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Numerical assessment of nonlocal dynamic stability of graded porous beams in thermal environment rested on elastic foundation

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Abstract. Numerical assessment of the dynamic stability behavior of nonlocal beams rested on elastic foundation has been provided in the present research. The beam is made of fucntional graded (FG) porous material and is exposed to thermal and humid environments. It is also consiered that the beam is subjected to axial periodic mechanical load which especific exitation frequency leading to its instability behavior. Beam modeling has been performed via a two-variable theory developed for thick beams. Then, nonlocal elasticity has been used to establish the governing equation which are solved via Chebyshev-Ritz-Bolotin method. Temperature and moisture variation showed notable effects on stability boundaries of the beam. Also, the stability boundaries are affected by the amount of porosities inside the material.

Keywords: beam theory; dynamic stability; nonlocal elasticity; porosities; thermal load

1. Introduction

Functionally graded materials (FGMs) contains two phases which are often ceramic and metal in a combined form. In fact, the material formation is graded between ceramic and metal and porosities will produce due to imperfect combination of the two phases. The material characteristics of FGMs may be described based on the portion of these two phases and also porosity amount (Ahmed *et al*. 2021a, b). However, providing many outstanding properties, FG materials will be used in different engineering sections including mechanical and civil engineering (Ahmed *et al*. 2018, Faleh *et al*. 2018, Thai *et al.* 2014).

Recent studies focus on engineering structures at nano-scales due to their involvement in nano-mechanical systems or devices. However, the main issue in these studies is to select an appropriate elasticity theory accounting for small scale impacts. The impact of size-dependency might be considered with the help of a scale parameter involved in non-local theory of elasticity Eringen (1983). The word "non-local" means that the stresses are not local anymore. This is because we are talking about a stress field of nano-scale structure. Many authors are aware of these facts and they are using this theory to analysis mechanical characteristics of small size engineering structures. Recently, engineering structures have been used in the production of nano-size devices and objects. Some of these structures have beam or plate shapes having nano dimensions. The most important issue about these structures is understanding their mechanical characteristics such as dynamic behaviors. However, such an investigation needs refined continuum mechanics due to nano dimension effects since classical mechanics is impotent to express such effects. To this end, nonlocal elasticity theory (Eringen 1983) is proposed to make the researches able to analysis mechanical characteristics of nano-size structures. The theory proposed a modified stress-strain relation based on nonlocal parameter in order to formulate nano-structures. This relation is used by many researched to provide suitable formulations for nano-structures (Al-Toki *et al.* 2021, Ahouel *et al*. 2016, Bellifa *et al*. 2017, Faleh *et al.* 2020, Fenjan *et al*. 2020a-e, Kunbar *et al.* 2020). The theory is also applicable for nano-structures made of FGMs. Natarajan *et al*. (2012) explored finite element based vibrational response of graded nanosize plates with simply-supported and clamped edges. Belkorissat *et al*. (2015) examined vibrational characteristics of graded nanosize plates by introducing a size-dependent four-unknown plate theory. They showed that the classical plate model cannot consider the shear deformation mechanism and proposed a more accurate theory containing a shear stress function. So, higher order theories such as third-order (Reddy 1990), 4-unknown and 3-unknown theories are more applicable for thick plates (Taj *et al.* 2013, Mahmoudi et al. 2018, Houari *et al*. 2016, Belabed *et al*. 2018). The theory can be extended for beam and then it can be called a two-variable beam theoy which is suitbale for thicker beams. Mechab *et al.* (2016) presented the size-dependent and porosity-dependent analysis of FG nanoplates lying on an elastic substrate.

In this study, a numerical assessment of the dynamic stability behavior of nonlocal beams rested on elastic foundation has been provided. The beam is made of fucntional graded porous material and is exposed to thermal and humid environments. It is also consiered that the beam is subjected to axial periodic mechanical load which

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especific exitation frequency leading to its instability behavior. Beam modeling has been performed via a two-variable theory developed for thick beams. Then, nonlocal elasticity has been used to establish the governing equation which are solved via Chebyshev-Ritz-Bolotin method. Temperature and moisture variation showed notable effects on stability boundaries of the beam. Also, the stability boundaries are affected by the amount of porosities inside the material.

2. Governing equations

2.1 Modeling of FG nanobeams

Assume a FG nanobeam with thickness h as illustrated in Fig. 1. A FG material may be described according to the varying volume fraction. Utilizing a modified power-law technique, elastic modulus E, density ρ and other material properties may be calculated by (Yahia *et al*. 2015)

$$E(z)=(E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+E_m-\frac{\xi}{2}(E_c+E_m) \tag{1a}$$

$$\rho(z)=(\rho_c-\rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\rho_m-\frac{\xi}{2}(\rho_c+\rho_m) \tag{1b}$$

$$\alpha(z)=(\alpha_c-\alpha_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\alpha_m-\frac{\xi}{2}(\alpha_c+\alpha_m) \tag{1c}$$

$$d(z)=(d_c-d_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p+d_m-\frac{\xi}{2}(d_c+d_m) \tag{1d}$$

It must be highlighted that ξ introduces porosity factors; p introduces material gradient index; **c** and **m** denote ceramic and metallic ingredients. Note that $\alpha(z)$ and $d(z)$ are thermal expansion and moisture coefficients.

Based on nonlocal elasticity theory, the nonlocality of stress field can be incorporated into the stress-strain relationship as

$$(1-(e_0a)\nabla^2)\sigma_{ij}=\varepsilon_{ij} \tag{2}$$

in which ∇^2 denotes the Laplacian operator and $e_0 a$ is the scale factor which considers the small size impact. Finally, the nonlocal constitutive relations based on plane stress plate model may be established as

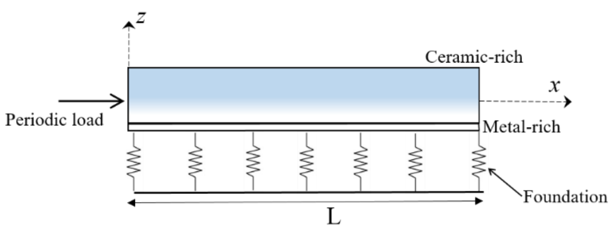


Fig. 1 Configuration of a beam under axial periodic load

$$(1-\mu\nabla^2)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix}=\begin{pmatrix}T_{11}&T_{12}&0&0&0\\ T_{12}&T_{22}&0&0&0\\ 0&0&T_{66}&0&0\\ 0&0&0&T_{44}&0\\ 0&0&0&0&T_{55}\end{pmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{3}$$

where

$$T_{11}=T_{22}=\frac{E(z)}{1-\upsilon^2(z)},$$
$$T_{12}=\upsilon(z)T_{11},$$
$$T_{44}=T_{55}=T_{66}=\frac{E(z)}{2(1+\upsilon(z))}$$

Modeling of the nanobeam is performed employing a 2-unknown beam theory which has fewer field unknowns compared with the refined and also first order beam theory. The displacement field of the 2-unknown beam model may be established as

$$d_1(x,z,t)=u(x,t)-zw_{,x}-H(z)w_{,xxx} \tag{4}$$

$$d_3(x,z,t)=w(x,t) \tag{5}$$

Here, u is in-plane displacement and w denotes the transverse displacement. For more accurate modeling of FGM structures, it is crucial to consider the exact positions of neutral surface. Generally, there is coupling between in-plane and out-of-plane displacements of FGM beams, as it can be seen in Eqs. (4) and (5). By considering the exact position of neutral surface, it is possible to eliminate this coupling. So, the displacement field of 2-unknown beam model can be reduced to the following form

$$d_1(x,z,t)=-(z-z^*)w_{,x}-(H(z)-z^{**})w_{,xxx} \tag{6}$$

$$d_3(x,z,t)=w(x,t) \tag{7}$$

It is evident that the displacement field is reduced to a single-unknown model and

$$z^*=\frac{\int_{-h/2}^{h/2}E(z)\,zdz}{\int_{-h/2}^{h/2}E(z)\,dz}$$
$$z^{**}=\frac{\int_{-h/2}^{h/2}E(z)\,H(z)dz}{\int_{-h/2}^{h/2}E(z)\,dz} \tag{8}$$

For the introduced theory, a shear function in below form has been selected

$$H(z)=\frac{1}{2}z\left[\frac{h^2}{4}-\frac{1}{3}z^2\right] \tag{9}$$

Finally, the strains based on the two-unknown beam model are obtained as

$$\left\{\varepsilon_x\right\} = \left\{\varepsilon_x^0\right\} + z\left\{\kappa_x\right\} + H(z)\left\{\eta_x\right\}$$
$$,\gamma_{xz} = g(z)\gamma_{xz} \tag{10}$$

where $g(z) = H'(z)$ and

$$\left\{\varepsilon_x^0\right\} = \left\{\frac{\partial u}{\partial x}\right\},$$
$$\left\{\kappa_x\right\} = \left\{-\frac{\partial^2 w}{\partial x^2}\right\},$$
$$\left\{\eta_x\right\} = \left\{-\frac{\partial^4 w}{\partial x^2}\right\}, \tag{11}$$
$$\left\{\gamma_{xz}\right\} = \left\{\frac{\partial^3 w}{\partial x^3}\right\}$$

Now, the below form has been selected to define the strain energy

$$U = 0.5\int_V \sigma_{ij}\,\varepsilon_{ij}\,dV =$$
$$0.5\int_V (\sigma_x\,\varepsilon_x + \sigma_{xz}\,\gamma_{xz})\,dV \tag{12}$$

Inserting Eqs. (10) and (11) into Eq. (12) yields

$$U = 0.5\int_{-0.5L}^{0.5L}[A_{11}\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} - 2B_{11}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}$$
$$-2B_{11}^s\frac{\partial u}{\partial x}\frac{\partial^4 w}{\partial x^4} + 2D_{11}^s\frac{\partial^2 w}{\partial x^2}\frac{\partial^4 w}{\partial x^4}$$
$$+D_{11}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + H_{11}^s\frac{\partial^4 w}{\partial x^4}\frac{\partial^4 w}{\partial x^4} \tag{13}$$
$$+A_{44}^s\frac{\partial^3 w}{\partial x^3}\frac{\partial^3 w}{\partial x^3}]dx$$

The work of non-conservative forces is expressed by

$$V = 0.5\int_{-0.5L}^{0.5L}[N_x^0\frac{\partial w}{\partial x}\frac{\partial w}{\partial x} + \mu N_x^0\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2}$$
$$-k_w w - \mu k_w\frac{\partial^2 w}{\partial x^2})dx \tag{14}$$

Note that $N_x^0 = N^T + N^D$ are induced loads due to temperature and moisture: $N^T = \int_{-0.5h}^{0.5h}\alpha(z)T_{11}(z)\Delta T dz$ and $N^D = \int_{-0.5h}^{0.5h} d(z)T_{11}(z)\Delta C dz$ where ΔT and ΔC are temperature and moisture variation, respectively. Moreover, k_w is the foundation parameter. Also, the kinetic energy is obtained as

$$K = 0.5\int_{-0.5L}^{0.5L}[I_0(\frac{\partial u}{\partial t}\frac{\partial u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial w}{\partial t})$$
$$+\mu I_0(\frac{\partial^2 u}{\partial x\partial t}\frac{\partial^2 u}{\partial x\partial t} + \frac{\partial^2 u}{\partial y\partial t}\frac{\partial^2 u}{\partial y\partial t} + \frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 w}{\partial x\partial t}$$
$$+\frac{\partial^2 w}{\partial y\partial t}\frac{\partial^2 w}{\partial y\partial t}) - I_1(\frac{\partial u}{\partial t}\frac{\partial^2 w}{\partial x\partial t}) - \mu I_1(\frac{\partial^2 u}{\partial x\partial t}\frac{\partial^3 w}{\partial x^2\partial t})$$
$$-J_1(\frac{\partial u}{\partial t}\frac{\partial^4 w}{\partial x^3\partial t}) - \mu J_1(\frac{\partial^2 u}{\partial x\partial t}\frac{\partial^5 w}{\partial x^4\partial t}) + I_2(\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 w}{\partial x\partial t}) \tag{15}$$
$$+\mu I_2(\frac{\partial^3 w}{\partial x^2\partial t}\frac{\partial^3 w}{\partial x^2\partial t}) + K_2(\frac{\partial^4 w}{\partial x^3\partial t}\frac{\partial^4 w}{\partial x^3\partial t})$$
$$+\mu K_2(\frac{\partial^5 w}{\partial x^4\partial t}\frac{\partial^5 w}{\partial x^4\partial t}) + J_2(\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^4 w}{\partial x^3\partial t})$$
$$+\mu J_2(\frac{\partial^3 w}{\partial x^2\partial t}\frac{\partial^5 w}{\partial x^4\partial t})]dx$$

where

$$(I_0, I_1, J_1, I_2, J_2, K_2)$$
$$= \int_{-h/2}^{h/2}(1, z, H, z^2, zH, H^2)\rho(z)dz \tag{16}$$

and

$$\left\{A_{11}, B_{11}, B_{11}^s, D_{11}, D_{11}^s, H_{11}^s\right\}$$
$$= \int_{-h/2}^{h/2} T_{11}\,(1, z, H, z^2, zH, H^2)dz \tag{17}$$

$$A_{44}^s = \int_{-h/2}^{h/2} g^2\frac{E(z)}{2(1+\nu)}dz \tag{18}$$

As mentioned, bending-extension coupling eliminates with considering the neutral surface location. Eqs. (13) and (15) can be reduced in term of *w* by discarding *u* and *v* as

$$U = 0.5\int_{-0.5L}^{0.5L}[+2\tilde{D}_{11}^s\frac{\partial^2 w}{\partial x^2}\frac{\partial^4 w}{\partial x^4} + \tilde{D}_{11}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2}$$
$$+\tilde{H}_{11}^s\frac{\partial^4 w}{\partial x^4}\frac{\partial^4 w}{\partial x^4} + A_{44}^s\frac{\partial^3 w}{\partial x^3}\frac{\partial^3 w}{\partial x^3}]dx \tag{19}$$

$$K = 0.5\int_{-0.5L}^{0.5L}[\tilde{I}_0(\frac{\partial w}{\partial t}\frac{\partial w}{\partial t}) + \mu\tilde{I}_0(\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 w}{\partial x\partial t})$$
$$+\tilde{I}_2(\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^2 w}{\partial x\partial t}) + \mu\tilde{I}_2(\frac{\partial^3 w}{\partial x^2\partial t}\frac{\partial^3 w}{\partial x^2\partial t})$$
$$+\tilde{K}_2(\frac{\partial^4 w}{\partial x^3\partial t}\frac{\partial^4 w}{\partial x^3\partial t}) + \mu\tilde{K}_2(\frac{\partial^5 w}{\partial x^4\partial t}\frac{\partial^5 w}{\partial x^4\partial t}) \tag{20}$$
$$+\tilde{J}_2(\frac{\partial^2 w}{\partial x\partial t}\frac{\partial^4 w}{\partial x^3\partial t}) + \mu\tilde{J}_2(\frac{\partial^3 w}{\partial x^2\partial t}\frac{\partial^5 w}{\partial x^4\partial t}]dx$$

in which

$$\left\{\tilde{A}_{11}, \tilde{D}_{11}, \tilde{D}_{11}^s, \tilde{H}_{11}^s\right\}$$
$$= \int_{-h/2}^{h/2} T_{11}\,(1, (z - z^*)^2, (z - z^*)(H - z^{**}), (H - z^{**})^2)dz \tag{21}$$

$$(\tilde{I}_0, \tilde{I}_2, \tilde{J}_2, \tilde{K}_2)$$
$$= \int_{-h/2}^{h/2}(1, (z - z^*)^2, (z - z^*)(H - z^{**}), (H - z^{**})^2)\,\rho(z)dz \tag{22}$$

3. Solution approach

Based on Chebyshev-Ritz method, the dynamic buckling problem of a porous FG nanobeam will be solved in this section. First, the field components may be assumed in the following form

$$u(x,t) = R^u(x)\sum_{n=1}^{\infty} U_{nm}P_m(x)e^{i\omega_n t} \tag{23}$$

$$w(x,t) = R^w(x)\sum_{n=1}^{\infty} W_{mn}P_n(x)e^{i\omega_n t} \tag{24}$$

Consider the following essential boundary conditions:
Simply-supported (S):

$w = \dfrac{\partial^2 w}{\partial x^2} = 0$ at x=+0.5*L*, -0.5*L*

Also, P_m(x) is the *n*-th Chebyshev polynomials of the first kind may be expressed as

$$P_m(x) = \cos[(m-1)\arccos(\frac{2x}{L})] \quad (25)$$

Note that R^i (i=u, w) denotes functions associated with the essential boundary conditions. Also, the functions may be introduced in below form

$$R^i(x) = (1+\frac{2x}{L})^{p^*}(1-\frac{2x}{L})^{q^*} \quad (26)$$

where p^*, q^* are affected by the type of edge conditions. For SS edges $p^*= q^*= 1$. Substituting Eqs. (23) and (24) in $\Pi = (U + V - K) = 0$ together with its minimization to the indefinite variables U_{mn}, and W_{mn}, the below equation results in simultaneous algebraic equations with respect to unknown coefficients.

$$\frac{\partial \Pi}{\partial U_{mn}} = \frac{\partial \Pi}{\partial W_{mn}} = 0 \quad (27)$$

The governing equations of periodically loaded FG beam in matrix form may be expressed as

$$[M]\{\ddot{W}_{mn}\} + [[K] + N_0(t)[G]]\{\ddot{W}_{mn}\} = 0 \quad (28)$$

in which [**M**] , [**K**] and [**G**] respectively introduce the mass, stiffness and geometrical matrices.

The periodic force with excitation frequency ϖ may be defined as $N_0(t) = -[\alpha + \beta\cos(\varpi t)]N_{cr}$, then the equations become

$$[M]\{\ddot{W}_{mn}\} + [[K] - \{\alpha + \beta\cos(\varpi t)\}N_{cr}[G]]\{W_{mn}\} = 0 \quad (29)$$

Here, static and dynamic force components have been denoted by α and β and the excitation frequency and foundation paramter are normalized as

$$\Omega = \varpi h\sqrt{\frac{\rho_c}{E_c}}, \quad K_w = \frac{k_w L^4}{D_0}, D_0 = \frac{E_c h^3}{12(1-v^2)} \quad (30)$$

By assuming periodic coefficients of Mathieu–Hill kind, the solution becomes

$$[[K] - N_{cr}\{\alpha \pm 0.5\beta\}[G] - 0.25\varpi[M]]\{W_{mn}\} = 0 \quad (31)$$

Above equation must be solved numerically to derive instability regions.

5. Findings and discussions

This section contains obtained results for instability region of periodically loaded FG nanobeams having porosities. First, the instability regions of FG structures without porosities have been verified with those obtained by Han *et al.* (2015), as reported in Table 1. This table confirms that the proposed solution and plate formulation is correct. Then, based on 2-variable beam theory and proposed solution, the new findings have been provided. Further investigations are based on 4 terms in the solution and the material properties are:

- $E_c = 380\ GPa$, $\rho_c = 3800\ kg/m^3$, $v_c = 0.3$, α_c=7*10^{-6}, d_c=0.0.001
- $E_m = 70\ GPa$, $\rho_m = 2707\ kg/m^3$, $v_m = 0.3$, α_m=23*10^{-6}, d_m=0.44

In Figs. 2 and 3, the changing of normalized excitation frequency according to dynamical force component (β) based on various temperature (ΔT) and moisture (ΔC) variation has been studied for simply-supported nano-size beam assuming p =1, α=0.3 and μ=0.2. It must be explained that increasing the temperature or moisture induces greater normalized excitation frequencies. Actually, by increasing in temperature or moisture, the beam stiffness will decrease. However, porosities inside the material may change the dynamic behavior of a structure. Indeed, thermal expansion coefficient is dependent on the porosity factor. As a conclusion, combined effects of moisture/temperature and porosity have notable impacts on stability boundaries of a porous FGM nano-size beam.

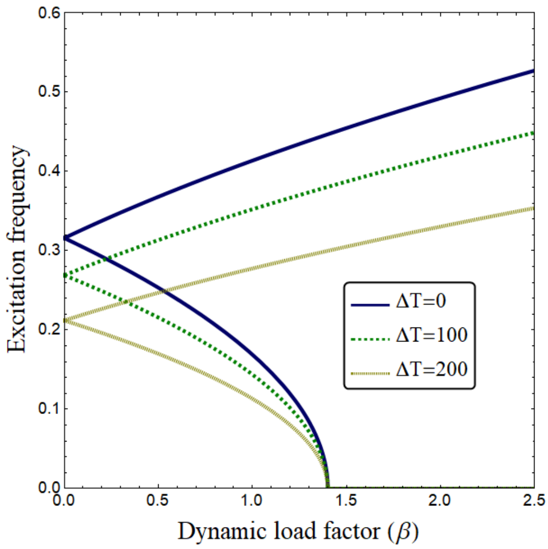


Fig. 2 Normalized frequency according to dynamic force component based on various temperature variation (p=1, ξ=0, a=10h, ΔC=0, α=0.3)

Table 1 Verification of normalized excitation frequencies of a FG structure at β=0.5

		α=0.3	
		Han *et al.* (2015)	present
$[\bar{K}]-(0.5\beta)N_{cr}[G]$	p=0.1	2.3325	2.33267
	p=1	2.3072	2.30725
	p=10	2.2746	2.2747
$[\bar{K}]+(0.5\beta)N_{cr}[G]$	p=0.1	3.389	3.3892
	p=1	3.3522	3.3523
	p=10	3.3049	3.30504

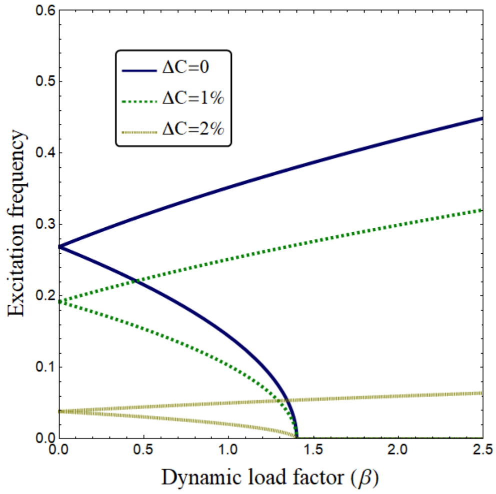


Fig. 3 Normalized frequency according to dynamic force component based on various moisture variation (p=1, ξ=0, a=10h, ΔT=100, α=0.3)

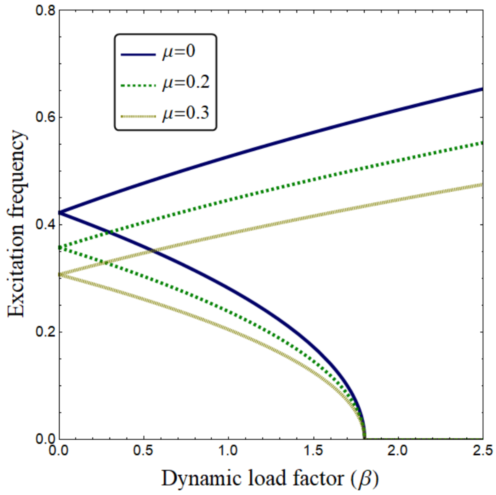
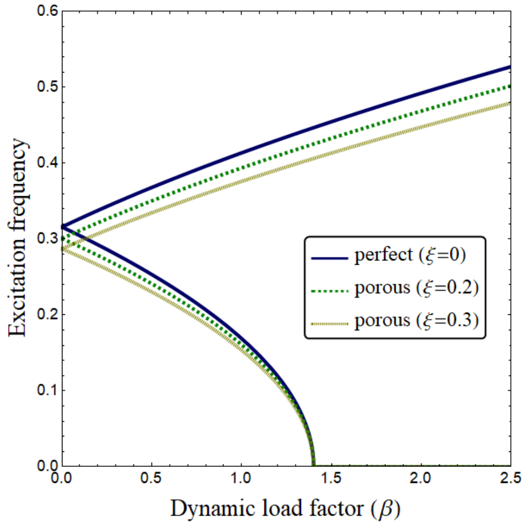


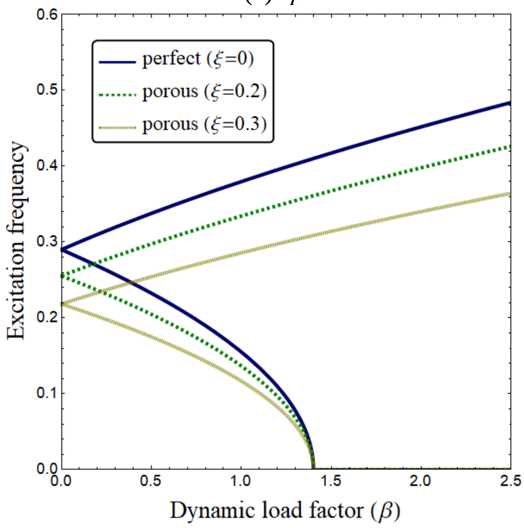
Fig. 4 Normalized frequency according to dynamic force component based on various nonlocal factors (α=0.1, p=1, ξ=0)

Figs. 4 and 5 illustrates the impact of static force component (α) and nonlocal constant (μ) on dynamical instability boundaries of porous FGM nanoplates when L/h=10, p=1. One can find from the figure that as the static force component growths, the boundaries of dynamical stability will decrease at prescribed nonlocal coefficient. Also, it is clear that by increase of nonlocal coefficient, the dynamical stability boundary will be diminished. Moreover, the start point (β=0) will be decreased as nonlocal coefficient rises. This is because of beam stiffness reduction at nano-dimension interaction.

Porosities amount influence on instability boundary of FGM nanobeam according to dynamical force component has been plotted in Fig. 6 when μ=0.2, α=*0.3* based of various material exponents. Increasing in porosities amount yields smaller vibration frequency due to decreasing the stiffness of nano-dimension beams. Moreover, the instability boundary gets smaller by the increasing of porosities amount. Thus, a porous FGM beam exposed



(a) p=1



(b) p=2

Fig. 6 Normalized frequency according to dynamic force component based on various material index and porosity amount (a/h=10, α=*0.3,* ΔT=0)

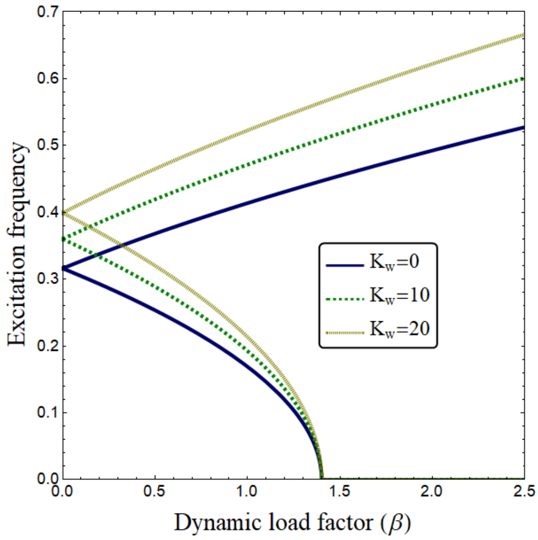


Fig. 7 Normalized frequency according to dynamic force component based on various foundation parameters (α=0.3, p=1, ξ=0)

periodic forces becomes more stable than non-porous beam. One can also find that by the decreasing of material exponent the instability region will increase. Actually, by the increment in material exponent value, the vibration natural frequency has been reduced.

Fig. 7 indicated the impacts of foundation parameter on instability boundary of FGM nanobeam when μ=0.2, $\alpha=0.3$. Increasing in foundation parameter yields larger vibration frequency due to increasing the stiffness of nano-dimension beam. It can be concluded that the beams are more stable under dynamic loads when they are rested on elastic foundation.

6. Conclusions

This paper presented new results for instability regions of periodically loaded FG nanobeams having porosities based on a 2-variable beam theory. The solution was based on Chebyshev-Ritz-Bolotin method. It was seen that as the static force component raised, the boundaries of dynamical stability will decrease at prescribed nonlocal coefficient. Also, it was understood that by increase of nonlocal coefficient, the dynamical stability boundary diminished. Increasing in porosities amount led to smaller vibration frequency due to decreasing the stiffness of nano-dimension beams. Moreover, the instability boundary get smaller by the increasing of porosities amount. As a conclusion, combined effects of moisture/temperature and porosity have notable impacts on stability boundaries of a porous FGM nano-size beam.

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