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Parallel tunnel settlement characteristics: a theoretical calculation approach and adaptation analysis

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Abstract. Settlement evaluation is important for shallow tunnels in big cities to estimate the settlement that occurs due to the excavation of twin tunnels. The majority of earlier research on analytical solutions, on the other hand, concentrated on calculating the settlement for a single tunnel. This research introduces a procedure to evaluate the settlement induced by the excavation of twin tunnels (two parallel tunnels). In this study, a series of numerical analysis were performed to validate the analytical solution results. Two geological conditions were considered to derive the settlement depending on each case. The analytical and numerical methods were compared, which involved considering many sections and conducting a parametric study; the results have good agreement. Moreover, a comparison of the 3D flat model and 2D (FEM) with the analytical solution shows that in the fill soil, the maximum settlement values were obtained by the analytical solution. In contrast, the values obtained by the analytical solution in the rock is more conservative than those in the fill. Finally, this method was shown to be appropriate for twin tunnels dug side by side by utilizing finite element analysis 3D and 2D (PLAXIS 3D and PLAXIS 2D) to verify the analytical equations. Eventually, it will be possible to use this approach to predict settlement troughs over twin tunnels.

Keywords: analytical methods; FEM; ground movement; settlement; twin tunnel

1. Introduction

Traffic in big cities has increased due to the growing population, thus requiring road tunnels to be constructed in many cases. Even though these tunnels have many advantages, tunnel construction may induce undesired settlement, affecting other structures. Safe tunnel design and construction require stability, surface deformation control, and effective supports. Thus, assessing ground settlements and their effects on structures above the tunnel is essential for tunnel projects (Terzaghi 1946).

Shallow metro tunnels are rapidly being constructed in large cities in China to decrease the traffic congestion arising from urbanization (Zhang and Huang 2014, Zhang *et al*. 2020). In general, surface settlement of soil due to tunneling has two essential causes: the stress relief mechanism and the subsidence because of removal support due to excavation (Fattah *et al*. 2013). Moreover, many parameters influence surface settlement, such as cohesion, friction angle, tunnel depth, tunnel diameter, Poisson's ratio, modulus of elasticity, face support pressure, and surface load. Each parameter or a combination of these parameters will affect the value of the subsidence (Chakeri and Ünver 2014).

Ground movements arise from changes in soil stresses around the tunnel face and the over-excavation of the tunnel cavity; this condition is often referred to as ground loss (Pinto and Whittle 2014). Thus, estimating the mechanism that causes ground movement is not easy especially because a complex relation exists. This situation has encouraged the use of numerical method analyses, such as finite element analysis (FEM) and finite difference method (FDM), over more than 30 years (Gioda and Swoboda 1999, Melis *et al*. 2002, Ocak 2009, Ercelebi *et al*. 2011, Yang *et al*. 2017, Khoo *et al*. 2018, Chortis and Kavvadas 2021). Despite the widespread availability of numerical methods, some scholars used empirical methods such as Potts and Addenbrooke (1997) who used an empirical equation to predict tunnel ground movements However, there are three typical techniques for predicting tunnel settlement: the first is the previously stated numerical simulation study. (2) second empirical methods, which are based on formulas captured from observations and can be applied for some field measurements (Peck 1969, Attewell and Farmer 1974, Atkinson and Potts 1977, O'Reilly and New 1982, Mair 1983, Herzog 1985, Vermeer 1991, Hamza *et al.* 1999, Macklin 1999). (3) analytical solutions.

There are some equations which can be introduced as an example of the empirical methods such as, Peck (1969) and Attewell and Taylor (1984) gathered surface settlement data from a number of large projects and used a Gaussian curve to analyze the data. They confirmed that the ground vertical displacement can be fitted to Gaussian or normal

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distribution curve. An empirical approach has been adopted for computing the ground deformation based on the following equation (Eq. (1))

$$u_y = u_{y0} \exp\left(-\frac{x^2}{2xi^2}\right) \tag{1}$$

Where:

u_{yo}	the maximum settlement above the tunnel axis,
x	the horizontal distance from the center line,
i	distance from tunnel axis to the point of inflection.

According to many previous researchers and scholars, the maximum surface settlement affects the shape of the settlement more than any other factor, as shown in Fig. 1. Mair (1983) suggested an equation to calculate the maximum value of surface settlement

$$S_{max} = \frac{V_s}{\sqrt{2\pi i}} = 1.252\frac{V_L.R^2}{i} \tag{2}$$

Where:

V_s	The volume loss,
V_L	the percentage of volume loss if the soil is incompressible,
R	the tunnel radius.

Macklin (1999) used the load factor parameter (LF = N/Nc) to predict the value of volume loss

$$V_l = 0.23exp\left(4.4\frac{N}{N_c}\right) \tag{3}$$

Where:

N	the stability number,
N_c	the critical stability number (dimensionless parameter).

Dimmock and Mair (2007) used a graph to estimate the value of Nc depending on two variables, namely, (P/D) and (H/D). In the curve shown in Fig. 2, P is the length of the unsupported tunnel, which is equal to zero in the tunnel boring machine method; H is the overburden thickness; and D is the tunnel diameter.

Following that, there is an analytical analysis technique that may be utilized to determine short-term surface settlement. However, these methods did not use the influence of many parameters (Lo and Rowe 1984, Loganathan and Poulos 1998, Chi *et al.* 2001).

Some equations may be used as examples of the analytical approach, such as those utilized by Attewell and Taylor (1984) to construct an implicit equation using homogeneous, isotropic, and half-space equations. This formula, which was first proposed by Sagaseta (1987), includes the influence of tunnel deformation and the ability of soil to compress. Compressible soil requires a Poisson's ratio μ less than 0.5. Verruijt and Booker (1998) evaluated the settlement by using the following equation

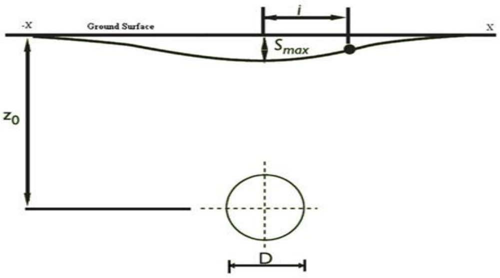


Fig. 1 Maximum surface settlement for a single tunnel

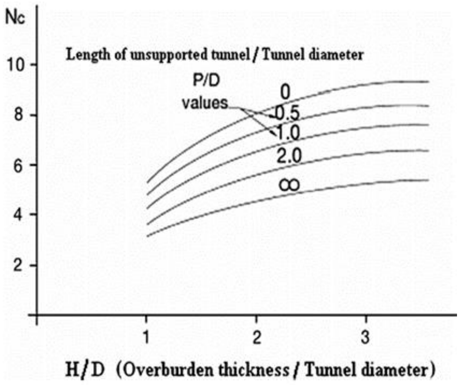


Fig. 2 Estimating N_c value

$$S_v = 4\varepsilon R^2(1-\Psi)\frac{Z_0}{x^2+{Z_0}^2} - 2\delta R^2\frac{Z_0(x^2-{Z_0}^2)}{(x^2+{Z_0}^2)^2} \tag{4}$$

Where:

R	the tunnel radius (m),
Z_0	the depth of tunnel axis (m),
ν	The Poisson's ratio,
ε	equivalent undrained ground loss,
Δ	the shape ovalization,
Ψ	Dilation angel
x	the distance from the central line of a tunnel (m).

Loganathan and Poulos (1998) used Verruijt and Booker's (1998) estimation and suggested the following analytical method

$$S_v = 4V_l(1-\Psi)R^2\frac{Z_0}{{Z_0}^2+x^2}\exp\left(-\frac{1.38x^2}{\left({Z_0}^2+R^2\right)^2}\right) \tag{5}$$

In general, few scholars dealt with twin tunnels, especially when using analytical formulas. Divall (2013) predicted the settlement for twin tunnels by using a semi-empirical equation and applied the superposition principle to create the following equation

$$S_v = S_{max}.\exp\left(-\frac{{X_a}^2}{2i^2}\right) + \exp\left(\frac{(X_a-d)^2}{2i^2}\right) \tag{6}$$

Where:

d	the horizontal distance between the center lines of two tunnels.
X_a	the lateral distance from the center line of the first excavated tunnel.

Many previous studies used the analytical solution to estimate the 2D distribution of ground movements for shallow tunnels in soft ground (Sagaseta 1987, Verruijt and Booker 1998, Verruijt 1997, Gonzalez and Sagaseta 2001). However, all of these previously mentioned analytical equations have been concentrated on prediction one tunnel settlement, and simplified the assumption concerning the soil parameters. In general, this type of analysis depends on a symmetrical framework to explain the horizontal and vertical components of ground deformation more than the empirical solution, which depends on Gaussian curve. Moreover, this analysis can be used to evaluate the accuracy of the numerical solution. However, very few analytical solutions have been used to estimate the settlement due to excavate twin parallel tunnels, resulting in a greater reliance on numerical simulation. As a result, it is critical to develop a new equation to estimate this type of settlement, to use it in estimating the accuracy of the numerical simulation, and to develop a framework to evaluate the entire settlement shape.

In this work, a novel theoretical method to estimating surface settling due to twin tunnel excavation is presented. In this approach a superposition method is used to describe surface settlement troughs over twin tunnels considering that each tunnel has its own settlement and the maximum settlement will be above the tunnel, while in the region between the two tunnels will be overlapping between the settlement (depends on the spacing between the two tunnels) and this overlapping shall be calculated using a superposition method and to sum the settlement induced from each tunnel. A case study, 2D FEM, and 3D FEM with a parametric study back up the findings of the suggested approach. Tunneling excavation is modeled by removing the element inside the tunnel boundary. Fill soil and rock are considered two separate cases to confirm the validation of the new method to predict settlement.

2. Analytical method to predict twin tunnel-induced surface settlement

In many cases and conditions, tunnels such as road tunnels and subway tunnels are driven in parallel. In this case, estimating the value of volume loss will be complicated. We have two kinds of settlements: short-term settlement, which is related to the construction of the first tunnel, and long-term settlement, which is due to both tunnels (Fang *et al.* 1994). Addenbrooke and Potts (2001) conducted a series of finite element analysis (FEM) to investigate the effect of excavating a new tunnel in an existing tunnel by using different geometrical arrangements and discovered that the settlement value increases as the central spacing between two parallel tunnels decreases. Hunt (2005) performed a series of 2D FEM and then extracted a modification factor, which can be applied to the overlapping region. The modifying settlement can be expressed as

$$S_{mod} = F.S_v \tag{7}$$

Where:

S_v	the horizontal distance between the center lines of two tunnels,
F	the modification factor, which is expressed as the following:

$$F = 1 + m\left(1 - \frac{d + x}{AK_1(Z_0 - Z)}\right) \tag{8}$$

Where:

A	the trough width coefficient (usually taken as 2.5 or 3),
d	the center to center horizontal distance of tunnels,
K_1	the zone of the lead tunnel,
M	the greatest modification factor specified by scholar.

The superposition technique is suggested to assess the surface settling caused by the excavation of twin tunnels. This settlement prediction method depends on a single-tunnel prediction formula. The second tunnel is assumed to have a similar diameter, size, and depth as the first tunnel. Two kinds of settlement—uniform convergence and ovalization—are included in this analytical formulae. The convergence and the ovalization modes (Fig. 3) could be determined in Eqs. (9)-(10) and (11)-(12) respectively.

Convergence mode

$$u_{xc} = u_{\varepsilon}\frac{xR}{x^2 + y^2} \tag{9}$$

$$u_{yc} = u_{\varepsilon}\frac{yR}{x^2 + y^2} \tag{10}$$

Ovalization mode

$$\mathrm{u_{yo}} = \mathrm{u_{\delta}}\frac{\mathrm{YR}}{3 - 4\mu}\frac{(3 - 4\mu)((X^2 + Y^2)^2}{(X^2 + Y^2)^3} \frac{-(3X^2 - Y^2)(X^2 + Y^2 - R^2)}{(X^2 + Y^2)^3} \tag{11}$$

$$u_{yo} = u_{\delta}\frac{YR}{3 - 4\mu}\frac{(3 - 4\mu)((X^2 + Y^2)^2}{(X^2 + Y^2)^3} \frac{-(3X^2 - Y^2)(X^2 + Y^2 - R^2)}{(X^2 + Y^2)^3} \tag{12}$$

Where:

R	the radius of the tunnel,
H	the tunnel depth to the spring line,
μ	The Poisson's ratio,
X	the vertical distance of the section
u_{δ}	max.deformation due to ovalization occurs at the tunnel cavity
u_{ε}	max.deformation due to convergence occurs at the tunnel cavity
y	the horizontal distance of the section.

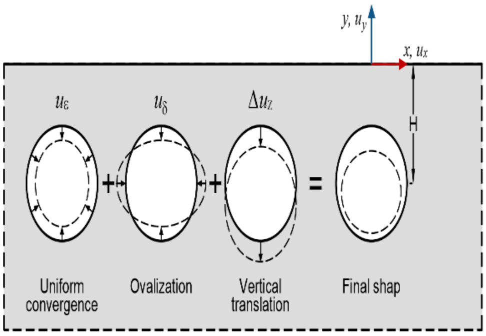


Fig. 3 Tunnels deformation modes (González and Sagaseta 2001)

The settlement for two tunnels may be stated as follows using the superposition principle

$$U_{Ytot} = U_{Y1} + U_{Y2} \tag{13}$$

$$U_{xtot} = U_{x1} + U_{x2} \tag{14}$$

Where:

U_{y1}	$(U_{Y1} = U_{yc} + U_{yo})$: the first tunnel’s vertical settlement (mm),
U_{y2}	$(U_{Y2} = U_{yc} + U_{yo})$: the second tunnel’s vertical settlement (mm),
U_{yc}	Settlement due to the convergencemode (mm),
U_{yc}	Settlement due to the ovalization mode (mm),
U_{x1}	*(Ux1=Uxc+Uxo): the first tunnel’s* horizontal settlement (mm),
U_{X2}	*(Ux2=Uxc+Uxo):* the second tunnel’s horizontal settlement (mm),
U_{xc}	Settlement due to the convergence mode (mm),
U_{xo}	Settlement due to the ovalization mode (mm).

The programming codes which express the equation

$$\mathrm{ux} = u_{max} * \mathrm{x} * \mathrm{self}.\frac{\mathrm{R}}{(\mathrm{x} ** 2) + (\mathrm{y} ** 2)} \tag{15}$$

$$uy \;=\; u_{max} * y * self.\frac{R}{(x ** 2) + (y ** 2)} \tag{16}$$

$$\begin{aligned} ux \;=\; & (u\_max * x * R/(3 - (4 * nu))) * (((3 - (4 * nu)) \\ & * (x2 + y2) ** 2) - (((3 * y2) - x2) \\ & * (x2 + y2 - R2)))/(x2 + y2) ** 3 \end{aligned} \tag{17}$$

$$\begin{aligned} uy \;= & -(u\_max * y * R/(3 - (4 * nu))) * (((3 - (4 * nu)) \\ & * (x2 + y2) ** 2) - (((3 * x2) - y2) \\ & * (x2 + y2 - R2)))/(x2 + y2) ** 3 \end{aligned} \tag{18}$$

3. Verification by case study

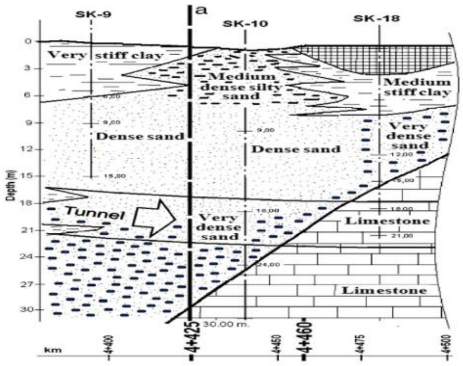
The proposed analytical solution (superposition method) needs to be validated to predict the surface settlement related to a case study. The Istanbul Metro was chosen as a case study to apply this equation on twin tunnel settlement. The results exhibited good agreement, as shown in Fig. 5. The spacing between the two tunnels is 14.4 m, and the depth from the surface to the tunnel center is 19.88 m, the diameter of each tunnel is 6.56 m.

Each site has its own geological conditions, Chen and Lee (2020) proved that the geological conditions is an effective factor on the deformation of the tunnel. The first construction stage of the Istanbul Metro Line began in 1992 and was open to the public in 2000. This line is being extended gradually, and additions and other extensions are being constructed in other locations. One of these metro lines is the twin line between Otogar and Kirazlı-1 (5.77 km). The metro line consists of a 3.87 km tunnel, a 0.62 km cut-and-cover station, and a 1.28 km at-grade crossing. The excavation of this section began in May 2006 and was completed in June 2008. This metro line integrates the Kirazlı-1–Basaksehir–Olimpiyat Koyu Metro Project, with a length of 15.8 km. The Otogar and Kirazlı-1 Metro Line will integrate the Aksaray–Ataturk Airport light metro line that is now in service. Mahmutolu (2011) and Ocak (2012) provide the full data of the Otogar and Kirazl-1 Metro Lines, including geological conditions, surface settlement measures, and soil characteristics (2009).

The metro lines in the study area were excavated by a Herrenknecht EPBM in the left tube and a Lovat EPBM in the right tube. One tube excavation followed around 100 m behind the other tube. About 200 m of this metro line was selected as the study area. This part is located between km 4+300 and 4+500 of the Otogar and Kirazlı-1 Metro Line. Extensive site investigation was performed to determine the ground condition. Previous information about the geological conditions of the site was revised and updated by (Ayson 2005). The subsurface soil was characterized by geotechnical drilling and in situ data and laboratory test results. The soil layers were grouped into four main categories depending on the soil classification and geotechnical test results to establish a geotechnical model for the project area. The geology of the study area is given in (Fig. 4). The geotechnical properties, layer sequence, and thickness of each layer used in the numerical model are given in Table 1.

where *c* is the cohesion, *φ* is the friction angle, E is the elasticity modulus, *ν* is the Poisson’s ratio, and *γ* is the unit weight. The proposed approach was found to be suitable for twin tunnels based on the aforementioned data and by putting the average parameters of the previous mentioned layers in the proposed equation, and by comparing measured data from Istanbul Metro with the proposed analytical method as shown in (Fig. 5). Till now this superposition technique is basically descriptive. Eventually it possible to use this approach to predict and evaluate settlement troughs due to the execution of twin tunnels.

(a) Main route and location



(b) Main route and geological section

Fig.4 Main route of Otogar–Kirazlı 1 metro line

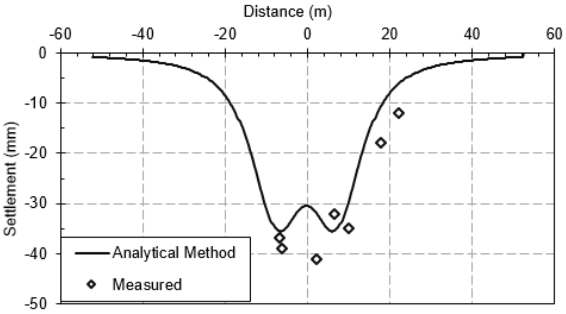


Fig. 5 Surface settlement curves for observed data and analytical solution results

Table 1 Soil grouping and geotechnical design data in the project area (Ayson 2005)

Soil	Thick. (m)	Υ kN/m^3	C (kPa)	Φ Deg.	E (MPa)	υ
stiff Clay	6.0	18.2	20	9	51	0.35
dense Sand	10.6	19	1	35	24	0.25
very dense Sand	--	19.5	1	35	30	0.30

Table 2 Geotechnical parameters for each layer in the numerical simulation

Soil	γ(kN/m^3)	c (kPa)	Φ Deg.	E' (MPa)	ν
Fill	21.5	24	10	30	0.4
Rock	24	200	33	600	0.2

Table 3 Lining and shotcrete modelling parameters

Element	Simulation type	Thickness (mm)	Υ (kN/m^3)	E (MPa)	ν
Lining	Volume (Linear Elastic)	950	25	28000	0.2
Shotcrete	Plate (Elastic)	280	25	10500	0.2

4. Finite element analysis modelling and geotechnical parameters

The Mohr–Coulomb (denoted by MC) model which was introduced by Mohr 1914 is typically used for soil and rock mechanics (Zhao 2000). The MC model was utilized as an elastic completely plastic constitutive model for both the fill and the rock in this study. The parameters of the soil are shown in the Table 2. The lining simulated as a solid (volume) element that formed with lining actual geometry. the lining material response is interpreted using liner elastic material. Shotcrete is simulated using thin plate member. The concrete material considered in analysis for both lining and shotcrete is included in Table 3.

4.1 Numerical models

A series of 2D and 3D numerical simulations based on the PLAXIS software package were performed to evaluate the effectiveness of the suggested approach in verifying the applicability of the proposed analytical solution for forecasting tunnel settlement. The first two numerical models used were 2D numerical models, the second two were 3D numerical models, and the third was a 3D numerical model with a slope. The 2D and 3D models have two kinds of soil, namely, rock and fill. The slope model consists of limestone rock.

All the models were laterally extended to a distance of 120 m and extended vertically to 56 m, and the diameter of the tunnel is 13 m. The displacement which is perpendicular to the lateral boundaries was restrained, while vertical displacement was permitted. There was no vertical or horizontal displacement along the model's bottom limits, while the top bounds were free to move. The mesh size was medium as well.

4.2 Stage of construction

For 2D Models, the first stage is the initial phase, which involves analyzing the soil without any load or excavation. This initial phase was at rest because no inclination in the soil was found. After the excavation began and was simulated by removing the soil from the boundary of the tunnel, and after the whole part was excavated, the plat was activated as a first support, and then the lining was used as a secondary support.

For 3D analysis, the first stage is the initial phase, which involves analyzing the soil without any load or excavation. Afterward, the excavation began and was simulated by removing the soil from the boundary of the tunnel. After the whole part was excavated, the plat was activated as a first support, and then the lining was activated as a secondary support.

5. FEM analysis result

5.1 Analysis results for fill soil models

To compare the 2D and 3D numerical analyses with the analytical solution, and to validate the generalizability of this approach, three cross sections at various levels were examined. (Fig. 9) shows the settlement directly above the tunnel, showing good agreement between the numerical simulation and the analytical solution. The maximum settlement in the 2D and analytical solution was almost about 85 mm; the 3D numerical simulation shows less settlement of almost 75 mm, and the settlement in the analytical solution was 88 mm. (Fig. 10) shows the settlement 4 m above the tunnel. The maximum settlement in the analytical solution was almost 82 mm, while it was more than 70 mm in the 2D numerical simulation. The 3D numerical simulation obtained a lower value of almost 70 mm. (Fig. 11) shows that the analytical solution's value was

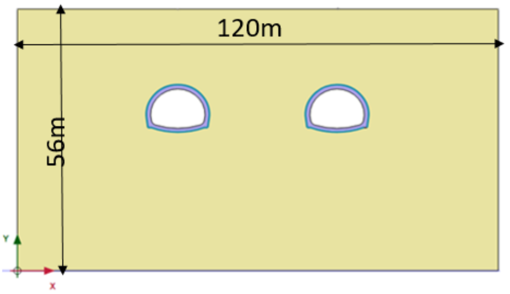


Fig. 6 Model configuration for 2D analysis

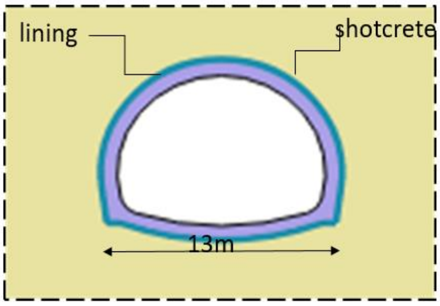
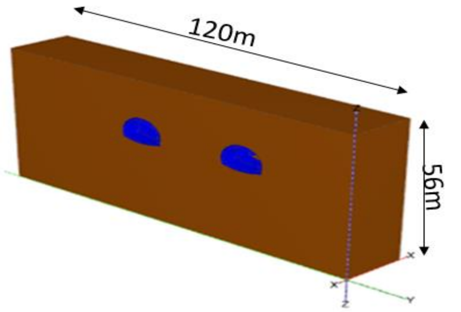
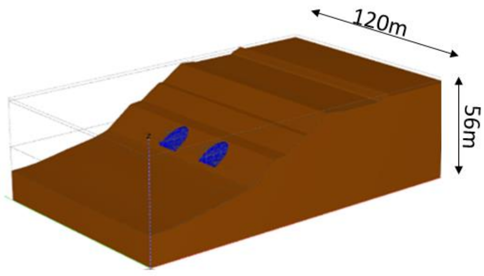


Fig. 7 Tunnel geometry



(a) Flat model



(b) Slope model

Fig. 8 Model configuration for 3D analysis

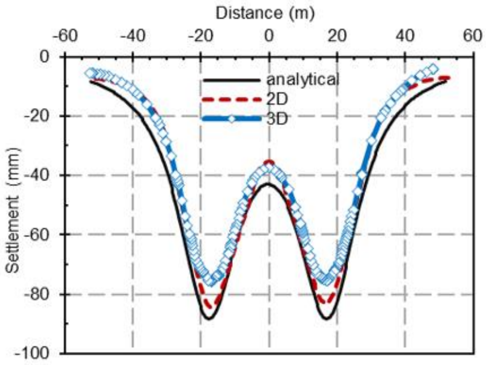


Fig. 9 Settlement results directly above tunnels for analytical, 2D and 3D analysis

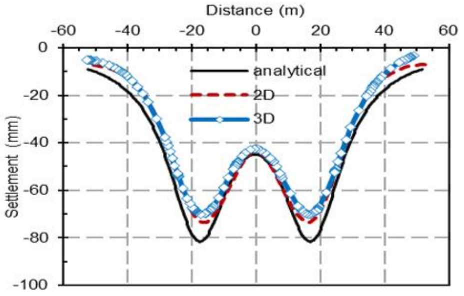


Fig. 10 Settlement results 4.0 above tunnels crown for analytical, 2D and 3D analysis

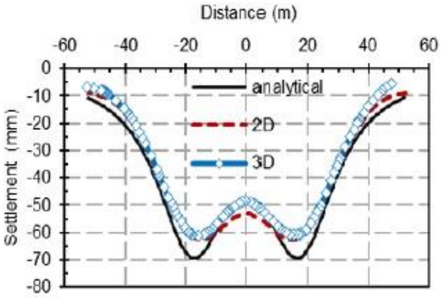


Fig. 11 Settlement results 14.0 above tunnels crown for analytical, 2D and 3D analysis

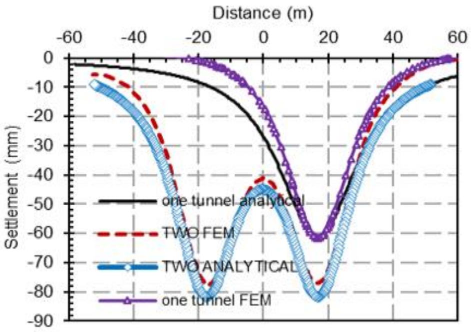


Fig. 12 the difference in settlement between single and twin tunnel 4m above the tunnel

70 mm while that of the 3D and 2D numerical simulation had a value of 60 mm. These findings indicate that the settlement is reduced as one moves higher above the tunnel, and the analytical solution gave the maximum settlement value. As a conclusion to these results, the analytical solution in fill soil shows more settlement than the FEM results, and the 2D FEM shows more settlement than the 3D FEM.

A comparison of the settlement in a single tunnel and twin tunnels has been done to demonstrate the impact of twin tunnel interaction in fill soil. The settlement has been taken 4m above the tunnel in both the 3D finite element analysis (FEM) and the analytical solution (new equation) as shown in (Fig. 12).

5.2 Analysis results for rock soil models

Comparison between 3D numerical analysis (in two cases, namely, flat land and slope), 2D numerical analysis, and the analytical solution is considered. The 3D numerical analysis includes a slope from limestone rock. Three cross sections above the tunnel were considered. The first cross section is directly above the tunnel, the second is 4 m above the tunnel, and the third one is 14 m above the tunnel. The analytical solution and the numerical simulations show good agreement. The following figures show the settlement directly above the tunnel where the maximum settlement value (vertical displacement) is found. The settlement in the 2D method, the analytical solution, and the 3D FEM is 7.5, 6.2, and 5.8 mm, respectively.

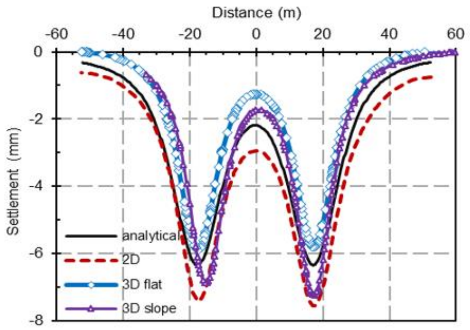


Fig. 13 Settlement results directly above tunnels for analytical, 2D and 3D analysis

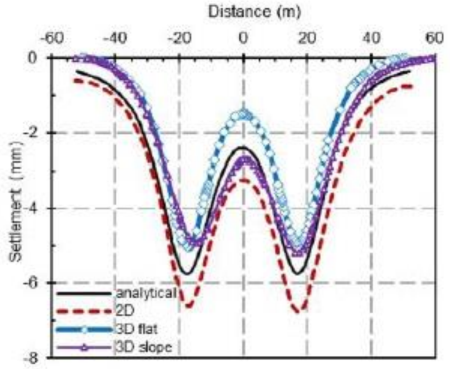


Fig. 14 Settlement results 4.0 above tunnels crown for analytical, 2D and 3D analysis

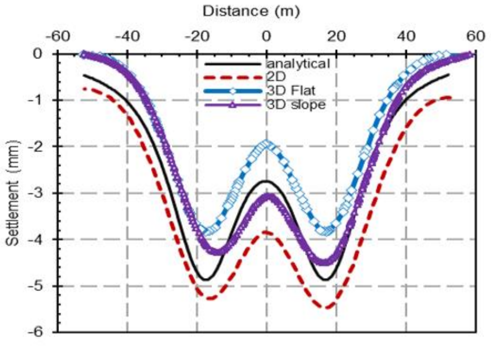


Fig. 15 Settlement results 14.0 above tunnels crown for analytical, 2D and 3D analysis

Based on the following figures, the center line of the maximum settlement in the 3D FEM in the slope deviates from the center line in the analytical solution and the flat soil because of the effect of the slope, which causes the center line of the settlement to deviate. Given the close values of the analytical and numerical methods in the slope, make the use of the proposed method for the slope is acceptable, with the deviation taken into consideration.

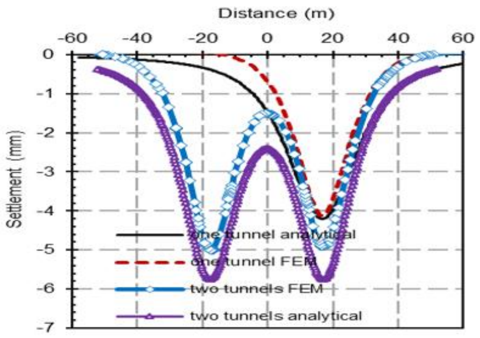


Fig. 16 The difference in settlement between single and twin tunnel 4m above the tunnel

A comparison of the settlement in a single tunnel and twin tunnels has been done to show the effect of twin tunnel interaction in rock soil. The settlement has been taken 4m above the tunnel in both the 3D finite element analysis (FEM) and the analytical solution (new equation), as shown in (Fig. 16).

6. Parametric study

Many parameters affect surface settlement in FEM analysis such as geotechnical factors (cohesion(C), friction angel (PHI), modulus of elasticity(E)) and the diameter of the tunnel (D) in the case of the single tunnel, and also related to the distance between the two tunnels in the case of two parallel tunnels. recognizing such effects can facilitate the measurement of settlement and present a better empirical method for settlement measurements. A series of parametric studies for the rock for the 3D FEM and the analytical solution, which includes the suggested equation, were performed in this part to examine the effect of various factors on the ground surface settlement.

The effects of five parameters on the settlement value were studied in terms of cohesion (C), modulus of elasticity (E), friction angle (phi), Poisson's ratio (u), and the distance between tunnels. In examining the influence of each parameter, the value of one parameter is changed whereas the other parameters are kept constant.

6.1 Effect of cohesion

Three cohesion values were considered in this parametric study, as shown in Fig. 17. The first value was c=100 kPa, and the numerical and analytical methods recorded a settlement value more than 6 mm. The second value was c = 200 kPa, and the settlement was more than 6 mm for the analytical solution and almost 5.8 mm for the numerical analysis. For c = 250 kPa, the settlement value for the analytical solution and the numerical analysis remained the same. For c = 400 kPa, the settlement values for both the numerical analysis and the analytical solution were almost the same. Two details emerge from the parametric study:

- When the cohesion is increased to more than a specific value, the settlement will not change in the numerical simulation (Fig. 17).
- The constant settlement value in the analytical solution shows the limitation of this method when taking different cohesion values, which means changing the cohesion will not influence this method.

6.2 Effect of modulus of elasticity

Three values of the modulus of elasticity were considered in this parametric study, as shown in Fig. 19. The first value was 400 MPa; the settlement value in the numerical analysis was almost 8 mm and more than 8 mm in the analytical solution. For E = 600 MPa, the settlement value was 6.2 mm in the numerical simulation and 5.8 mm in the analytical solution. For E = 800 MPa, the settlement value in the analytical solution and in the numerical simulation was equal to 4.8 mm. These findings show that the analytical solution is more conservative than the numerical simulation methods and has lower settlement values. Fig. 20 shows the relation between the maximum settlement and the modulus of elasticity. As the modulus of elasticity increases, the settlement decreases in both

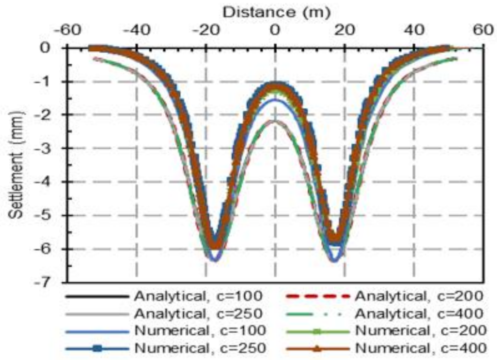


Fig. 17 Effect of changing the cohesion on the settlement from both the numerical and the analytical analysis

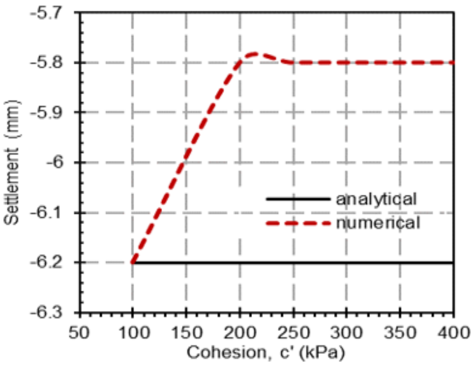


Fig. 18 Relation between the cohesion and the maximum settlement

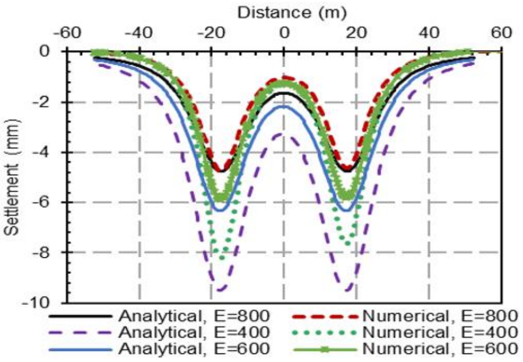


Fig. 19 Effect of modulus of elasticity on the settlement from the numerical method and the analytical solution

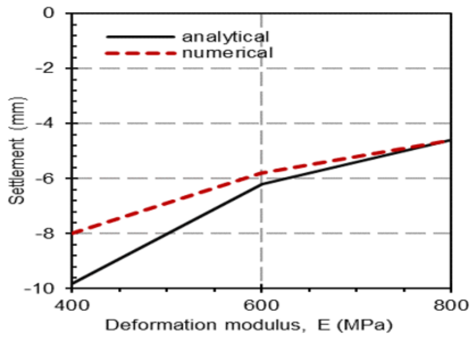


Fig. 20 Relation between settlement and deformation modulus

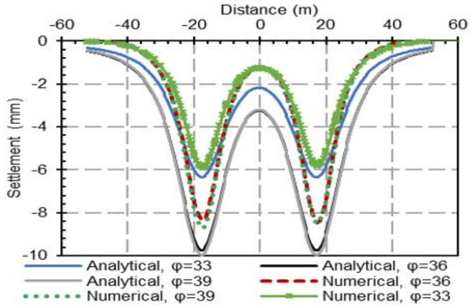


Fig. 21 Effect of friction angle on the settlement from the numerical method and the analytical solution

numerical and analytical methods; this finding is consistent with many previous research results (Li and Ji 2017). Furthermore, the sensitivity of this analytical solution (equation) to the modulus of elasticity is shown by altering the settlement values while changing the modulus of elasticity.

6.3 Effect of friction angle

Three friction angle values were considered, as shown in Fig. 21. The first value was 33°, and the difference between the analytical solution and the numerical simulation was less than 1 mm. The second value was 36°, and the difference between the two settlement values increased to 1.6 mm. The third value was 39°, and the difference between the numerical and analytical solutions is almost 1.8 mm. This finding leads us to consider that increasing the friction angle causes the difference between the analytical solution and FEM to increase. Moreover, the results show that the analytical solution is more conservative than the numerical simulation in most cases.

The settlement value increased with the increasing friction angle, as noted in both numerical and analytical methods. This result is linked to the fact that the settlement rises as the friction angle increases for high cohesion values (c=100, 200) kPa. At low cohesion values (c=25, 30) kPa, the settlement decreases with increasing friction angle until it reaches a specific value, then there will be a turning point where the settlement will increase as the friction angle increases (Fig. 22). This situation occurs because the shear strain increases as the friction angle increases in hard soil such as dense sand or rock. Similar results were observed by Ghiasi and Koushki (2020), who found that it is not necessary to get decreasing in the settlement with increasing the friction angel value.

6.4 Effect of poisson's ratio

This parametric study was conducted in terms of Poisson's ratio, as shown in Fig. 23. Three (3) Poisson's ratios were considered, namely, $\nu = 0.2, 0.25, 0.4$. As the Poisson's ratio increases, the settlement value in the numerical solution also increases; this finding is in agreement with that of Liu and Zhang (2017). In the analytical solution, the constant settlement value in comparison with that in the numerical analysis leads us to consider that the sensitivity to this parameter is less than that in the numerical simulation (Fig. 24).

6.5 Effect of tunnel spacing

The clear spacing between two adjacent tunnels is one of the most important factors that control soil movement. Nawel and Salah (2015) investigated the interaction between two parallel tunnels by numerical simulation. Do *et al*. (2014) concluded that an increase in the surface settlement can be

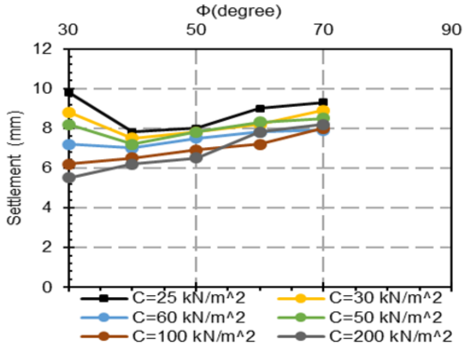


Fig. 22 Relation between friction angle and maximum settlement

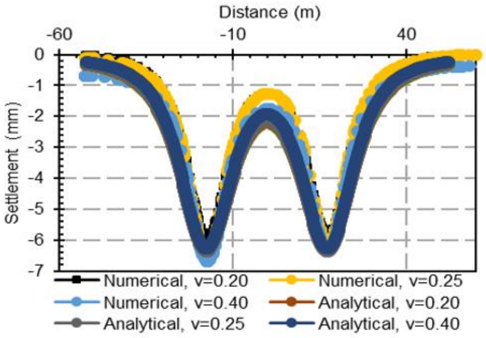


Fig. 23 Effect of Poisson's ratio on the settlement from the numerical method and the analytical solution

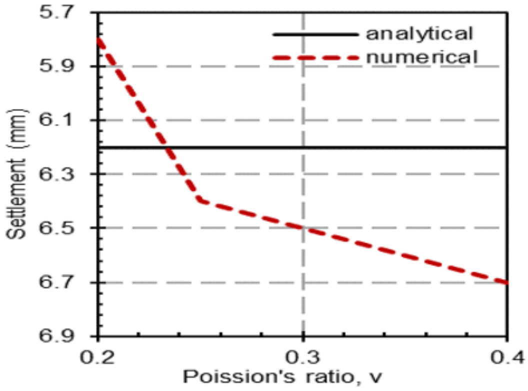


Fig. 24 Relation between Poisson's ratio and maximum surface settlement

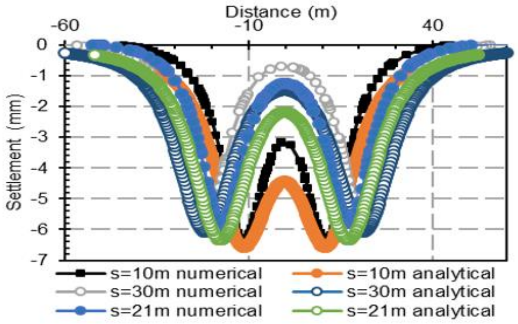


Fig. 25 Effect of changing distance on the settlement value from numerical and analytical methods

expected and compared with that induced above a single tunnel because of the interaction of twin tunnels. Chehade and Shahrour (2008) observed that the settlement profile and magnitude of the settlement depend on the distance between tunnels. They found that if the distance between the tunnels is at least three times the diameter of the tunnel, there would be no significant impact. Chakeri *et al.* (2011) investigated the interactions between twin tunnels by conducting a numerical analysis study and showed that as the horizontal spacing between two parallel tunnels increased, the settlement in the centerline of the tunnel decreased. Do *et al.* (2014) studied the interaction between twin tunnels in 2D numerical simulation.

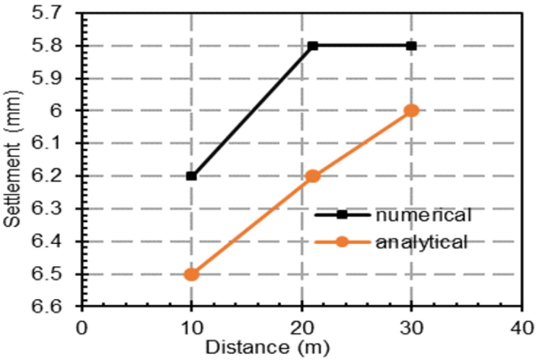


Fig. 26 Relation between maximum settlement and distance between tunnels

Fig. 25 shows the effect of the distance on the settlement, with three distances taken into consideration, namely, 10, 21, and 30 m. In both analytical and numerical solutions, the settlement reduced as the distance between two tunnels grew (Fig. 26). For example, when the distance between the twin tunnels was equal to 10 m, the settlement in the analytical solution and the numerical analysis was 6.4 and 6.2 mm, respectively. The analytical and numerical solutions settled at 6.2 and 5.8 mm, respectively, as the distance grew to 21 m. In summary, this parametric study showed that the numerical simulation (FEM) obtained greater settlement values than the analytical solution did.

7. Conclusions

The prediction and control of surface settlement are among the most critical problems in tunneling excavation because of the effect of different parameters. In this paper, twin parallel tunnel-induced surface settlement is mainly controlled by geotechnical factors (modulus of elasticity, cohesion, friction angle, Poisson's ratio) and engineering factors (tunnel depth, tunnel diameter, and distance between the twin tunnels). Two materials, namely, fill and rock, were considered to confirm the ability of the proposed analytical solution equation to predict the surface settlement above twin tunnels. In the fill soil material, three cases were considered: 3D FEM, 2D FEM, and the new equation of the analytical solution. In the rock material, four cases were considered: 3D FEM, 2D FEM, 3D FEM for a slope, and the new equation of the analytical solution. A case study was validated with a parametric study for the rock material, which includes five cases: cohesion (C), modulus of elasticity (E), friction angle (phi), Poisson's ratio (u), and the distance between the twin tunnels. These five cases were compared in two ways, namely, 3D FEM and the new equation in the analytical solution.

The modulus of elasticity, the friction angle, and the distance between the two tunnels seem to be the most effective variables that affect the settlement value, according to the new equation's sensitivity analysis findings, while the equation is not sensitive to Poisson ratio or to cohesion variation values.

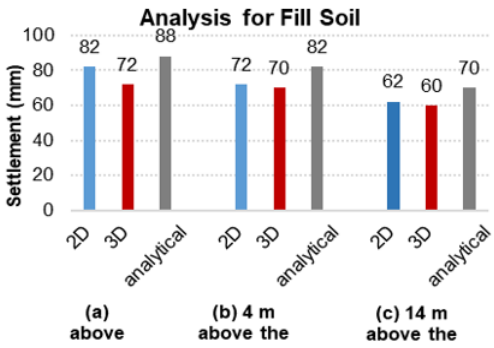


Fig. 27 Comparison between the settlement values in fill soil at different depth

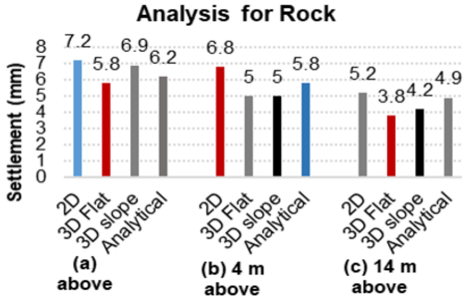


Fig. 28 Comparison between the settlement values in rock at different depth

And the difference between the finite element analysis method (FEM) and the analytical solution may due to the different sensitivity for each parameter. However, the two approaches seem to be in excellent agreement, leading to the conclusion that the two methods are suitable for engineering projects with certain analytical solution constraints.

Fig. 27 shows the difference among the three methods in fill soil at different levels above the tunnel, while Fig. 28 shows the settlement in the rock in the three methods at three different levels above the tunnel.

A few empirical equations can be used to predict the maximum surface settlement value. These equations include many crucial factors that influence maximum surface settlement values. Investigating the effect of each factor can help in understanding and applying these relationships or in suggesting a new formula for evaluating the maximum surface settlement.

In this paper, a new modified equation was compared with a series of 2D and 3D numerical analyses to determine the applicability of this analytical solution for many cases. The results show the following:

- This new analytical solution shows good agreement with the 2D and 3D FEM.
- A comparison between the 3D flat model and 2D with the analytical solution shows that in the fill soil, the maximum settlement values are obtained by the analytical solution, while the settlement values obtained by the analytical solution in the rock are between those of the 2D and 3D numerical analyses, thereby leading us to consider that the analytical solution in rock is more conservative than that in the fill.
- The analytical solution may be used in a slope situation, however there are certain restrictions owing to the slope effect's departure of the maximum settlement center line in the numerical solution.
- The parametric study shows that cohesion and Poisson's ratio do not affect the value of the settlement in the analytical solution, which can also be considered one of the limitations of this method.
- Another limitation of this method is its weak efficiency when applied in the case of different soil layers with a large range of parameters, such as fill soil and rock.
- Finally, the limits of this equation are focused on three major points: less sensitivity to cohesion fluctuation, less sensitivity to poison ratio values, and poor efficiency in layered soil with high parameter variation.
- It is recommended to make more improvement for this analytical equation in next coming research related to tunnel settlement to include more engineering projects.

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