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A generalized viscoelastic model and the corresponding parameter conversion method

Shuling Huang*[1], Xiuli Ding[1], Xiaohua Huang[2], Jun He[1] and Yuting Zhang[1]

[1]*Key Laboratory of Geotechnical Mechanics and Engineering of Ministry of Water Resources, Changjiang River Scientific Research Institute, Wuhan, Hubei 430010, P. R. China*
[2]*College of Civil Engineering and Architecture, Guangxi University, Nanning, Guangxi 530004, P. R. China*

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Abstract. Obtaining applicable rheological model and corresponding rheological parameters are the key issues of the long-term stability analysis of engineering rock mass. In this study, a generalized viscoelastic combination model with considering the effects of stress level is proposed. The proposed model is composed of a brittle viscous body and several Kelvin bodies in series, which unites the generalized Kelvin attenuated creep model and the generalized Burgers non-attenuated creep model. In addition, the tension-compression parameters and the shear parameters are used to express the proposed model, respectively. As these two types of parameters are often converted in the creep tests and engineering applications or change occurs to parameter types when extend the creep model from one-dimensional to three-dimensional. Thus, based on the assumption of constant volumetric modulus, a new conversion equation between the tension-compression parameters and the shear parameters is created for the proposed generalized viscoelastic combination model. Based on the new conversion equation, the three-dimensional extension of the generalized viscoelastic combination model expressed by both the tension-compression parameters and the shear parameters are derived. The proposed creep model and parameter conversion equation are then verified by the laboratory uniaxial compression test and triaxial compression test. The above proposed creep model and parameter conversion equation are applied to the example of rock foundation age deformation. Based on the application, potential problems caused by parameter conversion during rheological numerical simulations are discussed. Based on the discussion, the superiority of the parameter conversion method proposed in this study is fully illustrated.

Keywords: generalized viscoelastic combination model; parameter conversion; rock creep; shear parameter; tension-compression parameter

1. Introduction

One key factor affecting the long-term stabilities of geotechnical projects is the rheological phenomenon of rock materials, which is an inherent mechanical property of rock (Amitrano and Helmstetter 2006). To predict the long-term stability of rock engineering from rheological perspective, the rheological phenomenon of rock materials should be reasonably represented in theory. On this subject, different types of rheological constitutive models have been developed by scholars. One type of rheological constitutive model is the empirical constitutive equations that directly fit the test data of rock materials (Costin, 1988, Haupt, 1991). Another kind of rheological constitutive equations is the rheological combination models established based on rheological theories, i.e. the visco-elastoplastic theory (Fahimifar *et al.* 2015, Huang *et al.* 2020, Huang *et al.* 2021, Sterpi and Gioda, 2007, Wang *et al.* 2015, Zhang *et al.* 2019). The rheological combination model is a combination of various of rheological components that reflect different rheological phenomena of rock, such as the linear spring for the elastic deformation, the dashpot for the visco deformation and the plastic component for the plastic deformation, *et al*. Because of its straightforward and intuitive features, the rheological combination models have been extensively used in geotechnical engineering and have successfully reflect various rheological properties of rock, such as the creep effects, the stress relaxation effects, the elastic aftereffects and so on.

Apart from developing the rheological constitutive models themselves, determining the optimal rheological constitutive model and the corresponding model parameters that meet the requirements of a specific complex rock engineering are another two important fields of the rock rheological mechanics research (Bozzano *et al.* 2012, Khaledi *et al.* 2016, Huang *et al*. 2020). If the model and corresponding parameters are not properly determined, the final prediction of the long-term stability of rock engineering will be greatly deviated from the reality. Generally, the rheological model and parameter identification method can be divided into two categories. The first type of model and parameter identification method is the conventional analysis and parameter determination method based on classical mechanics (Cornelius and Scott, 1993, Haupt, 1991, Miura *et al.* 2003, Park and Schapery, 1999, Tschoegl, 1989). The second type of model and parameter identification method is the model and parameter

*Corresponding author, Professor, Ph.D.
E-mail: huangsl_2014@163.com

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inversion identification method based on indoor test results or field measured data (Gavrus *et al.* 1996). In the first approach, the rheological models and corresponding parameters are often obtained based on test results, empirical judgments as well as a variety of assumptions and simplifications. However, the engineering rock mass is a very complex system with various of uncertainties. In addition, these uncertainties are mutual interactive, cross effective and may altered constantly with the changes of engineering environments, construction conditions and also over time. Therefore, the rheological models and parameters identified by using the conventional analysis method are often quite different from the reality. For this reason, many scholars have explored the rheological behaviors of rocks from the perspective of reverse thinking and system thinking. The rheological model and parameter reverse identification method are the products of these kinds of thinking. Model identification refers to pick out the optimal model from a model set with certain attributes that can most accurately describe the system response behavior (Gavrus *et al.* 1996). Though the model identification is an important aspect in the reverse problem research of rock mechanics, no significant progress has made for parameter reverse analysis of rock materials. As viewed from existing research achievements, physical parameter inverse analyses are mostly conducted for rock rheological models (Malan 1999, Munson 1997).

In the rheological model and parameter identification process for specific rock materials, a group of parameters are generally required to judge whether the constitutive model expresses the mechanical properties of rock well or not. Generally, the rheological combination models can be expressed in two ways depending on the types of parameters used. One type of rheological combination model is expressed based on the tension-compression parameter class, i.e. the tension-compression modulus, tension-compression viscosity coefficient, *et al* (Callahan *et al.* 1998, Chen and Pande 1994, Park and Schapery 1999, Zhao *et al.* 2017). Another type of rheological combination model is expressed based on the shear parameter class, i.e. the shear modulus, shear viscosity coefficient, *et al* (Fahimifar *et al.* 2015, Khaledi *et al.* 2016, Xu *et al.* 2018). During rheological model and parameter identification, conversion between these two parameter classes unavoidably exists (Gavrus *et al.* 1996, Yang *et al.* 2019). For example, it is more convenient to express the rheological model parameters obtained from field displacement monitoring or rheological indoor tests by the tension-compression parameter class, while in the three-dimensional rheological numerical simulations, the shear parameter class are usually adopted as the rheological combination model parameters(Yang *et al.* 2019, Zhang *et al.* 2014). In fact, these two classes of parameters in the rheological combination models reflect different physical concepts of the rock materials. In practical applications, conversions from the tension-compression parameter class to the shear parameter class is common for the rheological combination model. It is worth noting that the shear viscosity coefficient and the tension-compression viscosity coefficient are not necessarily equal to each other. However, this issue is not usually concerned during the rheological parameter conversions, which can result in great discrepancy of the final expressions of the rock rheological mechanism. In addition, existing research achievements (Brantut *et al.* 2014, Callahan *et al.* 1998, Malan 1999, Tsai *et al.* 2008, Xie *et al.* 2011, Xu *et al.* 2012) show that stress level has significant impact on identifying the rock viscoelastic model and parameters. In the low stress levels, rock materials are mainly exhibit as attenuate rheology, which can be described by attenuate rheological models, such as the Kelvin-Voigt model. However, in the high stress levels, rock materials are mainly exhibit as non-attenuate rheology, which can be described by non-attenuate rheological models, such as the Burgers model. Nevertheless, the viscoelastic model identified based on laboratory rheological tests or reverse identification of field displacement data are often irrelevant with stress levels. In other words, the identified rheological models are generally only able to describe the rheological behaviors of rock mass under certain stress level (Amitrano and Helmstetter 2006). However, in practical engineering, the disturbed stress field due to rock excavation leads to significant differences in stress level at different depths of surrounding rocks. Thus, it is worth further investigating the reasonability of using the rheological model identified by a certain stress level for the whole surrounding rocks to predict the long-term stability. Therefore, further research is still required on the above issues to ensure the reasonability of the identified rock creep model and parameters as well as the reliability of their use in predicting the long-term stability of rock engineering.

In this study, a generalized viscoelastic combination model considering the effects of stress level is proposed. In addition, based on the assumption of constant volume modulus, a new conversion relationship between the tension-compression parameter class and shear parameter class of the combination model is created. Then, the effectiveness of the proposed combination model and the parameter conversion relationship are validated according to the optimization inversion analysis based on a uniaxial compression creep test and a triaxial compression creep test respectively. Finally, a simple application of the proposed generalized viscoelastic combination model and the parameter identification method are implemented. Based on this application, the potential problems exist during parameter conversion between the two parameter classes of viscoelastic combination models as well as in applying the model for numerical simulations are illustrated.

2. Generalized viscoelastic combination model and the relationship between the two types of parameters

2.1 Generalized viscoelastic combination model

In order to accurately describe the rheological properties of rock, a large number of viscoelastic combination models based on the rheological components have been proposed. A majority of these viscoelastic combination models can be classified as the generalized Maxwell model or the

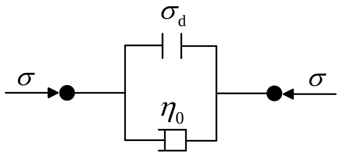


Fig. 1 Brittle-viscous body

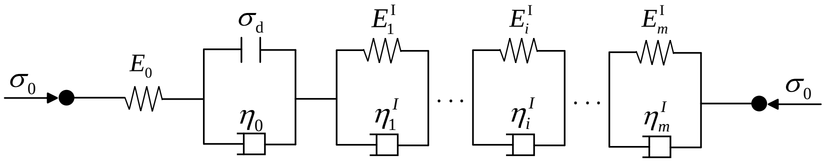


Fig. 2 Generalized viscoelastic combination model expressed by tension-compression parameters

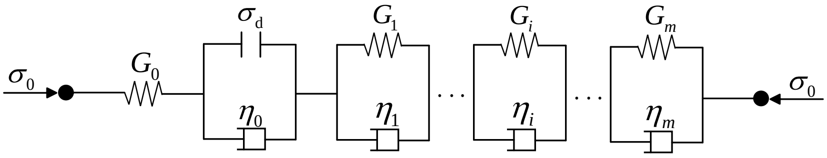


Fig. 3 Generalized viscoelastic combination model expressed by shear parameters

generalized Burgers model, the equivalence between which have been proved by the viscoelastic equivalent principle (Christensen 1982). Based on the viscoelastic equivalent principle, it has been verified that most viscoelastic combination models are the conversion of the generalized Burgers model, which indicates that the generalized Burgers model is of universality. However, existing rock creep tests indicate that (Itô and Sasajima 1987, Maranini and Brignoli 1999, Okubo *et al.* 1991) rock mainly show steady state creep when the stress level is greater than a certain stress state, while shows attenuate creep when the stress level is less than this state. Obviously, as a non-attenuate rheological combination model, the generalized Burgers model is unable to directly describe such phenomena. To overcome this shortcoming, a generalized viscoelastic combination model is proposed in this study. Firstly, a brittle viscous body is adopted to reflect the effects of stress level by parallel connecting a brittle element and a viscous element. Then, the brittle viscous body is connected with the generalized Kelvin model in series to form the generalized viscoelastic combination model.

The one-dimensional structural model and corresponding stress-strain relationship of brittle viscous body in the generalized viscoelastic combination model are shown in Fig. 1 and Eq. (1), respectively.

$$\varepsilon = \begin{cases} 0, & \sigma < \sigma_d \\ \int \dfrac{\sigma}{\eta_0} dt, & \sigma \geq \sigma_d \end{cases} \quad (1)$$

It can be known in this equation that the main role of the brittle element is to restrict the effect of the dashpot in parallel, i.e., in the case of $\sigma < \sigma_d$, the brittle element plays a role equivalent to a rigid body, and no deformation occurs to the loop. In the meantime, the dashpot in parallel does not work, and the viscoelastic combination rheological model can be degraded to the generalized Kelvin model composed by m Kelvin pieces, in order to describe the attenuation rheology. In the case of $\sigma \geq \sigma_d$, the brittle element is broken, failure and no longer exposed to any stress, then the deformation of the brittle loop is just the deformation of the dashpot. In the meantime, generalized viscoelastic combination rheological model can be degraded to the generalized Burgers model composed by m Kelvin pieces, in order to describe the non-attenuation rheology. The σ_d herein is the threshold stress of transiting from the attenuation rheology to the non-attenuation rheology, which can be solved by isochronous stress-strain curve set.

Figs. 2 and 3 show the one-dimensional structure model of the generalized viscoelastic combination model expressed by the tension-compression parameter class and the shear parameter class respectively. The generalized viscoelastic combination model expressed by the tension-compression moduli and the tension-compression viscosity coefficients is denoted as parameter type I, while that expressed by the shear moduli and shear viscosity coefficients is denoted as parameter type II.

For parameter type I, the creep equation of the generalized viscoelastic combination model can be obtained from Fig. 2:

$$\varepsilon^I(t) = \begin{cases} \left(\dfrac{1}{E_0} + \displaystyle\sum_{i=1}^{m} \dfrac{1}{E_i^I}\left(1 - exp\left(-\dfrac{t}{\tau_i^I}\right)\right)\right)\sigma_0, & \sigma_0 < \sigma_d \\ \left(\dfrac{1}{E_0} + \dfrac{t}{\eta_0^I} + \displaystyle\sum_{i=1}^{m} \dfrac{1}{E_i^I}\left(1 - exp\left(-\dfrac{t}{\tau_i^I}\right)\right)\right)\sigma_0, & \sigma_0 \geq \sigma_d \end{cases} \quad (2)$$

where E_0 is the elastic modulus of the independent spring, E_i^I is the tension-compression elastic modulus of the i-th Kelvin body $(i = 1, 2, \cdots, m)$, η_i is the tension-compression viscosity coefficient of the i-th Kelvin body, τ_i^I is the delay time of i-th Kelvin body $(\tau_i^I = \eta_i^I / E_i)$.

For parameter type II, based on the assumption of constant volumetric modulus, the creep equation of the generalized viscoelastic combination model can be obtained from Fig. 3:

$$\varepsilon^{II}(t)=\begin{cases}\left(\frac{2}{3}J_1'(t)+\frac{1}{9K}\right)\sigma_0, & \sigma_0<\sigma_d\\ \left(\frac{2}{3}J_1(t)+\frac{1}{9K}\right)\sigma_0, & \sigma_0\geq\sigma_d\end{cases} \tag{3}$$

In which, the shear creep compliance is shown as below.

$$\begin{aligned}J_1'(t)&=\frac{1}{2G_0}+\sum_{i=1}^{m}\frac{1}{2G_i}\left(1-exp\left(-\frac{t}{\tau_i}\right)\right)\\ J_1(t)&=\frac{1}{2G_0}+\frac{t}{2\eta_0}+\sum_{i=1}^{m}\frac{1}{2G_i}\left(1-exp\left(-\frac{t}{\tau_i}\right)\right)\end{aligned} \tag{4}$$

where K is the volumetric modulus; G_0 is the shear elasticity modulus of the independent spring; G_i is the shear elasticity modulus of the i-th Kelvin body $(i=1,\ 2\ \cdots,m)$; η_i is the viscosity coefficient of the i-th Kelvin body; τ_i is the delay time of i-th Kelvin body $(\tau_i=\eta_i/G_i)$. Thus, the following is obtained by substituting Eq. (4) into Eq. (3).

$$\varepsilon^{\mathrm{II}}(t)=\begin{cases}\left(\frac{1}{3G_0}+\frac{1}{9K}+\sum_{i=1}^{m}\frac{1}{3G_i}\left(1-exp\left(-\frac{t}{\tau_i}\right)\right)\right)\cdot\sigma_0, & \sigma_0<\sigma_d\\ \left(\frac{1}{3G_0}+\frac{1}{9K}+\frac{t}{3\eta_0}+\sum_{i=1}^{m}\frac{1}{3G_i}\left(1-exp\left(-\frac{t}{\tau_i}\right)\right)\right)\cdot\sigma_0, & \sigma_0\geq\sigma_d\end{cases} \tag{5}$$

It can be seen from the above analysis that through considering the viscous brittle body, the generalized viscoelastic combination model is able to unite the generalized Kelvin attenuation model and the generalized Burgers non-attenuation model. Apparently, according to viscoelastic equivalence principle or element quantity change method, the generalized viscoelastic combination model can be simplified as some combination models commonly used in rock engineering. If only considering a Kelvin body, when the stress level is below the threshold, then this combination model can be degraded to the Kelvin-Voigt model, i.e., the Poying-Tomson model is obtained (it can be known from the viscoelastic equivalence principle that the Kelvin-Voigt model and Poying-Tomson model are equivalent). When the stress level is greater than the threshold, then this combination model can be degraded to the Burgers model. Accordingly, the viscoelastic combination model with five elements can be obtained, which unites the Kelvin-Voigt model or Poying-Tomson model with the Burgers model. If considering two Kelvin bodies, then viscoelastic combination model with seven elements can be obtained. More complex viscoelastic model can also be converted to Kelvin body chain gradually according to the viscoelastic equivalence principle or the element quantity change method (Christensen 1982). However, it should be noted in the conversion process that, for the model expressed by tension-compression parameters, two elastic elements in series or in parallel with elastic modulus of E_i and E_j can be equivalent to those with elastic modulus of $E_iE_j/(E_i+E_j)$ and E_i+E_j; two viscous elements in series or in parallel with viscous coefficient of η_i and η_j can be equivalent to those with viscous coefficient of $\eta_i\eta_j/(\eta_i+\eta_j)$ and $\eta_i+\eta_j$. Therefore, the generalized viscoelastic combination model also has certain degree of universality to meet the needs of most rock engineering.

2.2 The relationship between tension-compression parameters and shear parameters in the combination model

The three-dimensional extension of viscoelastic combined model is generally believed to be dependent on certain assumptions, such as the constant Poisson's ratio assumption and the constant volumetric modulus assumption. Therefore, in the three-dimensional stress state, whether the tension-compression parameter class and the shear parameter class of the viscoelastic combination model can be interconverted or not and how to correctly convert the two parameter types depend on these assumptions. In fact, based on the constant volumetric modulus assumption, the viscous deformation is usually regarded as shear deformation without causing any volume change, i.e., the volume deformation is elastic. In this section, a new conversion relationship between the tension-compression parameters and the shear parameters is established based on the constant volumetric modulus assumption, so as to ensure the equivalence of the mechanical behaviors of the viscoelastic combination model that expressed by the two parameter types.

Let the following be

$$\varepsilon^{I}(t)=\varepsilon^{\mathrm{II}}(t) \tag{6}$$

Thus, the relationship between the corresponding physical values of the two types of parameters in the generalized viscoelastic combination model can be obtained.

$$\left.\begin{aligned}&K=\frac{E_0}{3(1-2\mu)}\\ &G_0=\frac{E_0}{2(1+\mu)}\quad, \quad \eta_0=\frac{\eta_0^I}{3}\\ &G_i=\frac{E_i^I}{3},\qquad\qquad \eta_i=\frac{\eta_i^I}{3}\end{aligned}\right\}(i=1,2\ \cdots,m) \tag{7}$$

where μ is the Poisson's ratio.

It can be seen from the above results that under the constant volumetric modulus assumption, the parameter conversion of the generalized viscoelastic combination model presents the following three characteristics. Firstly, for independent elastic element, the conversion from the tension-compression modulus to the shear modulus and the volumetric modulus are consistent with convention. Secondly, for the elastic element in the Kelvin body, the tension-compression modulus is three times over the shear modulus, which is independent of material characteristics. Thirdly, for the brittle viscous body in the model and the viscous element in the Kelvin body, the tension-compression viscosity coefficient is three times over the shear viscosity coefficient, also independent of material characteristics.

For the three-dimensional extension of the generalized viscoelastic combination model, under the constant volumetric modulus assumption, the expression of the total creep strain $\varepsilon_{ij}(t)$ can be obtained according to Eq. (3) and three-dimensional stress state, i.e.,

$$\varepsilon_{ij}(t)=\begin{cases}J_1'(t)s_{ij}+\frac{1}{9}\sigma_{kk}\delta_{ij}, & \lambda<\lambda_d\\ J_1(t)s_{ij}+\frac{1}{9}\sigma_{kk}\delta_{ij}, & \lambda\geq\lambda_d\end{cases} \tag{8}$$

where λ is the yielding approach index function. The threshold σ_d in Eq. (3) is replaced by λ_d in the three-dimensional condition. In fact, in the three-dimensional stress state, the yielding approach index function λ can be used to express the rock stress state, i.e., $\lambda = 1 - YAI$, where *YAI* is the yielding approach index (Zhang *et al.* 2011). The *YAI* describes the ratio between the distance of a point in the spatial stress state away from the yield surface along the most unfavorable stress path and the distance of the corresponding most stable reference point in the same direction of Lode's angle away from the yield surface along the most unfavorable stress path. If Mohr-Coulomb is taken as strength criterion, then the yielding approach index *YAI* will be the following.

$$YAI=\begin{cases}\dfrac{\alpha\sigma_\pi+\beta\tau_\pi+\gamma}{\alpha\sigma_\pi+\gamma}, & \dfrac{\sigma_1+\sigma_3}{2}\leq\sigma_R\\ \dfrac{\sigma_t-\sigma_1}{\sigma_t-\sigma_R}, & \dfrac{\sigma_1+\sigma_3}{2}>\sigma_R\end{cases} \tag{9}$$

where $\alpha = \sin\varphi/\sqrt{3}$, $\beta(\theta_\sigma) = (\cos\theta_\sigma - \sin\theta_\sigma \sin\varphi/\sqrt{3})/\sqrt{2}$, $\gamma = -c\cos\varphi$, $\tau_\pi = \sqrt{2J_2}$, $\sigma_\pi = (\sigma_1 + \sigma_2 + \sigma_3)/\sqrt{3}$, $\tan\theta_\sigma = (2\sigma_2 - \sigma_1 - \sigma_3)/\sqrt{3}/(\sigma_1 - \sigma_3)$, $\sigma_R = (\sigma_t - c\cos\varphi)/(1 - \sin\varphi)$. The range of yielding approach index *YAI* is [0, 1]. When $YAI = 0$, the stress point is on the yield surface; when $YAI = 1$, the stress point is along the isoclinal line. Definitely, through degradation, the yielding approach index function λ can also be used for the creep equation of viscoelastic combination model in the one-dimensional stress state. Eq. (4) is substituted into Eq. (8) to obtain the three-dimensional extension of the generalized viscoelastic combination model expressed by shear parameters:

$$\varepsilon_{ij}(t)=\begin{cases}\left(\frac{1}{2G_0}+\sum_{i=1}^{m}\frac{1}{2G_i}\left(1-exp\left(-\frac{t}{\tau_i}\right)\right)\right)s_{ij}+\frac{1}{9K}\sigma_{kk}\delta_{ij}, & \lambda<\lambda_d\\ \left(\frac{1}{2G_0}+\frac{t}{2\eta_0}+\sum_{i=1}^{m}\frac{1}{2G_i}\left(1-exp\left(-\frac{t}{\tau_i}\right)\right)\right)s_{ij}+\frac{1}{9K}\sigma_{kk}\delta_{ij}, & \lambda\geq\lambda_d\end{cases} \tag{10}$$

For the foregoing one-dimensional creep equation with three-dimensional assumptions, since the model parameters are identical to those used in the three-dimensional creep equation, the conversion relationship of the viscosity parameters will remain unchanged in the three-dimensional creep model. Therefore, Eq. (7) is substituted into Eq. (10) to obtain the creep equation of three-dimensional generalized viscoelastic combination model expressed by tension-compression parameters:

Similarly, under the assumption of constant Poisson's ratio, the conversion relationship also exists between the two types of parameters. In accordance with the same method mentioned above, the relationship between the tension-compression parameters and the shear parameters can also be obtained, and the three-dimensional creep equation of the generalized viscoelastic combination model can be available under such assumptions.

In fact, for rock viscoelastic behaviors, the two parameter types of the above model express different physical concepts. When parameter type I is used for the viscoelastic combination model, it is to represent the tension-compression effects of the rock material. However, when parameter type II is used for the viscoelastic combination model, it is to represent the shear effects of the rock material. When the elastic parameters of one-dimensional rheological model are tension-compression modulus, it is to describe the rheological behavior in the three-dimensional principal stress (or axial) direction. However, under the three-dimensional assumption, especially under the assumption of three-dimensional constant volumetric modulus, this rheological model is to describe the rheological behavior in three-dimensional partial stress (or distortion) direction, and meanwhile, the elastic parameters of this model are shear modulus. Therefore, the conversion expression between the tension-compression modulus of elastic parameters in one-dimensional rheological model and the shear modulus of elastic parameters in three-dimensional rheological model is essentially the conversion expression between the tension-compression modulus and the shear modulus of elastic parameters in three-dimensional rheological model. The difference in one-dimensional and three-dimensional viscosity parameter of the model is essentially the difference in the rheological rate along the axial direction and partial stress direction.

However, for the constant volumetric modulus assumption, the viscosity coefficient of one-dimensional model is usually directly applied to three-dimensional state with its value unchanged, or the elastic modulus of Kelvin body is converted by $E_i/(2(1+\mu))$ and applied to three-dimensional state. Obviously, this customary practice of model parameter conversion will cause more serious consequences. During the conversion from one-dimensional to three-dimensional, if the tension-compression modulus is converted according to three times over the shear modulus while the viscosity coefficient is directly applied to three-dimensional state, then the delay time τ_i will be magnified three times. If the tension-compression modulus is converted to shear modulus according to $E_i/(2(1+\mu))$, while the viscosity coefficient is directly applied to three-dimensional state, then the delay time τ_i will be magnified $2(1+\mu)$ times. If this incorrect relationship is applied to the rheological numerical computation of rock engineering,

$$\varepsilon_{ij}(t)=\begin{cases}\left(\frac{1+\mu}{E_0}+\sum_{i=1}^{m}\frac{3}{2E_i^I}\left(1-exp\left(-\frac{t}{\tau_i^I}\right)\right)\right)s_{ij}+\frac{(1-2\mu)}{3E_0}\sigma_{kk}\delta_{ij}, & \lambda<\lambda_d\\ \left(\frac{1+\mu}{E_0}+\frac{3}{2\eta_0}t+\sum_{i=1}^{m}\frac{3}{2E_i^I}\left(1-exp\left(-\frac{t}{\tau_i^I}\right)\right)\right)s_{ij}+\frac{(1-2\mu)}{3E_0}\sigma_{kk}\delta_{ij}, & \lambda\geq\lambda_d\end{cases} \tag{11}$$

it will inevitably lead to the following two results. Firstly, in the rheological numerical normal analysis, the solved relationship between the rheological displacement and time is irrational and unable to really reflect the rheological properties of rock mass. Secondly, in rheological numerical inverse analysis, the obtained model parameters may not be the actual rock rheological parameters.

In summary, conversion relationship exists between different types of rheological parameters, and the three-dimensional creep model dependent on specific hypotheses. Therefore, in the parameter identification of rock viscoelastic combination model, correctly understanding the above questions is the premise and foundation to obtain real creep model parameters and rock aging deformation laws.

3. Creep model and parameter identification based on laboratory tests

3.1 Creep model and parameter identification method

The rheological parameter identification of the rheological model based on the laboratory rock creep test curves can be implemented according to a two-step inverse analysis method. Firstly, the instantaneous elastic tension-compression parameters (i.e., the elastic modulus E_0) and shear parameters (i.e., the volumetric modulus K_0 and shear modulus G_0) of the viscoelastic combination model are identified according to the mutation strains after load of the creep test curves. During the instantaneous mutation displacements, only the independent spring works and the creep time is equal to zero, thus the instantaneous elastic tension-compression parameters and shear parameters can be easily obtained by Eqs. (2) and (5) or Eqs. (10) and (11) respectively. Secondly, based on the identified elastic parameters of the above independent spring, the particle swarm and other optimization inversion methods can be used in the test creep curves to identify the creep model parameters.

The essence of using optimization method for back analysis of creep model parameters is to find a set of parameters, so that the calculated value of the model parameters approaches the measured value. Therefore, the objective function of the back analysis can be set as the residual sum of squares between the measured values and the calculated values of the creep model. When the minimum is taken from the objective function, the optimal parameters of the creep model is identified. The objective function can be written in the following form:

$$f(x) = \sum_{i=1}^{n} [\varepsilon_i(X, t_i) - \bar{\varepsilon}_i(t_i)]^2 \tag{12}$$

where X is the parameter vector to be identified, such as $X = (\eta_0, G_1, \eta_1, G_2, \eta_2, \cdots, G_m, \eta_m)$. $\varepsilon_i(X, t_i)$ is the strains obtained from the viscoelastic combination model. $\bar{\varepsilon}_i(t_i)$ is the measured strains of creep test, n is the number of measured displacements at the measuring point.

In order to simplify the above optimization problems and reduce the computational workload, according to the elastic inversion results of the first step and engineering experience analogy, the potential upper and lower limits of the creep parameters can be given.

$$a_i \le X_i \le b_i \quad (i = 1,2,\cdots,l) \tag{13}$$

where X_i is the i-th parameter, b_i and a_i are the upper and lower limits of X_i, l is the number of parameters to be identified.

The particle swarm optimization algorithm (Eberhart and Kennedy, 1995) is adopted for parameter identification of rock creep model (Fig. 4), the processes of which are shown as follows:

(1) The particle swarm parameters, the number of Kelvin body adopted in the combination model and the range of parameters to be identified are initialized.

(2) The group of parameters to be identified is regarded as a particle, and several particles are regarded as a population. The position, which is the value of the parameters for identification, and the velocity of each particle are then randomly initialized in the range of the set interval according to priori knowledge, so as to evenly distribute them in the solution space.

(3) The stress state is determined according to the yielding approach index function λ. Then, the number of Kelvin bodies and corresponding mechanical parameters are substituted into the creep model under the corresponding stress state, i.e., Eq. (2) and Eq. (5) or Eq. (10) and Eq. (11). By calculation, the rock aging deformation can be obtained.

(4) The adaptive values p of each particle, which is the parameter evaluation criteria, are calculated according to Eq. (12).

(5) The current adaptive value p of each particle is compared with its own historical optimal adaptive value p_{id}. If p is better than p_{id}, then $p_{id} = p$.

(6) The optimal fitness value p of each particle is compared with the optimal fitness values of all particles p_{gd}. If p is better than p_{gd}, then $p_{gd} = p$.

(7) Change the position and moving speed of each particle according to Eq. (14).

$$\left.\begin{array}{l} v_{id} = wv_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}) \\ x_{id} = x_{id} + v_{id} \end{array}\right\} \tag{14}$$

where w is the inertia weight, c_1 and c_2 are learning factors of non-negative constant, r_1 and r_2 are the random number in the range of [0,1], x_{id} is the position of the particle or the parameter vector for inversion.

(8) If the iterative termination condition is satisfied (the maximum iteration number and the currently searched optimal position of the particle swarm satisfying the threshold of the adaptive value are usually adopted to be the termination condition), the iteration is terminated and the optimal position of particles are output. Otherwise, the parameter identification process returns to step (3).

3.2 Uniaxial compression creep test

The gray-green claystone specimens of Goupitan Hydropower Station are used for laboratory uniaxial compression creep test to validate the effectiveness of the proposed generalized viscoelastic combination model and

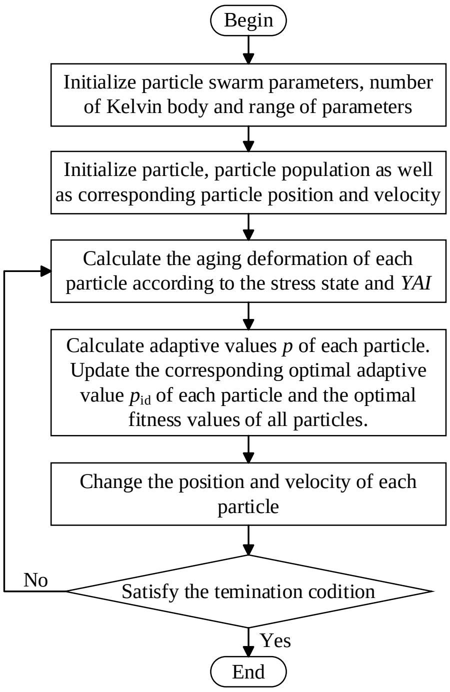


Fig. 4 Particle swarm optimization algorithm for parameter identification

Table 1 Identification results of the proposed model parameters for two different parameter types under uniaxial stress (axial pressure: 3.57MPa)

Tension-compression parameters	Value	Shear parameters	Value
E_0 /MPa	357	K /MPa	396.67
E_1^I /MPa	2772	G_0 /MPa	132.22
η_1^I /MPa.h	59600	G_1 /MPa	924
E_2^I /MPa	4725	η_1 /MPa.h	19866.67
η_2^I /MPa.h	20600	G_2 /MPa	1575
		η_2 /MPa.h	6866.67

Table 2 Identification results of the proposed model parameters for two different parameter types under uniaxial stress (axial pressure: 5.39MPa)

Tension-compression parameter	Value	Shear parameter	Value
E_0 /MPa	381.30	K /MPa	423.66
E_1^I /MPa	4238	G_0 /MPa	141.22
η_1^I /MPa.h	68600	G_1 /MPa	1412.67
E_2^I /MPa	7921	η_1 /MPa.h	22866.67
η_2^I /MPa.h	30520	G_2 /MPa	2640.33
η_0^I /MPa.h	1820600	η_2 /MPa.h	10173.33
		η_0 /MPa.h	606866.67

the parameter conversion method. The Poisson's ratio of the gray-green claystone specimens is 0.35. Fig. 5 shows the axial strain-time curve of the specimens. In the uniaxial stress state, the generalized viscoelastic creep models for inversion analysis are Eq. (2) and Eq. (5). During parameter identification, the rock threshold λ_d is set as 0.7.

The above particle swarm optimization algorithm is used for creep model and parameter identification to obtain the optimal creep curve. The identified model parameters are shown in Table 1 and Table 2. The identified creep parameters are substituted into the creep equations of the generalized viscoelastic combination model in the case of $m = 2$, i.e., the generalized viscoelastic combination model containing two Kelvin bodies. Thereby, the comparison of test curve and calculated curve with two different identification parameter types in the proposed model under uniaxial stress state is shown in Fig. 5. It can be seen from Fig. 5 that for either the attenuation creep in the lower stress state and the non-attenuation creep in the higher stress state, the calculated curves obtained under the two types of parameters consist well with the test curve, indicating that the generalized viscoelastic combination model and its two types of parameters obtained by identification can well describe the aging deformation characteristics of the gray-green claystone. As viewed from Table 1 and Table 2, the identified tension-compression parameters and shear parameters of the creep model basically satisfy the conversion relationship between given by Eq. (7), which indicates that, in the uniaxial stress state, the conversion relationship between the tension-compression parameters and the shear parameters of viscoelastic combination creep model given in Eq. (7) is reasonable.

3.3 Triaxial compression creep test

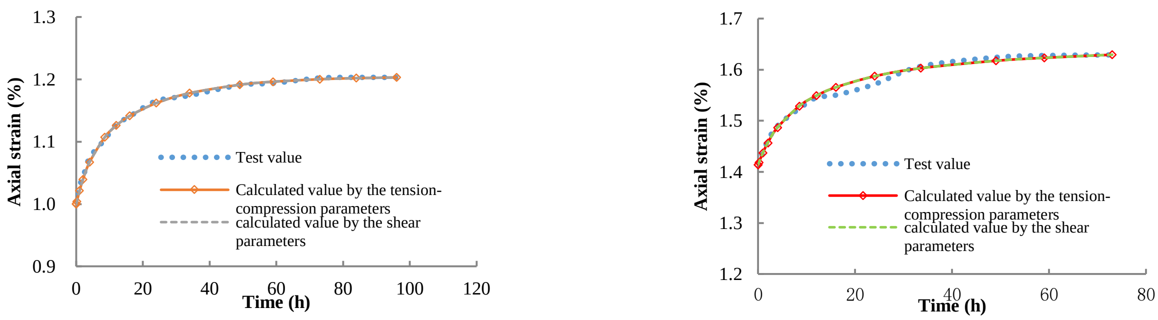
The quartz-mica schist specimens of Danba Hydropower Station are used for conventional triaxial compression creep tests to validate the effectiveness of the proposed generalized viscoelastic combination model and the parameter conversion method in three-dimensional condition. The confining pressure of the creep test is 10MPa. The Poisson's ratio of the quartz-mica schist specimens is 0.25. Fig. 6 shows the axial strain-time curve of the specimens.

In the conventional triaxial stress state, i.e., $\sigma_2 = \sigma_3$, $\sigma_{kk} = \sigma_1 + 2\sigma_3$ and $s_{11} = 2(\sigma_1 - \sigma_3)/3$. Therefore, according to Eq. (10) and Eq. (11), the axial strain expressions based on the two types of parameters can be obtained. When the shear parameters are used for expression, the axial strain is shown as below.

$$\varepsilon_{11}(t) = \begin{cases} \dfrac{\sigma_1 - \sigma_3}{3G_0} + \dfrac{\sigma_1 + 2\sigma_3}{9K} + \sum_{i=1}^{m} \dfrac{\sigma_1 - \sigma_3}{3G_i}\left(1 - exp\left(-\dfrac{t}{\tau_i}\right)\right), & \lambda < \lambda_d \\ \dfrac{\sigma_1 - \sigma_3}{3G_0} + \dfrac{\sigma_1 + 2\sigma_3}{9K} + \dfrac{\sigma_1 - \sigma_3}{3\eta_0}t + \sum_{i=1}^{m} \dfrac{\sigma_1 - \sigma_3}{3G_i}\left(1 - exp\left(-\dfrac{t}{\tau_i}\right)\right), & \lambda \geq \lambda_d \end{cases} \tag{15}$$

When the tension-compression parameters are used for expression, the axial strain expression is shown as below.

$$\varepsilon_{11}(t) = \begin{cases} \dfrac{2(1+\mu)}{3E_0}(\sigma_1 - \sigma_3) + \dfrac{(1-2\mu)}{3E_0}(\sigma_1 + 2\sigma_3) + \sum_{i=1}^{m} \dfrac{\sigma_1 - \sigma_3}{E_i^I}\left(1 - exp\left(-\dfrac{t}{\tau_i}\right)\right), \lambda < \lambda_d \\ \dfrac{2(1+\mu)}{3E_0}(\sigma_1 - \sigma_3) + \dfrac{(1-2\mu)}{3E_0}(\sigma_1 + 2\sigma_3) + \dfrac{\sigma_1 - \sigma_3}{\eta_0}t + \sum_{i=1}^{m} \dfrac{\sigma_1 - \sigma_3}{E_i^I}\left(1 - exp\left(-\dfrac{t}{\tau_i}\right)\right), \lambda \geq \lambda_d \end{cases} \tag{16}$$



(a) Axial pressure:3.57MPa, the attenuation mode creep (b) Axial pressure:5.39MPa, the non-attenuation mode creep

Fig. 5 Comparison of test curve and calculated curve for two different identification parameter types in the proposed model under uniaxial stress state

Table 3 Identification results of the proposed model parameters for two different parameter types under triaxial stress state (axial pressure: 40MPa)

Tension-compression parameter	Value	Shear parameter	Value
E_0 /GPa	32.67	K /GPa	21.78
E_1^I /GPa	1052.77	G_0 /GPa	13.07
η_1^I /GPa.h	12.71	G_1 /GPa	350.92
E_2^I /GPa	1864.73	η_1 /GPa.h	4.24
η_2^I /GPa.h	418.63	G_2 /GPa	621.58
		η_2 /GPa.h	139.54

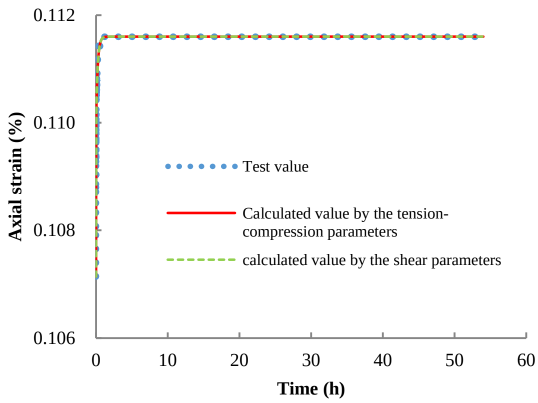
Table 4 Identification results of the proposed model parameters for two different parameter types under triaxial stress state (axial pressure: 60MPa)

Tension-compression parameter	Value	Shear parameter	Value
E_0 /GPa	19.41	K /GPa	12.94
E_1^I /GPa	272.76	G_0 /GPa	7.76
η_1^I /GPa.h	159.67	G_1 /GPa	90.92
E_2^I /GPa	864.79	η_1 /GPa.h	53.22
η_2^I /GPa.h	5201.71	G_2 /GPa	288.26
		η_2 /GPa.h	1733.90

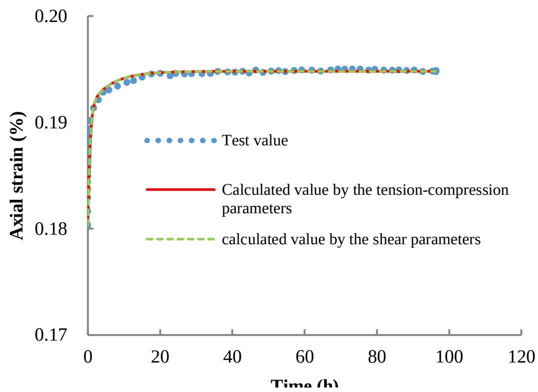
Table 5 Identification results of the proposed model parameters for two different parameter types under triaxial stress state (axial pressure: 70MPa)

Tension-compression parameter	Value	Shear parameter	Value
E_0 /MPa	15.48	K /MPa	10.32
E_1^I /MPa	186.73	G_0 /MPa	6.19
η_1^I /MPa.h	151.67	G_1 /MPa	62.24
E_2^I /MPa	852.58	η_1 /MPa.h	50.56
η_2^I /MPa.h	9229.84	G_2 /MPa	284.19
η_0^I /MPa.h	88081.67	η_2 /MPa.h	3076.61
		η_0 /MPa.h	29360.56

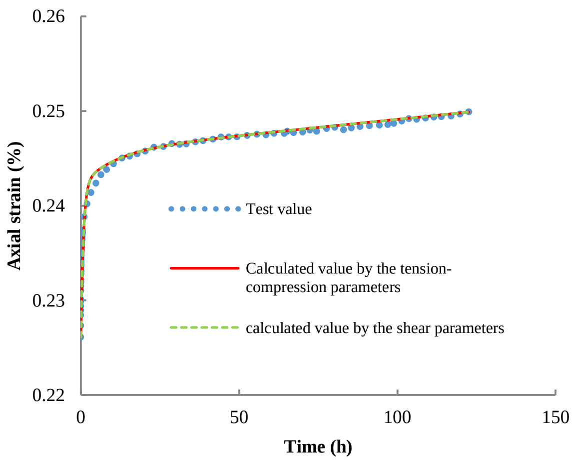
Therefore, in this conventional triaxial stress state, the generalized viscoelastic combination models in Eq. (15) and Eq. (16), i.e., the generalized viscoelastic combination model containing two Kelvin bodies, are used for inversion analysis. During parameter identification, the rock threshold λ_d is set as 0.72. The above particle swarm optimization algorithm is used for creep model and parameter identification to obtain the optimal creep curve. The identified model parameters are shown in Table 3 and Table



(a) Axial pressure:40MPa, the attenuation mode creep



(b) Axial pressure:60MPa, the attenuation mode creep



(c) Axial pressure:70MPa, the non-attenuation mode creep

Fig. 6 Comparison of test curve and calculated curve for two different identification parameter types in the proposed model under triaxial stress state

4 and Table 5. The identified creep parameters are substituted into the creep equations of generalized viscoelastic creep combination model in Eq. (15) and Eq. (16). Thereby, the comparison between the test curve and the calculated curves based on two different identification parameter types in the proposed model under triaxial stress state are shown in Fig. 6. It can be seen from Fig. 6 that, the attenuation creep in lower stress state and the non-attenuation creep in higher stress state of the calculated curves obtained under both the two types of parameters consist well with the test curve and the aging deformation development trend are basically the same. Moreover, the aging deformation curve obtained by the tension-compression parameters are completely overlapped with that obtained by the shear parameters. The above analysis validate that the generalized viscoelastic combination model and its two types of parameters obtained by identification can well describe the aging deformation characteristics of the quartz-mica schist. In the meantime, the conversion relationship between the tension-compression parameters and the shear parameters of the proposed creep model in the one-dimensional state also equally adapts to the three-dimensional stress state, which confirms the correctness and universality of the conversion relationship between the two types of parameters given by Eq. (7).

Table 6 Parameters of viscoelastic model

General parameter	value	Tension-compression parameters	Value	Shear parameters	Value
μ	0.25	E_0 /GPa	30	G_0 /GPa	12
K/GPa	20	E_1^I /GPa	180	G_1 /GPa	60
		η_1^I /GPa.d	2100	η_1 /GPa.d	700

4. Application analysis

Fig. 7 shows a rock foundation with square cross-section, the side length of which is 1m. The rock foundation extends long in the y direction. Neglecting the physical impacts, a uniform-distributed compressive stress, i.e., $P =$ 100MPa, occurs suddenly at $z = 1$m on the top surface of the rock foundation. The bottom side ($z = 0$m), the left side ($x = 0$m) and the right side ($x = 1$m) of the rock foundation are fixed in their normal direction, respectively. The rock mass at this position are elastic-viscoelastic materials, shown as attenuation creep under constant uniform-distribution compressive stress. To represent this rheologic characteristic, the generalized viscoelastic combination model with $m = 1$ is used, i.e., a model composed by an independent elastic member and a Kelvin body. The two

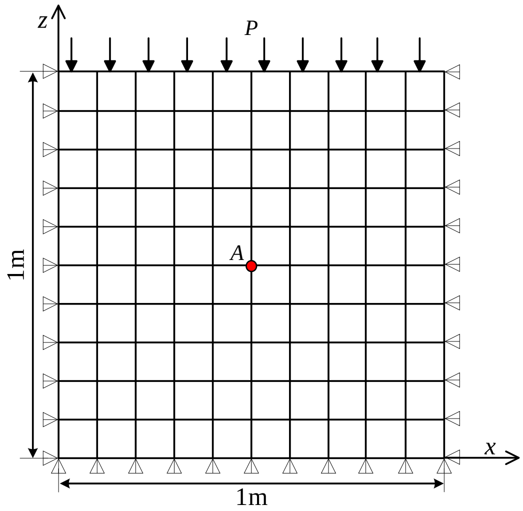


Fig. 7 Rock foundation (extends long in the y direction)

types of parameters in the model are shown in Table 6. In order to facilitate rheological numerical calculation, the constant volumetric modulus assumption is adopted herein, which is the same to the assumption in FLAC numerical calculation method.

4.1 Analytical solution of rock foundation aging deformation

The viscoelastic solution of rock foundation aging deformation is obtained according to the elastic-viscoelastic correspondence principle. The viscoelastic analytical solutions of rock foundation under external loads are given as below. Under external loads, the above elastic solution of rock foundation is shown as below.

$$\left.\begin{aligned} w(z) &= -\frac{zP}{K_0 + 4G_0/3} \\ \varepsilon_z(x,z) &= -\frac{P}{K_0 + 4G_0/3} \\ \sigma_x = \sigma_y &= -\frac{\lambda_0 P}{K_0 + 4G_0/3} \\ \sigma_z(x,z) &= -P \end{aligned}\right\} \quad (17)$$

where K_0 is the volumetric elastic modulus ($K_0 = E_0/(3(1-2\mu))$), G_0 is the shear elastic modulus ($G_0 = E_0/(2(1+\mu))$), E_0 is the elastic modulus, λ_0 is the Lamé coefficients ($\lambda_0 = (3K_0 - 2G_0)/3$).

Under the constant volumetric modulus assumption, the following relationship can be obtained through Laplace conversion:

$$\left.\begin{aligned} \bar{P}'(s) &= 1 + \frac{\eta_1}{G_0 + G_1} s \\ \bar{Q}'(s) &= \frac{2G_0G_1}{G_0 + G_1} + \frac{2G_0\eta_1}{G_0 + G_1} s \\ \bar{P}''(s) &= 1 \\ \bar{Q}''(s) &= K \end{aligned}\right\} \quad (18)$$

According to the elastic-viscoelastic correspondence theorem, the following can be obtained based on Eq. (17) and Eq. (18):

$$\left.\begin{aligned} \bar{\varepsilon}_z(s) &= -\frac{\frac{P}{s}}{K + \frac{2\bar{Q}'(s)}{3\bar{P}'(s)}} \\ \bar{\sigma}_z(s) &= -\frac{P}{s} \\ \bar{\sigma}_x(s) = \bar{\sigma}_y(s) &= -\frac{K - \frac{\bar{Q}'(s)}{3\bar{P}'(s)}}{K + \frac{2\bar{Q}'(s)}{3\bar{P}'(s)}} \frac{P}{s} \end{aligned}\right\} \quad (19)$$

The relevant parameters are then substituted into Eq. (19) after inverse transformation to obtain the displacement and strain equations of rock foundation:

$$\left.\begin{aligned} \varepsilon_z(t) &= P[-\frac{B}{A} + (\frac{B}{A} - C)e^{-At}] \\ w(z,t) &= \varepsilon_z(t)z = Pz[-\frac{B}{A} + (\frac{B}{A} - C)e^{-At}] \end{aligned}\right\} \quad (20)$$

where $A = \frac{3KG_0 + 3KG_1 + 4G_0G_1}{3K\eta_1 + 4G_0\eta_1}$, $B = \frac{3G_0 + 3G_1}{3K\eta_1 + 4G_0\eta_1}$, $C = \frac{3}{3K + 4G_0}$.

Accordingly, relevant parameters are substituted into Eq. (20) to obtain the viscoelasticity analytical solutions of the displacement at any point of rock foundation. Based on the analytical solution of rock foundation aging deformation, the creep model parameters identification method and FLAC numerical simulation are used to calculate the aging deformation of rock foundation with the proposed creep model expressed by both the tension-compression parameters and the shear parameters in the following sections.

4.2 Creep model parameter identification

As the above analytical solution is expressed by the shear parameters, when tension-compression parameters are adopted for the viscoelastic combination model, parameter conversion between two types of parameters exist during the rheological model parameter identification processes. To illustrate the superiority of the proposed parameter conversion method, several types of parameter identification conditions are set for discussing under the constant volumetric modulus assumptions:

(1) Condition 1: the elastic modulus of Kelvin body is converted according to $G_1 = E_1/(2(1+\mu))$, while the viscosity coefficient of Kelvin body is not converted but inherit directly;

(2) Condition 2: the elastic modulus of Kelvin body is converted according to $G_1 = E_1/3$ in Eq. (7), while the viscosity coefficient of Kelvin body is not converted but inherit directly;

(3) Condition 3: the elastic modulus and viscosity coefficient of Kelvin body are converted according to Eq. (7);

(4) Condition 4: model parameters are expressed by using shear modulus, shear viscosity coefficient and other shear parameters.

During calculation, the above four parameter identification conditions are used for parameter identification of the viscoelastic combination model based on the analytic solution of rock foundation aging deformation. In the first three conditions, the tension-compression parameters are adopted for the viscoelastic combination model, thus parameter conversion between the two types of parameters is necessary and are implemented by the corresponding parameter conversion methods. While in condition 4, the shear parameters are adopted, thus parameter conversion is not required. The particle swarm optimization algorithm is adopted as parameter identification method of the creep model. However, as the creep model is specified in this case, the creep model identification is not necessary herein. Therefore, it is only required to identify the creep model parameters and replace the aging strain by aging displacement in Eq. (12), i.e.,

$$f(x)=\sum_{i=1}^{n}[w_i(X,t_i)-\bar{w}_i(t_i)]^2 \tag{21}$$

where: $w_i(X,t_i)$ is the calculated value of aging displacement, $\bar{w}_i(t_i)$ is the analytic solution of aging displacement, n is the number of measured displacements at measuring point. Eq. (21) is taken as the objective function. Meanwhile, in the creep model parameter inversion process, step (3) needs to be changed into following form. The mechanical parameters, initial conditions and boundary conditions are substituted into FLAC numerical model, so as to obtain the aging displacement at critical point. Other steps are the same as mentioned in the above section. Moreover, under the premise of not affecting the aging deformation laws, the results of the numerical simulations and theoretical solutions in this section and the next section are all taken positive for correlation analysis.

The analytic solution of aging displacement at rock foundation central point A is taken as the inversion analysis objective herein. In the first three conditions, the tension-compression modulus and tension-compression viscosity coefficients, i.e., elastic modulus E_0, viscoelastic modulus E_1 and viscosity coefficient η_1^I are taken as the model parameters to be identified. In Condition 4, the shear modulus and shear viscosity coefficients, i.e., shear modulus G_0, viscoelastic shear modulus G_1 and viscosity coefficient η_1 are taken as model parameters to be identified. Table 7 lists the parameter identification results of the rheological model, and the aging displacement curves of rock foundation obtained based on these results are shown in Fig. 8.

As illustrated in Fig. 8, the convergence time, trend and final convergent results of the displacements obtained by different parameter conditions consist well with that obtained by the theoretical analysis. To be more specific, for the first three conditions, the tension-compression parameters obtained by inversion can be converted to the shear parameters according to the corresponding parameter conversion methods respectively and the conversed shear parameters are just equal to the shear parameters provided in Table 6. For condition 4, the shear parameters are also equal to the shear parameters provided in Table 6. Thus, for all the four conditions, the aging deformation obtained by the obtained parameters coincide with that obtained by the theoretical analysis well. However, in essence, these parameters obtained by inverse analysis are not all the real viscoelastic parameters of the rock mass (see Table 7). Compared with the real viscoelastic parameters (see Table 6), the results of Condition 1 show that the elastic modulus E_0 is equal to the corresponding true value while the viscoelastic modulus E_1 and viscous coefficient η_1^I of Kelvin body are quite different from the corresponding true value. The results of Condition 2 are better than Condition 1, in which the elastic modulus E_0 and viscoelastic modulus E_1 are both equal to the corresponding true value, while the viscosity coefficient η_1^I is quite different from the corresponding true value. The inverse analysis results of Condition 3 and Condition 4 are equal to the corresponding true parameter values, not only indicating the correctness of the established conversion equation between tension-compression parameters and shear parameters, but also showing the reasonability in the proposed parameter identification method of creep model.

The above analysis indicates that, even if the aging displacement evolutionary laws and analytical solutions are consistent in Condition 1 and Condition 2, the irrationality still exists on the model parameters obtained from these conditions. It should be noted that, in essences, the problem described herein is not the problem of the uniqueness of the solution, but the problem of whether the two types of parameters are converted or whether the conversion is reasonable or not. However, as pointed in the above analysis, the parameters obtained in Condition 1 and Condition 2 are incorrect. Therefore, in viscoelastic parameter identification, it is undoubtedly a better choice to use the method in Condition 3 or Condition 4.

Table 7 Results of parameter inversion for viscoelastic model

Parameter	Condition 1	Condition 2	Condition 3	Parameter	Condition 4
E_0 / GPa	30	30	30	G_0 / GPa	12
E_1 / GPa	150	180	180	G_1 / GPa	60
η_1^I/ GPa.d	700	700	2100	η_1 / GPa.d	700

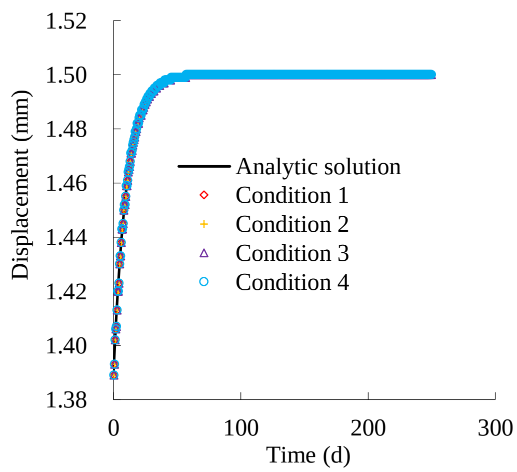
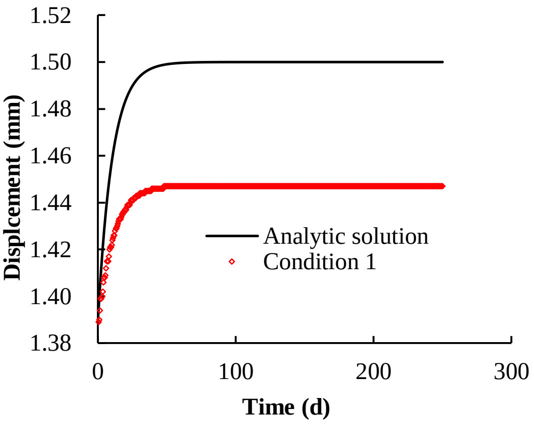
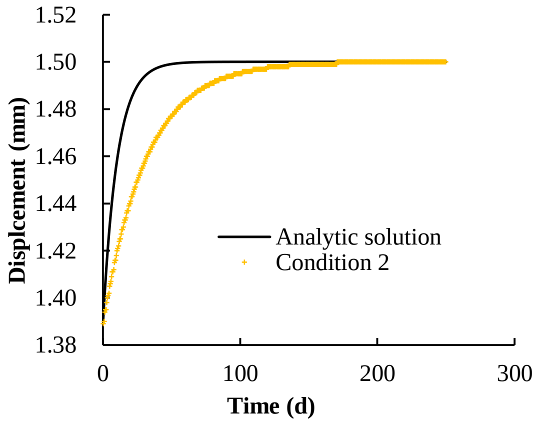


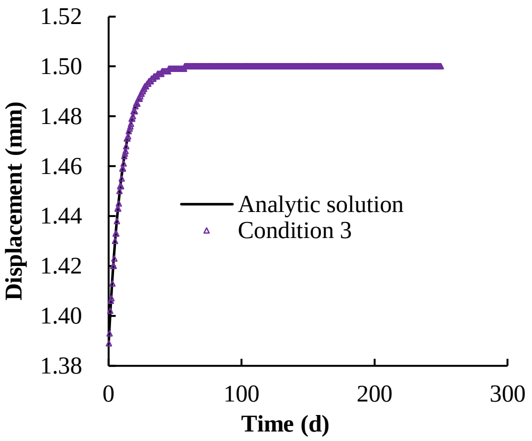
Fig. 8 Relationship between numerical solution calculated by inversed parameters and analytic solution of viscoelasticity



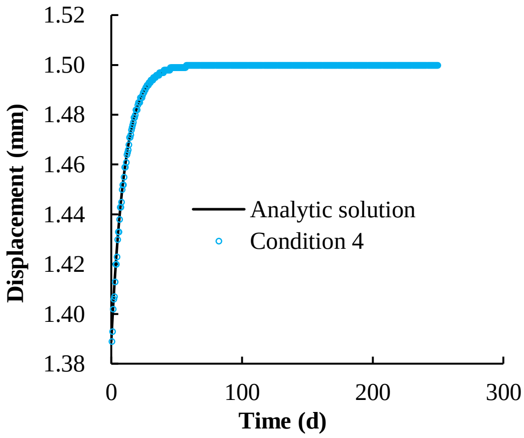
(a) Condition 1



(b) Condition 2



(c) Condition 3



(d) Condition 4

Fig. 9 Relationship between numerical solution and analytic solution of viscoelasticity in center A location of rock foundation

Table 8 Shear parameters after parameter conversion for different conditions

Parameter	Condition 1	Condition 2	Condition 3	Condition 4
G_0 / GPa	12	12	12	12
G_1 / GPa	72	60	60	60
η_1 / GPa.d	2100	2100	700	700

4.3 Aging deformation of rock foundation

Based on the above four parameter conditions, the Kelvin-Voigt model of FLAC is used to calculate the numerical solution of the rock foundation aging displacement (Point A in Fig. 7). The basic model parameters used during the numerical simulation are shown in Table 6, where the tension-compression parameters and general parameters in the table are adopted as the initial input parameters for the first three conditions and the shear parameters and general parameters in the table are adopted as the initial input parameter for Condition 4. During the numerical simulation, the elastic modulus of independent spring in the rheological model of the first three conditions is converted according to the first two equations of Eq. (7), while the viscoelastic modulus and viscosity coefficient in Kelvin body are converted according to the above four parameter conditions respectively. The final shear parameters after conversion for numerical simulation are provided in Table 8.

After the above process, the aging deformation curves of the rock foundation obtained based on the four different parameter conditions and the analytical solutions are compared in Fig. 9. It can be seen that, in Condition 1, as both the shear modulus and viscosity coefficient are different with the real shear parameters in Table 6, starkly difference exist between the simulated aging deformation curves and the theoretical analytical values, which appears as the difference in the convergence time, aging displacement evolutionary laws and final convergence displacement (see Fig. 9(a)). In Condition 2, as only the viscosity coefficient is different with the real shear parameters in Table 6, the simulated final convergence displacement is equal to the analytical value, but the convergence time and the aging displacement evolutionary laws between the simulated aging deformation curves and the theoretical analytical values have significant differences (see Fig. 9(b)). The simulated curve before the displacement converges is quite different from that of the analytical curve, while the deformation gradually converges to the analytical value as time goes on. The final convergence time of the simulated curve is about three times the convergence time of the analytical solution, because the viscosity coefficient obtained by parameter conversion are just three times of the viscosity coefficient of the shear parameters in Table 6. In Condition 3 and Condition 4 (see Fig. 9(c) and 9(d)), the convergence time, development trend and final convergent results are all consistent well with those of the analytical value, which indicate that the derived conversion equations between the tension-compression parameters and the shear parameters are reasonable.

From the above analysis, it can be seen that in Condition 1 and Condition 2, the essence of the above phenomenon lies in the unreasonable model parameters. No conversion or unreasonable conversion of the relevant parameters can lead to unreasonable aging deformations. In Condition 3, the tension-compression parameters are converted totally according to Eq. (7) in the calculation process. The converted rheological model parameters are essentially identical to that of Condition 4, so the two numerical results are equal.

5. Conclusions

The generalized viscoelastic combination model considering the effect of stress level is proposed in this study, which unites the generalized Kelvin attenuation model and the generalized Burgers non-attenuation model. Thus, the proposed generalized viscoelastic combination model has certain universality to meet the requirements of most rock engineering. Moreover, based on the constant volumetric modulus assumption, a new conversion equation between the tension-compression parameters and the shear parameters of the proposed generalized viscoelastic combination model is created. And the one-dimensional and three-dimensional expressions of the generalized viscoelastic combination model based on the tension-compression parameters and the shear parameters are derived. In addition, a two-step optimization inversion analysis method for creep model and parameter identification is proposed, which provides a theoretical basis for the parameter identification of viscoelastic combination model derived.

Based on the uniaxial and triaxial compression creep tests, the creep model and its parameters of the gray-green claystone at Goupitan Hydropower Station and those of the quartz-mica schist at Danba Hydropower Station are identified. Results indicate that the proposed generalized viscoelastic combination model, the conversion relationship between the two parameter types as well as the proposed two-step optimization inversion analysis method are reasonable and applicable. The examples also prove that the conversion relationship between the tension-compression parameters and the shear parameters obtained in one-dimensional state is applicable to three-dimensional stress state. Besides, the application analysis of rock foundation engineering also indicates the correctness of the conversion relationship between the tension-compression parameters and the shear parameters. No conversion or unreasonable conversion of the relevant parameters can lead to unreasonable aging deformations. However, it is worth noting that, as multiple Kelvin bodies are used in the proposed generalized viscoelastic combination model, it is often necessary to determine the number of Kelvin bodies used in advance during model and parameter identification. This may bring additional computational efforts to model and parameter identification. Therefore, in practical applications, only 1 or 2 Kelvin bodies are adopted in the model.

Generally, the viscoelastic combination model and parameter identification can be done in three ways. Firstly, the creep model parameters are obtained according to the one-dimensional creep equation and directly applied to the three-dimensional case. Secondly, when the tension-compression parameters are used for the creep model, it is necessary to convert the model parameters for normal analysis or back analysis. Thirdly, when the shear parameters are used for the creep model, no parameter conversion is required for normal analysis or back analysis. Similar problems also exist in model parameter identification process and engineering application for the viscoelastic combination models based other assumptions, the solution ideas of which should be the similar to this study.

Acknowledgments

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