

# An analytical approach to estimate the mechanical state of roof strata in underground longwall mining

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**Abstract.** The movement and collapse of roof strata in underground longwall mining is a key trigger factor for the occurrence of dynamic disasters. An accuracy estimation of roof strata mechanical state is critical for the prediction and control of dynamic disaster, such as coal burst and coal-and-gas outburst. An analytical approach is proposed in this work to estimate the mechanical state of roof strata in underground longwall mining. To do so, the unit width of roof strata is considered as a beam structure. A system of 4 simulations differential equations is proposed with 4 local slope data as input parameters to derive the mechanical expression of suspending roof strata. A differential evolution algorithm is further adapted to solve the equation system. In addition, a set of verification tests is carried out to showcase the feasibility and robustness of the proposed method. The results show that the average relative errors of 10 independent tests reach a high accuracy, which is less than 1% for the strata mechanical state control parameters. By using the estimated results, the slope, bending moment and shear force of suspending strata are derived. Moreover, the slope data sampling strategy is also devised. The parameters bound determination method is also proposed to ensure the calculation convergence. The local slope based analytical method proposed in this paper is a feasible approach to estimate the mechanical state of suspended roof strata before first weighting.

**Keywords:** analytical model; beam slope; differential evolution; local acceleration monitoring; longwall mining strata mechanical state

## 1. Introduction

The collapse of roof strata in underground longwall mining is a dramatic strata movement mainly caused by underground pressure redistribution (Hou *et al.* 2019, Jiang *et al.* 2016, Mirenkov *et al.* 2017). The strata collapse could be a trigger that contributes to the occurrence of catastrophic dynamic disasters such as coal burst and coal-gas outburst (Keykhosropour *et al.* 2018, Li *et al.* 2017). An accurate estimation of the mechanical state of roof strata is therefore crucial for disaster early-warning, prevention and mitigation.

Numerous analytical models were proposed to explore the behaviour of roof strata. Voussoir beam as one of the most widely used models was established by Evans (1941). Brady *et al.* (2012) and Diederichs *et al.* (1999) had made their contributions to the voussoir beam model. Whittaker

and Reddish (1989) derived the maximum bending moment expression of roof strata based on a simple beam model. Peng (2017) conducted extensive research on strata control in longwall mining, in which an estimation method for first and period weighting distance is proposed. Galvin (2016) suggested a series of beam models to calculate the span distance of strata. Carla (2017) found a new method to identify impending failure in rock slopes. Cetin *et al.* (2012) adopted an iterative procedure to predict the deflections at the road surface under dynamic load and designed the thickness of pavement with acceptable deflection limit. Pan *et al.* (2013) adopted the model of beam on an elastic foundation to analyse the strata behaviour and could provide a more precise formulation to indicate the loading conditions. Stolecki (2021) introduced the velocity of roof deflection as an indicator of underground workings stability. Ji *et al.* (2021) derived the mechanical state of hard roof strata by superposition method. More theories for stability analysis of beam on elastic foundations have been conducted over past decades (Koiter and van der Heijden, 2008; Timoshenko *et al.* 1962).

In relation to the monitoring methods of strata movement, the bed separation measurement method has been widely adopted in field applications (Gui *et al.* 2018). However, this method is only designated to collect data at the designed point and can be applied for a specific rock strata movement stage. That is, the bed separation measurement errors caused by unreliable reference objects are unavoidable. Matured in recent years, microseismic (MS) monitoring (Konicek 2019, Sun *et al.* 2012, Will *et al.*

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2014), electromagnetic radiation (EMR) monitoring (Song *et al.* 2017) and acoustic emission (AE) (Hesser *et al.* 2015; Kong *et al.* 2017; Sellers *et al.* 2003) monitoring methods are feasible alternative solutions to evaluate the stability of strata. Even so, the possible unreliable and untimely result, that caused by complex data processing algorithms, as well as the exorbitant devices could pose an obstacle to large-scale adoption of those methods (Mojtaba *et al.* 2018). To the best of the authors' knowledge, there is not yet a feasible method to directly monitor the movement of roof strata.

The Local Acceleration Monitoring (LAM) (Ji *et al.* 2020, Zhang *et al.* 2018) technique, a low-cost local monitoring method that has been successfully adopted in recent years to monitor strata stability. LAM uses inertial sensors to analyse the stability of roof strata from the view of local inclinations and vibrations. The sensor chip model is MPU-6050, and it has a 3-axis gyroscope and a 3-axis accelerometer to monitor displacement and vibration. The measuring range of the accelerometer is  $\pm 16$  g and the gyroscope of which is  $\pm 2000^\circ/\text{s}$ . The sensor sensitivity is as high as  $6.1 \times 10^{-5}$  g within the scale range of  $\pm 2$  g, and the signal frequency can be monitored from 4 Hz to 200 Hz. Since LAM is based on inertial measurements, the monitoring data will not be affected by underground reference objects, which is important for increasing the accuracy of monitoring processes. For the advantage of LAM, there is a possibility to estimate the mechanical state of suspended roof strata based on the local inclination data. The objective of this paper is to design a scheme to estimate the strata mechanical state by using local inclination data. This scheme simplifies the strata as a beam model. The local inclination data are designated as input parameters to inverse the analysis of the mechanical state of strata. In addition, a verification test is carried out by a numerical case study to examine the robustness of the proposed method.

## 2. The mechanical mode of hard roof strata

### 2.1 The structure of roof stratum

According to the plate and shell mechanism, the roof caving configuration can be simplified as shown in Fig. 1. The highlighted area in Fig. 1 at the middle of the

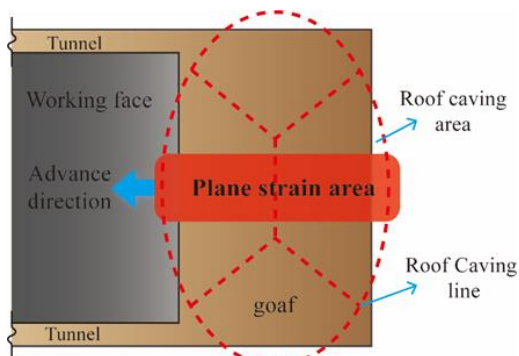


Fig. 1 Top view of roof strata caving configuration model

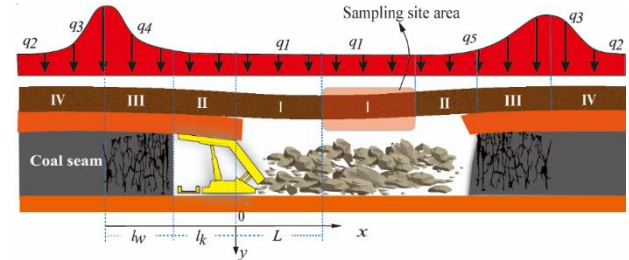


Fig. 2 Rock beam with loading and foundation mechanics model

Table 1 Rock beam mechanics conditions

Rock beam	Loading conditions	Support conditions
I	Uniformly distributed load- $q_1$	None
II	Increasing load- $q_4$ or $q_5$	Hydraulic support and/or residual roof
III	increasing load- $q_4$ or $q_5$	Yielded coal bed
IV	Uniformly distributed load- $q_2$ and decreasing load- $q_3$	Unyielded coal bed

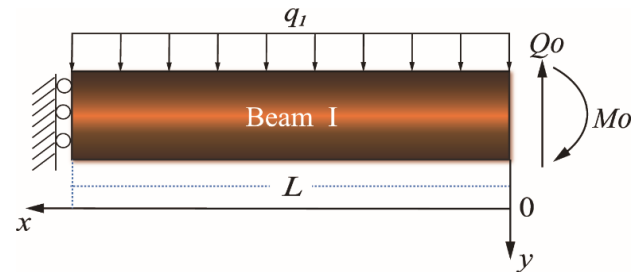


Fig. 3 Loading and boundary conditions of rock beam I

excavation is commonly regarded as a plane strain mechanical model. Therefore, the unit width of the roof strata, within the plane strain area, can be simplified as a beam structure (Yu 2016, Zhang *et al.* 2019).

The rock beam with corresponding boundary, foundations and loading conditions are displayed in Fig. 2. According to the differences in foundation support conditions, the rock beam can be divided into 4 parts, namely beam I to IV.

Based on the location of the local coordinate system shown in Fig. 2,  $x = L$  is the middle point of beam I, which can be considered as a symmetric structure. Beam I-IV with corresponding loading and support conditions are listed in Table 1.

Beam I is a suspended structure over the excavation space. Considering the field application feasibility, installation of LAM sensors in the beam I area is the most efficient scheme without too much drilling work. Therefore, beam I is selected as the slope data sampling area in this research work. The mechanical model of beam I is presented in the next section.

### 2.2 Mechanical model of beam I

Assuming the span of beam I is  $2L$ ,  $x=L$  is the mid-span section, the mechanical model of beam I can be simplified as a half structure. The nomination of beam I and thereafter is referred to the half structure of the whole suspended

beam. The mechanical conditions of beam I on the left side are shown in Fig. 3. The left end is a guided-support that allows deflection without any inclination. The right side is the intersection of beam I and II that represented as shear force  $Q_0$  and bending moment  $M_0$ . The overloading is a uniformly distributed loading  $q_1$ .

In view of the loading and boundary conditions of beam I, using simple equilibrium concepts, the bending moment  $M(x)$  of beam I can be derived as:

$$M(x) = M_0 - Q_0x + \frac{1}{2}q_1x^2 \quad (0 \leq x \leq L) \quad (1)$$

The slope  $\theta(x)$  of beam I is:

$$\theta(x) = \frac{1}{EI} (M_0x - \frac{1}{2}Q_0x^2 + \frac{1}{6}q_1x^3) + C \quad (0 \leq x \leq L) \quad (2)$$

where E is the plan strain modulus of strata beam, I is the area moment of inertia, C is a constant of integration. As beam I is a symmetric structure; when  $x=L$  the slope ( $\theta$ ) is 0 degrees and the deflection becomes the maximum, it means  $\theta(L) = 0$ . According to Eq. (2), the constant of integration C can therefore be calculated as:

$$C = -\frac{1}{EI} (M_0x - \frac{1}{2}Q_0x^2 + \frac{1}{6}q_1x^3) \quad (3)$$

Substituting the constant C into Eq. (2), the expression of  $\theta$  can be established by:

$$\theta(x) = \frac{1}{EI} (M_0x - \frac{1}{2}Q_0x^2 + \frac{1}{6}q_1x^3 - M_0L + \frac{1}{2}Q_0L^2 - \frac{1}{6}q_1L^3) \quad (0 \leq x \leq L) \quad (4)$$

Eq. (4) is the analytical solution of slope in beam I, which indicates the functional relationship among the slope  $\theta$ , the span L, the problem boundary conditions, and the bending moment M0 and shear force Q0.

### 3. Mechanical state estimation by inclination data

#### 3.1 Solving mechanical system of beam I

According to Eq. (4), the mechanical state of beam I can be estimated by the solving of the L, M0, and Q0 by slope data  $\theta$ .

For a certain excavation scenario, i.e. with fixed L, M0, and Q0, the slope  $\theta$  can be derived by the position x. In practice, slope  $\theta$  is measurable data. The sampling position x can be expressed as:

$$x = x_1 + (n-1)\Delta l_i \quad (5)$$

where parameter  $x_1$  is the distance between the first sampling site and  $x = 0$  (the interface of rock beam I and II),  $\Delta l_i$  is the distances between the sampling sites, and n is the order of sampling site (see also Fig. 4).

Since the position x is represented by Eq. (5), there are 4 unknowns  $x_1$ , L, M0 and Q0 in Eq. (4). To solve the unknowns, a system of 4 simultaneous equations is required, i.e. at least 4 inclination sensors should be installed within the beam I area. It means,

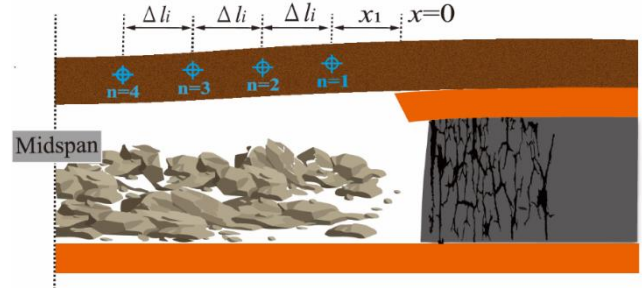


Fig. 4 The position x of sampling site

$$\begin{cases} \theta_1 = \frac{1}{EI} (M_0x_1 - \frac{1}{2}Q_0x_1^2 + \frac{1}{6}q_1x_1^3 - M_0L + \frac{1}{2}Q_0L^2 - \frac{1}{6}q_1L^3) \\ \theta_2 = \frac{1}{EI} (M_0(x_1 + \Delta l_i) - \frac{1}{2}Q_0(x_1 + \Delta l_i)^2 + \frac{1}{6}q_1(x_1 + \Delta l_i)^3 - M_0L + \frac{1}{2}Q_0L^2 - \frac{1}{6}q_1L^3) \\ \theta_3 = \frac{1}{EI} (M_0(x_1 + 2\Delta l_i) - \frac{1}{2}Q_0(x_1 + 2\Delta l_i)^2 + \frac{1}{6}q_1(x_1 + 2\Delta l_i)^3 - M_0L + \frac{1}{2}Q_0L^2 - \frac{1}{6}q_1L^3) \\ \theta_4 = \frac{1}{EI} (M_0(x_1 + 3\Delta l_i) - \frac{1}{2}Q_0(x_1 + 3\Delta l_i)^2 + \frac{1}{6}q_1(x_1 + 3\Delta l_i)^3 - M_0L + \frac{1}{2}Q_0L^2 - \frac{1}{6}q_1L^3) \end{cases} \quad (6)$$

Solving Eq. (6) by 4 slope data, the mechanical state of beam I can be achieved. For solving systems of nonlinear equations, a number of approaches have been presented. In this study, the global optimization algorithm differential evolution (DE) is adopted for its capability of solving complex equations. More discussion on the DE algorithm is presented in section 4.

#### 3.2 Sampling strategy

Eq. (6) is derived base on the mechanical structure showed in Fig. 4, which is half of the symmetric structure. Consequently, the discussion on sampling strategy should be located on one side of the mid-span section. The mid-span section of beam I is dynamically moving forward as the span distance of beam I is increasing due to the advancement of the working face. Therefore, the sampling sites located on the left side of the mid-span should also cross the mid-span to its right side. Otherwise, it will be an invalidated site that cannot be adopted to solve equation (6). To overcome the issue, one needs to set the sampling sites to the right side of the mid-span.

In terms of the sampling interval, the theoretical ultimate span of strata (the first weighting distance) is an ideal reference length to place the sampling sites. Several theories and methods have been developed to calculate the first weighting distance. The field geological condition could be complex and the strata may collapse at a distance that is less than analytical calculated results (Boothukuri et al. 2019, Ning et al. 2020). To avoid the sampling sites becoming invalid, a conservative algorithm (Song 1988) is adopted in this study according to Eq. (7).

$$L_{\max} = \sqrt{\frac{2h_k^2\sigma_t}{(h_k\gamma_k + \sum_{i=1}^n h_i\gamma_i)}} \quad (7)$$

where  $L_{\max}$  is the ultimate span of the key stratum,  $h_i$  and  $\gamma_i$  are the thickness and bulk density of overlying strata that

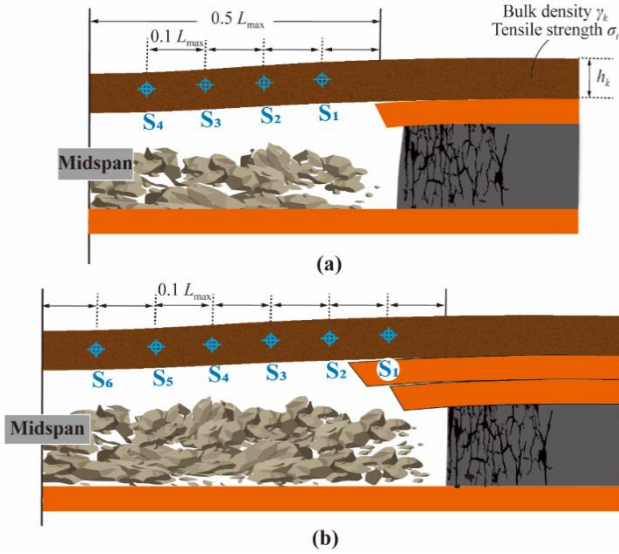


Fig. 5 Sampling sites distribution strategy

are supported by the key stratum, and  $\sigma_t$ ,  $h_k$  and  $\gamma_k$  are the tensile strength, thickness and bulk density of key stratum, respectively. As discussed above and given Eq. (6) is comprised of 4 independent equations, hence at least 4 sampling sites are needed to be placed. In theory, the different positions of the 4 sampling sites can be chosen arbitrarily and random, however, considering the measured errors and monitoring accuracy in practice, a uniform distribution strategy is suggested. That is, the half span of  $0.5 L_{max}$  should be divided evenly into 5 sections.

The distance between each adjacent sampling site is therefore  $0.1 L_{max}$  as shown in Figure 5a. It is worth noting that the distribution strategy showed in Figure 5a is an ideal model, and in reality, the strata caving configuration is more complex. The interface of beam I and II is unknown, hence one or more sampling sites might be located within the beam II domain, for instance the S1 site in Figure 5b. More sampling locations should be sited to overcome this difficulty, as demonstrated in Figure 5b. For this case, 6 sampling sites are suggested to be divided into 3 groups: group 1 (S1, S2, S3, S4), group 2 (S2, S3, S4, S5) and group 3 (S3, S4, S5, S6).

### 3.3 Bounds of the unknowns

The bounds of the 4 unknowns should be given as the search range of the differential evolution algorithm. The appropriate bounds of unknowns are very essential for DE to reach the expected result.

- Bounds of  $x_1$ : According to the sampling strategy illustrated in Figure 5, the intersection of beam I and II is located between: (1) S1 and coal seam, and (2) S1 and S2. In both these two scenarios, the maximum  $x$  is  $0.1 L_{max}$ , the minimum  $x$  is 0, thus the bound of  $x$  is  $[0, 0.1 L_{max}]$ .

- Bounds of  $L$ : The length of beam I is smaller or equal to the working face advance distance, hence the maximum value of  $L$  can be expressed as  $0.5 l_a$ , where  $l_a$  is the coal face advance distance. The bound of  $L$  is  $[0, 0.5 l_a]$ .

- Bounds of M0 and Q0: Determining the bounds of bending moment M0 and shear force Q0 are complex since beam II-IV and the corresponding loading, boundary and foundation conditions should be considered carefully. In this study, analytical solutions of M0 and Q0 proposed in a peer-reviewed study (Pan *et al.* 2013) is adopted as the centre values (Mcen and Qcen) of both bounds:

$$M_{cen} = f_{c1} x_{c1}^2 \left[ \left( \frac{l_w + x_{c1}}{x_{c1}} + 2 \right) e^{-\frac{l_w + x_{c1}}{x_{c1}}} - \left[ \frac{L + l_k + l_w + x_{c1}}{x_{c1}} + 2 + \frac{L + l_k}{x_{c1}} \left( \frac{L + l_k + l_w + x_{c1}}{x_{c1}} + 1 \right) \right] e^{-\frac{L + l_k + l_w + x_{c1}}{x_{c1}}} \right] + \frac{q_4(L + l_k)^2}{2} - \frac{p_z l_k^2}{3} \quad (8)$$

$$Q_{cen} = f_{c1} x_{c1} e \left[ \left( \frac{l_w + x_{c1}}{x_{c1}} + 1 \right) e^{-\frac{l_w + x_{c1}}{x_{c1}}} - \left( \frac{L + l_k + l_w + x_{c1}}{x_{c1}} + 1 \right) e^{-\frac{L + l_k + l_w + x_{c1}}{x_{c1}}} \right] + q_4(L + l_k) - p_z l_k \quad (9)$$

where  $f_{c1}$  and  $x_{c1}$  are the peak value adjusting parameter and the increasing rate adjusting parameter, respectively, of the increasing load  $q_4$ ,  $l_k$  and  $l_w$  are the length of beam III and rock beam II. The  $p_z$  is the average supporting force. In addition, a bias of centre value 0.8 is introduced to derive the bounds with a reasonable tolerance. The bounds of M0 and Q0 are  $[(1-0.8) M_{cen}, (1+0.8) M_{cen}]$  and  $[(1-0.8) Q_{cen}, (1+0.8) Q_{cen}]$ , respectively.

## 4. Differential evolution algorithm

Differential Evolution is a branch of evolutionary algorithms developed by Rainer Storn and Kenneth Price. It is a population-based direct search algorithm for global optimization capable of handling non-differentiable, non-linear and multi-modal objective functions, with few, easily chosen, control parameters (Mohamed *et al.* 2012, Omidi and Mazaheri 2020, Rout *et al.* 2013). For the advantages of the DE algorithm, it is adopted to solve the equation (6). The basic computation procedures of DE are briefly introduced in this section. In this study, a Python library, Scipy is adopted to perform the DE optimization. In terms of DE control parameters, the default setting of the Scipy DE algorithm is adopted.

### 4.1 Initialization

The evolution strategy of the DE algorithm starts with an initial population containing NP D-dimensional vectors. Every individual vector in the population is a set of possible values of  $x_1$ ,  $L$ , M0 and Q0. An individual vector is symbolized by  $x_{i,g}$  with  $i = 0, 1, \dots, NP-1$  referring to the order of a vector in a population, and  $g$  referring to the index of generation. One individual element in the vector is represented by  $x_{j,i,g}$  with  $j = 0, 1, \dots, D-1$  referring to the order of an element in a vector.

The initialization of a population ( $g = 0$ ) is to choose a set of random values for each element within the given range. In this study, an element is a trial value of  $x_1$ ,  $L$ , M0 and Q0, and the corresponding element range is the parameter bound ( $b_{j,i,low}$ ,  $b_{j,i,up}$ ). It means,

$$x_{i,0} = (x_{j,i,0}) = \text{rand}_j(0,1) (b_{j,i,up} - b_{j,i,low}) + b_{j,i,low} \quad (10)$$

where  $\text{rand}_j(0,1)$  is a random number generator that returns a random number within the range  $[0,1)$ . By this random number, a new random value for each element is generated,

the initial population is generated as well.

#### 4.2 Mutation

Since the initialization population is created, the DE algorithm mutates and recombines current vectors to create NP intermediary vectors, symbolized by  $v_{i,g}$ :

$$\mathbf{v}_{i,g} = \mathbf{x}_{i0,g} + F(\mathbf{x}_{i1,g} - \mathbf{x}_{i2,g}) \quad (11)$$

where  $\mathbf{x}_{i0,g}$ ,  $\mathbf{x}_{i1,g}$  and  $\mathbf{x}_{i2,g}$  are vectors, chosen randomly or follow certain strategy, in last generation population,  $F$  is the mutation scale factor that controls the amplification of differential variation ( $\mathbf{x}_{i1,g} - \mathbf{x}_{i2,g}$ ) which also influences the convergence speed. As mutated vectors, the intermediary vectors  $v_{i,g}$  not only inherit from a parent vector but also introduce differences between parent vectors.

#### 4.3 Crossover

In order to further increase the randomness and diversity of the new vectors is introduced. Crossover operation aims to produce a trial population consist of NP trial vectors  $u_{i,g}$ :

$$\mathbf{u}_{i,g} = u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if } (\text{rand}_j(0,1) < Cr \text{ or } j = j_{\text{rand}}) \\ x_{j,i,g} & \text{otherwise} \end{cases} \quad (12)$$

where  $Cr$  is the crossover probability in the range  $[0,1]$ , and  $\text{rand}_j(0,1)$  is a random number generator for the  $j$ th parameter in a vector. Increasing the value of  $Cr$  allows more parameters of intermediary vectors to progress into the next generation. Decreasing the value of  $Cr$  allows more parameters to remain unchanged. The  $j_{\text{rand}}$ , a randomly chosen number in the range  $[0, D-1]$  to ensure the trial vectors  $u_{i,g}$  will get at least one parameter from the intermediary vectors  $v_{i,g}$ . As a further mutation operation, the trial vectors  $u_{i,g}$  inherit elements from both current vectors  $\mathbf{x}_{i,g}$  and intermediary vectors  $v_{i,g}$ .

#### 4.4 Selection

Given the trial vectors  $u_{i,g}$  and the current vector  $\mathbf{x}_{i,g}$ , the comparison will be carried out by substituting the parameters of the trail and current vector into the objective function. If the trial vector has an equal or better performance, the trail vector will replace the current vector in the next generation:

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g} & \text{if } f(\mathbf{u}_{i,g}) \leq f(\mathbf{x}_{i,g}) \\ \mathbf{x}_{i,g} & \text{otherwise.} \end{cases} \quad (13)$$

In this study, by repeating the mutation and competition cycle, the solved  $x_1$ ,  $L$ ,  $M_0$  and  $Q_0$  will keep approaching to the target value, until the desired result is achieved.

### 5. Case study

#### 5.1 Numerical example

A set of verification tests are carried out in this section

Table 2 Model parameters setting

Input parameter	Validation testing
Modulus of strata beam $E$ (GPa)	25
Unit width of strata beam $b_w$ (m)	1
Thickness of strata beam $h_k$ (m)	6
Bulk density $\gamma_k$ (kN/m <sup>3</sup> )	25
Tensile strength of strata beam $\sigma_t$ (MPa)	15
Half span of strata beam $l_g$ (m)	20
Peak loading adjusting parameter $f_{c1}$ (N/m)	$59.55 \times 10^6$
Increasing adjusting parameter $x_{c1}$ (m)	2
Length of rock beam III $l_w$ (m)	10
Length of rock beam II $l_k$ (m)	0

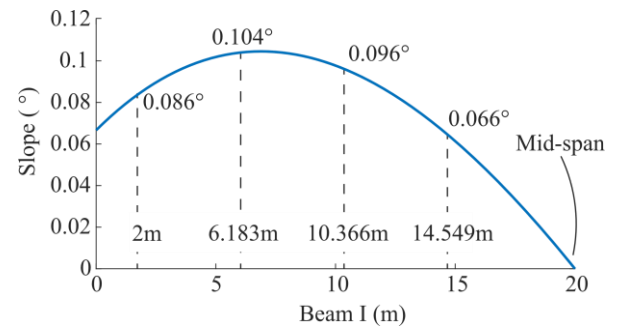


Fig. 6 Slope curve of beam I

Table 3 Parameter bounds

Parameters	Bounds	Target
$x_1$ (m)	$[0, 4.183]$	2
$L$ (m)	$[0, 40]$	20
$M_0$ (N·m)	$[1.8e^7, 1.62e^8]$	$9e^7$
$Q_0$ (N)	$[2.9e^6, 2.6e^7]$	$1.46e^7$

to showcase the feasibility and robustness of the proposed method. The parameter setting reported in a peer-reviewed study (Pan *et al.* 2013) is adopted as the prototype (see also Table 2).

By assuming the suspended distance of strata as 40 m, the slope curve of beam I can be derived by Eqs. (4), (8) and (9), see Fig. 6.

#### 5.2 Verification test

For the proposed mechanical state estimation approach, the input parameters are the 4 local inclination data, and the output parameters are the  $x_1$ ,  $L$ ,  $M_0$  and  $Q_0$ .

In terms of input parameters, following the sampling strategy proposed in section 3.2, the 4 local slope data can be determined by the analytical slope solution showed in Fig. 6. As suggested by the sampling strategy, the theoretical first weighting distance is adopted to calculate the interval of sampling sites (Vecchia *et al.* 2016). In this case, by substituting the  $\sigma_t$ ,  $h_k$ ,  $\gamma_k$ ,  $h_i$  and  $\gamma_i$  into equation (7) the theoretical first weighting distance  $L_{\text{max}} = 41.83$  m, i.e., the interval of sampling sites is 4.183 m. The  $x_1$  can be

Table 4 Estimation results

Group	$x$ (m)	$L$ (m)	$M_0$ (N·m)	$Q_0$ (N)
1	2.06	20.14	$8.99e^7$	$1.45e^7$
2	2.48	20.55	$9.61e^7$	$1.47e^7$
3	1.80	19.87	$8.62e^7$	$1.44e^7$
4	2.09	20.16	$9.03e^7$	$1.45e^7$
5	2.09	20.16	$9.03e^7$	$1.45e^7$
6	2.37	20.44	$9.43e^7$	$1.47e^7$
7	1.89	19.96	$8.75e^7$	$1.44e^7$
8	1.97	20.04	$8.87e^7$	$1.45e^7$
9	2.10	20.18	$9.05e^7$	$1.45e^7$
10	2.15	20.22	$9.13e^7$	$1.46e^7$
Average	2.10	20.17	$9.05e^7$	$1.45e^7$
Target	2.00	20.00	$9.00e^7$	$1.46e^7$
Error	5.0%	0.85%	0.56%	0.68%

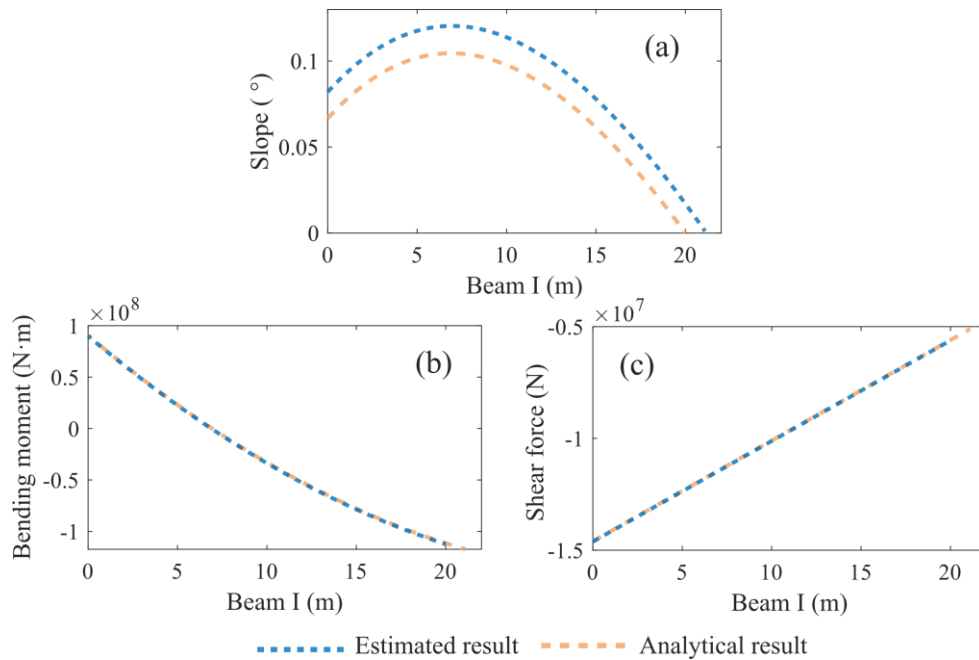


Fig. 6 Slope curve of beam I

chosen randomly ranges from 0 to 4.183 m that set to 2 m in this case study. Thus, the 4 local slope sampling sites are easily determined (see Fig. 6). To verify the performance of the proposed method, large bounds are adopted for  $L$ ,  $M_0$  and  $Q_0$  as listed in Table 3.

By substituting the 4 local slope data and the output parameter bounds into the DE algorithm, a group of estimated results is derived. To eliminate the influence of singular solution, the estimation calculation is repeated 10 times, the solutions are summarized in Table 4.

The low relative error of estimated  $x$ ,  $L$ ,  $M_0$  and  $Q_0$  with its target value indicate the proposed approach is capable to estimate the mechanical state of beam I, the suspended strata. Moreover, the relatively large parameter bound showcases the robustness of the proposed method. By substituting the estimated  $L$ ,  $M_0$  and  $Q_0$  into Eq.

(1) and (2), the slope, bending moment and shear force (a derivative of  $M(x)$ ) of beam I can be derived (see Figure 7).

### 5.3 Research limitations

The verification test shows an ideal result, whereas the proposed mechanical state estimation method is based on a simplified beam model. In practice, the overburden structure is complex having a large variation of stratum thickness and geological structure. The strata behaviour is subjected to be influenced by complex geologic conditions. A model with consideration of fault is suggested for further study.

The failure position of strata can be either in front of the working face or over goaf. The proposed method is capable to estimate the strata failure behaviour when the failure

occurs over goaf. For a comprehensive understanding of the strata movement, the mechanical model of strata over coal seam (in front of working face) is suggested to be further investigated in the further study.

The slope data are analytically calculated without errors. The resolution of slope data reaches  $0.001^\circ$ . However, in practice, the slope data are monitored by sensors, the errors of data caused by systematic errors and random errors are unavoidable which could pose an undesirable effect to the method. Latest studies have reported the high precision inclination sensors have been developed (Guo *et al.* 2019, Łuczak and Ekwińska 2021). A tilt sensor with  $2.86 \times 10^{-5}$  degrees resolution based on silicon-on-insulator (SOI) MEMS technology has been reported (Zou *et al.* 2014), as well as a wireless micro-electro-mechanical systems (MEMS) inclination sensor with  $2.5 \times 10^{-3}$  degrees resolution, has been applied in structural health monitoring engineering (Yu *et al.* 2009). By utilizing the latest sensor technology, the data errors are hopefully be minimized.

## 6. Conclusions

This paper proposed a local slope data-based analytical approach to estimate the mechanical state of roof strata. This method uses local slope data to estimate suspended span, boundary bending moment and shear force. A set of verification tests were conducted to verify the effectiveness and robustness of the proposed method. The following conclusions can be drawn from the present study. An equations system was derived for the estimation of strata mechanical behaviour. The input parameters are four (4) local slope data of strata with fixed sampling intervals. The output parameters are the sampling site positions  $x_1$ , half span of suspended strata  $L$ , bending moment  $M_0$  and shear force  $Q_0$  of strata at the support boundary. The differential evolution algorithm is adopted to solve the equations system. The corresponding parameter bounds are suggested to ensure the calculation convergence. The performance of the proposed method is verified by 10 independent tests. The average relative errors of  $L$ ,  $M_0$  and  $Q_0$  are less than 1%. By adopting the proposed method, the mechanical state of suspended strata, slope, bending moment and shear force can be estimated to higher accuracy.

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