

# Reliability analysis of external and internal stability of reinforced soil under static and seismic loads

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**Abstract.** In this study, the reliability analysis of internal and external stabilities of Reinforced Soil Walls (RSWs) under static and seismic loads are investigated so that it can help the geotechnical engineers to perform the design more realistically. The effect of various variables such as angle of internal soil friction, soil specific gravity, tensile strength of the reinforcements, base friction, surcharge load and finally horizontal earthquake acceleration are examined assuming the variables uncertainties. Also, the correlation coefficient impact between variables, sensitivity analysis, mean change, coefficient of variation and type of probability distribution function were evaluated. In this research, external stability (sliding, overturning and bearing capacity) and internal stability (tensile rupture and pull out) in both static and seismic conditions were investigated. Results of this study indicated sliding as the predominant failure mode in the external stability and reinforcing rupture in the internal stability. First-Order Reliability Method (FORM) are applied to estimate the reliability index (or failure probability) and results are validated using the Monte Carlo Simulation (MCS) method. The results showed among all variables, the internal friction angle and horizontal earthquake acceleration have dominant impact on the both reinforced soil wall internal and external stabilities limit states. Also, the type of probability distribution function affects the reliability index significantly and coefficient of variation of internal friction angle has the greatest influence in the static and seismic limits states compared to the other variables.

**Keywords:** random variables; reinforced soil; reliability; seismic; stability

## 1. Introduction

Soil is one of the important and complicated materials used in geotechnical structures and researchers always try to improve the mechanical properties of soil using various reinforcement methods (Carlos *et al.* 2016). From a mechanical point of view, the reinforced soil is actually a combination of soil and reinforcing material that improves the soil-reinforcement composite material properties. The reinforcements have different types such as strips, nets, rods, fibers, and sheets that provide the soil texture with tensile strength upon loading. The reinforced soil was introduced for the first time by Vidal in 1969 (Vidal 1969). At the beginning of the 1970s, the researchers found a potential in developing a new reinforcing method used for the weak soil layers. After the "Polymer Grid Reinforcement" conference in 1984, the reinforcing soil wall instructions were presented by Jones (1985). In Iran, the first reinforced soil walls were implemented in 1976, which has been applicable and developed so far. In recent

years, due to the uncertainty in soil parameters, using probabilistic and reliability analysis has been considered by researchers and engineers in designing various geotechnical structures. Tobutt (1982) was the first researcher using the Monte Carlo (MC) method to analyze the slope stability reliability analysis. In this study, Bishop stability analysis method was used by taking into account variables with Normal and log Normal distribution functions. The results showed that soil adhesion and internal friction angle are most effective parameters in the stability analysis. Chalermyanont and Benson (2004) conducted a parametric study using Monte Carlo simulation to assess how uncertainty in design parameters affects the mechanically stabilized earth (MSE) walls internal failure probability.

The simulation techniques were used in developing a set of reliability-based design (RBD) estimations for the MSE walls internal stability. Sayed *et al.* (2008) performed the static and dynamic analysis for RSWs using three Methods of First-Order Second-Moment method (FOSM), Point Estimate Method (PEM) and First-Order Reliability Method (FORM). Basha and Babu (2014) have investigated the RSWs reliability with the FORM method to determine the appropriate load and resistance coefficient parameters. The results showed there are various amounts of resistance against sliding, overturning, and bearing failure capacity. Also, the friction angle has important effect on the resistance, but variation of the friction angle has a significant impact on the wall stability states. Jingyu *et al.* (2016) evaluated the internal and external stabilities of

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reinforced retaining wall based on the Checking Point (CP) method and MCS method. The results showed that the reinforced retaining wall internal stability (reinforced materials tensile and anti-uplift) is the key index in reliability analysis; in a large scale, the anti-uplift reliability index and the vertical distance from the reinforcement strip to the wall top are basically inversely proportional; the reinforcement strip anti-uplift reliability index in a wall toe is minimum, so it is the key index; it must be noticed in calculation that the reinforcement strip tensile reliability index present to the retaining walls middle range. Michel (2017) evaluated RSWs internal and external stabilities under static loads, results showed that the internal friction angle is the most effective parameter. Also, the correlation between random variables was investigated in a static condition by Powers.

Zevgolis and Bourdeau (2017) presented a new methodology for reinforced soil walls internal stability reliability assessment taking into account the highly strength-redundant character of these structures. Result showed that with the updated reliability of the reinforced soil walls it can be quantified as failure propagates among different reinforcement layers. GuhaRay *et al.* (2018) examined the geotextile-reinforced walls stability reliabilities using the first order reliability (FORM) and first-order second-moment reliability (FOSM) methods, respectively. Hamrouni *et al.* (2018) presented a reliability study of mechanically stabilized earth walls reinforced by geosynthetic strips. The response surface methodology optimized by a genetic algorithm was used to find the reliability index. Muktinutalapati and GuhaRay (2020) propose the usage of building derived materials as a partial replacement for sand and backfill materials for the retaining walls. Experimental results indicated that earth pressures were not significantly enhanced by building derived materials addition to sand, which suggests that building derived materials could be used as effective lightweight backfills. Chehade *et al.* (2021) presented a thorough study of earth retaining walls seismic behavior using upper bound theorem of limit analysis in combination with discretization technique. The pseudo-static approach is adopted to model the seismic effect.

Ahmed and Basha (2021) proposed a three-wedge failure mechanism to determine active earth pressure exerted by narrow backfills on gravity retaining walls. This study recommends employing an appropriate failure mechanism to develop shear bands behind retaining wall based on the depth and the width of narrow backfill. It also indicated that significant savings in the cross-sectional area may be accomplished as compared to a classical design by Coulomb's theory. Pang *et al.* (2021) proposed generalized probability density evolution method to formulate the system reliability of complex slopes with consideration the uncertainty in multiple slope parameters and ground motions. The results indicate that randomness in ground motions have a much larger influence than randomness in slope parameters, but randomness in slope parameters cannot be completely neglected.

Based on the previous researches, the evaluation of different variables' effect on the RSWs reliability index has

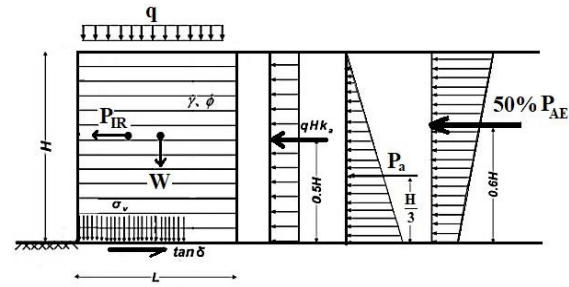


Fig. 1 Forces on the reinforced soil wall (Basha and Babu 2014)

been investigated, so, the internal friction angle, soil specific weight, soil reinforcement, loading, base friction, and earthquake coefficient are random variables that are taken in the RSWs reliability analysis in this research. In contrast to previous studies where the internal and external stabilities of the reinforced soil has been less studied, the seismic and static effects have been also studied simultaneously here.

## 2. Reinforced soil wall stability in static and seismic conditions

RSWs stabilities consist of external and internal stabilities controlling. External stability includes several controls such as sliding, overturning, and bearing capacity.

On the other hand, rupture and pulling out of the reinforcement are another effective factor on internal stabilities (Jones 1985). The analytical relationships for external and internal stabilities for the two cases of static and dynamic states are shown in Fig. 1. Two  $P_{AE}$  and  $P_{IR}$  forces are added to static loads while assessing the reinforced soil wall dynamic stability during an earthquake. Each  $P_{AE}$  represents the horizontal seismic force applied to the RSWs back, and  $P_{IR}$  represents the horizontal seismic force applied to the reinforcement, based on FHWA\* instruction and is used to examine dynamic stability. Traditional limit equilibrium pseudo-static approach has been extensively applied for the seismic design of earth retaining structures due to its theoretical simplicity and readily implementation. Within this framework, the Mononobe-Okabe theory (1926) is the most popular and typical approach for dynamic earth pressure estimation under a ground motion defined by a horizontal seismic coefficient of  $a_h$  and a vertical seismic coefficient of  $a_v$ . It should be noted that in the seismic forces calculations based on FHWA recommendation, the Mononobe Okabe method has been used. In this study, bearing capacity of the soil is considered according to the Vesic theory (Vesic 1962), geotextile-reinforced type, and the granular soil embankment. The Vesic bearing capacity calculation method was developed as an improvement and an extension of the earlier Terzaghi method with slightly more accurate shear failure surface.

\* Federal Highway Administration

Table 1 Analytical equations to estimate the RSWs external stabilities (Choudhury *et al.* 2007)

$f_s = \frac{\tan\delta(\gamma HL + qL)}{k_a \gamma \frac{H^2}{2} + k_a qH}$	(1)	Sliding	
$f_s = \frac{\gamma \frac{HL^2}{2} + q \frac{L^2}{2}}{k_a \gamma \frac{H^3}{6} + k_a q \frac{H^2}{2}}$	(2)	Overturning	External Static Stability
$f_s = \frac{q_{ult}}{\sigma_v} = \frac{0.5\gamma L N_\gamma}{\gamma H + q}$	(3)	Bearing Capacity	
$f_s = \frac{\text{Reinforcement tensile strength}}{\text{Force applied to any reinforcement}} = \frac{T}{\sigma_v k_a V}$	(4)	Rupture	Internal Static Stability
$f_s = \frac{2\tan\mu(\gamma z + q)L_e}{\sigma_v k_a V}$	(5)	Pullout	Internal Static Stability
$f_s = \frac{\tan\delta(\gamma HL + qL)}{k_a \gamma \frac{H^2}{2} + k_a qH + 0.5P_{AE} + P_{IR}}$	(6)	Sliding	External Seismic Stability
$f_s = \frac{\gamma \frac{HL^2}{2} + q \frac{L^2}{2}}{k_a \gamma \frac{H^3}{6} + k_a q \frac{H^2}{2} + 0.5P_{AE} 0.6H + P_{IR} \frac{H}{3}}$	(7)	Overturning	External Seismic Stability
$k_a = \tan^2\left(45 - \frac{\varphi}{2}\right)$			(8)
$P_{AE} = 0.375k_{ae}\gamma H^2$			(9)
$P_{IR} = 0.5k_{ae}\gamma H^2$			(10)
$k_{ae} = \frac{\cos^2(\varnothing - \theta_w - \psi)}{\cos\psi \cos^2\theta \cos(\delta + \theta_w + \psi) \left[1 - \sqrt{\frac{\sin(\delta + \varnothing) \sin(\varnothing - \beta - \psi)}{\cos(\delta + \theta_w + \psi) \cos(\beta - \theta_w)}}\right]}$			(11)
$\psi = \tan^{-1} \frac{k_h}{1 \pm k_v}$			(12)

In these equations,  $\varphi$  is the internal soil friction angles,  $\gamma$  is the specific weight,  $\delta$  is the friction coefficient between backfill and base,  $\mu$  is the friction coefficient between soil and geosynthetic,  $H$  is the wall height,  $L$  is the reinforcement length,  $k_a$  is the active pressure coefficient,  $T$  is the long term design strength of geosynthetic reinforcements,  $N_\gamma$  represents bearing capacity factor,  $V$  is the effective geosynthetic surface,  $q$  is the surcharge loading,  $\sigma_v$  is the vertical pressure,  $L_e$  is the effective length of geosynthetic,  $P_{AE}$  represents the horizontal seismic force applied to the RSWs back,  $P_{IR}$  represents the overall horizontal seismic force applied to the whole reinforcement layers,  $k_{ae}$  is the Mononobe-Okabe seismic active pressure,  $k_h$  is the horizontal acceleration,  $k_v$  is the vertical acceleration,  $\theta_w$  is the reinforced soil wall angle relative to the vertical line,  $\beta$  is the wall face angle relative to the horizon and  $\psi$  is the resultant earthquake force angle relative to the vertical line.

### 3. Reliability in the structural design

At first, the reliability index ( $\beta$ ) is expressed as the ratio of mean ( $\mu_g$ ) to the standard deviation ( $\sigma_g$ ) of the limit state function ( $g$ ), which is called the Cornell reliability index (Melchers and Beck 2018).

$$\beta_c = \frac{\mu_g}{\sigma_g} \quad (13)$$

Since this index could be changed by altering the form of the limit state function ( $g$ ), Hasofer and Lind presented a new method to solve this problem. They introduced this new reliability index as the smallest distance from the origin to the limit state function in a Normal standard space (Griffiths and Fenton, 2007). Generally, reliability analysis methods are divided into three types of analytical methods, First and Second-Order Reliability (FORM and SORM) methods and simulation-based techniques. In designs, balance between safety and construction costs should be considered. It is possible to design when having a precise prediction of performance, but it is not feasible in reality, therefore confidence boundaries must be considered. These boundaries are often considered in the form of an increased safety factor. Also, increasing the confidence level leads to more conservative design and higher costs. Hence, the safety coefficient cannot be considered as an ideal criterion for important projects. Since the safety factor is expressed as a fixed number, it does not take any tolerance in executive matters and the inherent uncertainties existing in the soil into account. In geotechnical problems, the indeterminacy and variability in soil resistance parameters are seen frequently, and using the deterministic analysis does not seem to be very reasonable due to the deterministic parameters. Therefore, using random variables and reliability methods is the best way to express these uncertainties. These uncertainties are defined by random variables that have special probability distribution functions (their statistical parameters). The parameters selection in the geotechnical engineering field, has been one of the

Table 2 Limit functions used to study the reinforced soil walls' internal and external stabilities (Choudhury *et al.* 2007)

$g_1 = R - Q = (\tan\delta(\gamma HL + qL)) - (k_a\gamma \frac{H^2}{2} + k_a qH)$	(18)	Sliding	
$g_2 = R - Q = \left( \gamma \frac{HL^2}{2} + q \frac{L^2}{2} \right) - \left( k_a\gamma \frac{H^3}{6} + k_a q \frac{H^2}{2} \right)$	(19)	Overturning	External Static Stability
$g_3 = R - Q = (0.5\gamma LN_\gamma) - (\gamma H + q)$	(20)	Bearing Capacity	
$g_4 = R - Q = T - \sigma_v k_a V$	(21)	Rupture	Internal Static Stability
$g_5 = R - Q = (2\tan\mu(\gamma z + q)L_e) - (\sigma_v k_a V)$	(22)	Pullout	Internal Static Stability
$g_4 = R - Q = (\tan\delta(\gamma HL + qL)) - \left( k_a\gamma \frac{H^2}{2} + k_a qH + 0.5P_{AE} + P_{IR} \right)$	(23)	Sliding	
$g_5 = R - Q = \left( \gamma \frac{HL^2}{2} + q \frac{L^2}{2} \right) - \left( k_a\gamma \frac{H^3}{6} + k_a q \frac{H^2}{2} + 0.5P_{AE} 0.6H + P_{IR} \frac{H}{3} \right)$	(24)	Overturning	External Seismic Stability

challenges faced by engineers. In geotechnical engineering, the correlation between random variables is irrefutable, while ignoring this effect in the soil can affect the answer to the problem. The correlation factor is one of the important parameters in reliability calculations. Correlation coefficients can be calculated empirically from statistical equations according to the following formula.

$$\rho_{xy} = \frac{CoV(X,Y)}{\sqrt{\sigma^2(X)\sigma^2(Y)}} \quad -1 \leq \rho_{xy} \leq +1 \quad (14)$$

In this equation,  $CoV(X, Y)$  is a covariance between two random variables as a measure of dependence between the both  $X$  and  $Y$ , which is measurable from the following equation:

$$CoV(X, Y) = E[(X - \mu_x)(Y - \mu_y)] \quad (15)$$

where  $E$  is the expectation value.

#### 4. The limit state function of the RSWs internal and external stabilities

By defining the limit state function in reliability, we can describe the destruction and safety extent, and in fact, it shows the boundary between the desirable and undesirable performances in a specific structural condition. The final limit state function can be expressed simply as follows

$$g(R, Q) = R - Q \quad (16)$$

In the problem, three following conditions can occur for the limit state function:

$$\begin{cases} g(R, Q) = R - Q > 0 \rightarrow \text{safety} \\ g(R, Q) = R - Q = 0 \rightarrow \text{Limit state} \\ g(R, Q) = R - Q < 0 \rightarrow \text{Failure} \end{cases}$$

where  $R$  corresponds to the resistance or capacity parameters effects and  $Q$  corresponds to the load effect parameters, and the failure probability can be achieved as follows (Phoon and Ching 2018)

$$P_f = P(R \leq Q) \quad (17)$$

Thus, regarding the above concept, the limiting

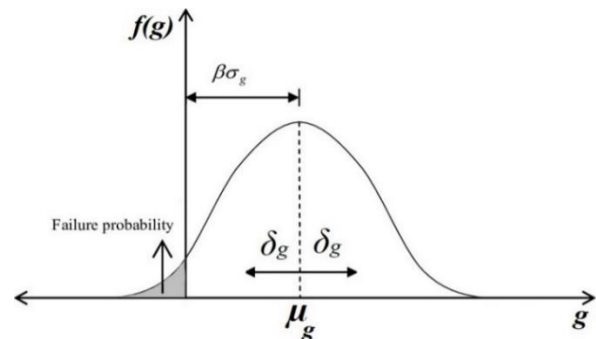


Fig. 2 Probability density for the limit State Function (Phoon and Ching 2015)

functions used in this study are considered separately in Table 2.

#### 5. Reliability index

Reliability analysis includes a set of methods that consider uncertainties using some statistical variables, and the probabilistic distribution type for these variables based on the limit state function. Reliability analysis has two main components, namely random variables expressing uncertainty and also the rupture or failure limit function to calculate the failure probability. In reliability analysis, there is an important quantity called the reliability index. The reliability index ( $\beta$ ) is expressed by the ratio between the mean ( $\mu_g$ ) and standard deviation ( $\sigma_g$ ) of the limit state function ( $g$ ). Fig. 2, shows a geometric explanation of the reliability index in a one-dimensional space. The main idea of the Reliability Index is the distance from the measurement point of the mean to the level of the limit state, which is equal to the difference of the standard deviation to limit function, the longer the distance, the safer the structure will be (Melchers and Beck 2018).

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{(\sigma_R^2 + \sigma_Q^2)}} \quad (25)$$

Table 3 Expected performance index based on reliability index (Phoon and Ching 2015)

Failure Probability	Reliability Index( $\beta$ )	Expected Performance
0.16	1	Dangerous
0.07	1.5	Unsuitable
0.023	2	Bad
0.006	2.5	Less than Average
0.001	3	More than Average
0.00003	4	Good
0.000003	5	Excellent

reliability index is more than 3.

Table 4 Citation data in this research

Variable	Average	Variation Coefficient (Percentage)	Distribution Type	Reference and Data Source
$\phi$ (degree)	30-45	5-20	Log norm Normal	Chun <i>et al.</i> (2014), Sayed <i>et al.</i> (2008), Basha and Babu (2014), Basha and Babu (2010), Jingyu <i>et al.</i> (2016), Powers (2017), Pang <i>et al.</i> (2018)
$\gamma$ (kN/m <sup>3</sup> )	16-18	3-10	Normal	Genske <i>et al.</i> (1991), Sayed <i>et al.</i> (2008), Basha and Babu (2014), Basha and Babu (2010), Jingyu <i>et al.</i> (2016), Powers (2017), Pang <i>et al.</i> (2018)
$q$ (kPa)	12-20	0-20	Normal	Basha and Babu (2014), Jingyu <i>et al.</i> (2016)
$\delta$ (degree)	20-23	5-20	Log norm Normal	Chun <i>et al.</i> (2014), Sayed <i>et al.</i> (2008), Basha and Babu (2014), Basha and Babu (2010), Jingyu <i>et al.</i> (2016)
$T$ (kN)	45-50	0-10	Normal	Chun <i>et al.</i> (2014), Jingyu <i>et al.</i> (2016), Powers (2017), Pang <i>et al.</i> (2018)
$k_h$	0-0.3	0-30	Normal	Sayed <i>et al.</i> (2008), Basha and Babu (2014), Jingyu <i>et al.</i> (2016)
$\rho_{\phi\gamma}$		0.5-0.8		Sayed <i>et al.</i> (2008), Basha and Babu (2014), Jingyu <i>et al.</i> (2016)
$L$ (m)		5-10		Fixed
$H$ (m)		6-10		Fixed
<ul style="list-style-type: none"> <li>Assume a correlation between random variables</li> <li>Wall height 6 to 10 meters - Reinforcement length and wall width 5 to 10 meters</li> </ul>				Research assumptions: <ul style="list-style-type: none"> <li>Cohesionless Granular Soil</li> <li>Fill and Foundation Soil</li> </ul>

### 6. The failure probability and reliability index calculation

Calculating the failure probability allows the engineering judgment to be according to the location, conditions, and the structures' importance. In fact, reliability analysis methods transfer the uncertainties of input parameters to output values of the function. In other words, the input parameters uncertainty effect on the output values will apply. Table 3, presents the expected performance indicator for evaluation by the US Army Corps of Engineers regarding expected structural performance and the degree of reliability index or failure probability (Pang *et al.* 2018). According to this table, the structure type will be safe if the

### 7. Introducing model and random variables for RSWs stabilities

In this research, first by introducing random variables, statistical parameters including mean, standard deviation, variation coefficient and probability distribution function type are introduced and then the reliability index will be calculated. In order to investigate each variable's effect, sensitivity analysis is carried out to determine what effect each variable has on the reliability index and the failure probability. In the first stage, the limit condition functions, i.e., the boundary between health and failure, must be determined based on failure modes. In this research, external and internal stabilities with limit functions are

Table 5 Input hypotheses limit on functions for calculating reliability index and failure probability

Variables	Average	Variation Coefficient (Percentage)	Distribution Type
$\phi$	30	20	Normal
$\gamma$	16	10	Normal
$q$	20	15	Normal
$\delta$	23	10	Normal
$T$	50	10	Normal
$K_h$	0.3	30	Normal
$L$		5 m	
$H$		6 m	
$\rho_{\phi\gamma}$		0.8	

Table 6 External stabilities reliability index under static conditions without correlation between variables

	Sliding		Overturning		Bearing Capacity	
	$\beta$	$P_f$	$\beta$	$P_f$	$\beta$	$P_f$
FORM	2.29	0.0108	6.96	$1.69 \times 10^{-12}$	2.39	0.0083
MCS	2.27	0.0115	6.95	$1.71 \times 10^{-12}$	2.38	0.0085

Table 7 External stabilities reliability index under static conditions with correlation between variables

	Sliding		Overturning		Bearing Capacity	
	$\beta$	$P_f$	$\beta$	$P_f$	$\beta$	$P_f$
FORM	2.21	0.013	6.16	$3.45 \times 10^{-10}$	2.34	0.0094
MCS	2.19	0.014	6.17	$3.32 \times 10^{-10}$	2.34	0.0095

Table 8 Pullout reliability index under static conditions (without correlation between variables)

Layer Number	Geosynthetic Length	Placement Depth	FORM		MCS	
			$\beta$	$P_f$	$\beta$	$P_f$
1	3.27	0.5	3.85	$5.74 \times 10^{-5}$	3.86	$5.53 \times 10^{-5}$
2	3.27	1.5	3.85	$5.74 \times 10^{-5}$	3.86	$5.53 \times 10^{-5}$
3	3.27	2.5	3.85	$5.74 \times 10^{-5}$	3.86	$5.53 \times 10^{-5}$
4	3.55	3.5	3.93	$4.24 \times 10^{-5}$	3.95	$3.95 \times 10^{-5}$
5	4.13	4.5	4.05	$2.50 \times 10^{-5}$	4.02	$2.8 \times 10^{-5}$
6	4.71	5.5	4.15	$1.637 \times 10^{-5}$	4.10	$2 \times 10^{-5}$

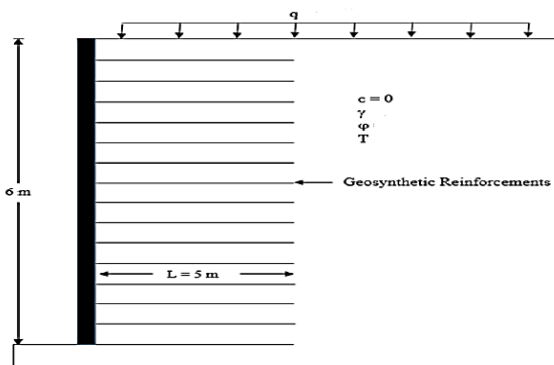


Fig. 3 The reinforced soil wall geometry

considered according to Tables 1 and 2. The hypotheses of this research are presented in Fig. 3 and Table 4. In this table, the variation coefficient and mean value and the variables distribution type are cited according to various sources (the last column of Table 4).

## 8. Results and discussion

The random variables and their statistical characteristics considered in our research are presented in Table 4. Also, reliability analysis is carried out based on input data presented in Table 5. The calculations performed by MATLAB 9.1 and RT software in reliability analysis both with and without parameters correlation are shown in Table 6-13. In the second validation step, the first-order reliability analysis (FORM) results were also compared with the Monte Carlo simulation method. In the Monte Carlo method, the calculation become more accurate with increasing iterations. The most important advantage of the first-order reliability analysis (FORM) method is to reduce the iterations and costs of calculations. In this research, for each limit function, the number of iterations starts from 1000 and then continues by increasing the iteration numbers until the reliability index remains constant. The results showed very little differences.

Table 9 Pullout reliability index under static conditions (with correlation between variables)

Layer Number	Geosynthetic Length	Placement Depth	FORM		MCS	
			$\beta$	$P_f$	$\beta$	$P_f$
1	3.27	0.5	3.85	$5.74 \times 10^{-5}$	3.86	$5.53 \times 10^{-5}$
2	3.27	1.5	3.85	$5.74 \times 10^{-5}$	3.86	$5.53 \times 10^{-5}$
3	3.27	2.5	3.85	$5.74 \times 10^{-5}$	3.86	$5.53 \times 10^{-5}$
4	3.55	3.5	3.93	$4.24 \times 10^{-5}$	3.95	$3.95 \times 10^{-5}$
5	4.13	4.5	4.05	$2.50 \times 10^{-5}$	4.02	$2.8 \times 10^{-5}$
6	4.71	5.5	4.15	$1.637 \times 10^{-5}$	4.10	$2 \times 10^{-5}$

Table 10 Rupture reliability index under static conditions (without correlation between variables)

Layer Number	Geosynthetic Length	Placement Depth	FORM		MCS	
			$\beta$	$P_f$	$\beta$	$P_f$
1	3.27	0.5	2.49	0.006	2.49	0.006
2	3.27	1.5	2.41	0.007	2.41	0.007
3	3.27	2.5	2.36	0.009	2.36	0.009
4	3.55	3.5	2.31	0.0102	2.31	0.0102
5	4.13	4.5	2.28	0.011	2.28	0.011
6	4.71	5.5	2.25	0.012	2.25	0.012

Table 11 Rupture reliability index under static conditions (with correlation between variables)

Layer Number	Geosynthetic Length	Placement Depth	FORM		MCS	
			$\beta$	$P_f$	$\beta$	$P_f$
1	3.27	0.5	2.49	0.006	2.49	0.006
2	3.27	1.5	2.41	0.007	2.41	0.007
3	3.27	2.5	2.36	0.009	2.36	0.009
4	3.55	3.5	2.31	0.0102	2.31	0.0102
5	4.13	4.5	2.28	0.011	2.28	0.011
6	4.71	5.5	2.25	0.012	2.25	0.012

Table 12 External stability reliability index in seismic conditions without correlation between random variables

	Sliding		Overturning	
	$\beta$	$P_f$	$\beta$	$P_f$
MCS	0.961	0.1680	1.589	0.0559

Table 13 External stability reliability index in seismic conditions with correlation between random variables

	Sliding		Overturning	
	$\beta$	$P_f$	$\beta$	$P_f$
MCS	0.959	0.1687	1.588	0.0560

Parameter correlation comparisons in Tables 6 and 7 shows that the failure probability increases and the reliability index decreases. It is observed that the failure probability due to slipping is reduced by 4%, overturning by 12% and bearing capacity by up to 2%. The predominant failure mode is the slip mode in the external stability study.

In internal stability study of reinforced soil in a static state, the reinforcement material pulling and rupture from

the maximum tension point (collision with the slip line) is evaluated by considering the reinforcement length on both sides of the slip line.

Results were evaluated by applying the parameters both with and without correlations for which the results are presented in Tables 8-11. Rupture is the dominant failure mode in internal stability conditions. In the reinforced soil walls internal stability study, six layers of geosynthetic were used with one meter intervals in which the length  $L_e$  can be different. According to the results, it can be stated that in lower geosynthetic reinforcements, due to the increase in altitude, and the increase in surcharge load, the greater resistance and extraction possibility are reduced while the failure probability will decrease. Also, the rupture reinforcement assessment results in Tables 10 and 11 show that in higher geotextiles, the reliability index increases, and the failure probability decreases, in other words, in the high geotextiles reinforcements, the failure probability is lower.

Seismic stability reliability assessment of the reinforced soil, as well as the static method, with a difference that the forces caused by the earthquake are introduced in the equation. In this study, in order to seismically evaluate the

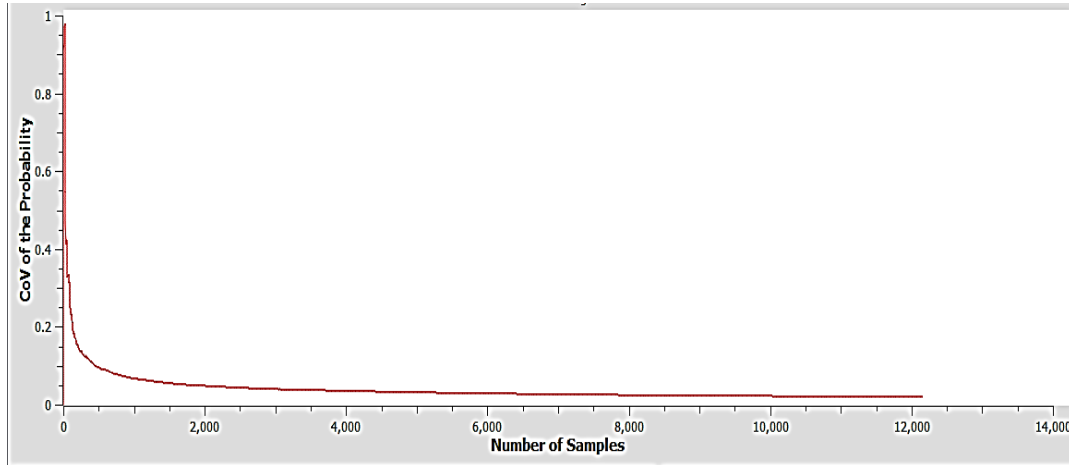


Fig. 4 Iterations for sliding functions under seismic conditions

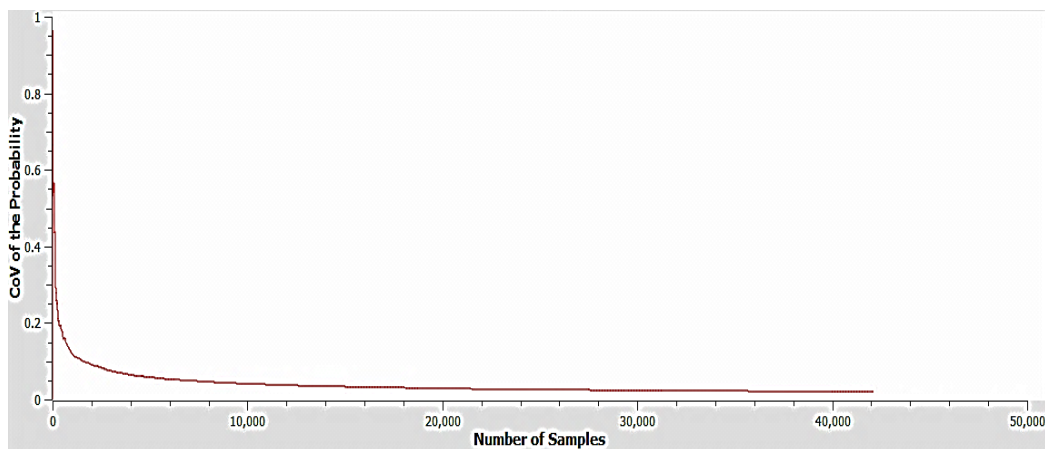


Fig. 5 Iterations for overturning functions under seismic conditions

external stability, the sliding and overturning reliability indices comparisons with the static state are shown in Tables 12 and 13. Results show that the reinforced soil reliability index in seismic conditions are decreased significantly, thus according to the "danger zone" in Table 3, the geometry and technical characteristics of the reinforced soil should be analyzed and designed in a safe range in seismic conditions.

The results of Figs. 4 and 5 show that in Monte Carlo method, the covariance (COV) of seismic limit state function for sliding minimized at 12321 iterations and for overturning it is minimized in 422112 iterations and then stopped. The criterion for stopping computational operations is the convergence of limit functions.

**8.1 External static stability**

In this research, in order to investigate the reinforced soil behavior and to identify effective variables a parametric study has been done by analyzing internal and external stabilities. Figs. 6 and 7 represent the surcharge loading and internal friction angle effects on the reliability index in external stability, respectively. These results indicate that by increasing surcharge load, the reliability index is decreased for sliding, overturning and bearing capacity by 17%, 10%

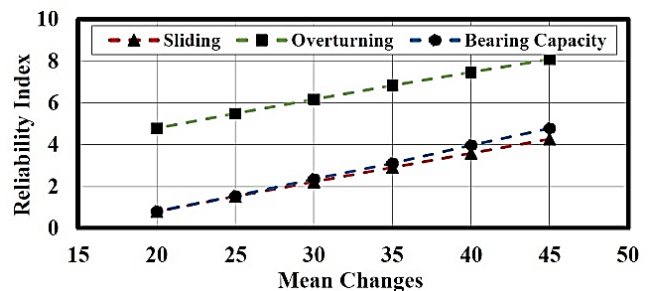


Fig. 6 Reliability index changes against mean internal friction angle changes

and 6%, respectively. These results showed that increasing surcharge load, the failure probability increases and is effective on the sliding and overturning failure modes. These results also indicate the linear incremental effect of the internal friction angle on the reliability index; sliding 81%, overturning 40% and bearing capacity 83%. Figure 8 shows the soil specific weight effects on external stability and reliability index. Soil specific weight varies from 16 to 21  $kN/m^3$ . Results show that by increasing the specific weight, the external stability reliability index increases, especially for overturning.

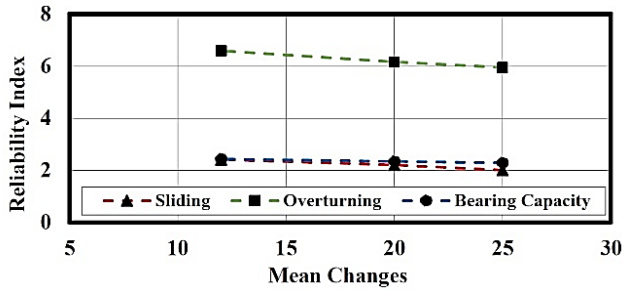


Fig. 7 Reliability index changes against mean surcharge changes

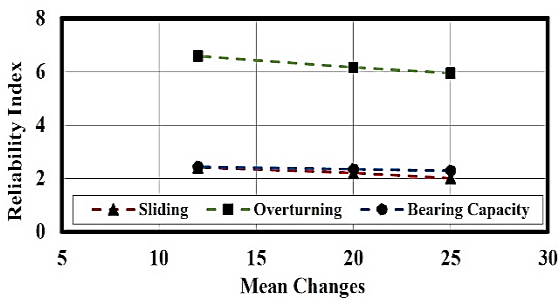


Fig. 8 Reliability index changes vs. mean specific weight changes

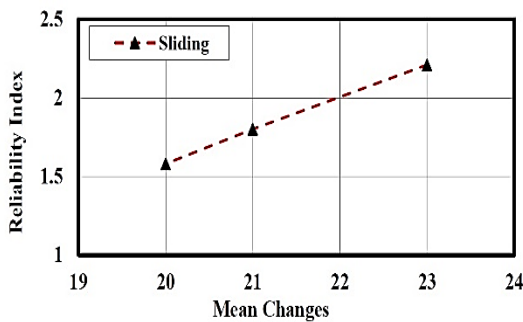


Fig. 9 Reliability index changes vs. mean friction average changes

Fig. 9 shows that the mean variation of base friction ( $\delta$ ) only effects on sliding stability. The positive gradient and increase of this variable in comparison to other variables represent that this parameter has the most effect on the sliding stability after the internal friction angle.

### 8.2 Mean variable changes in static internal stability

In this part, the parametric analysis of the reinforced soil internal stability, including the reinforcement rupture and pulling out are investigated in static condition. It is necessary to select and analyze different geosynthetic reinforcement layers performances. For this purpose, a layer of reinforcement at a 2 meters of depth from the embankment surface is selected (4th reinforced layer is selected) and variables effects are investigated. The reinforcement internal stability is affected by various variables such as surcharge load, soil specific gravity, internal friction angle, base friction and also the tensile strength of the reinforcement, were selected as variables

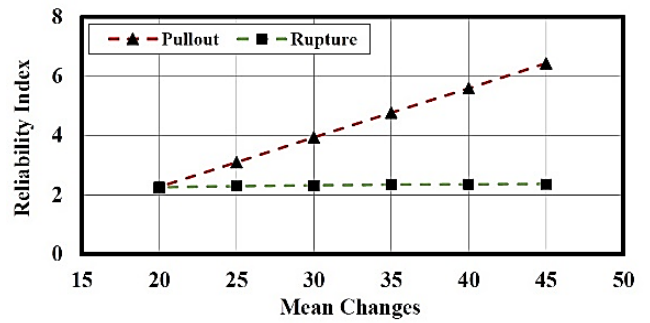


Fig. 10 Reliability index vs. internal friction average angle changes

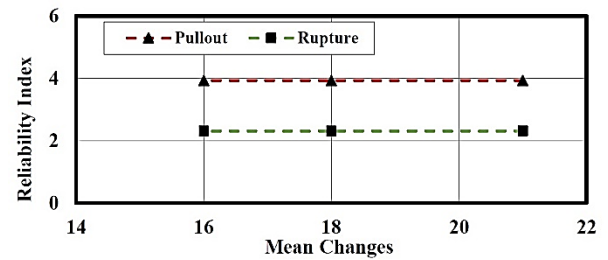


Fig. 11 Reliability index relative to mean specific weight changes

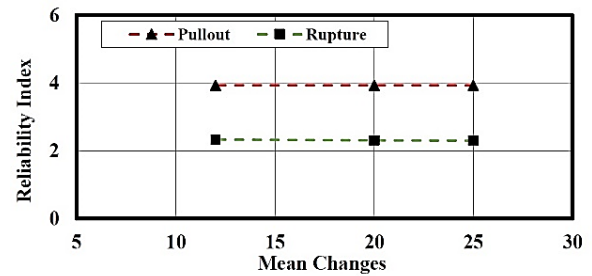


Fig. 12 Reliability index relative to mean surcharge changes

and the reliability index changes relative to their mean changes.

The statistical analysis results are shown in Figs. 10 to 12. Fig. 10 shows the significant effect of internal friction angle on the internal reliability and stability indices, thus by increasing internal friction angle, the reliability index will be very noticeable in the pull-out mode and will increase up to 64% and in the rupture mode will be very small and will increase up to 5%.

It should be noted that the internal friction angle effect on external stability is also tangible and high. Fig. 11 shows that changes in the mean specific gravity ( $\gamma$ ) have little effect on reliability index, so, as the specific gravity increases, the reliability index increases slightly in both pulling out and rupture modes. It should be noted that these results and studies have been performed for a specific depth of the armature. Fig. 12 shows that the change in the mean surcharge load ( $q$ ) on the reliability index changes very little, so, as the average surcharge load increases, the reliability index decreases in both pulling out and rupture modes.

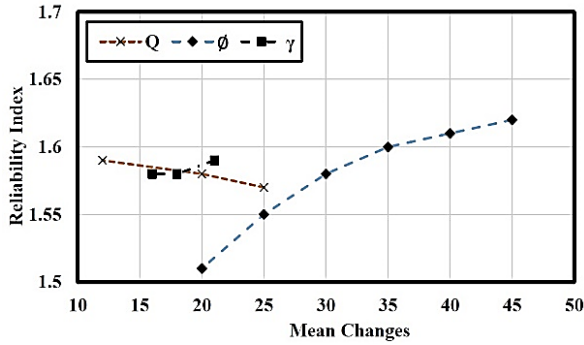


Fig. 13 Reliability index relative to the random variable means in seismic sliding stability

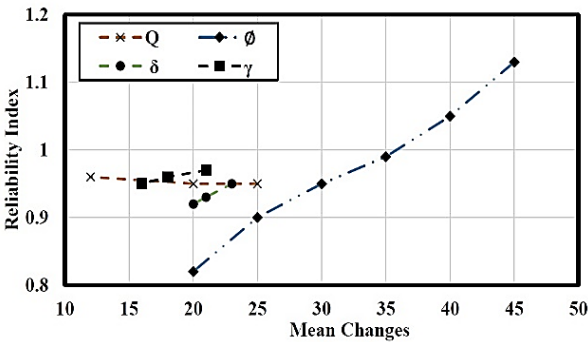


Fig. 14 Reliability index relative to the random variable means in seismic overturning stability

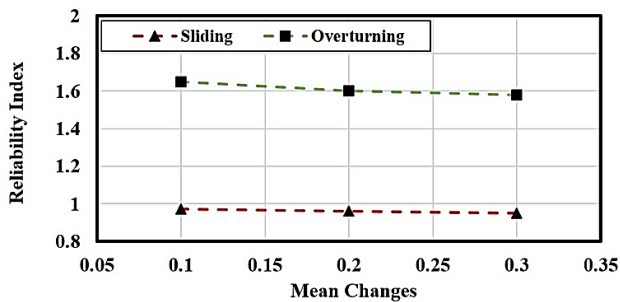


Fig. 15 Reliability index changes relative to  $k_h$  average external seismic stability

### 8.3 Seismic external stability variation review

It was noted that due to the seismic forces of earthquake, reliability index decreases and failure probability increases. In order to investigate these effects, the change in reliability indices relative to the variable means are shown in Figs. 13 and 14.

These results show that in seismic conditions by increasing surcharge load changes, the sliding and overturning reliability indices decrease and failure probability increases.

In seismic analysis, the internal friction angle has the greatest effect on the reliability index and, as a result, its effect is similar to the static external stability state.

Figs. 13 and 14 show that while the internal friction angle increases, the reliability indices in sliding and overturning increase in return. Results also show that the specific gravity has little effects on the reliability index in

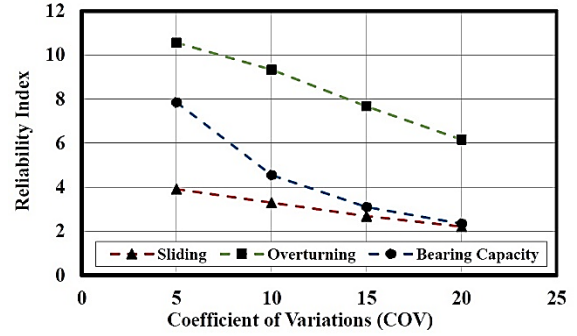


Fig. 16 Reliability index changes due to coefficient of variations (COV) for internal friction angle

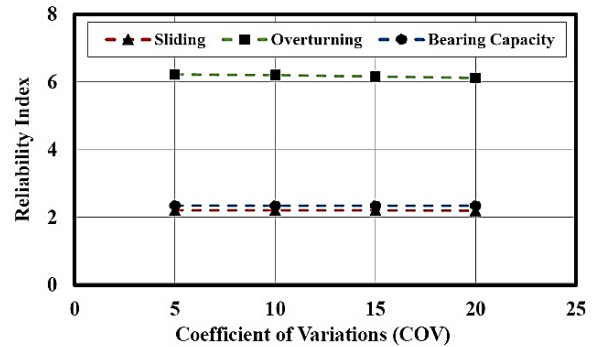


Fig. 17 Reliability index changes due to coefficient of variations (COV) for soil specific gravity

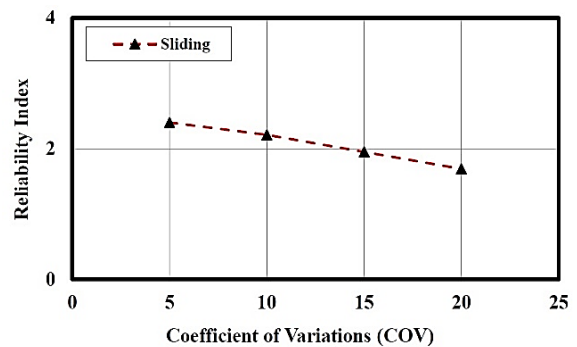


Fig. 18 Reliability index changes due to coefficient of variations (COV) for base friction

seismic conditions and with its increase, the reliability indices in the sliding and overturning modes get a slight increase.

As in the static state in seismic conditions, the base friction is very effective on sliding stability and increases the reliability index. Horizontal earthquake acceleration is one of the factors affecting the analysis and reinforced soil reliability in seismic analysis. In this research, quasi-static coefficients of 0.1, 0.2 and 0.3 have been used. Obviously, by increasing the earthquake acceleration, reliability index decreases and failure probability increases. Results of this analysis on sliding and overturning stability are shown in Fig. 15.

### 8.4 Coefficient of Variation Evaluation (COV) on static external stability

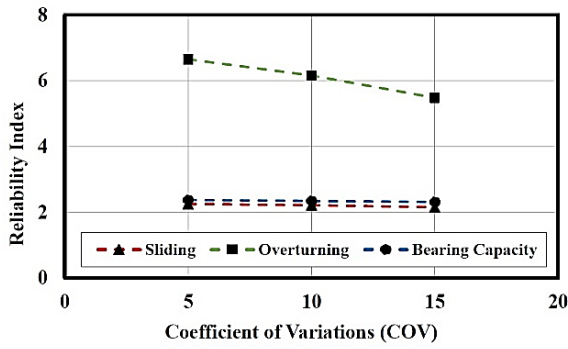


Fig. 19 Reliability index changes due to coefficient of variations (COV) for surcharge load

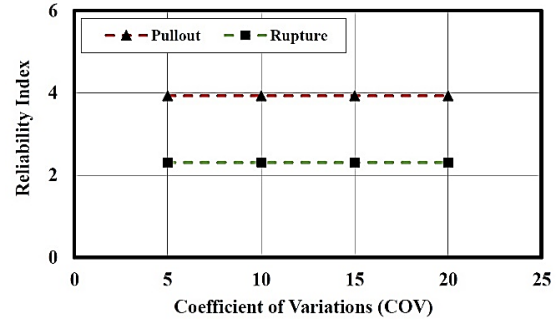


Fig. 21 Reliability index changes due to coefficient of variations (COV) for specific weight

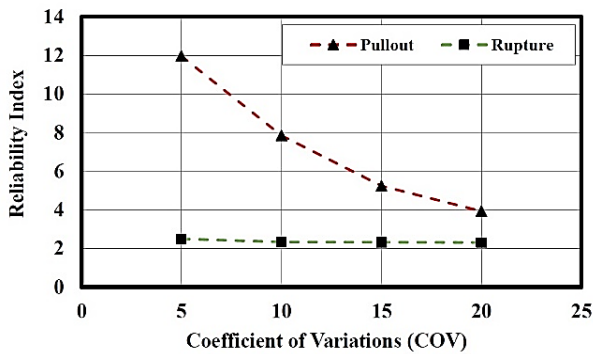


Fig. 20 Reliability index changes due to coefficient of variations (COV) for internal friction angle

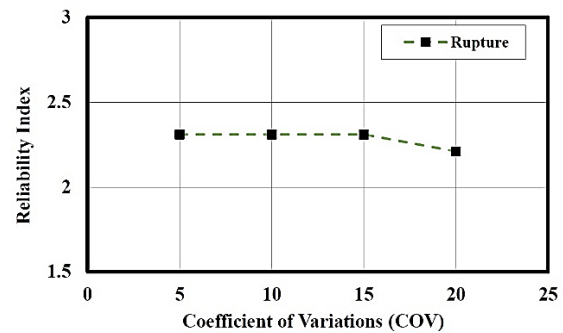


Fig. 22 Reliability index changes due to coefficient of variations (COV) for reinforced tensile strength

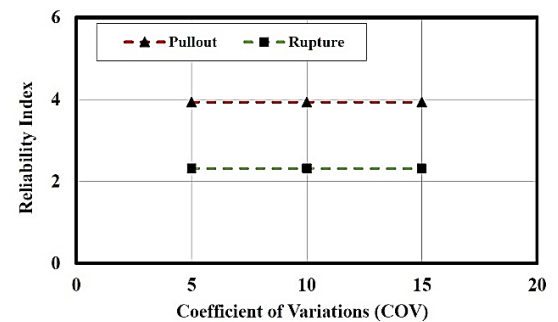


Fig. 23 Reliability index changes due to coefficient of variations (COV) for surcharge load

Uncertainties in various parameters and information are always present in geotechnical engineering. As the variable uncertainty increases, its coefficient of variation will also increase. In this part, the variable coefficient of variation effect on the reliability index is evaluated. The reinforced soil wall external stability results of this study for sliding, overturning and bearing capacity are shown in Figs.16 to 19.

As can be observed from Fig. 16, the internal friction angle coefficient of variation (COV) has the greatest effect on sliding, and the reliability index decreases from 3.9 to 2.2.

The internal friction angle coefficient of variation is very effective on the overturning failure mode and decreases the reliability index from 10 to 6. This coefficient of variation effect on bearing capacity is about 30% reduction.

Fig. 17 shows the little effect of the soil specific gravity coefficient of variation on stability, which shows that this variable's effect on the overturning failure mode is much more noticeable. The effects are so small that are indistinguishable in figure. Also, the base friction coefficient of variation effect in Fig. 18 shows a decreasing trend. Fig. 19 also shows that the surcharge load coefficient variation doesn't have a significant effect on the reliability index for sliding and bearing capacity compared to overturning.

### 8.5 Coefficient of Variation (COV) evaluation in the static internal stability

The of variable uncertainty effects from the coefficient of variation (COV) on reinforced soil wall internal stability, including the reinforcement rupture and pulling out failure modes have also been evaluated in this study. Results are shown in Figs. 20 to 23. Results show that the internal friction angle coefficient of variation has a very significant effect on the pulling out reliability index and reduces it. Results also show a small effect of other variables on reinforced soil walls internal stabilities reliability index.

## 9. Reinforcement type

In the reinforced soil wall internal stability analysis, the type and mechanical characteristics of the reinforcement have a significant effect on stability index and failure probability. In this study, to investigate the materials effect, two different reinforcement types were evaluated in terms

Table 14 Reliability Index Changes relative to reinforcement type variations in the fractional limit function

Reinforcement Type	$T$ (kN/m)	$V$	$\beta$	$P_f$	Layer Number
SWG40	40	0.4	2.31	0.0102	4
SWG50	50	0.5	2.32	0.009	4

Table 15 Changes in probability distribution type on reliability index

Limit Function	$\phi$	$\gamma$	$q$	$\delta$	$T$	$\beta$ with Correlation	$\beta$ without Correlation
Sliding (Static)	Normal	Normal	Normal	Normal	-	2.21	2.29
	Log-Normal	Normal	Normal	Normal	-	2.48	2.58
	Normal	Normal	Normal	Log-Normal	-	2.211	2.30
Overturning (Static)	Normal	Normal	Normal	-	-	6.16	6.96
	Log-Normal	Normal	Normal	-	-	9.42	10.89
Bearing Capacity (Static)	Normal	Normal	Normal	-	-	2.34	2.39
	Log-normal	Normal	Normal	-	-	3.06	3.11
Pullout Layer 4 (Static)	Normal	Normal	Normal	-	-	3.93	3.93
	Log-Normal	Normal	Normal	-	-	6.68	6.68
Rupture Layer 4 (Static)	Normal	Normal	Normal	-	Normal	2.31	2.31
	Log-Normal	Normal	Normal	-	Normal	2.33	2.34
	Normal	Normal	Normal	-	Log-Normal	2.31	2.31
Overturning (Seismic)	Normal	Normal	Normal	Normal	-	1.588	1.589
	Log-Normal	Normal	Normal	Normal	-	1.576	1.577
Sliding (Seismic)	Normal	Normal	Normal	Normal	-	0.95	0.96
	Log-Normal	Normal	Normal	Normal	-	0.96	0.97
	Normal	Normal	Normal	Log-Normal	-	0.96	0.97

of mechanical properties (tensile strength of geotextiles). These two have different tensile strengths of 40 kN/m and 50 kN/m and are introduced under SWG40 and SWG50 brands (Jingyu *et al.* 2016).

Results of this analysis (Table 14) show that by reducing the tensile strength of the reinforcement as well as its effective cross section, failure probability increases and reliability index decreases.

## 10. Investigation of the probability distribution type effect

Here, the probability distribution function type effect is investigated. The variables studied in this research are shown in Table 4 and two types of Normal and Log-Normal distribution are proposed for them. In previous studies of this research, variables were considered with Normal distribution and in this section, the reliability index is evaluated with Log-Normal distribution for variables. The data in Table 15 show the tangible effect of the variables probability distribution function type on the reliability index.

The probability distribution function effect on the overturning limit state function, which has a lower

probability failure index, is also very noticeable. Again, it can also be seen that the effective parameter of internal soil friction has the greatest effect on the results.

## 11. $\phi$ and $\gamma$ correlation changes study

Here,  $\phi$  and  $\gamma$  correlation changes are evaluated. In the references mentioned in Table 4, the correlation between these two variables is reported from 0.5 to 0.8. The results of this analysis are shown in Fig. 24 for reinforced soil wall external and internal stabilities. This figure shows that in external stability, reliability index decreases slightly by increasing correlation coefficient, in other words, the reliability index and correlation coefficient are inversely related. Fig. 24 also shows that the internal stability correlation coefficient has a much smaller effect on the reliability index than the external stability

## 12. Wall reinforcement height and length effect

In this paper, the effect of changes in constant wall parameters including reinforcement height and length (H and L) on the reinforced soil wall external stability in static

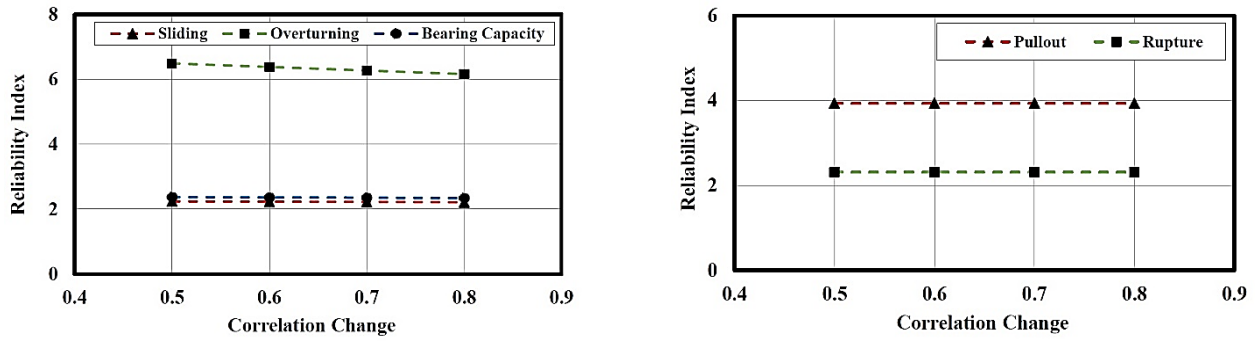


Fig. 24 Investigation of correlation changes between  $\phi$  and  $\gamma$  with respect to reliability index

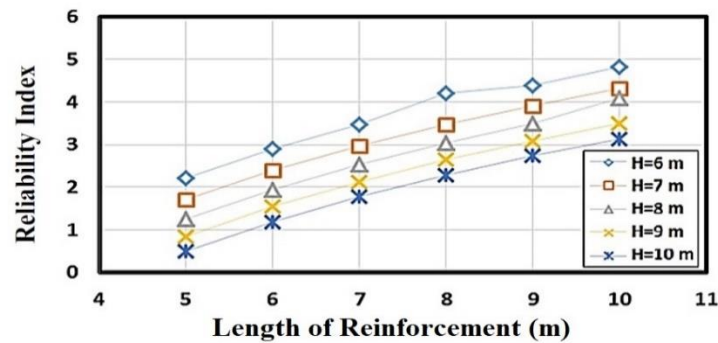


Fig. 25 Changes in wall height and reinforcement length vs. sliding stability reliability index

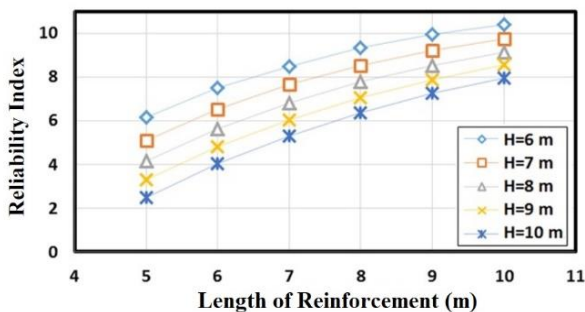


Fig. 26 Changes in wall height and length of reinforcement vs. overturning reliability index

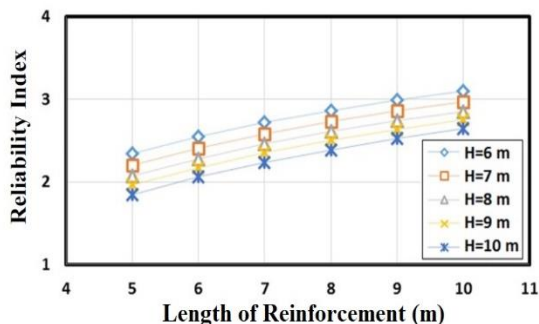


Fig. 27 Changes in wall height and length of reinforcement vs. bearing capacity reliability index

increases. Also, in conditions with a constant wall height, with increasing the reinforcement length, reliability index increases and failure probability decreases. According to the information in Table 3, the expected performance index of reinforced soil wall based on the reliability index should be about 3, hence, based on this reliability index value, the appropriate H and L values can be identified and introduced in the wall design. As an example, the sliding stability of a reinforced soil wall with a height of 8 meters and a reinforcing length of 8 meters results in a reliability index of 3, and, suitable values can be selected for overturning and bearing capacity in the same way

### 13. Sensitivity analysis

Sensitivity analysis means examining the reliability index variability with respect to statistical parameters such as mean and standard deviation. The importance of sensitivity analysis is in the variables ranking so that variables of high importance will have more changes in sensitivity analysis. Also, less important variables can be removed from the probability analysis and decided about them with certainty.

Sensitivity analysis can be studied in relation to various statistical parameters. In the continuation of the research, the sensitivity analysis types and their relationships will be introduced. The E1 factor indicates the variable effect on the limit function without considering the correlation coefficient and indicates the relative share of each variable on the failure probability and can be expressed by the

conditions are investigated. Results are shown in Figs. 25 to 27. Evaluation of these results shows that at constant length conditions for the reinforcement, by increasing wall height, reliability index decreases and failure probability

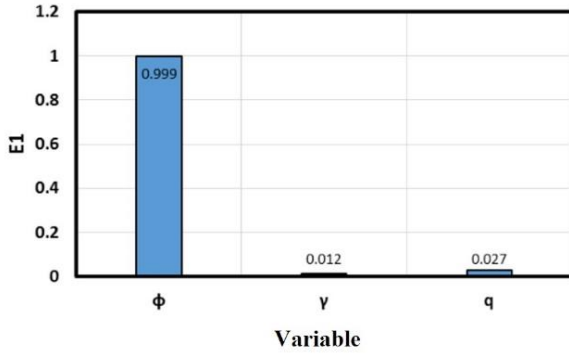


Fig. 28 Sensitivity analysis for sliding limit function

following equation

$$E_1 = \frac{-\frac{\partial g}{\partial Z}}{\sqrt{\sum_{i=1}^n (\frac{\partial g}{\partial Z_i})^2}} \quad (26)$$

E2 factor indicates the variable's effect on the limit function by considering the correlation coefficient and indicates the relative share of each variable on the failure probability and can be expressed by the following equation

$$E_2 = \frac{-\frac{\partial g}{\partial Z}}{\sqrt{\sum_{j=1}^n \sum_{i=1}^n \frac{\partial g}{\partial Z_i} \frac{\partial g}{\partial Z_j} \rho_{ij}}} \quad (27)$$

These two factors (E1 and E2) indicate the variable's effect on the reliability index and its positive and negative signs indicate the relationship between the function and the variable. A positive sign means that the variable is of the load kind and a negative sign means that the variable is of the resistance kind. Alternatively, positive sensitivity coefficient means a decrease in the reliability index by increasing the variable value and a negative sensitivity coefficient means an increase in the reliability index by increasing the variable value.

E3 factor indicates the effect of the variable's mean on the reliability index and is expressed by the following equation

$$E_3 = \frac{\partial \beta}{\partial \mu_x} = \frac{\beta_2 - \beta_1}{\Delta \mu_x} \quad (28)$$

The E4 factor represents the variable's standard deviation effect on the reliability index which is expressed by the following equation

$$E_4 = \frac{\partial \beta}{\partial \sigma_x} = \frac{\beta_2 - \beta_1}{\Delta \sigma_x} \quad (29)$$

This factor is always negative and indicates that as the standard deviation increases, the probability distribution function becomes wider and the failure probability increases. Results of variables sensitivity analysis are shown in Table 16.

The data in Table 16 shows the sensitivity of variables used in this study to each of the limit functions. Figs. 28 to

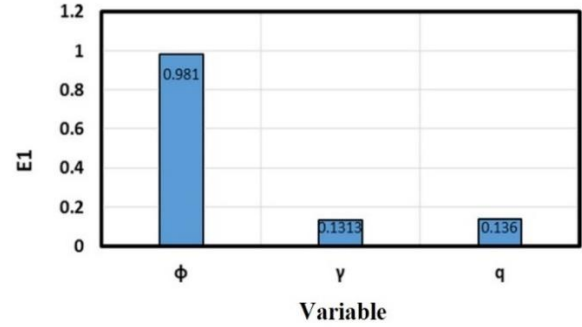


Fig. 29 Sensitivity analysis for overturning limit function

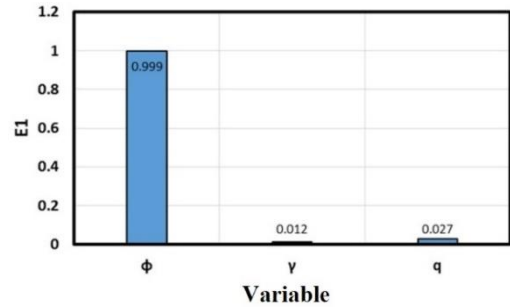


Fig. 30 Sensitivity analysis for bearing capacity limit function

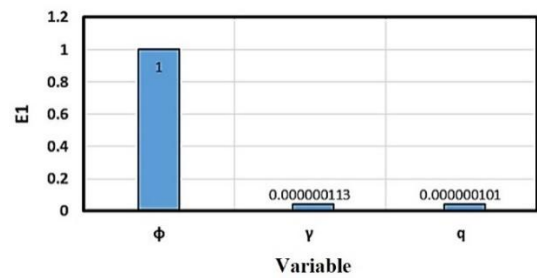


Fig. 31 Sensitivity analysis for pullout limit function

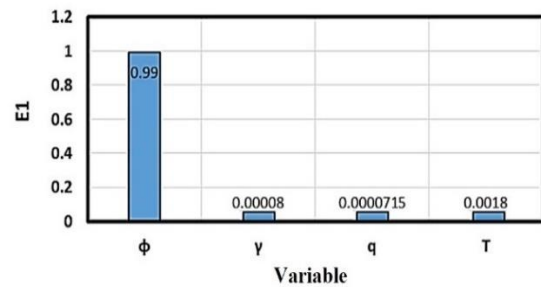


Fig. 32 Sensitivity analysis for rupture limit function

32 show the sensitivity analysis of the E1 factor as a column chart for better understanding.

These results show that for the sliding limit function, the internal friction angle and then the base friction are the most sensitive among variables. Also, for the overturning limit function, the internal friction angle is the most sensitive among variables and the soil surcharge load and specific gravity are approximately equal to the overturning stability. The bearing capacity limit function also has the greatest effect on the internal friction angle variable.

Figs. 30 and 31 also show that for the internal stability

Table 16 Sensitivity analysis of random variables

Limit Function		$\phi$	$\gamma$	$q$	$\delta$	T
Sliding (Static)	$E_1$	-0.883	-0.036	0.074	-0.46	-
	$E_2$	-0.871	-0.063	0.077	-0.48	-
	$E_3$	0.835	0.060	-0.074	0.46	-
	$E_4$	-1.63	-0.097	-0.012	-0.46	-
Overturning (Static)	$E_1$	-0.981	-0.1313	0.136	-	-
	$E_2$	-0.952	-0.258	0.160	-	-
	$E_3$	0.80	0.218	-0.136	-	-
	$E_4$	-4.88	-1.16	-0.114	-	-
Bearing Capacity (Static)	$E_1$	-0.999	-0.012	0.027	-	-
	$E_2$	-0.999	-0.021	0.028	-	-
	$E_3$	0.98	0.021	-0.027	-	-
	$E_4$	-1.63	-0.028	-0.001	-	-
Pullout Layer 4 (Static)	$E_1$	-1	$-1.13 \times 10^{-7}$	$1.01 \times 10^{-7}$	-	-
	$E_2$	-1	$-1.88 \times 10^{-7}$	$1.01 \times 10^{-7}$	-	-
	$E_3$	0.999	$1.88 \times 10^{-7}$	$1.01 \times 10^{-7}$	-	-
	$E_4$	-3.93	$-5.93 \times 10^{-7}$	$-2.29 \times 10^{-11}$	-	-
Rupture Layer 4 (Static)	$E_1$	0.999-	$8 \times 10^{-5}$	$7.15 \times 10^{-5}$	-	-0.00018
	$E_2$	0.999-	0.00013	$7.15 \times 10^{-5}$	-	-0.00018
	$E_3$	0.999	0.0013	$-7.15 \times 10^{-5}$	-	0.00018
	$E_4$	-0.046	$-4.9 \times 10^{-6}$	$2.38 \times 10^{-10}$	-	$-1.5 \times 10^{-9}$

limit functions, the internal friction angle is the most effective variable, thus other variables can be considered constant in studying the pulling out and rupturing reinforcement limiting functions.

#### 14. Conclusions

In this paper, the reliability analysis of the reinforced soil walls internal and external stabilities in static and seismic conditions were investigated by the first-order reliability method and the data validation was performed by the Monte Carlo simulation method. One of the results of a design based on reliability analysis is the quantification of reliability in the geotechnical structure design, which can be expressed by the reliability index. The innovation in this research is the use of first-order reliability analysis (FORM) and Monte Carlo simulation (MCS) methods in reliability assessment. Also, the mean changes, correlation and coefficient of different variables variation in this study have been evaluated to calculate the most effective variable according to sensitivity analysis, which in previous studies a limited number of variables have been considered. Also, the seismic and static effects have been studied simultaneously, which in previous studies, the reinforced soil internal and external stabilities has been less studied.

The main results of this research can be expressed as follows:

- The predominant failure mode for the reinforced soil wall is slip-type in external stability and reinforcing rupture in internal stability, and the reliability index in these two failure modes has the lowest value, which needs to be evaluated more carefully by engineers in design.
- The correlation coefficient between variables is very important on the results that should be considered in modeling. In this study, it was observed that the reliability indices functions in external stability decreases with increasing the variables correlation coefficient, hence, the reliability index in sliding stability decreased by approximately 4%, overturning stability by 12% and bearing capacity stability by 2%. The correlation coefficient of some variables may have a small effect on the reliability index. Also, in this study it was found that the correlation coefficient effect between variables on internal stability was greater than the external.
- Sensitivity analysis showed that the first and second variables are effective in internal friction angle and base friction sliding stability. Also, the sensitivity analysis of overturning stability, internal friction angle and surcharge load are effective variables. Internal stability sensitivity analysis also showed that the internal friction angle has the greatest effect on the reliability index. These results indicate that due to high sensitivity, more care should be taken in geotechnical studies in determining the internal friction angle.
- Changes in the variable means showed that with increasing surcharge load, the reliability index in static

and seismic analysis decreases, but in other variables with increasing the mean, the reliability index increases and the failure probability decreases.

- The results of this study showed that the probability distribution function affects the reliability index. The probability distribution function effect on overturning stability is significant. Just changing the internal friction angle distribution function type has a great effect on reinforced soil wall stability.
- The results of this study show that the internal friction angle coefficient of variation has the greatest effect on the reliability index in static and seismic analysis compared to other variables and by increasing the coefficient of variation, the reliability index decreases.
- Wall height and reinforcement length evaluation showed that by increasing the wall height, the reliability index decreases with the constant reinforcement length and constant wall height as well, the reliability index increases with increasing the length of the reinforcement.
- Earthquake force is one of the most influential factors on the reliability index and with increasing the horizontal acceleration of the earthquake, the reliability index decreases.

## References

- Ahmed, S.M. and Basha, B.M. (2021), "External stability analysis of narrow backfilled gravity retaining walls", *Geotech. Geol. Eng.*, **39**(2), 1603-1620. <https://doi.org/10.1007/s10706-020-01580-3>.
- Basha, B.M. and Babu, G.S. (2010), "Optimum design for external seismic stability of geosynthetic reinforced soil walls: reliability based approach", *J. Geotech. Geoenviron. Eng.*, **136**(6), 797-812. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0000289](https://doi.org/10.1061/(ASCE)GT.1943-5606.0000289).
- Basha, B.M. and Babu, G.S. (2012), "Target reliability-based optimisation for internal seismic stability of reinforced soil structures", *Geotechnique*, **62**(1), 55-68. <https://doi.org/10.1680/geot.8.P.076>.
- Basha, B.M. and Babu, G.S. (2014), "Reliability-based load and resistance factor design approach for external seismic stability of reinforced soil walls", *Soil Dyn. Earthq. Eng.*, **60**, 8-21. <https://doi.org/10.1016/j.soildyn.2014.01.013>.
- Carlos, D.M., Pinho-Lopes, M. and Lopes, M.L. (2016), "Effect of geosynthetic reinforcement inclusion on the strength parameters and bearing ratio of a fine soil", *Proc. Eng.*, **143**, 34-41. <https://doi.org/10.1016/j.proeng.2016.06.005>.
- Chalermyanont, T. and Benson, C.H. (2004). "Reliability-based design for internal stability of mechanically stabilized earth walls", *Geotech. Geoenviron. Eng.*, **130**(2), 163-176. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2004\)130:2\(163\)](https://doi.org/10.1061/(ASCE)1090-0241(2004)130:2(163)).
- Chehade, H.A., Dias, D., Sadek, M., Jenck, O. and Chehade, F.H. (2021), "Pseudo-static analysis of reinforced earth retaining walls", *Acta Geotechnica*, 1-15. <https://doi.org/10.1007/s11440-021-01148-2>.
- Choudhury, D., Nimbalkar, S.S. and Mandal, J.N. (2007), "External stability of reinforced soil walls under seismic conditions", *Geosynthetics Int.*, **14**(4), 211-218. <https://doi.org/10.1680/gein.2007.14.4.211>.
- Genske, D.D., Adachi, T. and Sugito, M. (1991), "Reliability analysis of reinforced earth retaining structures subjected to earthquake loading", *Soils Found.*, **31**(4), 48-60. [https://doi.org/10.3208/sandf1972.31.4\\_48](https://doi.org/10.3208/sandf1972.31.4_48).
- Griffiths, D.V. and Fenton, G.A. (Eds.). (2007), Probabilistic methods in geotechnical engineering (Vol. **491**), Springer Science & Business Media.
- GuhaRay, A., Mondal, S. and Mohiuddin, H.H. (2018), "Reliability analysis of retaining walls subjected to blast loading by finite element approach", *J. Inst. Engineers (India): Series A*, **99**(1), 95-102. <https://doi.org/10.1007/s40030-018-0273-z>.
- Hamrouni, A., Dias, D. and Sbartai, B., (2018), "Reliability analysis of a mechanically stabilized earth wall using the surface response methodology optimized by a genetic algorithm", *Geomech. Eng.*, **15**(4), 937-945. <https://doi.org/10.12989/gae.2018.15.4.937>.
- Jingyu, C., Zhihong, N., Lianheng, Z. and Wei, L. (2016), "Case study on the typical failure modes and reliability of reinforced-earth retaining wall", *Electronic J. Geotech. Eng.*, **21**(1), 305-317. <http://www.ejge.com/2016/Ppr2016.0032ma.pdf>.
- Jones, C.J. (1985), Earth reinforcement and soil structures. Elsevier.
- Melchers, R.E. and Beck, A.T. (2018), Structural reliability analysis and prediction. John Wiley & sons.
- Mononobe, N. and Matsuo, H. (1929), "On the determination of earth pressure during earthquakes", *Proceedings of the World Engineering Conf.*, Vol. 9, str. 176.
- Okabe, S. (1924), "General theory on earth pressure and seismic stability of retaining wall and dam", *Proc. Civil Engrg. Soc., Japan*, **10**(6), 1277-1323.
- Pang, R., Xu, B., Kong, X., Zou, D. and Zhou, Y. (2018), "Seismic reliability assessment of earth-rockfill dam slopes considering strain-softening of rockfill based on generalized probability density evolution method", *Soil Dyn. Earthq. Eng.*, **107**, 96-107. <https://doi.org/10.1016/j.soildyn.2018.01.020>.
- Pang, R., Xu, B., Zhou, Y. and Song, L. (2021), "Seismic time-history response and system reliability analysis of slopes considering uncertainty of multi-parameters and earthquake excitations", *Comput. Geotech.*, **136**, 104245. <https://doi.org/10.1016/j.compgeo.2021.104245>.
- Phoon, K.K. and Ching, J. (2Eds.) (2015), Risk and reliability in geotechnical engineering. CRC Press.
- Powers, M. (2017), Reliability Analysis of Geosynthetic Reinforced Soil Walls. Graduate Theses & Non-Theses, 136. [http://digitalcommons.mtech.edu/grad\\_rscl/136](http://digitalcommons.mtech.edu/grad_rscl/136).
- Muktinatalapati, J. and GuhaRay, A. (2021), "Earth pressures on retaining walls backfilled with sand admixed with building derived materials: Laboratory scale study", *Int. J. Geomech.*, **21**(6), 04021072. [https://doi.org/10.1061/\(ASCE\)GM.1943-5622.0002030](https://doi.org/10.1061/(ASCE)GM.1943-5622.0002030).
- Sayed, S., Dodagoudar, G.R. and Rajagopal, K. (2008), "Reliability analysis of reinforced soil walls under static and seismic forces", *Geosynthetics Int.*, **15**(4), 246-257. <https://doi.org/10.1680/gein.2008.15.4.246>.
- Tobutt, D.C. (1982), "Monte Carlo simulation methods for slope stability", *Comput. Geosci.*, **8**(2), 199-208. [https://doi.org/10.1016/0098-3004\(82\)90021-8](https://doi.org/10.1016/0098-3004(82)90021-8).
- Vesic, A.S. (1967), A study of the bearing capacity of deep foundations.
- Vidal, H. (1969), The principle of reinforced earth, Highway Research Board, No. 282, 1-16. <http://onlinepubs.trb.org/Onlinepubs/hrr/1969/282/282-001.pdf>.
- Zevgolis, I.E. and Bourdeau, P.L. (2017), "Reliability and redundancy of the internal stability of reinforced soil walls", *Comput. Geotech.*, **84**, 152-163. <https://doi.org/10.1016/j.compgeo.2016.11.022>