile total capacity with strength value 0.92 followed by diameter of pile (D) with strength value 0.81. The parameters, length(L) and pile set (PS) have a strength of about 0.72 and 0.66, respectively. Whereas, the drop height has the minimum effect on the capacity of pile, i.e., 0.54. It can be concluded that all the five parameters have higher influences on the pile bearing capacity and hence considered in predicting the output. In addition, a combined analysis was done, and the output of the findings is given in the form of a Doughnut diagram in Fig. 4, from which the degree of relevance of each parameter can be clearly deduced. The same things noticed here that, the significance of all the parameters lies between 25.17% and 14.67% which is almost equal to their average significance value.

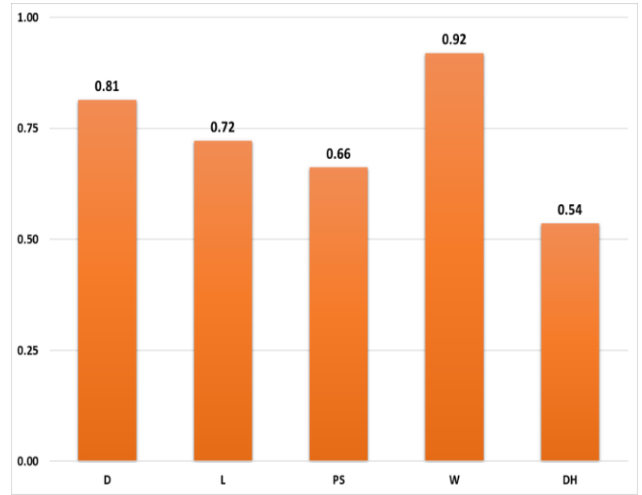


Fig. 3 Sensitivity Analysis using bar chart

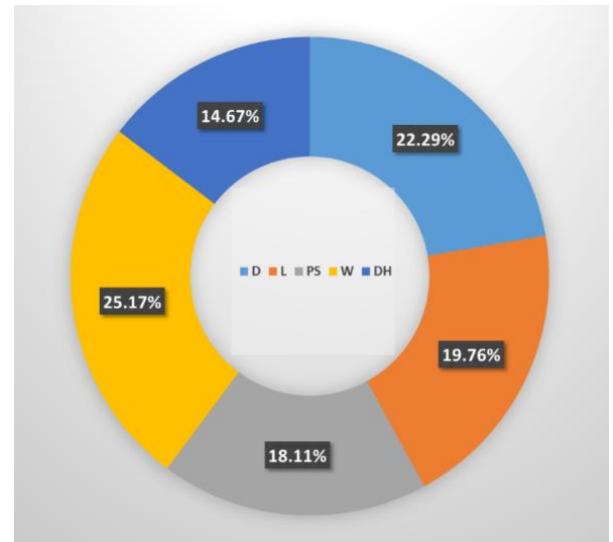


Fig. 4 Doughnut Diagram of SA

4.4 Performance parameters

For the performance assessment of the developed models, the following ten performance statistical parameters have been analyzed in this study namely, coefficient of determination (R^2), performance index (PI), Nash–Sutcliffe efficiency (NS), Willmott's index of agreement (WI), variance account factor (VAF), root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), root mean square error to observation's standard deviation ratio (RSR), weighted mean absolute percentage error (WMAPE) (Biswas *et al.* 2019, Chai and Draxler 2014, Kardani *et al.* 2021, Nash and Sutcliffe 1970, Pradeep *et al.* 2022, Srinivasulu and Jain 2006, Zhou *et al.* 2022)

$$R^2 = \frac{\sum_{i=1}^N (d_i - d_{mean})^2 - \sum_{i=1}^N (d_i - y_i)^2}{\sum_{i=1}^N (d_i - d_{mean})^2} \quad (15)$$

$$PI = adj.R^2 + (0.01 \times VAF) - RMSE \quad (16)$$

Table 2 Ideal values of performance indices

Indices	R ²	PI	NS	WI	VAF	RMSE	MAE	MAPE	RSR	WMAPE
Ideal value	1	2	1	1	100	0	0	0	0	0

$$NS = 1 - \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (y_i - y_{mean})^2} \quad (17)$$

$$WI = 1 - \left[\frac{\sum_{i=1}^N (d_i - y_i)^2}{\sum_{i=1}^N \{|y_i - d_{mean}| + |d_i - d_{mean}|\}^2} \right] \quad (18)$$

$$VAF = \left(1 - \frac{var(d_i - y_i)}{var(d_i)} \right) \times 100 \quad (19)$$

$$MSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (d_i - y_i)^2} \quad (20)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - d_i| \quad (21)$$

$$MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{d_i - y_i}{d_i} \right| \times 100 \quad (22)$$

$$RSR = \frac{RMSE}{\sqrt{\frac{1}{N} \sum_{i=1}^N (d_i - d_{mean})^2}} \quad (23)$$

$$WMAPE = \frac{\sum_{i=1}^N \left| \frac{d_i - y_i}{d_i} \right| \times d_i}{\sum_{i=1}^N d_i} \quad (24)$$

where d_i is the observed i^{th} value, y_i is the predicted i^{th} value, d_{mean} is the average of observed value, N is the number of data sample, Note that, for an ideal model, the values of these indices should be equal to their ideal values, the details of which are presented in Table 2

5. Results and discussion

5.1 Simulation of developed model

All the models were developed in MATLAB environment with MATLAB 2015a version and version with i3-8130U CPU @ 2.20 GHz, 12.00 GB RAM. The computational cost of the models was noted as 69.32 s (PSO-ANN), 79.2 s (PSO-ELM) and 75.3 s (PSO-ANFIS). The going through the trial-and-error method, the configuration of best performing developed models is provided in Table 3 where population size (p), the maximum number of iterations (i), lowerbound (lb), upper bound (ub) and the number of hidden neurons (n) are

Table 3 Configuration of optimum hybrid ANN models

Parameters	PSO-ANN	PSO-ELM	PSO-ANFIS
p	30	50	20
n	15	15	15
i	200	200	200
lb	-1	-1	-1
ub	+1	+1	+1
c_1	1.5	1	0.2
c_2	2.5	2	0.3
cost function	RMSE	RMSE	RMSE

mentioned for the ANN, ELM and ANFIS. The cognitive constant (c_1) and the social constants (c_2) of PSO optimization is also mentioned in the table while taking RMSE as a cost function.

After developing the models, their performance will be discussed in the following subsection. Rank analysis is the most straightforward and widely used method for determining the effectiveness of developed models and comparing their robustness. The statistical parameters are used to assigned the score value in this study, with their ideal values serving as the benchmark. It depends on how many models are used. The greatest score is given to the best performing results model, and vice versa. The ranking ratings for two models with the same outcomes may be the same. The overall score of a model is calculated by adding the scores value of training phase and testing phase. The equation used for calculate the total score is given as

$$Total\ score = \left[\sum_{i=1}^m X_i + \sum_{j=1}^n X_j \right] \quad (25)$$

Where X_i and X_j is the Score of the performance indicators for training and testing phase respectively. Number of performance indicators in the training and testing phase is represented by m and n respectively. For example, in this study ensemble unit of all three models achieve the highest score (62) after that ANFIS-PSO hybrid model achieve (59) score value followed by ELM-PSO (56) and ANN-PSO (23) hybrid model.

From table 4, it can be observed that, though the overall performance of ensemble unit was the best among all the other models but the ANFIS-PSO has given a slightly better result in testing ($R^2 = 0.8490$, $VAF = 84.8505$, $RMSE = 0.0994$, and $RSR = 0.3899$) and the ELM-PSO has given the better performance in training ($R^2 = 0.8857$, $VAF = 88.5683$, $RMSE = 0.0859$, and $RSR = 0.3381$). Similar conclusion can be derived from the score analysis where the score of ANFIS-PSO is the highest (35) in testing whereas the ELM PSO has the highest in Training with the value of 33. While with the overall score of 62, ensemble unit is the overall best performing. It is evident that the lowest performing model for is ANN-PSO for both training ($R^2 = 0.7899$, VAF

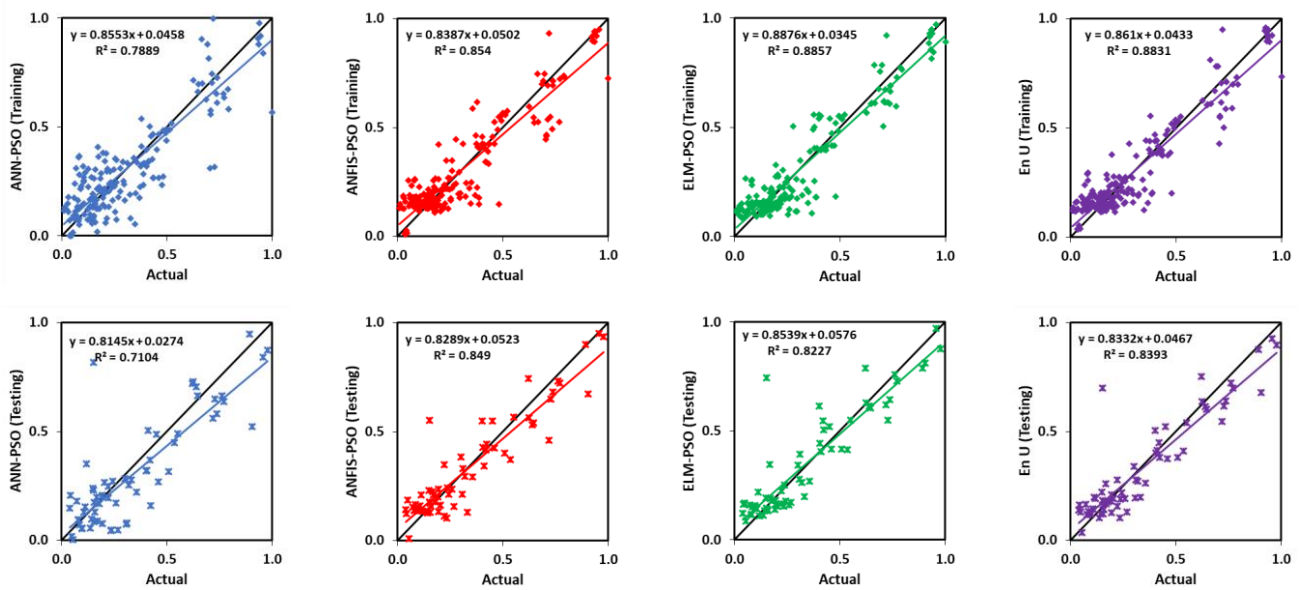


Fig. 5 Actual vs predicted graph of proposed models

Table 4 Performance evaluation of Developed model with Rank analysis

Parameter	ANN-PSO		ANFIS-PSO		ELM-PSO		En U		
	TR	TS	TR	TS	TR	TS	TR	TS	
R²	Value	0.7889	0.7104	0.8540	0.8490	0.8857	0.8227	0.8831	0.8393
	Score	1	1	2	4	4	2	3	3
WMAPE	Value	0.2817	0.3013	0.2246	0.2118	0.2090	0.2085	0.0733	0.2040
	Score	1	1	2	2	3	3	4	4
RMSE	Value	0.1183	0.1451	0.0972	0.0994	0.0859	0.1080	0.0503	0.1026
	Score	1	1	2	4	3	2	4	3
VAF	Value	78.3346	69.5173	85.3696	84.8505	88.5683	82.1488	88.1530	83.9287
	Score	1	1	2	4	4	2	3	3
PI	Value	1.4483	1.2355	1.6065	1.5851	1.6824	1.5209	1.7111	1.5621
	Score	1	1	2	4	3	2	4	3
RSR	Value	0.4655	0.5695	0.3825	0.3899	0.3381	0.4237	0.4537	0.4028
	Score	1	1	3	4	4	2	2	3
MAPE	Value	66.6401	50.4271	61.5207	40.260	55.7671	40.8096	30.8638	38.326
	Score	1	1	2	3	3	2	4	4
WI	Value	0.9412	0.9123	0.9585	0.9566	0.9690	0.9511	0.9326	0.9544
	Score	2	1	3	4	4	2	1	3
MAE	Value	0.0881	0.1022	0.0703	0.0718	0.0654	0.0707	0.0229	0.0692
	Score	1	1	2	2	3	3	4	4
MBE	Value	0.0005	-0.0356	-0.0003	-0.0057	-0.0007	0.0080	0.0017	-0.0099
	Score	3	1	4	4	2	3	1	2
Sub Total		13	10	24	35	33	23	30	32
Total Score		23		59		56		62	

= 78.33, $RMSE = 0.1183$, and $RSR = 0.4655$) and testing ($R^2 = 0.7104$, $VAF = 69.51$, $RMSE = 0.1451$, and $RSR = 0.5995$). From the above results, it can be concluded the results have been significantly improved by using different hybrid models.

Fig. 5 represent the scatterplot between the actual and predicted values of the proposed models for training and testing datasets separately. The performance of proposed

model will increase as the points get closer to regression line. The best model was selected by these values are more or less equivalent to one in this chart indicating a relationship between both axes and R^2 . Based on the performance of this curve the ELM-PSO hybrid model outperforms other models, while the ANN-PSO is the least performing model and ensemble unit performance was also better than all model's performance.

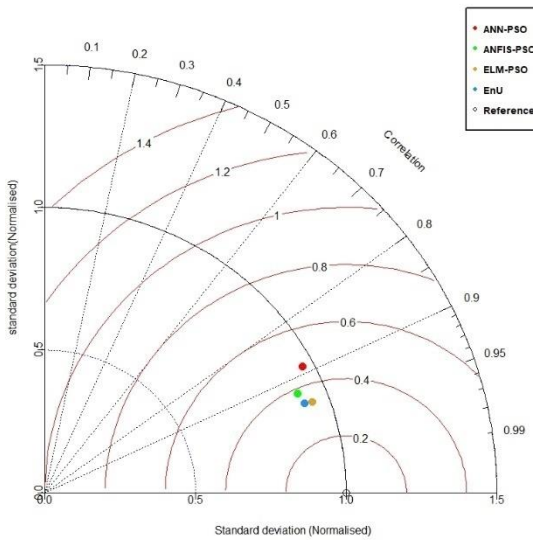


Fig. 6 Taylor diagram for training dataset

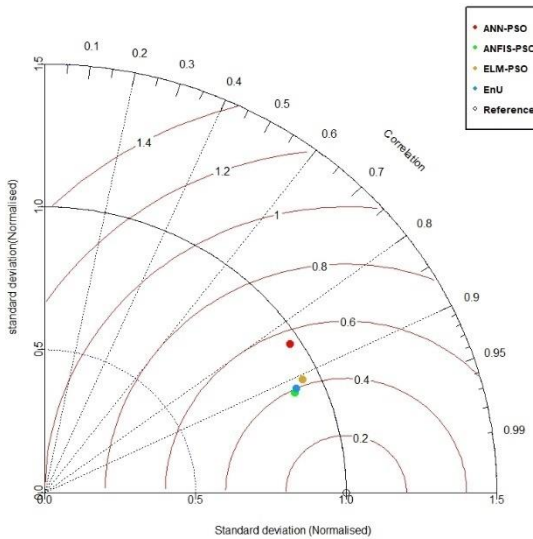


Fig.7 Taylor diagram for testing dataset

5.2 Taylor diagram

Taylor diagrams are a basic graphical representation of how anticipated values correlate to observed values, and they are used to compare the performance of various simulation models (Taylor 2001). It plots standard deviations, correlation coefficients, and root-mean square (RMS) difference in a 2-D graph to show statistical comparison of multiple models. The radial distance from the origin is used to represent the standard deviation. The RMS error is proportional to the gap between actual and predicted fields as measured in standard deviation units. The azimuthal angle represents the correlation coefficient. Taylor diagram for the developed models are represented in Figs. 6 and 7 for training and testing respectively. In the Fig. 6, the closest model for training is ANFIS-PSO followed by ensemble unit. While for Fig. 7, it can be

	ANN-PSO (TR)	ANFIS-PSO (TR)	ELM-PSO (TR)	En U (TR)	ANN-PSO (TS)	ANFIS-PSO (TS)	ELM-PSO (TS)	En U (TS)
R ²	21%	15%	11%	12%	29%	15%	18%	16%
WMAPE	28%	22%	21%	7%	30%	21%	21%	20%
RMSE	12%	10%	9%	5%	15%	10%	11%	10%
VAE	22%	15%	11%	12%	30%	15%	18%	16%
PI	28%	20%	16%	14%	38%	21%	24%	22%
RSR	47%	38%	34%	45%	57%	39%	42%	40%
MAPE	67%	62%	56%	31%	50%	40%	41%	38%
WI	6%	4%	3%	7%	9%	4%	5%	5%
MAE	9%	7%	7%	2%	10%	7%	7%	7%
MBE	0%	0%	0%	0%	4%	1%	1%	1%

Fig. 8 Error matrix

concluded that ENN-PSO is the best performing model in testing followed by Ensemble unit. It is evident from the Taylor diagram that the ensemble unit performs well for both the dataset, while ANN-PSO is the most underperforming model among the all.

5.3 Error matrix

The Error Matrix, is a tool for displaying the correctness of a model. Figure 8 depicts the amount of error associated with hybrid models based on numerous performance parameters in this section. Further details of error matrix are presented in (Pradeep *et al.* 2021). In this study the error values for indices R² in the range of 11% to 29%. Similarly, error value for indices RMSE, WMAPE, VAE, PI, RSR, MAPE, WI, MAE, and MBE are obtained in the range of 5% to 15%, 7% to 30%, 11% to 30%, 14% to 38%, 34% to 57%, 31% to 67%, 3% to 9%, 2% to 10%, and 0% to 4% respectively in models for training and testing dataset. Finally, in terms of total inaccuracy, which spans from 0% to 67%, all models are roughly comparable. Mostly, all models' errors were around nearly 19% when comparing purpose En U has been consider as a best one.

5.4 Uncertainty analysis

Uncertainty analysis is performed to analyse the uncertainty of the developed predictive hybrid models. In tis section the potential of predictive trained models has been analysed. The error mean \bar{e} of the prediction error are presented in Eq. (26).

$$\bar{e} = \frac{1}{n} \sum_{i=1}^n e_i \quad (26)$$

Where, e_i represent as each data error in prediction. The positive value of mean error represents the trained model is overestimate the actual values and vies-versa. Using the Wilson score technique without continuity correction, a confidence band can be generated around the predicted

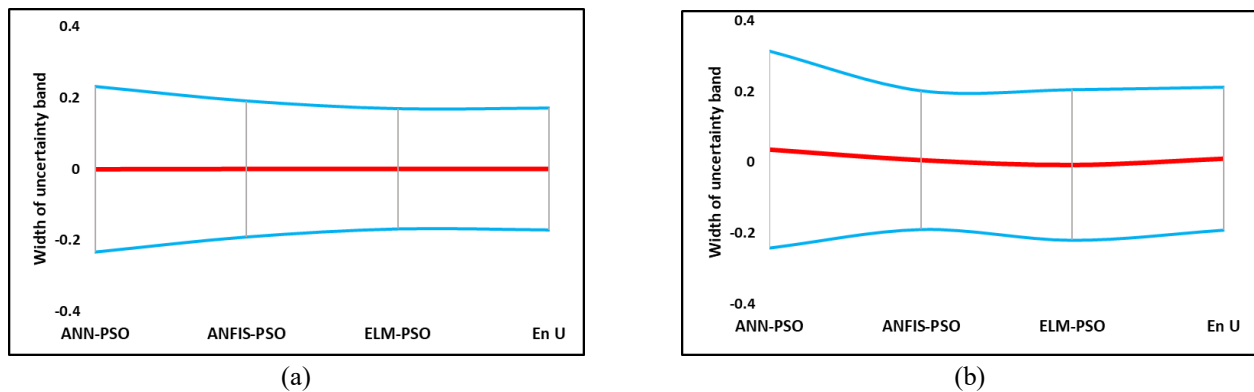


Fig. 9 Comparisons of uncertainty band width for all models in (a) training and (b) testing

Table 4 Performance evaluation of Developed model with Rank analysis

Model		Mean prediction error	Width of uncertainty band	95% prediction error interval
ANN-PSO	Train	-0.00054	0.232474	-0.23301 to 0.231937
	Test	0.03556	0.277972	-0.24241 to 0.313532
ANFIS-PSO	Train	0.000306	0.191038	-0.19073 to 0.191344
	Test	0.005739	0.195962	-0.19022 to 0.201701
ELM-PSO	Train	0.000709	0.168867	-0.16816 to 0.169576
	Test	-0.00803	0.21272	-0.22075 to 0.204692
En U	Train	0.000184	0.171197	-0.17101 to 0.171381
	Test	0.009878	0.201836	-0.19196 to 0.211714

values of an error using the error mean and standard deviation values. Table 5 represent the results of uncertainty analysis for the developed model and Fig. 9 represent the compression of uncertainty band width of all developed models including ensemble unit. The ANN-PSO attains higher uncertainty (0.2324 and 0.2779) for training and testing data compared to other two models including ensemble unit while the ensemble unit has the lowest amount of certainty with the range of -0.17101 to 0.171381.

6. Conclusions

The present study proposes soft-computing based alternative model for prediction of bearing capacity of pile foundation. It gives the comparative assessment of the performance of three hybrid regression models for prediction of bearing capacity of pile foundation and a novel ensemble learning model is proposed by combining the predictions of PSO-ANN, PSO-ANFIS and PSO-ELM. The models are trained and tested on the dataset of 257 PDA tests from literature. Data validation is carried out using descriptive statistics, sensitivity analysis and plotting correlation matrix and thus, the robustness of the data used is confirmed. Sensitivity analysis reveals that ram weight and pile diameter are the most influential parameter in dynamic pile test while drop height has least correlation with the output.

The simulation results of the models conclude that the ensemble unit ($R^2 = 0.84$, RMSE = 0.1, uncertainty = 0.2))

achieves the highest rank in the rank analysis performed using various popular performance parameters and lowest uncertainty value in the uncertainty analysis. Among hybrid models, PSO-ANFIS ($R^2 = 0.85$, RMSE = 0.097, uncertainty = 0.195) turns out to be the best performing model followed by PSO-ELM ($R^2 = 0.82$, RMSE = 0.01, uncertainty = 0.21) and PSO-ANN ($R^2 = 0.71$, RMSE = 0.14, uncertainty = 0.28). The performance of PSO-ANFIS is very encouraging while PSO-ANN is lagging. The ensemble learning model is proved to have advantage of better fitting and high generalization ability and is reliable to apply for the existing dataset. Further, the ensemble model, PSO-ANFIS and PSO-ELM can be extended to other foundation engineering applications once the validity of datasets is confirmed. ANFIS and ELM hybridized with other optimization techniques should be tested for prediction of bearing capacity of piles. Since the model was trained using relatively large sets of real data, the suggested model in this study can serve as a supplement to semi-empirical equations for predicting the bearing capacity when doing in-situ tests is costly and/or problematic.

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Appendix

Table 4 Comparisons of actual vs predicted values of the models used in the study

Training					
S.NO.	Actual	ANN-PSO	ANFIS-PSO	ELM-PSO	En U
1	555	522.8	739.9	794.2	691.2
2	536	591.5	821.8	759.8	728.2
3	850	849.3	828.4	772.1	815.2
4	318	684.3	720.7	672.2	692.3
5	858	722.9	747.9	617.0	694.2
6	979	738.5	751.5	723.2	737.5
7	533	738.5	751.5	723.2	737.5
8	586	1025.9	761.3	739.2	836.1
9	934	973.5	755.4	754.6	823.2
10	749	738.5	751.5	723.2	737.5
11	810	600.7	753.6	679.2	679.8
12	1480	951.2	1126.2	986.3	1022.6
13	1225	988.0	1107.4	1004.5	1034.1
14	829	1327.8	871.8	745.9	970.0
15	605	764.4	843.0	771.5	793.4
16	772	483.8	935.6	1056.5	837.0
17	781	875.7	1016.6	1030.0	977.3
18	500	1104.3	871.5	668.8	873.1
19	585	1025.7	905.6	761.5	892.6
20	603	1063.3	887.0	779.6	904.4
21	770	1201.4	881.0	746.7	933.9
22	748	618.3	903.4	829.4	788.6
23	363	701.4	719.8	626.3	681.3
24	1172	542.6	1034.7	879.9	826.9
25	789	744.7	786.5	805.5	780.1
26	910	1187.4	687.3	915.8	923.3
27	856	1021.1	712.1	968.9	898.5
28	1849	1191.0	1725.1	1698.9	1549.3
29	1230	1346.3	1794.4	2004.1	1728.1
30	1761	1191.0	1725.1	1698.9	1549.3
31	2680	1346.3	1794.4	2004.1	1728.1
32	1770	1191.0	1725.1	1698.9	1549.3
33	660	1276.0	824.7	799.3	956.6
34	790	946.9	930.7	821.8	897.6
35	648	618.3	903.4	829.4	788.6
36	817	848.2	894.5	807.1	849.4
37	553	848.2	894.5	807.1	849.4
38	970	1025.7	905.6	761.5	892.6
39	806	1180.1	835.9	815.2	936.1
40	1149	1496.5	871.6	635.9	984.0
41	862	357.6	980.9	701.6	688.6
42	997	489.5	981.3	1218.7	911.0
43	1106	844.0	1190.5	1131.7	1061.8
44	829	506.5	852.7	783.2	720.4

45	390	892.9	883.3	732.5	833.5
46	410	529.2	823.0	800.6	723.5
47	1300	843.3	949.7	683.7	823.3
48	1537	886.2	922.4	649.9	815.7
49	486	886.2	922.4	649.9	815.7
50	378	791.1	909.1	771.9	824.2
51	567	716.9	853.3	823.8	800.4
52	754	521.2	740.3	839.0	706.5
53	568	567.6	811.2	826.9	740.7
54	949	881.7	1169.7	1222.2	1098.2
55	469	347.2	1152.9	1194.7	916.2
56	718	1334.3	755.8	629.6	892.1
57	642	1132.3	761.3	598.5	820.0
58	1102	1069.0	713.3	873.2	880.3
59	950	771.6	771.2	814.7	786.5
60	2750	1363.8	2147.7	2375.4	1982.9
61	1008	568.3	705.5	741.3	675.2
62	884	714.5	649.1	1043.4	807.6
63	1282	1144.8	1278.4	1182.7	1203.2
64	389	664.1	709.3	710.9	695.8
65	792	578.2	721.5	758.4	689.7
66	626	1376.3	709.4	813.3	953.9
67	475	600.7	753.6	679.2	679.8
68	500	552.7	791.0	812.8	724.3
69	1468	471.7	797.3	817.8	702.9
70	746	851.7	838.0	786.8	824.3
71	783	731.8	798.9	795.7	776.8
72	505	677.2	869.1	807.4	787.6
73	700	542.2	825.0	814.7	733.1
74	751	567.6	811.2	826.9	740.7
75	831	791.1	909.1	771.9	824.2
76	568	464.2	774.0	822.1	694.2
77	1347	1423.2	1723.0	1477.6	1543.5
78	950	1585.9	1713.4	1399.9	1563.8
79	1141	1066.2	1180.1	1199.0	1151.2
80	1392	565.8	1109.3	1155.3	955.9
81	645	1205.1	670.5	920.8	925.0
82	640	796.6	717.1	805.9	773.0
83	1604	1257.2	784.3	901.7	973.0
84	1920	1218.1	787.0	908.7	964.1
85	1015	1295.4	1026.7	1062.7	1123.1
86	1125	1325.3	1009.3	1134.7	1151.8
87	1020	1330.1	1009.6	1061.4	1127.8
88	1150	666.2	846.1	893.5	806.6
89	885	1056.5	875.0	916.4	946.1
90	910	1034.8	1004.5	956.3	997.1
91	1095	1046.1	1018.4	956.6	1005.4
92	916	1260.3	1000.2	985.2	1076.1

93	1576	1404.4	1674.2	1663.7	1586.3
94	1612	1418.9	1654.6	1661.6	1583.5
95	1681	1441.0	1664.0	1637.1	1585.0
96	1750	1480.0	1603.1	1676.2	1590.3
97	1272	1446.0	941.3	687.1	1009.7
98	879	1522.6	1049.3	1021.4	1187.2
99	895	996.6	1187.2	804.3	993.6
100	867	1677.0	1151.7	977.0	1254.4
101	1005	1571.4	921.9	841.0	1096.2
102	328	733.2	781.5	712.0	742.1
103	1572	1682.4	1044.9	1155.3	1282.5
104	1450	1500.0	1032.4	1021.9	1174.6
105	980	1448.4	1003.6	1248.3	1228.1
106	1063	1290.4	1026.4	1318.5	1211.1
107	1058	1102.7	794.9	874.7	918.9
108	942	972.4	844.6	816.6	874.7
109	774	577.1	709.0	730.2	675.3
110	749	752.8	756.0	637.7	713.6
111	588	740.7	751.0	663.6	717.2
112	707	701.0	817.6	782.8	769.0
113	2790	2757.1	2699.0	2630.7	2693.2
114	2900	2440.0	2729.8	2760.8	2650.2
115	3430	3859.8	3335.8	3440.0	3535.8
116	3460	3272.6	3470.0	3501.8	3419.6
117	780	863.7	723.7	698.6	758.6
118	770	863.7	723.7	698.6	758.6
119	740	863.7	723.7	698.6	758.6
120	770	863.7	723.7	698.6	758.6
121	450	291.0	329.5	591.7	409.3
122	410	291.0	309.5	587.3	401.0
123	430	291.0	375.4	689.2	459.0
124	410	291.0	329.5	591.7	409.3
125	1290	1094.8	885.2	881.5	949.3
126	1600	1085.3	878.8	908.0	953.5
127	1380	933.6	900.7	930.1	921.2
128	3490	3407.0	3401.5	3148.4	3314.6
129	3450	3361.7	3417.6	3188.6	3320.0
130	2730	3668.9	3446.6	3410.1	3503.2
131	3470	3602.7	3463.4	3445.5	3500.6
132	3460	3800.2	3311.0	3056.5	3374.5
133	2960	2572.8	2785.0	2916.7	2764.9
134	3420	3799.3	3396.1	3291.4	3485.2
135	1500	1436.8	2271.2	1612.3	1780.1
136	3530	3132.2	3501.9	3577.6	3413.1
137	2790	2757.1	2699.0	2630.7	2693.2
138	3430	3822.7	3318.6	3388.2	3500.3
139	3460	3272.6	3470.0	3501.8	3419.6
140	1980	1938.6	2207.6	2155.9	2105.5

141	1850	1877.4	2065.6	1895.5	1947.3
142	1960	1896.3	2078.5	1930.9	1969.9
143	2020	1940.7	2172.8	2110.4	2078.5
144	1920	1937.0	2139.2	2057.4	2047.4
145	2550	2663.4	2067.5	2364.6	2357.5
146	2780	2504.4	1957.2	2357.6	2268.2
147	2490	2650.6	2049.1	2361.2	2346.1
148	2710	2671.1	2104.3	2373.7	2375.5
149	1650	1629.5	1834.6	2118.7	1870.1
150	2690	2181.6	1847.8	2390.5	2142.0
151	1820	1629.5	1834.6	2118.7	1870.1
152	2690	2244.1	1865.9	2381.8	2164.6
153	2890	2502.2	2061.4	2309.8	2286.0
154	2480	2528.3	2135.1	2354.5	2334.6
155	2410	2709.7	2304.3	2217.8	2400.5
156	1570	2108.4	2375.1	1993.8	2158.3
157	2660	3055.5	2808.0	2955.0	2936.7
158	2540	3351.4	2814.1	2952.2	3030.1
159	2630	3272.8	2704.0	2853.0	2933.6
160	2980	2265.3	2759.0	2887.6	2650.0
161	2710	2815.2	2637.0	2572.4	2670.0
162	2610	2417.1	2649.2	2583.7	2553.8
163	2790	2363.2	2630.8	2546.9	2517.9
164	570	1238.2	1165.8	1095.1	1163.6
165	570	1531.3	1173.4	1172.3	1284.7
166	560	1514.5	1155.7	1149.2	1265.4
167	520	1341.2	1068.7	903.1	1095.8
168	640	1152.5	1056.6	814.2	1001.6
169	710	1049.1	946.1	689.2	888.3
170	850	713.4	802.5	715.9	744.4
171	890	713.4	802.5	715.9	744.4
172	860	1023.9	968.2	851.4	944.7
173	1170	1473.2	1300.6	714.2	1149.0
174	1040	988.1	1086.0	908.0	993.1
175	850	713.4	802.5	715.9	744.4
176	890	713.4	802.5	715.9	744.4
177	1650	1368.6	1727.1	2179.8	1773.8
178	1680	1448.1	1708.6	2162.4	1786.3
179	1560	1368.6	1727.1	2179.8	1773.8
180	1040	739.9	731.9	802.1	759.0
181	1730	1842.8	1433.1	1640.7	1633.7
182	1760	1877.1	1414.7	1631.8	1635.0
183	1700	1995.5	1439.5	1667.8	1692.9
184	1900	1084.9	1941.6	1700.5	1589.9
185	1550	1050.8	1849.3	1957.6	1638.1
186	1170	1120.4	1461.5	1053.7	1212.3
187	1080	1120.4	1461.5	1053.7	1212.3
188	1200	894.5	1081.4	909.7	962.9
189	3680	2211.4	2740.5	3308.4	2774.3

190	2060	2039.7	2231.5	2179.9	2153.6
191	1970	1768.3	2170.1	2167.3	2043.8
192	980	953.9	677.8	837.9	820.0
Testing					
1	623	464.1	739.3	802.2	678.9
2	648	724.5	824.3	768.4	775.2
3	560	600.7	753.6	679.2	682.3
4	427	788.2	760.3	707.2	750.3
5	568	600.7	753.6	679.2	682.3
6	753	477.7	732.7	671.3	635.5
7	669	803.4	997.1	1034.5	952.3
8	452	346.0	914.9	859.6	726.1
9	811	1063.3	887.0	779.6	902.5
10	870	579.6	995.6	931.9	849.5
11	537	897.1	799.2	830.3	839.1
12	1849	1191.0	1725.1	1698.9	1556.8
13	1663	1368.6	1732.3	1791.3	1644.2
14	983	1180.1	835.9	815.2	931.4
15	1330	534.1	998.9	1228.2	939.5
16	956	545.7	728.4	839.3	712.2
17	625	476.4	782.2	826.2	706.2
18	694	1470.7	744.6	666.9	934.3
19	1650	1363.8	2147.7	2375.4	1992.6
20	438	978.7	706.2	842.2	834.4
21	1370	1144.8	1278.4	1182.7	1205.5
22	1501	1031.0	1277.5	1202.1	1177.9
23	819	578.2	721.5	758.4	691.5
24	821	875.8	863.2	766.8	833.7
25	575	521.7	838.8	800.2	730.9
26	825	722.8	825.1	738.8	764.8
27	1096	446.9	1100.7	1146.8	921.6
28	1419	1225.3	725.9	965.6	957.6
29	1194	441.5	812.6	872.1	722.5
30	898	851.4	882.0	900.4	879.2
31	799	1096.1	1061.6	945.3	1031.7
32	1113	1203.8	998.3	895.8	1024.2
33	854	952.9	1045.5	1463.9	1162.4
34	1341	551.2	1404.1	1624.6	1225.9
35	3530	3132.2	3501.9	3577.6	3417.8
36	780	613.5	714.0	742.9	694.0
37	480	291.0	309.5	587.3	399.9
38	3610	3241.7	3449.2	3262.2	3322.7
39	3320	3487.7	3335.9	2967.2	3253.8
40	2870	2533.4	2766.6	2865.0	2731.0
41	2900	2440.0	2729.8	2760.8	2654.1
42	2160	1938.6	2207.6	2155.9	2109.5
43	2730	2181.6	1847.8	2390.5	2134.8
44	2460	2670.3	2085.9	2368.8	2357.9

45	2480	2532.2	2116.7	2342.7	2318.7
46	2760	2486.5	2484.5	2154.7	2371.2
47	2400	2718.0	2801.9	2960.5	2831.6
48	2790	2250.9	2594.1	2476.4	2451.1
49	940	967.2	959.9	766.7	895.4
50	1170	859.1	1079.5	833.2	928.7
51	950	988.1	1086.0	908.0	995.3
52	810	3054.4	2154.4	2814.4	2650.7
53	1090	436.2	630.9	811.1	635.1
54	1310	1245.4	1587.4	1448.7	1437.5
55	1720	1529.2	1690.2	2144.5	1799.1
56	1680	1995.5	1439.5	1667.8	1684.2
57	1730	823.5	1782.3	2006.2	1573.7
58	1040	1120.4	1461.5	1053.7	1219.0
59	2410	2752.6	2190.6	2424.3	2438.9
60	3360	2055.6	2562.1	3046.6	2578.4
61	2110	1805.1	1542.7	1690.0	1671.8
62	2010	1352.8	1641.6	1706.0	1577.7
63	1040	940.6	659.4	850.7	809.3
64	1820	1937.0	2139.2	2057.4	2050.6
