

# Predicting the shear strength parameters of rock: A comprehensive intelligent approach

Hadi Fattahi<sup>1</sup> and Mahdi Hasanipanah<sup>\*2,3</sup>

<sup>1</sup>Faculty of Earth Sciences Engineering, Arak University of Technology, Arak, Iran

<sup>2</sup>Institute of Research and Development, Duy Tan University, Da Nang 550000, Vietnam

<sup>3</sup>Department of Mining Engineering, University of Kashan, Kashan, Iran

(Received April 13, 2021, Revised October 1, 2021, Accepted November 10, 2021)

**Abstract.** In the design of underground excavation, the shear strength (SS) is a key characteristic. It describes the way the rock material resists the shear stress-induced deformations. In general, the measurement of the parameters related to rock shear strength is done through laboratory experiments, which are costly, damaging, and time-consuming. Add to this the difficulty of preparing core samples of acceptable quality, particularly in case of highly weathered and fractured rock. This study applies rock index test to the indirect measurement of the SS parameters of shale. For this aim, two efficient artificial intelligence methods, namely (1) adaptive neuro-fuzzy inference system (ANFIS) implemented by subtractive clustering method (SCM) and (2) support vector regression (SVR) optimized by Harmony Search (HS) algorithm, are proposed. Note that, it is the first work that predicts the SS parameters of shale through ANFIS-SCM and SVR-HS hybrid models. In modeling processes of ANFIS-SCM and SVR-HS, the results obtained from the rock index tests were set as inputs, while the SS parameters were set as outputs. By reviewing the obtained results, it was found that both ANFIS-SCM and SVR-HS models can provide acceptable predictions for interlocking and friction angle parameters, however, ANFIS-SCM showed a better generalization capability.

**Keywords:** ANFIS; hybrid models; shear strength; SVR

## 1. Introduction

In geotechnical engineering field of study, especially underground excavation and slope design, there is a pressing need for estimating most reliably the rock strength as well as the deformation parameters. For the purpose of designing rock engineering structures, researchers make an extensive use of the parameters related to rock shear strength (SS). These parameters designate the way the rock material put up resistance to shear stress-induced deformations. In general, the above-noted resistance takes place owing to two internal mechanisms, namely interlocking ( $c$ ) and internal friction angle ( $\varphi$ ) (Alejano and Carranza-Torres 2011). On the other hand, in lab, researchers often face a number of parameters that make it very difficult for them to directly determine the rock shear parameters. Remember that it is not easy to provide high quality core samples, particularly in regard to rocks that are highly fractured, weathered, and weak. Regardless of this issue, to directly determine the parameters is commonly a destructive, costly, time-consuming task (Alejano and Carranza-Torres 2011, Jahed Armaghani *et al.* 2014). Therefore, it is more practical to indirectly estimate the parameters of SS using several rock index tests that are both simpler and more economical.

Literature is consisted of a great deal of research carried

out into rock SS (Barla *et al.* 2010, Amann *et al.* 2012, Asadi and Bagheripour 2013). It includes a number of studies conducted into the SS of sand and clay with taking into consideration a variety of rock particle mixtures. The scholars in this field generally maintain that the SS of the mixtures grows if the percentages of rock particles increases (Iannacchione and Vallejo 2000). In case of jointed rocks, Singh and Singh (2012) suggested an improved non-linear Mohr–Coulomb strength criterion and, at the same time, discussed two limitations of the criterion: 1) it has a linear strength response and 2) it overlooks the impacts of intermediate principal stress upon the strength behaviors. The proposed model was found applicable doing back analysis on the results of over 730 triaxial tests. In another study, Hajdarwish and Shakoor (2006) applied a variety of mudrock (which included 45 samples) for the establishment of a correlation between the engineering/geological properties and the SS parameters. To do this, they adopted techniques of bivariate and multiple regression. For each one of their samples, they determined numerous parameters such as dry density and absorption. Yazdani (2012), on the other hand, employed a new Hoek *et al.* (2002) failure criteria to carry out a comparative research on SS parameters of shale. The results showed that the failure envelope that was acquired by means of the new Hoek–Brown criterion better represented the shale under field conditions. Based on his results, shear behavior on the basis of the conventional Mohr–Coulomb criterion represented the intact rock behavior regardless of the existence of discontinuities in the rock mass. Islam and Skalle (2013) made an evaluation on the shale mechanical characteristics

\*Corresponding author, Ph.D.

E-mail: Hasanipanahmahdi@duytan.edu.vn

on the basis of variable confinement pressures. Having examined the SS criteria in cases of rock joints and rockfill, Barton (2013) maintained that the conventional Mohr–Columb model requires nonlinearity in order to show the behavior of intact rock strength more effectively.

Literature, in recent years, has emphasized the capacity of artificial intelligence techniques to be efficiently applied to the different engineering field (Zhang and Goh 2013, Taghavifar and Mardani 2014, Moayedi and Armaghani 2018, Bejarbaneh *et al.* 2018, Zhou *et al.* 2019, Asteris *et al.* 2018, 2019a, b, c, 2021, Asteris and Mokos 2019, Asteris and Nikoo 2019, Apostolopoulou *et al.* 2020, Zhang *et al.* 2020, 2021a, 2021b, Hasanipanah and Bakhshandeh Amnieh 2020a, b, Hasanipanah *et al.* 2020a, b, c, Bardhan *et al.* 2021, Harandizadeh *et al.* 2021) such as rock mechanics (Kainthola *et al.* 2015, Mishra *et al.* 2015, Jahed Armaghani *et al.* 2016, Dantas Neto *et al.* 2017, Khandelwal *et al.* 2018, Murlidhar *et al.* 2018, Jahed Armaghani *et al.* 2018, Armaghani *et al.* 2020a, Bai *et al.* 2021, Liu *et al.* 2021), in particular. An effective tool of artificial intelligence is artificial neural network (ANN) (Monjezi *et al.* 2013, Hasanipanah *et al.* 2016, Sharma *et al.* 2017). This type of network has been known as one of the most dynamic study areas in innovative and varied engineering applications. ANNs have been found capable of applying all influential factors to predictive models; though, they still suffer from a number of limitations, e.g., having a slow learning rate and being entrapped in local minima (Eberhart and Shi 1998, Adhikari and Agrawal 2011). To overcome these weaknesses, adaptive neuro-fuzzy inference system (ANFIS), as a hybridization of ANN and fuzzy system and presented by Jang (1993), has been applied in the literature (Singh *et al.* 2005, Iphar 2012, Singh *et al.* 2012, Bouayad and Emeriault 2017, Mottahedi *et al.* 2018). Murlidhar *et al.* (2018) offered a combination of ANFIS and genetic algorithm (GA) to predict  $c$  and compared the ANFIS-GA performance with the ANN. They indicated the effectiveness of ANFIS in estimating the rock interlocking compared ANN. In another study, Kainthola *et al.* (2015) predicted the  $\phi$  using ANFIS and linear regression models. In their study, 597 rock samples related to Himalayan rock types were used. They indicated the effectiveness of ANFIS compared to linear regression for prediction of  $\phi$ . The use of support vector regression (SVR) has been also highlighted in different mining and geotechnical fields. Chen *et al.* (2021) employed three hybrid models by using three optimization algorithm, i.e. firefly algorithm, GA and particle swarm optimization in combination with SVR model to predict blast-induced ground vibration. Their results indicated that the proposed SVR models were viable and effective in predicting ground vibration. In another study, a human learning optimization (HLO) algorithm was combined with the SVR model to predict concrete compressive strength by Huang *et al.* (2021). They showed the proposed HLO-SVR model was a powerful tool for the prediction aim and has the capacity to generalize. Also, Fattahi *et al.* (2017) combined the SVR with cultural algorithm to predict tensile strength of rocks. Their results also confirmed the use of hybrid SVR model in the field of tensile strength of rocks. The Harmony Search (HS) is also

recently used as an optimization algorithm in different subjects of geotechnical engineering. As an example, Fattahi (2020) used a combination of the relevance vector regression (RVR) with the cuckoo search (CS) and harmony search (HS) algorithms in predicting the uniaxial compressive strength (UCS) of weak rocks. The results indicated that the HS can be introduced as an effective algorithm for optimization in this field.

The present study introduces two hybrid machine learning models for the purpose of predicting efficiently the SS parameters of shale through the following: point load index ( $Is(50)$ ), dry density ( $DD$ ), Schmidt hammer rebound number ( $SHn$ ), ultrasonic velocity ( $Vp$ ), and Brazilian tensile strength ( $BTS$ ). For this work, this study presents an ANFIS implemented by subtractive clustering method (SCM) and an improved support vector regression (SVR) by using Harmony Search (HS) algorithm. Note that, it is the first work that predicts the SS parameters of shale through ANFIS-SCM and SVR-HS hybrid models.

## 2. Research significance

Precise prediction of rock SS parameters is an important subject in civil and geotechnical engineering fields such as excavation and slope design. Therefore, the aim of this study is to develop the accurate and practical models to predict SS parameters, including  $c$  and  $\phi$ , through two intelligent approaches, i.e. ANFIS-SCM and SVR-HS models. Additionally, selecting the proper and effective parameters on the intensity of  $c$  and  $\phi$  is an important task in the modeling process. For this aim, the present study investigates the effect of several parameters upon the  $c$  and  $\phi$  through a sensitivity analysis.

## 3. Methodology

### 3.1 Combination of SVR and HS

#### 3.1.1 SVR

A nonlinear mapping  $\varphi(\cdot): R^n \rightarrow R^{nh}$  is specified to map the input and output datasets  $\{(x_i, y_i)\}_{i=1}^N$  (where  $N$  is the total number;  $x_i$  is inputs and  $y_i$  is the actual value) into a so-called high dimensional feature space (which may have infinite dimensions),  $R^{nh}$ . Such a linear function, that is to say SVR function, is like Eq. (1),

$$f(x) = W^T \varphi(x) + b \quad (1)$$

where  $\varphi(x)$  is the inputs feature and both  $b$  and  $W$  are coefficients. In SVR approach, the coefficients ( $b$  and  $W$ ) are predicted by minimizing the empirical risk as Eq. (2),

$$R_{emp}(f) = \frac{1}{N} \sum_{i=1}^N \Theta_{\varepsilon}(y_i, W^T \varphi(x) + b) \quad (2)$$

that  $\Theta_{\varepsilon}(y_i, W^T \varphi(x) + b)$  is specified as Eq. (3),

$$\Theta_{\varepsilon}(y_i, W^T \varphi(x) + b) = \begin{cases} |W^T \varphi(x_i) + b - y_i| - \varepsilon, & \text{if } |W^T \varphi(x_i) + b - y_i| \geq \varepsilon \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

The SVR focuses on minimizing the training error between the  $\varepsilon$ -insensitive loss function and the training data and finding the optimum hyper plane (Hong 2011).

Minimize:

$$\text{Min}_{w,b,\xi^*,\xi} R_{\xi}(W, \xi^*, \xi) = \frac{1}{2} W^T W + C \sum_{i=1}^N (\xi_i^*, \xi_i) \quad (4)$$

with the constraints

$$y_i - W^T \varphi(x_i) - b \leq \varepsilon + \xi_i^*, \quad \xi_i^* \geq 0, \quad i = 1, 2, \dots, N$$

$$-y_i + W^T \varphi(x_i) + b \leq \varepsilon + \xi_i, \quad \xi_i \geq 0, \quad i = 1, 2, \dots, N$$

$\xi_i^*$ : training errors above  $\varepsilon$ ,  $\xi_i$ : training errors below  $-\varepsilon$  and C is parameter to trade off these two terms. The first term of Eq. (4), usage of the principle of maximizing the distance of two different training data, is utilized to penalize large weights, to maintain regression function flatness, and to regularize weight sizes. In addition, the parameter  $W$  (in Eq. (1)) is achieved, after the quadratic optimization problem (Fattahi 2016),

$$W = \sum_{i=1}^N (\beta_i^* - \beta_i) \varphi(x_i) \quad (5)$$

where  $\beta_i^*$ ,  $\beta_i$  are the Lagrangian multipliers. Finally, the SVR function is Eq. (6):

$$f(x) = \sum_{i=1}^N (\beta_i^* - \beta_i) K(x_i, x_j) + b \quad (6)$$

Here,  $K(x_i, x_j)$  is called the function of kernel that is,  $K(x_i, x_j) = \varphi(x_i) \varphi(x_j)$  (Fattahi and Bayat 2019). In this study, the radial basis kernel function (RBF)  $K(x_i, x_j) = \exp(-\|x_i - x_j\| / 2\sigma^2)$ ,  $\sigma > 0$  is utilized in the SVR model.

The SVR capability is extremely dependent upon its parameters, i.e., the error margin  $\varepsilon \in [0.01, 0.6]$ , and the regularization parameter  $C \in [2^{-5}, 2^{15}]$ , to be set correctly (Fattahi 2017). However, there are no common rules for choosing these parameters. Most researchers still use the Grid algorithm in a conventional approach (trial and error), constructing a few SVR models based on different parameter settings first, then evaluating them on the validation set to find the best parameters. However, this procedure is time consuming; we tried to use it, but were unable to converge at the global optimum. Consequently, we utilized a new metaheuristic algorithm (HS algorithm) to find best SVR parameters.

Also, the use of optimized SVR method has been highlighted in mining and rock mechanic fields. For example, Fattahi (2016) hybridized the SVR with particle swarm optimization algorithm and differential evolution algorithm in predicting the deformation modulus of a rock mass. In another studies, hybridizing the SVR with cultural algorithm is presented to predict tensile strength of rocks by Fattahi and Babanouri (2017).

The results of above studies confirmed the acceptability of optimized SVR methods in the mentioned fields.

```

Begin;
Define objective function  $f(x)$ ,  $x=(x_1, x_2, \dots, x_d)^T$ 
Define Harmony Memory Considering rate (HMCR)
Define Pitch adjusting rate (PAR) and other parameters
Generate Harmony Memory with random harmonies
while (t<max number of iterations)
    while (i<=number of variables)
        if (rand<HMCR),
            Choose a value from HM for the variable i
            if (rand<PAR),
                Adjust the value by adding certain amount
            end if
        else
            Choose a random value
        end if
    end while
    Accept the New Harmony (solution) if better
end while
Find the current best solution
end
    
```

Fig. 1 Pseudo-code for the HS

### 3.1.2 HS Algorithm in selecting parameters of the SVR

The HS algorithm is one of the most important metaheuristic algorithms for optimization aims and is based on natural musical performance processes (Geem 2009). The pseudo-code of HS algorithm is showed in Fig. 1. A detailed explanation of the HS algorithm can be found in Geem (2009) and Geem *et al.* (2001). In this paper, the HS algorithm is used to select the appropriate parameters ( $C, \sigma$  and  $\varepsilon$ ) of their SVR. The overall process of the proposed method is presented in Fig. 2.

### 3.2 ANFIS

ANFIS proposed by Jang (1993), is used to nonlinear estimation where past samples are utilized to estimate the sample ahead (Wu *et al.* 2009). A scheme of ANFIS is shown in Fig. 3.

An ANFIS network is performed based on five layers. The first and second layers are the fuzzification and product layers. The normalized process is done in the third layer, and the fourth layer is defuzzification, and finally in the fifth layer, the output model is introduced by summation of output in the fourth layer 4. More details about these five layers can be found in the study conducted by Kainthola *et al.* (2015) and Murlidhar *et al.* (2018). The ANFIS model has been widely used for predicting aims in the literature, especially in fields of rock mechanic and mining. For example, hybridizing the ANFIS with stochastic fractal search algorithm is presented to predict blast-induced air-overpressure (AOp) by Ye *et al.* (2020). In another study, Hasanipanah *et al.* (2020c) hybridized the ANFIS and firefly algorithm in predicting the tensile strength. Furthermore, the blast-induced ground vibration has been predicted using ANFIS by some researchers. The results of above studies indicate the effectiveness of ANFIS model for predicting aims.

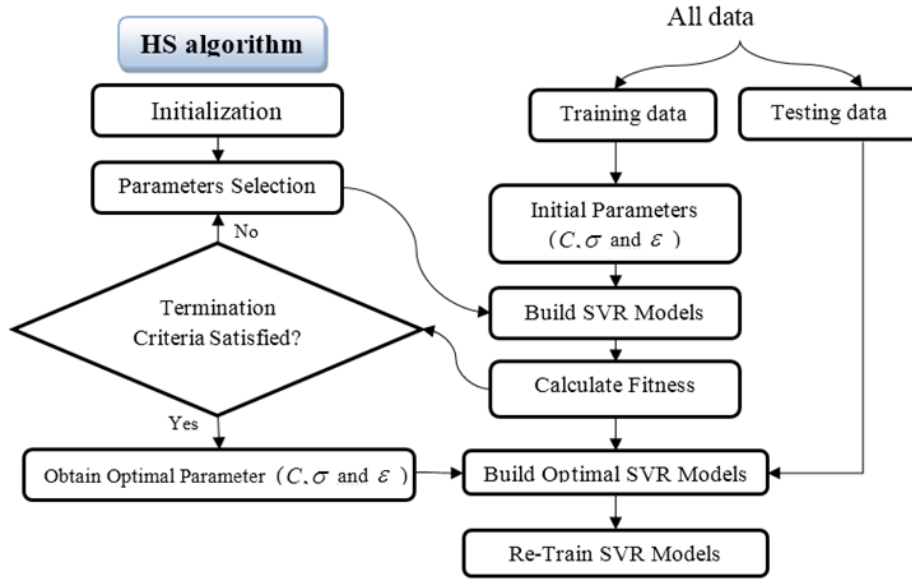


Fig. 2 SVR-HS process for optimization of SVR parameters

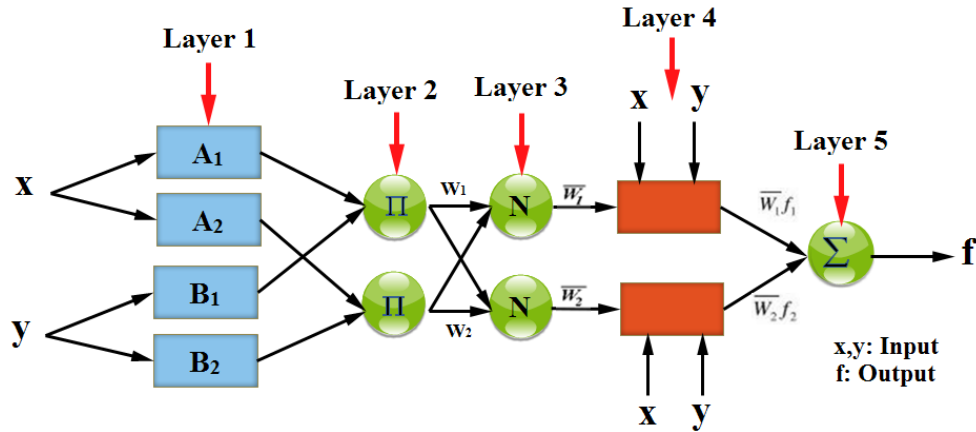


Fig. 3 ANFIS architecture for two-input (after Jang (1993))

3.2.1 SCM

Because it automatically estimates cluster number and cluster location, the SCM is an interesting solution for ANFIS network synthesis. In SCM, each sample point is considered a potential cluster center. While staying independent of the dimension problem, this approach makes calculating time linearly proportional to data size. The SCM was used to find the cluster center of all data. Using the number of subtractive centers, automated membership functions (MFs) and rule bases, as well as the position of MF inside dimensions, were constructed.

The SCM proposed by Chiu (1994) in which data sets are considered as the candidates for clusters center. The SCM is defined as:

$$D_i = \sum_{j=1}^n \exp \left( - \frac{\|x_i - x_j\|^2}{\left(\frac{r_a}{2}\right)^2} \right) \quad (12)$$

where, the radius  $r_a$  defines a neighborhood and is a positive constant. Next, the density measure for each data

point  $x_i$  is revised as follows:

$$D_i = D_i - D_{ci} \exp \left( - \frac{\|x_i - x_{ci}\|^2}{\left(\frac{r_b}{2}\right)^2} \right) \quad (13)$$

where,  $r_b$  is a positive constant. A detailed description of the subtractive clustering method can be found in Chiu (1994). In this paper, SCM is used to detect the antecedent MFs.

4. Prediction of SS parameters

4.1 Behaviour of shale rock under triaxial test

The SS parameters define the intact rock behavior in shear-failure condition. SS actually defines how rock material is strong against deformations brought about by shear stress (Jahed Armaghani *et al.* 2014). In general, such strength is known by two internal mechanisms, namely  $c$  and  $\varphi$ . When rock particles are contact on each other, an



Fig. 4 Universal servo-controlled 3000 KN testing machine (Jahed Armaghani *et al.* 2014)

internal friction is generated, which is measured by calculating the  $\varphi$  (Jahed Armaghani *et al.* 2014). Essentially, triaxial compression tests are able to determine the parameters of SS.

When the UCS of rock is a value between 0.5 and 25 MPa, the rock is known as weak or soft (ISRM 2007). The compaction and cementation of clay minerals is a long procedure that leads to the creation of a kind of rock termed sedimentary rock, e.g., shale. These types of rocks have a texture consisting of a characteristic lamination and a particular arrangement of minerals induced by the sedimentation process. As a result, along such small-scaled lamination planes, failure may simply take place, which is normally known as shear failure. Note that various anisotropy states exist with a variable degree of complexity (McLamore and Gray 1990, Jahed Armaghani *et al.* 2014). When we have rock materials with anisotropic behaviors exposed to triaxial compression conditions, the main stress difference ( $\sigma_1 - \sigma_3$ ) can be understood using the following equation (Brady 2004):

$$\sigma_1 - \sigma_3 = \frac{2C_w + \sigma_3 \tan \phi_w}{(1 - \tan \phi_w \cot \beta) \sin 2\beta} \quad (14)$$

where  $C_w$  signifies the  $c$  of fracture planes,  $\phi_w$  denotes the  $\varphi$  of the plane, and  $\beta$  stands for the weakness plane inclination angle. In case the angle of inclination equals  $45 + 0.5\phi_w$ , the strength value will be at the minimum level. As a result, the related value of the main stress difference is altered as:

$$\sigma_1 - \sigma_3 = 2(C_w + \sigma_3 \tan \phi_w) [(1 + \tan^2 \phi_w)^{\frac{1}{2}} + \tan \phi_w] \quad (15)$$

The  $\phi_w$  value in a compression test is ranged normally between 30 and 50. Consequently, the  $\beta$  value will be roughly between 60 and 70. It leads to the conclusion that small-scaled discontinuity planes like lamination can reduce the strength of rock material whereas the orientation of loading in respect of discontinuity plane is the most influential parameter in the rock strength reduction (Jaeger *et al.* 2009). More details can be found in Ghazvinian and Hadei (2012), and also Jaeger *et al.* (2009).

#### 4.2 Inputs and output data

To achieve the objective of this research in developing ANFIS-SCM and SVR-HS models, the requirement datasets were borrowed from Jahed Armaghani *et al.* (2014). The samples were gathered from the excavation area of Ayer Hitam located in Malaysia.

Generally, to carry out laboratory experiments in geotechnical fields, a key part of the procedure is sample preparation. For this purpose, in accordance with ISRM (2007), samples of definite size and shape were provided. As a result, in case of various experiments, trimming and coring the disc and cylindrical samples were done. Five triaxial experiments were carried out with  $\sigma_3$  of 1, 2, 4, 8, and 16 MPa for the purpose of determining the SS parameters in case of each  $c$  and  $\varphi$ . At the final step, considering the Mohr–Coulomb failure theory, the SS parameters were calculated. Totally 52  $c$  and  $\varphi$  values were obtained from the block samples.

Rock properties generally denote three factors: index strength, fundamental strength (i.e., tensile and compressive strengths), and physical properties (i.e.,  $V_p$  and  $DD$ ). In addition, two strength index tests, namely  $SHn$  and  $Is_{(50)}$  were defined through applying the L-type Schmidt's hammer and point-load test apparatus, respectively. Moreover, the Brazilian tensile strength ( $BTS$ ) was applied to all of the samples in order to reveal the indirect tensile strength of shale. Mechanical tests on core samples were conducted through a machine equipped with servo control capability. Fig. 4 shows the entire test system, including a controller, pump unit and load frame. Also, a specimen before and after triaxial test conduction with cell pressure of 8 MPa is shown in Fig. 5. In modeling of ANFIS-SCM and SVR-HS,  $Is_{(50)}$ ,  $SHn$ ,  $DD$ ,  $V_p$  and  $BTS$  were taken as the input parameters, while,  $\varphi$  and  $c$  as were taken as the output parameters. Starting with 52 data points, the ANFIS-SCM and SVR-HS were trained using 42 data points and the 10 data points as the testing set. Table 1 shows a descriptive statistics of the parameters used in the modeling. More details about measuring datasets can be found in Jahed Armaghani *et al.* (2014). Also, the relationship between the input and output parameters (a linear fitting)



Fig. 5 Specimen with 8 MPa cell pressure before and after test conduction (Jahed Armaghani *et al.* 2014)

Table 1 Descriptive statistics for modeling parameters

Parameter	Category	Unit	Mean	St. Error	St. Deviation	Kurtosis	Skewness	Min-Max
DD	Input	Kg/m <sup>3</sup>	2525	40.60	292.78	-1.42	0.11	2031-2990
SHn	Input	MPa	32.98	1.09	7.91	-0.19	0.43	19-54
BTS	Input	MPa	2.71	0.14	1.02	-1.54	0.27	1.3-4.5
Is <sub>(50)</sub>	Input	MPa	2.81	0.14	1.01	-1.34	0.22	1-4.6
Vp	Input	m/s	2556.69	49.03	353.56	-1.43	-0.003	1988-3112
$\varphi$	Output	°	30.081	0.85	6.17	-1.29	0.09	20.3-40.1
<i>c</i>	Output	MPa	7.16	0.21	1.53	-0.53	0.50	4.7-10.7

Table 2 relationship between the input and output parameters

	DD	SHn	BTS	Is 50	Vp	$\varphi$	<i>c</i>
DD	1	-	-	-	-	-	-
SHn	0.768	1	-	-	-	-	-
BTS	0.838	0.741	1	-	-	-	-
Is 50	0.817	0.746	0.883	1	-	-	-
Vp	0.767	0.692	0.865	0.863	1	-	-
$\varphi$	0.797	0.630	0.800	0.825	0.792	1	-
<i>c</i>	-0.547	-0.546	-0.550	-0.589	-0.670	-0.681	1

are indicated in Table 2. From this Table, it can be seen that the *DD*, *SHn*, *BTS*, *Is<sub>(50)</sub>* and *Vp* and have a good relationship with the  $\varphi$  parameter. Also, the relationship between the input parameters and *c* were varied in the range of 0.546-0.670.

#### 4.3 Models performance evaluation

To improve the stability of the training of the ANFIS-SCM and SVR-HS models, data of both output and inputs should be normalized. In this study, all data was converted into values between [0, 1] by the expression:

$$x_M = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad (16)$$

where *x*, *x<sub>M</sub>*, *x<sub>min</sub>* and *x<sub>max</sub>* are before being normalized, after being normalized, minimum value and maximum value respectively. Also, squared correlation coefficient (*R*<sup>2</sup>), variance account for (VAF), root-mean-squared-error

(RMSE) and mean-squared-error (MSE) were selected to be the measure of accuracy of the predictive models, as follows (Teymen and Mengüç 2020, Sadeghi *et al.* 2020, Huang *et al.* 2020, 2021, Jahed Armaghani *et al.* 2021, Le *et al.* 2021).

$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^n (t_k - \hat{t}_k)^2} \quad (17)$$

$$R^2 = 1 - \frac{\sum_{k=1}^n (t_k - \hat{t}_k)^2}{\sum_{k=1}^n t_k^2 - \frac{\sum_{i=1}^n \hat{t}_k^2}{n}} \quad (18)$$

$$MSE = \frac{1}{n} \sum_{k=1}^n (t_k - \hat{t}_k)^2 \quad (19)$$

$$VAF = \left[ 1 - \frac{\text{var}(t_k - \hat{t}_k)}{\text{var}(t_k)} \right] \quad (20)$$

where *t<sub>k</sub>*,  $\hat{t}_k$  and *n* are actual value, predicted value and observations number respectively.

#### 4.4 Prediction of SS parameters using SVR-HS model

In the current paper, the SVR-HS model is proposed to predict the SS parameters ( $\varphi$  and *c*) using MATLAB

Table 3 Regulated parameters for run the HS algorithm

Parameter	Values for prediction of	Values for prediction of
	$\varphi$	of $c$
Maximum number of iterations	50	150
Population number	25	30
Harmony memory consideration rate	0.4	0.9
Pitch adjustment rate	0.1	0.09
Number of new harmonies	15	15
Fret width damp ratio	0.995	0.995
Fitness	RMSE	RMSE

Table 4 SVR parameters by using HS algorithm

Parameters	Optimal value of	Optimal value of	Optimal value of
	$\sigma$	$C$	$\epsilon$
Values for prediction of $\varphi$	1.20322	3963.553	0.069659
Values for prediction of $c$	0.88609	1027.347	0.016051

Table 5 Different parameters used in ANFIS-SCM

Parameter	Values for prediction of	Values for prediction of
	$\varphi$	of $c$
Input MF	Gaussian	Gaussian
Output MF	Linear	Linear
No. of nodes	512	476
No. of linear parameters	252	234
No. of nonlinear parameters	420	390
Total No. of parameters	672	624
No. of training data pairs	42	42
No. of testing data pairs	10	10
No. of fuzzy rules	42	39

environment. To find the optimal value of SVR parameters, viz.  $\epsilon$ , regularization parameter ( $C$ ) and Kernel RBF parameter ( $\sigma$ ), the HS algorithm is used. Regulated parameters for run the HS algorithm is showed in Table 3. Also, the best values for SVR parameters are listed in Table 4.

4.5 Prediction of SS parameters using ANFIS-SCM model

For comparison aims, the ANFIS-SCM was also applied for the estimation of SS parameters ( $\varphi$  and  $c$ ). It should be noted that ANFIS-SCM model used to predict SS parameters was implemented with the help of MATLAB, based on the same data used in SVR-HS model.

Table 5 shows parameter types and their values used in the ANFIS-SCM. In Table 5, the generalized Gaussian MFs were used in the present model. MFs have been tested and it is important to mention that the used rules generally are based on the model and variables that are depended on user experience and trial and error methods. Furthermore, the shape of MFs depends on parameters, and changing these parameters will change the shape of the MF.

Also, Figs. 6 and 7 show the MFs of the input parameters for ANFIS-SCM. These curves (with different

Table 6 Performance indices for two hybrid intelligence models

Model		RMSE	MSE	$R^2$	VAF
SVR-HS	For prediction of $c$ Training datasets	0.0646	0.0041	0.9230	0.8406
	Testing datasets	0.1832	0.0335	0.8618	0.8007
	For prediction of $\varphi$ Training datasets	0.0923	0.0085	0.9082	0.8246
	Testing datasets	0.1340	0.0179	0.9055	0.8236
ANFIS-SCM	For prediction of $c$ Training datasets	0.0454	0.0020	0.9675	0.8811
	Testing datasets	0.1358	0.0184	0.9011	0.8206
	For prediction of $\varphi$ Training datasets	0.0474	0.0022	0.9761	0.8916
	Testing datasets	0.0826	0.0068	0.9344	0.8502

colors) represent different clusters. By using the SCM, the cluster center of all data was found out. Then the numbers of subtractive centers were used to generate automatic MFs and rule base, as well as the location of MF within dimensions.

5. Results and discussion

By means of five input parameters, i.e.,  $Is_{(50)}$ ,  $SHn$ ,  $DD$ ,  $Vp$  and  $BTS$ , the values of  $\varphi$  and  $c$  were predicted in this study. To this aim, the ANFIS-SCM and SVR-HS are developed. Performance indices (VAF, MSE, RMSE and  $R^2$ ) for two hybrid intelligence models is listed in Table 6. Note that the lowest MSE and RMSE and the highest VAF and  $R^2$  are the most ideal. From Table 6, the ANFIS-SCM with  $R^2 = 0.9011$ , MSE= 0.0184, VAF=0.8206 and RMSE = 0.1358 in testing phase, is more accurate than SVR-HS model with  $R^2 = 0.8618$ , MSE= 0.0335, VAF=0.8007 and RMSE = 0.1832 for predicting  $c$  parameter. On the other hand, the superiority of ANFIS-SCM model in predicting  $\varphi$  can be also observed from Table 6, so that the values of  $R^2$ , MSE, VAF and RMSE obtained from ANFIS-SCM model were equal to 0.9344, 0.0068, 0.8502 and 0.0826 respectively, while the values of  $R^2$ , MSE, VAF and RMSE obtained from SVR-HS model were equal to 0.9055, 0.0179, 0.8236 and 0.1340 respectively. Also, a correlation between estimated values of SS parameters ( $\varphi$  and  $c$ ) and measured values for all 52 data points is depicted in Figs. 8 to 11. These Figs indicate the ANFIS-SCM model provided a higher performance than the SVR-HS model for both  $\varphi$  and  $c$  predictions, since a very close agreement between the measured and the predicted were obtained. In addition, a comparison between predicted SS parameters ( $\varphi$  and  $c$ ) by the predictive models and actual values for only testing data points is depicted in Figs. 12 and 13. As it can be observed from these figures and table 6, both SVR-HS and ANFIS-SCM models were able and successful to estimate SS parameters ( $\varphi$  and  $c$ ), however, the ANFIS-SCM performs better than the SVR-HS.

To evaluate the effect of each input parameter on output parameters ( $\varphi$  and  $c$ ), a sensitivity analysis was also conducted in this study using Yang and Zang (1997) method, defined as follow:

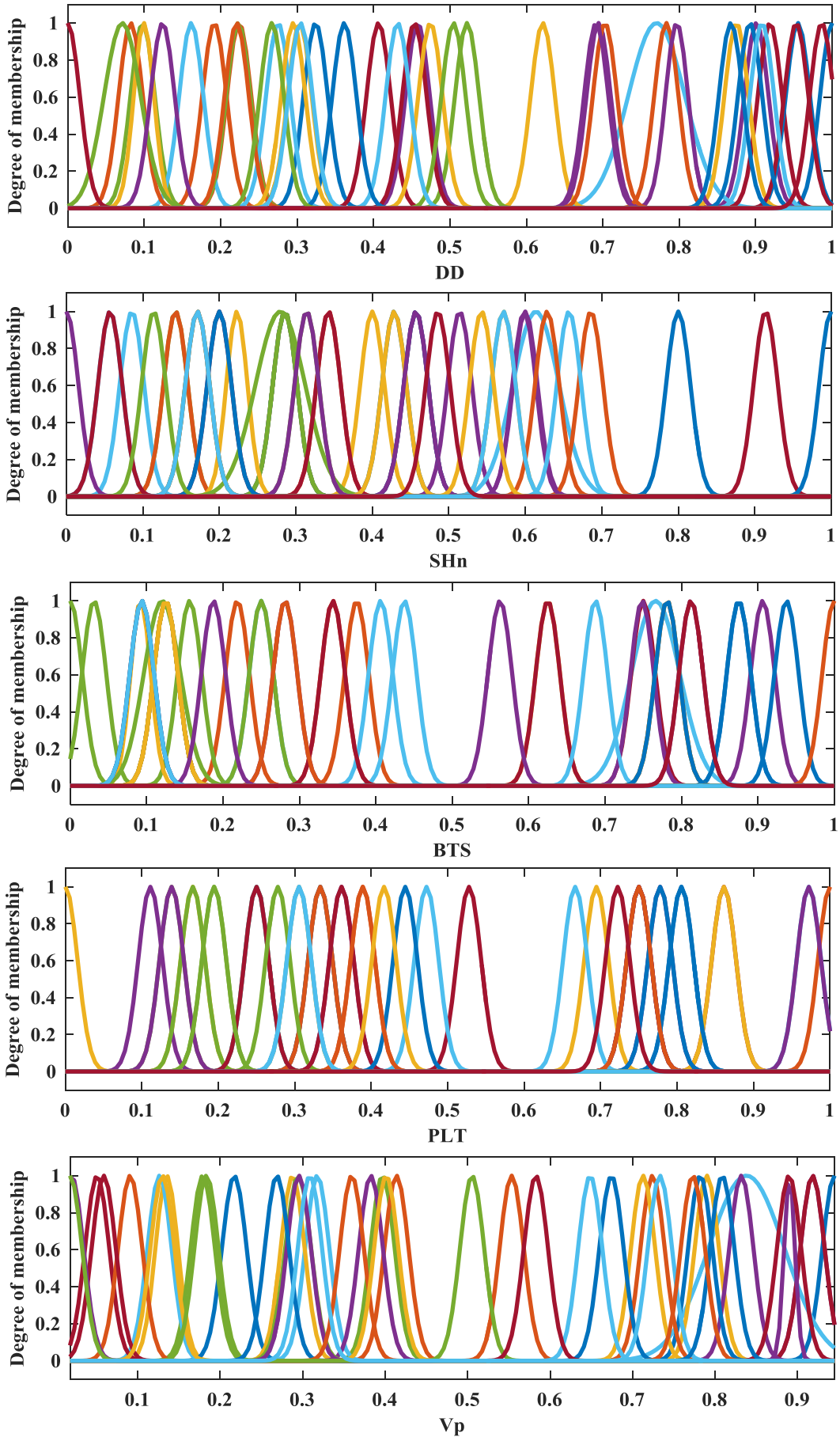


Fig. 6 MFs obtained by ANFIS-SCM for prediction of  $\varphi$

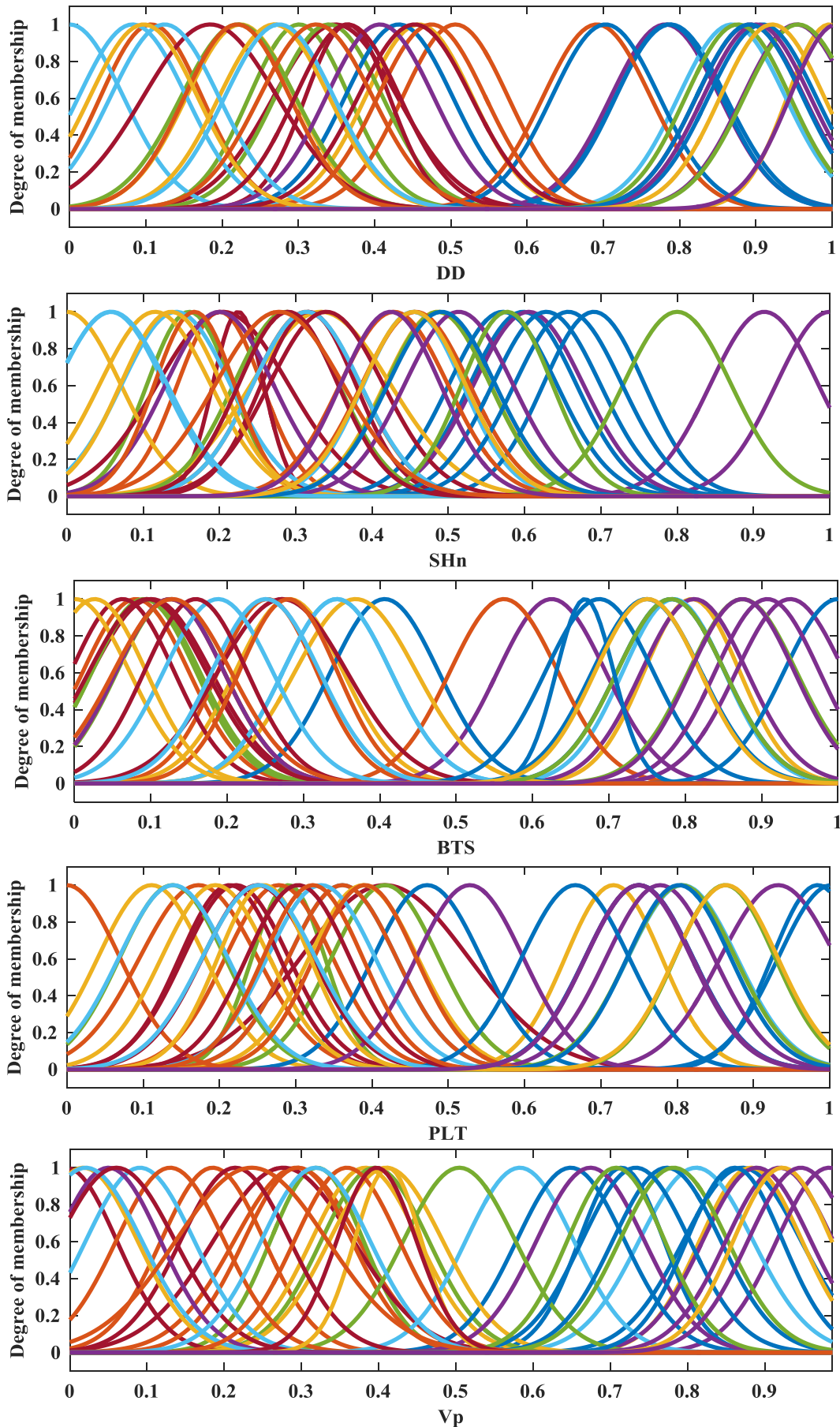


Fig. 7 MFs obtained by ANFIS-SCM for prediction of  $c$

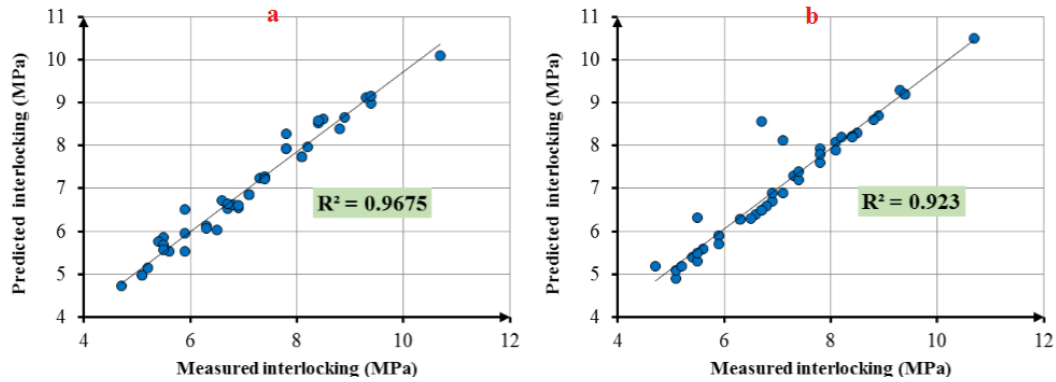


Fig. 8 Plot of the calculated values of interlocking ( $c$ ) versus the measured values for training datasets, a) ANFIS-SCM, b) SVR-HS

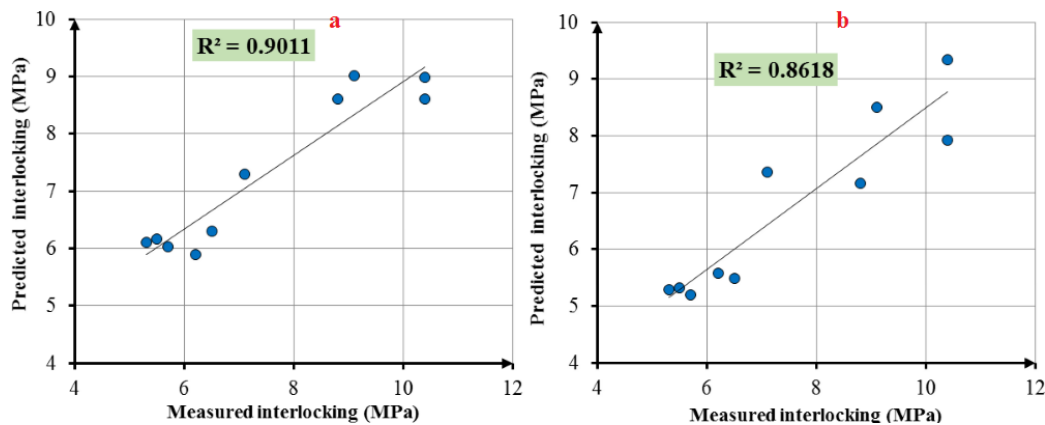


Fig. 9 Plot of the calculated values of interlocking ( $c$ ) versus the measured values for testing datasets, a) ANFIS-SCM, b) SVR-HS

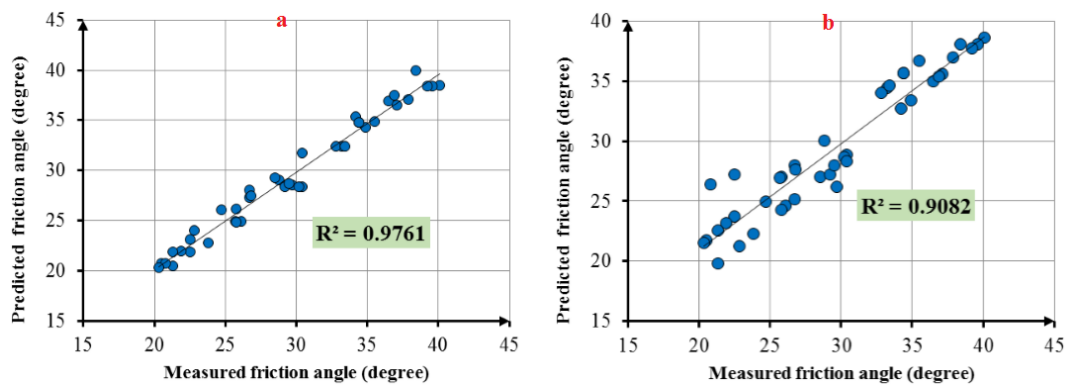


Fig. 10 Plot of the calculated values of  $\varphi$  versus the measured values for training datasets, a) ANFIS-SCM, b) SVR-HS

$$r_{ij} = \frac{\sum_{k=1}^n (y_{ik} \times y_{ok})}{\sqrt{\sum_{k=1}^n y_{ik}^2 \sum_{k=1}^n y_{ok}^2}} \quad (21)$$

where  $n$  is number of data, and  $y_{ik}$  and  $y_{ok}$  are considered as the input and output parameters, respectively. The amount of  $r_{ij}$  ranges between 0 and 1, so that the most effective input parameter has the highest  $r_{ij}$  amount. The results of sensitivity analysis for both  $\varphi$  and  $c$  parameters are shown in Fig. 14. Based on this Fig, it can be observed that the  $V_p$  and  $DD$  (with the highest  $r_{ij}$  amounts) were the most effective parameter of  $\varphi$  and  $c$ , respectively, in this study.

## 6. Conclusions

This study applies rock index test to the indirect measurement of the SS parameters of shale. In this regard,  $Is_{(50)}$ ,  $SHn$ ,  $DD$ ,  $V_p$  and  $BTS$  were taken as the input parameters, while,  $\varphi$  and  $c$  as were taken as the output parameters (SS parameters). The application of machine learning methods in different fields of engineering has been increasing as it gives an advantage of doing on non-linear tasks. Hence, two hybrid machine learning methods, namely ANFIS-SCM and SVR-HS, were proposed in this study to predict  $\varphi$  and  $c$ . Starting with 52 data points, the ANFIS-SCM and SVR-HS models were trained using 42 data

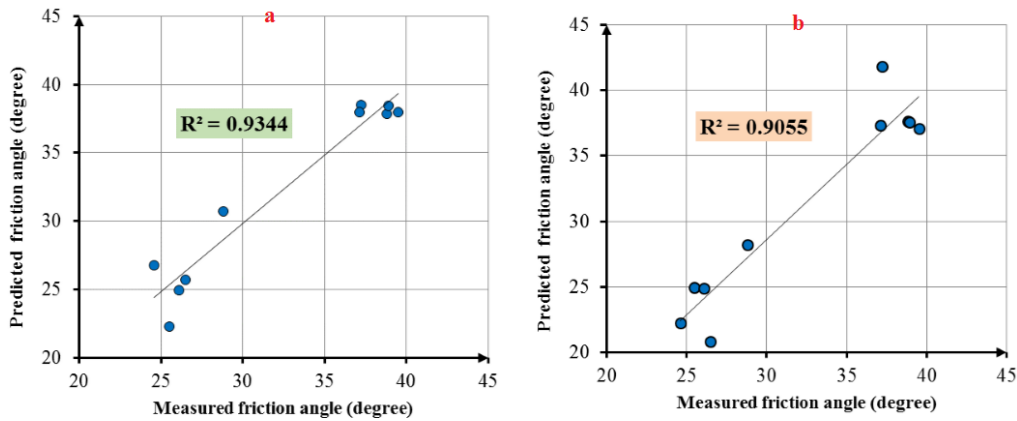


Fig. 11 Plot of the calculated values of  $\varphi$  versus the measured values for testing datasets, a) ANFIS-SCM, b) SVR-HS

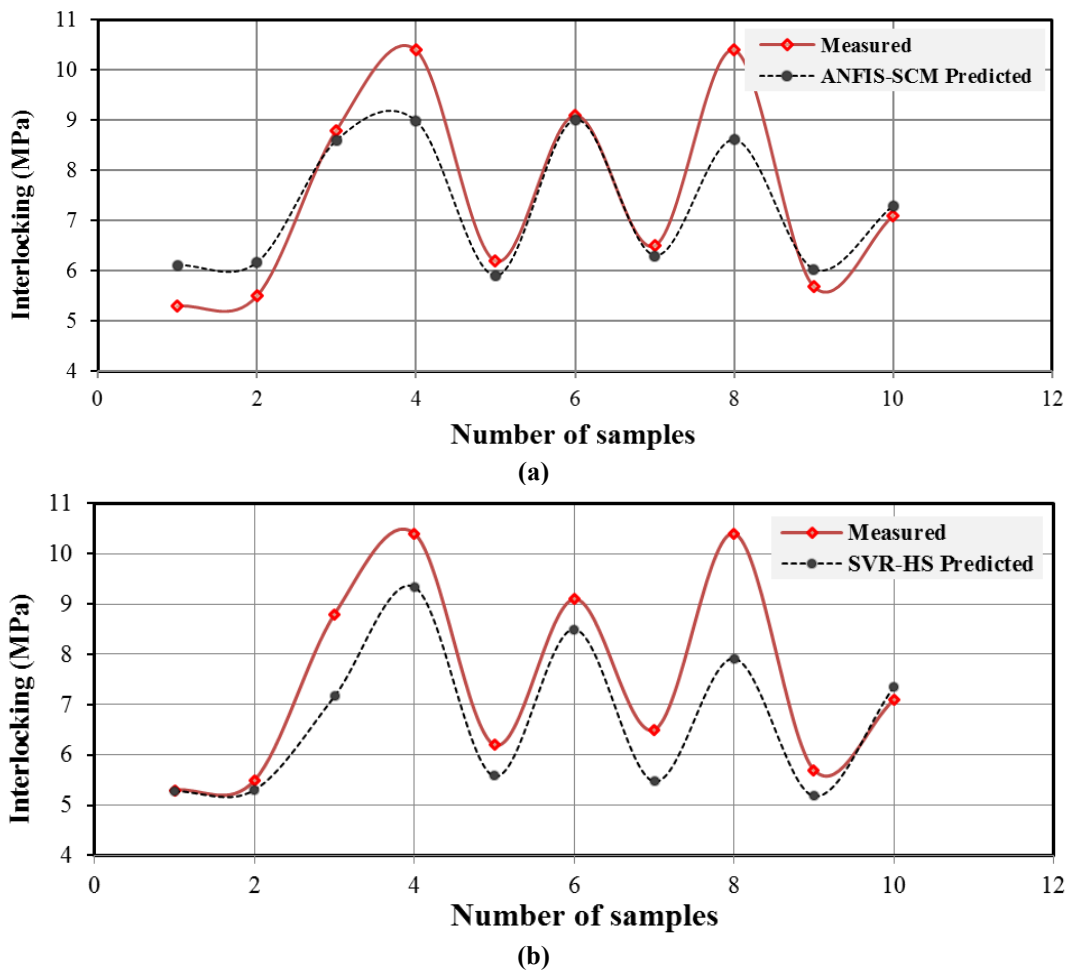


Fig. 12 Demonstrating the errors of  $c$  estimation for testing phase by a) ANFIS-SCM, b) SVR-HS

points and the remainder (10 data points) was used as the testing set. After modeling, the statistical criteria such as  $R^2$  and RMSE were used to check the performance of the proposed models. The conclusions of this study can be briefly listed as follows.

- Based on obtained results, both ANFIS-SCM and SVR-HS models were capable of predicting the  $\varphi$  and  $c$  with reasonable accuracy. Nevertheless, the acceptability and reliability of ANFIS-SCM model was better than SVR-HS model for both  $\varphi$  and  $c$  parameters.

- According to sensitivity analysis, it was demonstrated that  $Vp$  and  $DD$  were the most effective parameter of  $\varphi$  and  $c$ , respectively, in this case study.

**References**

Adhikari, R. and Agrawal, R.K. (2011), “Effectiveness of PSO based neural network for seasonal time series forecasting”, *Proceedings of the Indian International Conference on Artificial*

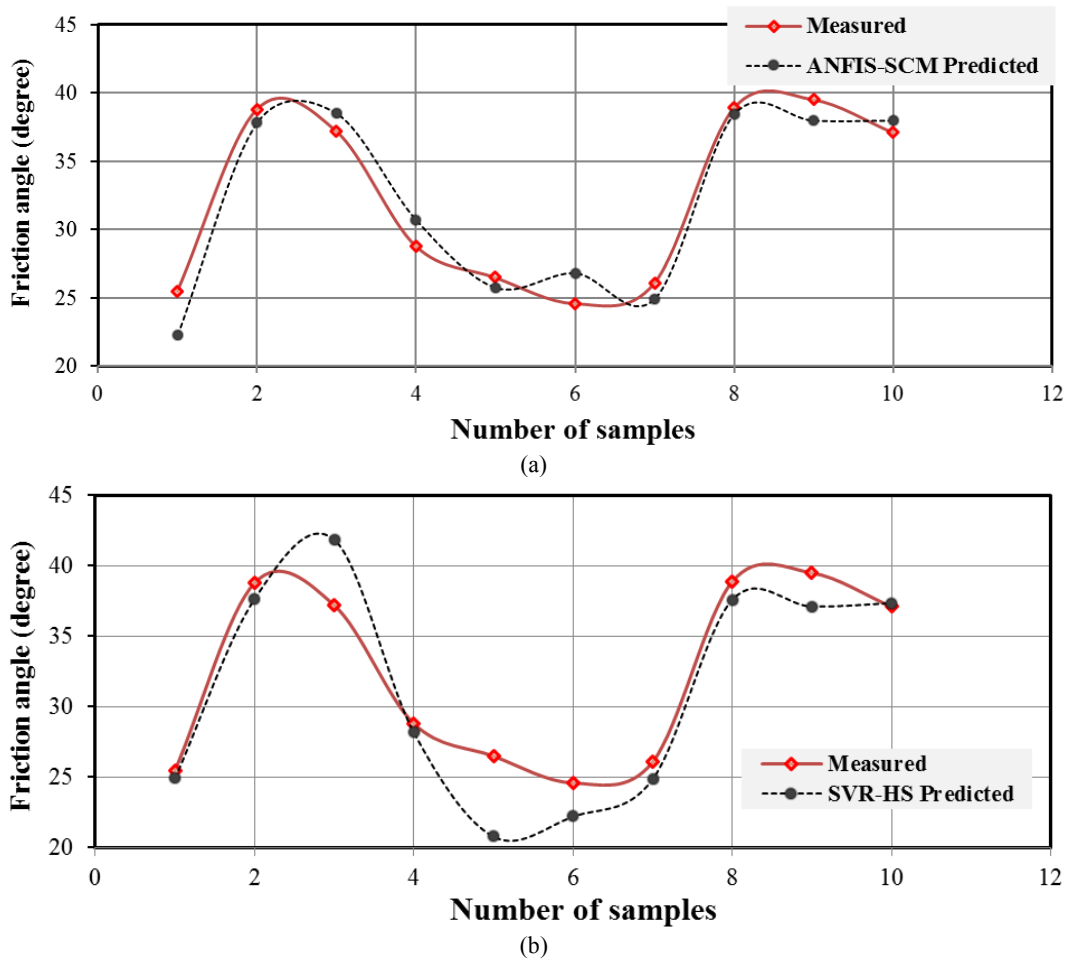


Fig. 13 Demonstrating the errors of  $\phi$  estimation for testing phase by a) ANFIS-SCM, b) SVR-HS

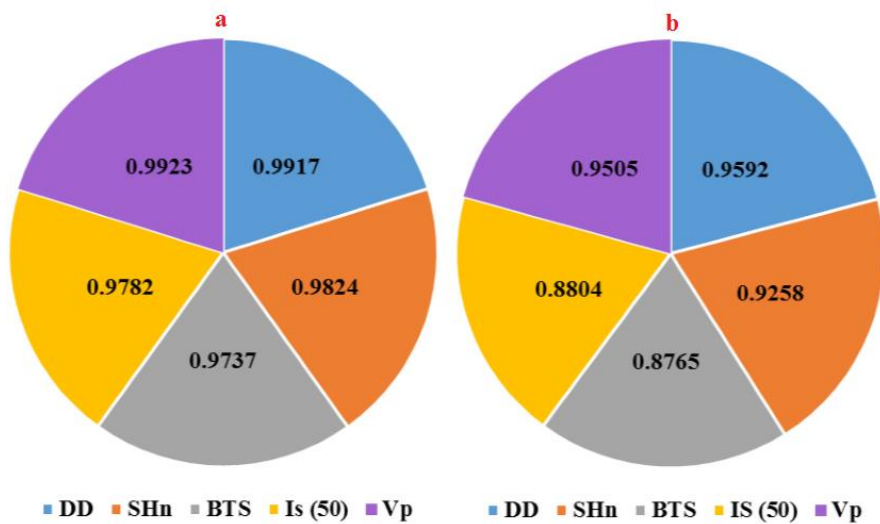


Fig. 14 Sensitivity analysis results for a)  $\phi$  and b)  $c$  parameters

*Intelligence (IICA)*, Tumkur, December.  
 Alejano, L.R. and Carranza-Torres, C. (2011), "An empirical approach for estimating shear strength of decomposed granites in Galicia, Spain", *Eng. Geol.*, **120**(1-4), 91-102. <https://doi.org/10.1016/j.enggeo.2011.04.003>  
 Amann, F., Kaiser, P. and Button, E.A. (2012), "Experimental

study of brittle behavior of clay shale in rapid triaxial compression", *Rock Mech. Rock Eng.*, **45**, 21-33. <https://doi.org/10.1007/s00603-011-0195-9>  
 Apostolopoulou, M., Asteris, P.G., Armaghani, D.J., Douvika, M.G., Lourenço, P.B., Cavaleri, L., Bakolas, A. and Moropoulou, A. (2020), "Mapping and holistic design of natural

- hydraulic lime mortars”, *Cem. Concr. Res.*, **136**, 106167, <https://doi.org/10.1016/j.cemconres.2020.106167>.
- Armaghani, D.J., Mirzaei, F., Toghrolji, A. and Shariati, A. (2020a), “Indirect measure of shear strength parameters of fiber-reinforced sandy soil using laboratory tests and intelligent systems”, *Geomech. Eng.*, **22**(5), 397-414. <http://dx.doi.org/10.12989/gae.2020.22.5.397>.
- Asadi, M. and Bagheripour, M.H. (2013), “Numerical and intelligent modeling of triaxial strength of anisotropic jointed rock specimens”, *Earth Sci. Inform.*, **7**, 165-172. <https://doi.org/10.1007/s12145-013-0137-z>.
- Asteris, P.G., Mamou, A., Hajihassani, M., Hasanipanah, M., Koopialipour, M., Le, T.T., Kardani, N. and Jahed Armaghani, D. (2021), “Soft computing based closed form equations correlating L and N-type Schmidt hammer rebound numbers of rocks”, *Transp. Geotech.*, **29**, 100588. <https://doi.org/10.1016/j.trgeo.2021.100588>.
- Asteris, P.G. and Mokos, V.G. (2019), “Concrete compressive strength using artificial neural networks”, *Neural Comput. Appl.* <https://doi.org/10.1007/s00521-019-04663-2>.
- Asteris, P.G. and Nikoo, M. (2019), “Artificial bee colony-based neural network for the prediction of the fundamental period of infilled frame structures”, *Neural Comput. Appl.*, **31**(9), 4837-4847. <https://doi.org/10.1007/s00521-018-03965-1>.
- Asteris, P.G., Apostolopoulou, M., Skentou, A.D. and Antonia Moropoulou, A. (2019a), “Application of artificial neural networks for the prediction of the compressive strength of cement-based mortars”, *Comput. Concr.*, **24**(4), 329-345. <http://dx.doi.org/10.12989/cac.2019.24.4.329>.
- Asteris, P.G., Argyropoulos, I., Cavaleri, L., Rodrigues, H., Varum, H., Thomas, J. and Lourenço, P.G. (2018), “Masonry compressive strength prediction using artificial neural networks”, *International Conference on Transdisciplinary Multispectral Modeling and Cooperation for the Preservation of Cultural Heritage*, Athens, October.
- Asteris, P.G., Armaghani, D.J., Hatzigeorgiou Karayannis, C.G. and Pilakoutas, K. (2019b), “Predicting the shear strength of reinforced concrete beams using artificial neural networks”, *Comput. Concr.*, **24**(5), 469-488. <http://dx.doi.org/10.12989/cac.2019.24.5.469>.
- Asteris, P.G., Moropoulou, A., Skentou, A.D., Apostolopoulou, M., Mohebkhah, A., Cavaleri, L., Rodrigues, H. and Varum, H. (2019c), “Stochastic vulnerability assessment of masonry structures: Concepts, modeling and restoration Aspects”, *Appl. Sci.*, **9**(2), 243. <https://doi.org/10.3390/app9020243>.
- Bai, X., Cheng, W., Ong, D.E.L. and Li, G. (2021), “Evaluation of geological conditions and clogging of tunneling using machine learning”, *Geomech. Eng.*, **25**(1), 59-73. <http://dx.doi.org/10.12989/gae.2021.25.1.059>.
- Bardhan, A., Gokceoglu, C., Burman, A., Samui, P. and Asteris, P.G. (2021), “Efficient computational techniques for predicting the California bearing ratio of soil in soaked conditions”, *Eng. Geol.*, **291**, 106239. <https://doi.org/10.1016/j.enggeo.2021.106239>.
- Barla, G., Barla, M. and Debernardi, D. (2010), “New triaxial apparatus for rocks”, *Rock Mech. Rock Eng.*, **43**, 225-230. <https://doi.org/10.1007/s00603-009-0076-7>.
- Barton, N. (2013), “Shear strength criteria for rock, rock joints, rockfill and rock masses: problems and some solutions”, *J. Rock Mech. Geotech. Eng.*, **5**(4), 249-261. <https://doi.org/10.1016/j.jrmge.2013.05.008>.
- Bejarbaneh, B.Y., Bejarbaneh, E.Y., Amin, M.F.M., Fahimifar, A., Jahed Armaghani, D. and Majid, M.Z.A. (2018), “Intelligent modelling of sandstone deformation behaviour using fuzzy logic and neural network systems”, *Bull. Eng. Geol. Environ.*, **77**, 345-361. <https://doi.org/10.1007/s10064-016-0983-2>.
- Bouayad, D. and Emeriault, F. (2017), “Modeling the relationship between ground surface settlements induced by shield tunneling and the operational and geological parameters based on the hybrid PCA/ANFIS method”, *Tunn. Undergr. Sp. Technol.*, **80**, 1-9. <https://doi.org/10.1016/j.tust.2017.03.011>.
- Brady, B.H. (2004), *Rock Mechanics: For Underground Mining*, Springer, Berlin, Germany.
- Chen, W., Hasanipanah, M., Nikafshan Rad, H., Jahed Armaghani, D. and Tahir, M.M. (2021), “A new design of evolutionary hybrid optimization of SVR model in predicting the blast-induced ground vibration”, *Eng. Comput.*, **37**, 1455-1471. <https://doi.org/10.1007/s00366-019-00895-x>.
- Chiu, S.L. (1994), “Fuzzy model identification based on cluster estimation”, *J. Intell. Fuzzy Syst.*, **2**, 267-278. <https://doi.org/10.3233/IFS-1994-2306>.
- Dantas Neto, S.A., Indraratna, B., Oliveira, D.A.F. and de Assis, A.P. (2017), “Modelling the shear behaviour of clean rock discontinuities using artificial neural networks”, *Rock Mech. Rock Eng.*, **50**, 1817-1831. <https://doi.org/10.1007/s00603-017-1197-z>.
- Eberhart, R.C. and Shi, Y. (1998), “Evolving artificial neural networks”, *Proceedings of the International Conference on Neural Networks and Brain*, PL5-PL13, Beijing, October.
- Fattahi, H. (2016), “Application of improved support vector regression model for prediction of deformation modulus of a rock mass”, *Eng. Comput.*, **32**, 567-580. <https://doi.org/10.1007/s00366-016-0433-6>.
- Fattahi, H. (2017), “Applying soft computing methods to predict the uniaxial compressive strength of rocks from schmidt hammer rebound values”, *Computat. Geosci.*, **21**, 665-681. <https://doi.org/10.1007/s10596-017-9642-3>.
- Fattahi, H. (2020), “A New Method for Forecasting of Uniaxial Compressive Strength of Weak Rocks”, *J. Mining Environ.*, **11**(2), 505-515. <https://doi.org/10.22044/jme.2020.9328.1835>.
- Fattahi, H. and Babanouri, N. (2017), “Predicting tensile strength of rocks from physical properties based on support vector regression optimized by cultural algorithm”, *J. Mining Environ.*, **8**(3), 467-474. <https://doi.org/10.22044/jme.2016.824>.
- Fattahi, H. and Bayat, N. (2019), “Forecasting of Rock Drillability Using a New Computational Intelligent Method”, *Geotech. Geol. Eng.*, **38**, 5693. <https://doi.org/10.1007/s10706-019-00971-5>.
- Geem, Z.W. (2009), *Music-inspired Harmony Search Algorithm: Theory and Applications*, Springer Verlag, Berlin, Germany.
- Geem, Z.W., Kim, J.H. and Loganathan, G.V. (2001), “A new heuristic optimization algorithm: harmony search simulation”, *Simulation*, **76**(2), 60-68. <https://doi.org/10.1177/003754970107600201>.
- Ghazvinian, A. and Hadei, M.R. (2012), “Effect of discontinuity orientation and confinement on the strength of jointed anisotropic rocks”, *Int. J. Rock Mech. Min. Sci.*, **55**, 117-124. <http://dx.doi.org/10.1016/j.ijrmms.2012.06.008>.
- Hajdarwish, A. and Shakoor, A. (2006), “Predicting the shear strength parameters of mudrocks”, *Geol. Soc. London*, **2**, 607.
- Harandizadeh, H., Armaghani, D.J., Asteris, P.G. and Gandomi, A.H. (2021), “TBM performance prediction developing a hybrid ANFIS-PNN predictive model optimized by imperialism competitive algorithm”, *Neural Comput. Appl.*, **33**(23), 16149-16179. <https://doi.org/10.1007/s00521-021-06217-x>.
- Hasanipanah, M. and Bakhshandeh Amnieh, H. (2020a), “A fuzzy rule based approach to address uncertainty in risk assessment and prediction of blast-induced flyrock in a quarry”, *Nat. Resour. Res.*, **29**(2), 669-689. <https://doi.org/10.1007/s11053-020-09616-4>.
- Hasanipanah, M. and Bakhshandeh Amnieh, H. (2020b), “Developing a new uncertain rule-based fuzzy approach for evaluating the blast-induced backbreak”, *Eng. Comput.*, **37**(3), 1879-1893. <https://doi.org/10.1007/s00366-019-00919-6>.

- Hasanipanah, M., Keshtegar, B., Thai, D.K. and Troung, N.T. (2020a), "An ANN-adaptive dynamical harmony search algorithm to approximate the flyrock resulting from blasting", *Eng. Comput.*, **2020**, 1-13. <https://doi.org/10.1007/s00366-020-01105-9>.
- Hasanipanah, M., Meng, D., Keshtegar, B., Trung, N.T. and Thai, D.K. (2020b), "Nonlinear models based on enhanced Kriging interpolation for prediction of rock joint shear strength", *Neural Comput. Appl.*, **33**(9), 4205-4215. <https://doi.org/10.1007/s00521-020-05252-4>.
- Hasanipanah, M., Noorian-Bidgoli, M., Jahed Armaghani, D. and Khamesi, H. (2016), "Feasibility of PSO-ANN model for predicting surface settlement caused by tunneling", *Eng. Comput.*, **32**, 705-715. <https://doi.org/10.1007/s00366-016-0447-0>.
- Hasanipanah, M., Zhang, W., Armaghani, D.J. and Rad, H.N. (2020c), "The potential application of a new intelligent based approach in predicting the tensile strength of rock", *IEEE Access*, **8**, 57148-57157. <https://doi.org/10.1109/ACCESS.2020.2980623>.
- Hoek, E., Carranza-Torres, C. and Corkum, B. (2002), "Hoek-Brown failure criterion—2002 edition", *Proceedings of the 5th North American Rock Mechanics Symposium and 17th Tunnelling Association of Canada Conference: University of Toronto*, Toronto, July.
- Hong, W.C. (2011), "Electric load forecasting by seasonal recurrent SVR (support vector regression) with chaotic artificial bee colony algorithm", *Energy*, **36**, 5568-5578. <https://doi.org/10.1016/j.energy.2011.07.015>.
- Huang, J., Duan, T., Zhang, Y., Liu, J., Zhang, J. and Lei, Y. (2020), "Predicting the permeability of pervious concrete based on the beetle antennae search algorithm and random forest model", *Adv. Civ. Eng.*, **2**, 8863181. <https://doi.org/10.1155/2020/8863181>.
- Huang, J., Sun, Y. and Zhang, J. (2021), "Reduction of computational error by optimizing SVR kernel coefficients to simulate concrete compressive strength through the use of a human learning optimization algorithm", *Eng. Comput.* <https://doi.org/10.1007/s00366-021-01305-x>.
- Iannacchione, A.T. and Vallejo, L.E. (2000), "Shear strength evaluation of clay-rock mixtures", *Slope Stability 2000, ASCE Geotechnical Special Publication 101*, 209-223, Denver, August.
- International Society for Rock Mechanics (ISRM). (2007), "The complete ISRM suggested methods for rock characterization, testing and monitoring: 1974–2006". *Ulusay R, Hudson JA (eds) Suggested methods prepared by the commission on testing methods, International Society for Rock Mechanics*, ISRM Turkish National Group, Ankara, Turkey.
- Iphar, M. (2012), "ANN and ANFIS performance prediction models for hydraulic impact hammers", *Tunn. Undergr. Sp. Technol.*, **27**, 23-29. <https://doi.org/10.1016/j.tust.2011.06.004>.
- Islam, M.A. and Skalle, P. (2013), "An experimental investigation of shale mechanical properties through drained and undrained test mechanisms", *Rock Mech. Rock Eng.*, **46**, 1391-1413. <https://doi.org/10.1007/s00603-013-0377-8>.
- Jaeger, J.C., Cook, N.G.W. and Zimmerman, R. (2009), *Fundamentals of Rock Mechanics*, Wiley-Blackwell, NJ, USA.
- Jahed Armaghani, D. and Asteris, P.G. (2021), "A comparative study of ANN and ANFIS models for the prediction of cement-based mortar materials compressive strength", *Neural Comput. Applic.*, **33**, 4501-4532. <https://doi.org/10.1007/s00521-020-05244-4>.
- Jahed Armaghani, D., Amin, M.F.M., Yagiz, S., Faradonbeh, R.S. and Abdullah, R.A. (2016), "Prediction of the uniaxial compressive strength of sandstone using various modeling techniques", *Int. J. Rock Mech. Min. Sci.*, **85**, 174-186. <https://doi.org/10.1016/j.ijrmms.2016.03.018>.
- Jahed Armaghani, D., Hajihassani, M., Yazdani Bejarbaneh, B., Marto, A. and Tonnizam Mohamad, E. (2014), "Indirect Measure of Shale Shear Strength Parameters by Means of Rock Index Tests through an Optimized Artificial Neural Network", *Measurement*, **55**, 487-498. <https://doi.org/10.1016/j.measurement.2014.06.001>.
- Jahed Armaghani, D., Safari, V., Fahimifar, A., Monjezi, M. and Mohammadi, M.A. (2018), "Uniaxial compressive strength prediction through a new technique based on gene expression programming", *Neural Comput. Appl.*, **30**(11), 3523-3532. <https://doi.org/10.1007/s00521-017-2939-2>.
- Jang, J.S.R. (1993), "ANFIS adaptive-network-based fuzzy inference system", *IEEE Trans. Syst. Man Cybern.*, **23**, 665-685. <https://doi.org/10.1109/21.256541>.
- Kainthola, A., Singh, P.K., Verma, D., Singh, R., Sarkar, K. and Singh, T.N. (2015), "Prediction of strength parameters of himalayan rocks: a statistical and ANFIS approach", *Geotech. Geol. Eng.*, **33**, 1255-1278. <https://doi.org/10.1007/s10706-015-9899-z>.
- Khandelwal, M., Marto, A., Fatemi, S.A., Ghorogi, M., Armaghani, D.J., Singh, T.N. and Tabrizi, O. (2018), "Implementing an ANN model optimized by genetic algorithm for estimating cohesion of limestone samples", *Eng. Comput.*, **34**(2), 307-317. <https://doi.org/10.1007/s00366-017-0541-y>.
- Le, T.T., Asteris, P.G. and Lemonis, M.E. (2021), "Prediction of axial load capacity of rectangular concrete-filled steel tube columns using machine learning techniques", *Eng. Comput.* <https://doi.org/10.1007/s00366-021-01461-0>.
- Liu, L., Yang, C. and Wang, X. (2021), "Landslide susceptibility assessment using feature selection-based machine learning models", *Geomech. Eng.*, **25**(1), 1-16. <http://dx.doi.org/10.12989/gae.2021.25.1.001>.
- Mclamore, R. and Gray, K. (1990), *The Mechanical Behaviour of Anisotropic Sedimentary Rocks*, *J. Eng. Ind.*, **89**(1), 62-73. <https://doi.org/10.1115/1.3610013>.
- Mishra, D.A., Srigan, M., Basu, A. and Rokade, P.J. (2015), "Soft computing methods for estimating the uniaxial compressive strength of intact rock from index tests", *Int. J. Rock Mech. Min. Sci.*, **80**, 418-424. <https://doi.org/10.1016/j.ijrmms.2015.10.012>.
- Moayed, H. and Armaghani, D.J. (2018), "Optimizing an ANN model with ICA for estimating bearing capacity of driven pile in cohesionless soil", *Eng. Comput.*, **34**(2), 347-356. <https://doi.org/10.1007/s00366-017-0545-7>.
- Monjezi, M., Hasanipanah, M. and Khandelwal, M. (2013), "Evaluation and prediction of blast-induced ground vibration at Shur River Dam Iran, by artificial neural network", *Neural Comput. Appl.*, **22**, 1637-1643. <https://doi.org/10.1007/s00521-012-0856-y>.
- Mottahedi, A., Sereshki, F. and Ataei, M. (2018), "Overbreak prediction in underground excavations using hybrid ANFIS-PSO model", *Tunn. Undergr. Sp. Technol.*, **68**, 142-152. <https://doi.org/10.1016/j.tust.2018.05.023>.
- Murlidhar, B.R., Ahmed M., Mavaluru D., Siddiqi A.F. and Mohamad E.T. (2018), "Prediction of rock interlocking by developing two hybrid models based on GA and fuzzy system", *Eng. Comput.*, **35**, 1419-1430. <https://doi.org/10.1007/s00366-018-0672-9>.
- Sadeghi, F., Monjezi, M. and Armaghani, D.J. (2020), "Evaluation and optimization of prediction of toe that arises from mine blasting operation using various soft computing techniques", *Nat. Resour. Res.*, **29**(2), 887-903. <https://doi.org/10.1007/s11053-019-09605-2>.
- Sharma, L.K., Vishal, V. and Singh, T.N. (2017), "Developing novel models using neural networks and fuzzy systems for the prediction of strength of rocks from key geomechanical properties", *Measurement*, **102**, 158-169.

- <https://doi.org/10.1016/j.measurement.2017.01.043>.
- Singh, M. and Singh, B. (2012), "Modified Mohr-Coulomb criterion for non-linear triaxial and polyaxial strength of jointed rocks", *Int. J. Rock Mech. Min. Sci.*, **51**, 43-52. <https://doi.org/10.1016/j.ijrmms.2011.12.007>.
- Singh, R., Kainthola, A. and Singh, T.N. (2012), "Estimation of elastic constant of rocks using an ANFIS approach", *Appl. Soft Comput.*, **12**(1), 40-45. <https://doi.org/10.1016/j.asoc.2011.09.010>.
- Singh, T.N., Kanchan, R., Verma, A.K. and Saigal, K. (2005), "A comparative study of ANN and Neuro-fuzzy for the prediction of dynamic constant of rockmass", *J. Earth Syst. Sci.*, **114**, 75-86. <https://doi.org/10.1007/BF02702010>.
- Taghavifar, H. and Mardani, A. (2014), "Prognostication of vertical stress transmission in soil profile by adaptive neuro-fuzzy inference system based modeling approach", *Measurement*, **50**, 152-159. <https://doi.org/10.1016/j.measurement.2013.12.035>.
- Teymen, A. and Mengüç, E.C. (2020), "Comparative evaluation of different statistical tools for the prediction of uniaxial compressive strength of rocks", *Int. J. Min. Sci. Technol.*, **30**(6), 785-797. <https://doi.org/10.1016/j.ijmst.2020.06.008>.
- Wu, J.D., Hsu, C.C. and Wu, G.Z. (2009), "Fault gear identification and classification using discrete wavelet transform and adaptive neuro-fuzzy inference", *Expert Syst. Appl.*, **36**, 6244-6255. <https://doi.org/10.1016/j.eswa.2008.07.023>.
- Yang, H., Nikafshan Rad, H., Hasanipanah, M., Bakhshandeh Amnieh, H. and Nekouie, A. (2020), "Prediction of Vibration Velocity Generated in Mine Blasting Using Support Vector Regression Improved by Optimization Algorithms", *Nat. Resour. Res.*, **29**, 807-830. <https://doi.org/10.1007/s11053-019-09597-z>.
- Yang, Y. and Zang, O. (1997), "A hierarchical analysis for rock engineering using artificial neural networks", *Rock Mech. Rock Eng.*, **30**, 207-222. <https://doi.org/10.1007/BF01045717>.
- Yazdani, B. (2012), "Shear Strength Parameters of Shale Based on Triaxial Compression Test", M.Sc. Dissertation, Universiti Teknologi Malaysia.
- Ye, J., Dalle, J., Nezami, R., Hasanipanah, M. and Armaghani, D.J. (2020), "Stochastic fractal search-tuned ANFIS model to predict blast-induced air overpressure", *Eng. Comput.*, 1-15. <https://doi.org/10.1007/s00366-020-01085-w>.
- Zhang, W. and Goh, A.T.C. (2013), "Multivariate adaptive regression splines for analysis of geotechnical engineering systems", *Comput. Geotech.*, **48**, 82-95. <https://doi.org/10.1016/j.compgeo.2012.09.016>.
- Zhang, W., Li, H., Li, Y., Liu, H., Chen, Y. and Ding, X. (2021a), "Application of deep learning algorithms in geotechnical engineering: A short critical review", *Artif. Intell. Rev.*, 1-41. <https://doi.org/10.1007/s10462-021-09967-1>.
- Zhang, W., Wu, C., Zhong, H., Li, Y. and Wang, L. (2021b), "Prediction of undrained shear strength using extreme gradient boosting and random forest based on Bayesian optimization", *Geosci. Front.*, **12**(1), 469-477. <https://doi.org/10.1016/j.gsf.2020.03.007>.
- Zhang, W., Zhang, R., Wu, C., Goh, A.T.C., Lacasse, S., Liu, Z. and Liu, H. (2020), "State-of-the-art review of soft computing applications in underground excavations", *Geosci. Front.*, **11**(4), 1095-1106. <https://doi.org/10.1016/j.gsf.2019.12.003>.
- Zhou, J., Li, E., Wang, M., Chen, X., Shi, X. and Jiang, L. (2019), "Feasibility of stochastic gradient boosting approach for evaluating seismic liquefaction potential based on SPT and CPT case histories", *J. Perform. Constr. Fac.*, **33**(3), 04019024. [https://doi.org/10.1061/\(ASCE\)CF.1943-5509.0001292](https://doi.org/10.1061/(ASCE)CF.1943-5509.0001292).