

Generalized magneto-thermo-microstretch elastic solid with finite element method under the effect of gravity via different theories

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Abstract. The present paper is aimed at studying the effect of gravity on the general model of the equations of generalized magneto-thermo-micro-stretch for a homogeneous isotropic elastic half-space solid. The problem is in the context of the Green-Lindsay (G-L) theories, as well as the coupled theory (CT). Finite element method is used to obtain the expressions for the displacement components, the force stresses, the temperature, the couple stresses, and the micro-stress distribution. Comparisons are made with the results in the presence and absence of gravity and magnetic field of a particular case for the generalized micropolar thermo-elasticity elastic medium (without micro-stretch constants) between the three theories.

Keywords: finite element method; gravity; magnetic field; thermo-microstretch; thermal relaxation

1. Introduction

In recent years, more attentions have been made to the interaction between elastic, magnetic field, a thermal field, gravitational, and micro-stretch because of their utilitarian aspects in a diverse field, especially, geophysics, geology, acoustics, engineering, and structures. Anya and Khan (2019) studied the reflection and propagation of plane waves at free surfaces of a rotating micropolar fiber-reinforced medium with voids. Bofill and Quintanilla (1995) introduced asymptotic behavior of solutions and an existence result for the linear theory of thermo-micro-stretch elastic solids. These problems are based on the more realistic elastic model since earth; moon and other planets all have a gravitational effect.

The theory of thermo-micro-stretch elastic solids was introduced by Eringen (1990). In the framework of the theory of thermo-micro-stretch solids, Eringen established a uniqueness theorem for the mixed initial-boundary value problem. The theory was illustrated through the solution of one-dimensional wave and comparing with lattice dynamical results. A reciprocal theorem and a representation

of Galerkin type were presented by De Cicco and Nappa (1999). De Cicco and Nappa (2000) extended the linear

theory of thermo-micro-stretch elastic solids to permit the transmission of heat as thermal waves at finite speed. Iesan and Nappa (2001) studied the problem of the plane strain of

microstretch elastic solids. The basic results and an extensive review of the theory of thermo-microstretch elastic solids can be found in Ref. Eringen (1999).

The classical uncoupled theory of thermoelasticity predicts two phenomena not compatible with physical observation. First, the equation of heat conduction of this theory doesn't contain any elastic terms contrary to the fact that elastic changes produce heat effects. Secondly, the heat equation is parabolic type predicting infinite speeds of propagation for heat waves.

Biot (1956) introduced the theory of coupled thermo-elasticity to overcome the first shortcoming. The governing equations for this theory are coupled, eliminating the first paradox of the classical theory. However, both theories share the second shortcoming since the heat equation for the coupled theory is also parabolic. Two generalizations of the coupled theory were introduced. The first is due to Lord and Shulman (1967), who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. This new law contains the heat flux vector as well as its time derivative. It also contains a new constant that acts as relaxation time. Since the heat equation of this theory is of the wave type, it automatically ensures finite speeds of propagation of heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and constitutive relations, remain the same as those for the coupled and the uncoupled theories. Saeed *et al.* (2020) applied a G-L model on thermoelastic interaction in a poroelastic material using finite element method. Othman (2002) studied the dependence of the

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modulus of elasticity on the reference temperature in a two dimensional generalized thermoelasticity with one relaxation time. The second generalization of the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity. Hobiny and Abbas (2020) studied the fractional order thermoelastic wave assessment in a two-dimension medium with voids.

A generalization of this inequality was proposed by Green and Laws (1972). In 1972 Green and Lindsay developed the theory of generalized thermoelasticity with two relaxation times, which, is based on a generalized inequality of thermo-dynamics. This theory does not violate the Fourier's law of heat conduction when the body under consideration has a center of symmetry. In this theory, both the equations of motion and of heat conduction are hyperbolic, but the equation of motion is modified and differs from that of the coupled thermoelasticity theory. The analysis on plane waves through magneto-thermoelastic micro-stretch rotating medium with temperature dependent elastic properties was studied by Othman *et al.* (2019). The influence of magnetic field, initial stress, and gravity in an isotropic material is discussed studied by Elnagar *et al.* (1994). Abd-Elaziz *et al.* (2019) investigated the effect of Thomson and initial stress in a thermo-porous elastic solid under G-N electromagnetic theory. Several relevant works studied the effect of gravity eminent researchers like Marin (1999), Jain *et al.* (2018), Toh *et al.* (2018), Baghban *et al.* (2019), Ghalndari *et al.* (2019), Ramezanizadeh *et al.* (2019), Salih *et al.* (2019), Khan *et al.* (2019), Othman *et al.* (2020), Lata and Singh (2020), Lin *et al.* (2020), Yu *et al.* (2020), Bhatti *et al.* (2020), Lata and Kaur (2019), Kaur and Lata (2020), Kaur, *et al.* (2020).

The exact solution of the generalized thermoelasticity theory governing equations for a coupled and non-linear/linear exists only for very special and simple initial and boundary problem. In view of calculating general problems, a numerical solution technique is to be used. For this reason, the finite element method is chosen. The method of weighted residuals offers us the formulation of the finite element equations and we obtain a better-approximated solution of linear and nonlinear ordinary and partial differential equations. Applying this method basically involves three steps. The first step is to assume the general behavior of the unknown field variables in such a way as to satisfy the given differential equations. Substitution of these approximating functions in the differential equations and boundary conditions result in some errors, called the residual. This residual has to vanish in an average sense over the solution domain. The second step is the integration time. The time derivatives of the unknown variables have to be determined by former results. The third step is to solve the equations resulting from the first and the second step by the solving algorithm of the finite element program (Zienkiewicz *et al.* (2013), Kumar *et al.* (2017), Eftekhari (2018), Lata and Kaur (2020)). Othman and Abbas (2014) discussed the effect of rotation on plane waves in generalized thermo-micro-stretch elastic solid comparison of different theories using finite element method. Abbas and Othman (2012) investigated the propagation of plane waves

in generalized thermo-micro-stretch elastic solid with thermal relaxation using finite element method. Marin *et al.* (2015) investigated some considerations on double porosity structure for micropolar bodies. The purpose of the present paper is to obtain the normal displacement, the temperature, the normal force stress and the tangential couple stress in a micro-stretch elastic solid under a magnetic field, and gravitational effect. The distributions of the considering variables are represented graphically. A comparison of the temperature, the stresses, and the displacements are carried out between the three theories for the propagation of waves in a semi-infinite micro-stretch elastic solid for different cases. The numerical results are calculated and presented graphically in the context of CT and G-L models.

2. Formulation of the problem

We obtain the constitutive and the field equations for a linear isotropic generalized thermo-micro-stretch elastic solid in the absence of body forces. We use a rectangular coordinate system (x,y,z) having originated on the surface $y=0$ and z -axis pointing vertically into the medium. A magnetic field with constant intensity $\mathbf{H} = (0,H_0,0)$, acting parallel to the boundary plane (taken as the direction of the y -axis). The surface of the half-space is subjected to a thermal shock which is a function of y and t . Thus, all the quantities considered will be functions of the time variable t , and of the coordinates x and z . We begin our consideration with linearized equations of electro-dynamics of slowly moving medium by Othman and Abd-Elaziz (2019).

$$\mathbf{J} = \text{curl } \mathbf{h} - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \quad (1)$$

$$\text{curl } \mathbf{E} = -\mu_0 \frac{\partial \mathbf{h}}{\partial t}, \quad (2)$$

$$\mathbf{E} = -\mu_0 \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H} \right), \quad (3)$$

$$\nabla \cdot \mathbf{h} = 0. \quad (4)$$

These equations are supplemented by the displacement equations of the theory of elasticity, taking into consideration the Lorentz force to give (Othman and Abd-Elaziz 2019).

$$\sigma_{i\ell,t} + F_i = \rho u_{i,tt}, \quad (5)$$

$$F_i = \mu_0 (J \times H)_i. \quad (6)$$

The basic governing equations of linear generalized magneto-thermoelasticity with gravity in the absence of body forces and heat sources are (Othman *et al.* 2019).

$$\begin{aligned} (\lambda + \mu) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right) + (\mu + k) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) - k \frac{\partial \phi_2}{\partial z} + \lambda_0 \frac{\partial \phi^*}{\partial x} \\ - \gamma (1 + \tau_1 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} + F_1 + \rho g \frac{\partial w}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \end{aligned} \quad (7)$$

$$(\lambda + \mu) \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2} \right) + (\mu + k) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + k \frac{\partial \phi_1}{\partial x} + \lambda_0 \frac{\partial \phi^*}{\partial z} \quad (8)$$

$$-\hat{\gamma}(1 + \tau_1) \frac{\partial T}{\partial t} \frac{\partial T}{\partial z} + F_3 - \rho g \frac{\partial u}{\partial x} = \rho \frac{\partial^2 w}{\partial t^2},$$

$$(\alpha + \beta + \gamma) \nabla(\nabla \cdot \boldsymbol{\phi})_i - \gamma \nabla \times (\nabla \times \boldsymbol{\phi})_i + k (\nabla \times \mathbf{u})_i - 2k \phi_1 = j\rho \frac{\partial^2 \phi_1}{\partial t^2}, \quad (9)$$

$$\alpha_0 \nabla^2 \phi^* - \frac{1}{3} \lambda_1 \phi^* - \frac{1}{3} \lambda_0 (\nabla \cdot \mathbf{u}) + \frac{1}{3} \hat{\gamma} T = \frac{3}{2} \rho j \frac{\partial^2 \phi^*}{\partial t^2}, \quad (10)$$

$$K \nabla^2 T = \frac{\rho C_E}{c_1} \frac{\partial}{\partial t} (c_2 + \tau_2) \frac{\partial T}{\partial t} + \frac{\hat{\gamma} T_0}{c_1} \frac{\partial}{\partial t} (c_2 + \delta \tau_2) \frac{\partial}{\partial t} \nabla \cdot \mathbf{u} + \hat{\gamma}_1 T_0 (c_2 + \delta \tau_2) \frac{\partial \phi^*}{\partial t}, \quad i = 1, 2, 3. \quad (11)$$

$$\sigma_{i\ell} = (\lambda_0 \phi^* + \lambda u_{,r}) \delta_{i\ell} + (\mu + k) u_{\ell,i} + \mu u_{i,\ell} - k \varepsilon_{i\ell r} \phi - \hat{\gamma} T \delta_{i\ell}, \quad (12)$$

$$m_{i\ell} = \alpha \phi_{,r} \delta_{i\ell} + \beta \phi_{,i} + \gamma \phi_{\ell,i}, \quad (13)$$

$$\lambda_i = \alpha_0 \phi_{,i}^* \quad (14)$$

$$\mathbf{e} = \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{w}}{\partial z}, \quad \omega_{ij} = \frac{1}{2} (u_{j,i} - u_{i,j}). \quad (15)$$

We study the above basic equations for the following four different theories

Theories	τ_1	τ_2	c_1	c_2	δ
(i) Classical and Dynamical coupled theory (CT)	0	0	1	1	0
(ii) Lord and Shulman's theory (L-S)	0	> 0	1	1	1
(iii) Green and Lindsay's theory (G-L)	> 0	> 0	1	1	0

The state of plane strain parallel to the xz -plane is defined by

$$u_1 = u(x, z, t), \quad u_2 = 0, \quad u_3 = w(x, z, t), \quad \phi_1 = \phi_3 = 0, \\ \phi_2 = \phi_2(x, z, t), \quad \phi^* = \phi^*(x, z, t), \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \quad (16) \\ \hat{\gamma} = (3\lambda + 2\mu + k) \alpha_{t_1}, \quad \hat{\gamma}_1 = (3\lambda + 2\mu + k) \alpha_{t_2}.$$

The constants $\hat{\gamma}$ and $\hat{\gamma}_1$ depend on the mechanical as well as the thermal properties of the body and the dot denote the partial derivative with respect to time, α_{t_1} and α_{t_2} are the coefficients of linear thermal expansions.

The components of the electric intensity and the current density are represented as,

$$\mathbf{E} = -\mu_0 H_0 \left(-\frac{\partial w}{\partial t}, 0, \frac{\partial u}{\partial t} \right), \quad \mathbf{h} = -H_0 \mathbf{e}, \\ \mathbf{J} = (H_0 \frac{\partial e}{\partial z}, 0, -H_0 \frac{\partial e}{\partial x}) - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \quad (17)$$

External forces can be represented as

$$\mathbf{F} = \mu_0 H_0^2 \left(\frac{\partial e}{\partial x} - \varepsilon_0 \mu_0 \frac{\partial^2 u}{\partial t^2}, 0, \frac{\partial e}{\partial z} - \varepsilon_0 \mu_0 \frac{\partial^2 w}{\partial t^2} \right). \quad (18)$$

The constitutive relation can be written as

$$\sigma_{xx} = \lambda_0 \phi^* + (\lambda + 2\mu + k) \frac{\partial u}{\partial x} + \lambda \frac{\partial w}{\partial z} - \hat{\gamma} (1 + \tau_1) \frac{\partial T}{\partial t}, \quad (19)$$

$$\sigma_{zz} = \lambda_0 \phi^* + (\lambda + 2\mu + k) \frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x} - \hat{\gamma} (1 + \tau_1) \frac{\partial T}{\partial t}, \quad (20)$$

$$\sigma_{xz} = \mu \frac{\partial u}{\partial z} + (\mu + k) \frac{\partial w}{\partial x} + k \phi_2, \quad (21)$$

$$\sigma_{zx} = \mu \frac{\partial w}{\partial x} + (\mu + k) \frac{\partial u}{\partial z} + k \phi_2, \quad (22)$$

$$m_{xy} = \gamma \frac{\partial \phi_2}{\partial x}, \quad (23)$$

$$m_{zy} = \gamma \frac{\partial \phi_2}{\partial z}. \quad (24)$$

For convenience, the following non-dimensional variables are used:

$$\bar{x}_i = \frac{\omega^*}{c_2} x_i, \quad \bar{u}_i = \frac{\rho c_2 \omega^*}{\hat{\gamma} T_0} u_i, \quad \bar{t} = \omega^* t, \quad \bar{\tau}_1 = \omega^* \tau_1, \\ \bar{\tau}_2 = \omega^* \tau_2, \quad \bar{T} = \frac{T}{T_0}, \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\hat{\gamma} T_0}, \quad \omega^* = \frac{\rho C_E c_2^2}{K}, \quad (25) \\ \{\bar{m}_{ij}, \bar{\lambda}_i\} = \frac{\omega^*}{c_2 \hat{\gamma} T_0} \{m_{ij}, \lambda_i\}, \quad \{\bar{\phi}_2, \bar{\phi}^*\} = \frac{\rho c_2^2}{\hat{\gamma} T_0} \{\phi_2, \phi^*\}, \\ \bar{g} = \frac{g}{c_2 \omega^*}, \quad c_2^2 = \frac{\lambda + 2\mu + k}{\rho}.$$

Using Eq. (25), Eqs. (7)-(11) become (dropping the dashed for convenience)

$$[\alpha \frac{\partial^2}{\partial x^2} + a_6 \frac{\partial^2}{\partial z^2} - \beta^2 \frac{\partial^2}{\partial t^2}] u + [(1 - a_6 + R_H) \frac{\partial^2}{\partial x \partial z} + g \frac{\partial}{\partial x}] w \\ - a_3 \frac{\partial \phi_2}{\partial z} + a_0 \frac{\partial \phi^*}{\partial x} - (1 + \tau_1) \frac{\partial T}{\partial t} \frac{\partial T}{\partial x} = 0, \quad (26)$$

$$[\alpha \frac{\partial^2}{\partial z^2} + a_6 \frac{\partial^2}{\partial x^2} - \beta^2 \frac{\partial^2}{\partial t^2}] w + [(1 - a_6 + R_H) \frac{\partial^2}{\partial x \partial z} - g \frac{\partial}{\partial x}] u \\ + a_3 \frac{\partial \phi_2}{\partial x} + a_0 \frac{\partial \phi^*}{\partial z} - (1 + \tau_1) \frac{\partial T}{\partial t} \frac{\partial T}{\partial z} = 0, \quad (27)$$

$$\nabla^2 \phi_2 = a_7 \frac{\partial^2 \phi_2}{\partial t^2} + 2a_8 \phi_2 - a_8 \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right), \quad (28)$$

$$\left(\frac{c_3^2}{c_2^2}\nabla^2 - \frac{c_4^2}{\omega^{*2}} - \frac{\partial^2}{\partial t^2}\right)\phi^* - \frac{c_5^2}{\omega^{*2}}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + a_9 T = 0, \quad (29)$$

$$\begin{aligned} \varepsilon_2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) &= \frac{1}{c_1} \frac{\partial}{\partial t} (c_2 + \tau_2 \frac{\partial}{\partial t}) T \\ &+ \frac{\varepsilon_1}{c_1} \frac{\partial}{\partial t} (c_2 + \delta\tau_2 \frac{\partial}{\partial t}) \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \varepsilon_3 (c_2 + \delta\tau_2 \frac{\partial}{\partial t}) \frac{\partial \phi^*}{\partial t}, \end{aligned} \quad (30)$$

$$\sigma_{xx} = a_0 \phi^* + \frac{\partial u}{\partial x} + a_1 \frac{\partial w}{\partial z} - (1 + \tau_1 \frac{\partial}{\partial t}) T, \quad (31)$$

$$\sigma_{zz} = a_0 \phi^* + \frac{\partial w}{\partial z} + a_1 \frac{\partial u}{\partial x} - (1 + \tau_1 \frac{\partial}{\partial t}) T, \quad (32)$$

$$\sigma_{xz} = a_2 \frac{\partial u}{\partial z} + (a_2 + a_3) \frac{\partial w}{\partial x} + a_3 \phi_2, \quad (33)$$

$$\sigma_{zx} = a_2 \frac{\partial w}{\partial x} + (a_2 + a_3) \frac{\partial u}{\partial z} - a_3 \phi_2, \quad (34)$$

$$m_{xy} = a_4 \frac{\partial \phi_2}{\partial x}, \quad (35)$$

$$m_{zy} = a_4 \frac{\partial \phi_2}{\partial z}, \quad (36)$$

$$\lambda_x = a_5 \frac{\partial \phi^*}{\partial x}, \quad (37)$$

$$\lambda_z = a_5 \frac{\partial \phi^*}{\partial z}, \quad (38)$$

where,

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}, \quad \beta^2 = \frac{V_A^2}{c^2} + 1, \quad V_A^2 = \frac{\mu_0 H_0^2}{\rho}, \quad R_H = \frac{V_A^2}{c^2},$$

$$c_3^2 = \frac{2a_0}{3\rho j}, \quad c_4^2 = \frac{2\lambda_1}{9\rho j}, \quad c_5^2 = \frac{2\lambda_0}{9\rho j},$$

$$(a_0, a_1, a_2, a_3) = \frac{1}{\rho c_2^2} (\lambda_0, \lambda, \mu, k), \quad (a_4, a_5) = \frac{\omega^{*2}}{\rho c_2^4} (\hat{\gamma}, \alpha_0),$$

$$a_6 = a_2 + a_3 = \frac{\mu + k}{\rho c_2^2}, \quad a_7 = \frac{j\rho c_2^2}{\gamma}, \quad a_8 = \frac{k c_2^2}{\gamma \omega^{*2}},$$

$$a_9 = \frac{2\hat{\gamma}c_2^2}{9\hat{\gamma}j\omega^{*2}}, \quad \varepsilon_1 = \frac{\hat{\gamma}^2 T_0}{\rho^2 C_E c_2^2}, \quad \varepsilon_2 = \frac{K \omega^*}{\rho C_E c_2^2}, \quad \varepsilon_3 = \frac{\hat{\gamma} \hat{\gamma}_1 T_0}{\rho^2 C_E c_2^2}.$$

3. Solution of the problem

The solution of the considering physical variables can

be decomposed in terms of normal mode as given in the following form:

$$\begin{aligned} &[u, w, \phi^*, \phi_2, \sigma_{il}, m_{il}, T, \lambda_z](x, z, t) \\ &= [\bar{u}, \bar{w}, \bar{\phi}^*, \bar{\phi}_2, \bar{\sigma}_{il}, \bar{m}_{il}, \bar{T}, \bar{\lambda}_z](x) \exp(\omega t + iaz), \end{aligned} \quad (39)$$

where, $[\bar{u}, \bar{w}, \bar{\phi}^*, \bar{\phi}_2, \bar{\sigma}_{il}, \bar{m}_{il}, \bar{T}, \bar{\lambda}_z](x)$ are the amplitude of the functions ω is a complex and a is the wave number in the z - direction.

Using Eq. (39), then Eqs. (26)-(30) become

$$D^2 u = A_1 u - A_2 D w + A_3 \phi_2 - A_4 D \phi^* + A_5 D T, \quad (40)$$

$$D^2 w = A_6 w - A_7 D u - A_8 \phi_2 - A_9 D \phi^* + A_{10} D T, \quad (41)$$

$$D^2 \phi_2 = A_{11} \phi_2 - A_{12} u + a_8 D w, \quad (42)$$

$$D^2 \phi^* = A_{13} \phi^* + A_{14} D u + A_{15} w + A_{16} T, \quad (43)$$

$$D^2 T = a^2 T + A_{17} T + A_{18} D u + A_{19} w + A_{20} \phi^*. \quad (44)$$

where,

$$A_1 = \frac{a^2 a_6 + \beta^2 \omega^2}{\alpha}, \quad A_2 = \frac{1}{\alpha} [ia(1 - a_6 + R_H) + g],$$

$$A_3 = \frac{iaa_3}{\alpha}, \quad A_4 = \frac{a_0}{\alpha}, \quad A_5 = \frac{(1 + \tau_1 \omega)}{\alpha},$$

$$A_6 = \frac{a^2 \alpha + \beta^2 \omega^2}{a_6}, \quad A_7 = \frac{1}{a_6} [ia(1 - a_6 + R_H) - g], \quad A_8 = \frac{a_3}{a_6},$$

$$A_9 = \frac{iaa_0}{a_6}, \quad A_{10} = \frac{(1 + \tau_1 \omega)}{a_6}, \quad A_{11} = a^2 + a_7 \omega^2 + 2a_8,$$

$$A_{13} = [a^2 + \frac{c_2^2}{c_3^2} (\frac{c_4^2}{\omega^{*2}} + \omega^2)], \quad A_{14} = \frac{c_2^2 c_5^2}{c_3^2 \omega^{*2}}, \quad A_{15} = \frac{iac_2^2 c_5^2}{c_3^2 \omega^{*2}},$$

$$A_{16} = \frac{a_9 c_2^2}{c_3^2}, \quad A_{17} = \frac{\omega}{c_1 \varepsilon_2} (c_2 + \tau_2 \omega), \quad A_{18} = \frac{\varepsilon_1 \omega}{c_1 \varepsilon_2} (c_2 + \delta\tau_2 \omega),$$

$$A_{19} = iaA_{18}, \quad A_{20} = \frac{\varepsilon_3 \omega}{\varepsilon_2} (c_2 + \delta\tau_2 \omega).$$

4. Boundary conditions

4.1 A mechanical boundary condition that the bounding plane to the surface $x=0$ has no traction, so we have

$$\begin{aligned} \sigma_{xx}(0, z, t) &= \sigma_{xz}(0, z, t) = 0, \\ m_{xy}(0, z, t) &= \lambda_x(0, z, t) = 0. \end{aligned} \quad (45)$$

4.2 The thermal boundary condition is (Othman and Song 2009)

$$uT - \frac{\partial T}{\partial x} = r(0, z, t), \quad (46)$$

where $\frac{\partial T}{\partial x}$ denotes the normal components of the heat

flux vector, ν is the Biot's number and $r(0,z,t)$ represents the intensity of the applied heat sources.

5. Finite element method

The finite element method is a powerful technique originally developed for the numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. This method is so general that it can be applied to a wide variety of engineering problems, including heat transfer, fluid, mechanical, chemical processing, etc. For the finite element method, one can refer to Abbas and his colleagues (2012, 2014). In this section, the governing equations of two-dimensional problems for the gravity effect on generalized magneto-thermo-micro-stretch elastic solid for different theories are summarized, using the corresponding finite element equations. In the finite element method, the eight-node isoparametric, quadrilateral element is used for displacement components, temperature, and micro-rotation calculations. The displacement components u , w , the temperature T , the micro-rotation ϕ_2 , and the micro-stretch ϕ^* are related to the corresponding nodal values by

$$u = \sum_{i=1}^m N_i u_i(t), \quad w = \sum_{i=1}^m N_i w_i(t), \quad T = \sum_{i=1}^m N_i T_i(t), \quad (47)$$

$$\phi_2 = \sum_{i=1}^m N_i \phi_{2i}(t), \quad \phi^* = \sum_{i=1}^m N_i \phi_i^*(t).$$

where m denotes the number of nodes per element, and N_i 's are the shape functions. The weighting functions and the shape functions coincide. Thus,

$$\begin{aligned} \delta u &= \sum_{i=1}^m N_i \delta u_i(t), \quad \delta w = \sum_{i=1}^m N_i \delta w_i(t), \quad \delta T = \sum_{i=1}^m N_i \delta T_i(t), \\ \delta \phi_2 &= \sum_{i=1}^m N_i \delta \phi_{2i}(t), \quad \delta \phi^* = \sum_{i=1}^m N_i \delta \phi_i^*(t). \end{aligned} \quad (48)$$

Thus, the finite element equations corresponding to Eqs. (23)-(25) can be obtained as

$$\begin{aligned} \int_0^\infty \frac{d\delta u}{dx} \frac{du}{dx} dx + \int_0^\infty \delta u (A_1 u - A_2 \frac{dw}{dx} + A_3 \phi_2 - A_4 \frac{d\phi^*}{dx} \\ + A_5 \frac{dT}{dx}) dx = \delta u \frac{du}{dx} \Big|_0^\infty, \end{aligned} \quad (49)$$

$$\begin{aligned} \int_0^\infty \frac{d\delta w}{dx} \frac{dw}{dx} dx + \int_0^\infty \delta w (A_6 w - A_7 \frac{du}{dx} - A_8 \phi_2 - A_9 \frac{d\phi^*}{dx} \\ + A_{10} \frac{dT}{dx}) dx = \delta w \frac{dw}{dx} \Big|_0^\infty, \end{aligned} \quad (50)$$

$$\begin{aligned} \int_0^\infty \frac{d\delta \phi_2}{dx} \frac{d\phi_2}{dx} dx + \int_0^\infty \delta \phi_2 (A_{11} \phi_2 - A_{12} u + a_8 \frac{dw}{dx}) dx \\ = \delta \phi_2 \frac{d\phi_2}{dx} \Big|_0^\infty, \end{aligned} \quad (51)$$

$$\begin{aligned} \int_0^\infty \frac{d\delta \phi_2}{dx} \frac{d\phi_2}{dx} dx + \int_0^\infty \delta \phi_2 (A_{11} \phi_2 - A_{12} u + a_8 \frac{dw}{dx}) dx \\ = \delta \phi_2 \frac{d\phi_2}{dx} \Big|_0^\infty, \end{aligned} \quad (51)$$

$$\begin{aligned} \int_0^\infty \frac{d\delta \phi^*}{dx} \frac{d\phi^*}{dx} dx + \int_0^\infty \delta \phi^* (A_{13} \phi^* + A_{14} \frac{du}{dx} + A_{15} w \\ + A_{16} T) dx = \delta \phi^* \frac{d\phi^*}{dx} \Big|_0^\infty, \end{aligned} \quad (52)$$

$$\begin{aligned} \int_0^\infty \frac{d\delta T}{dx} \frac{dT}{dx} dx + \int_0^\infty \delta T (a^2 T + A_{17} T + A_{18} \frac{du}{dx} + A_{19} w \\ + A_{20} \phi^*) dx = \delta T \frac{dT}{dx} \Big|_0^\infty. \end{aligned} \quad (53)$$

6. Results and discussion

With the view of illustrating the theoretical results obtained in the preceding sections and concerned with Classical-Dynamical and Green-Lindsay theories, we present some numerical results. The material chosen for this purpose is magnesium crystal (a micro-stretch thermoelastic solid). The micropolar parameters are following Othman (2002), Othman and Song (2009)

$$\begin{aligned} \rho &= 1.74 \times 10^3 \text{ kg.m}^{-3}, \quad j = 0.2 \times 10^{-19} \text{ m}^2, \\ \lambda &= 9.4 \times 10^{10} \text{ N.m}^{-2}, \quad \gamma = 0.779 \times 10^{-9} \text{ N}, \\ k &= 10^{10} \text{ N.m}^{-2}, \quad \mu = 4.0 \times 10^{10} \text{ N.m}^{-2}. \end{aligned}$$

The thermal characteristics were taken from are as follows:

$$\begin{aligned} \tau_0 &= 0.1, \quad T_0 = 298 \text{ K}, \quad \alpha_{t_1} = 0.05 \times 10^{-3} \text{ K}^{-1}, \quad \alpha_{t_2} = 0.04 \times 10^{-3} \text{ K}^{-1}, \\ K &= 1.7 \times 10^2 \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}, \quad C_E = 1.04 \times 10^3 \text{ J.kg}^{-1} \text{ K}^{-1}. \end{aligned}$$

The stretch parameters from

$$\begin{aligned} \lambda_0 &= 2.1 \times 10^{10} \text{ N.m}^{-2}, \quad \lambda_1 = 0.7 \times 10^{10} \text{ N.m}^{-2}, \quad j_0 = 0.19 \times 10^{-19} \text{ m}^2, \\ \alpha_0 &= 0.779 \times 10^{-9} \text{ N}, \quad b_0 = 0.9 \times 10^{-9} \text{ N}. \end{aligned}$$

Case 1:

$$\begin{aligned} t &= 0.5, \quad \omega_0 = 2, \quad \tau_1 = 0.1, \quad \tau_2 = 2, \quad \nu = 50, \quad r = 100, \\ H_0 &= 10^8, \quad g = 0.0, \quad g = 1. \end{aligned}$$

Case 2:

$$\begin{aligned} t &= 0.5, \quad \omega_0 = 2, \quad \tau_1 = 0.1, \quad \tau_2 = 2, \quad \nu = 50, \quad r = 100, \quad g = 1, \\ H_0 &= 0, \quad H_0 = 10^8. \end{aligned}$$

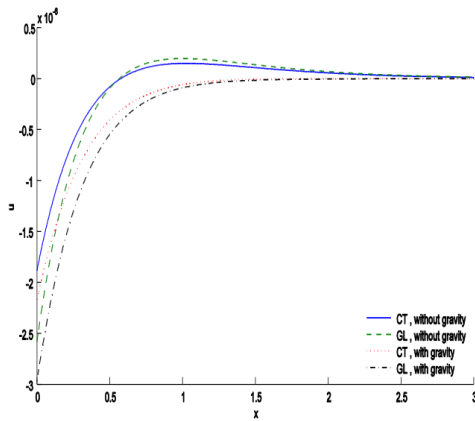


Fig. 1 The variation of CT and G-L models on u for different values of x with and without gravity

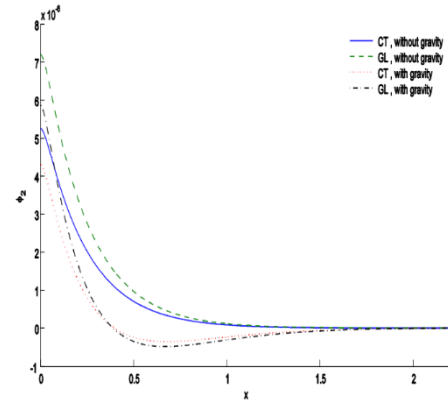


Fig. 4 The variation of CT and G-L models on ϕ_2 for different values of x with and without gravity.

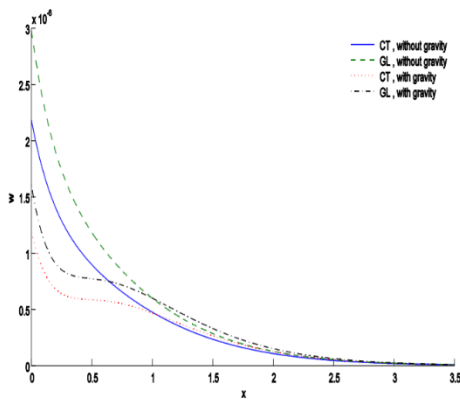


Fig. 2 The variation of CT and G-L models on w for different values of x with and without gravity

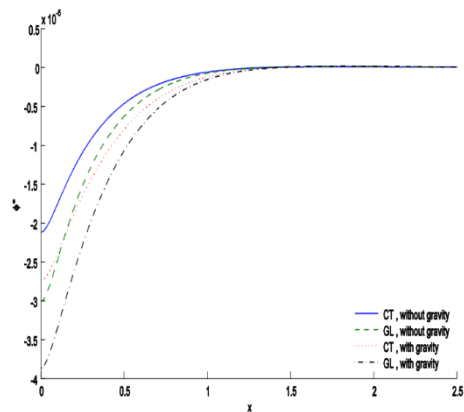


Fig. 5 The variation of CT and G-L models on ϕ^* for different values of x with and without gravity

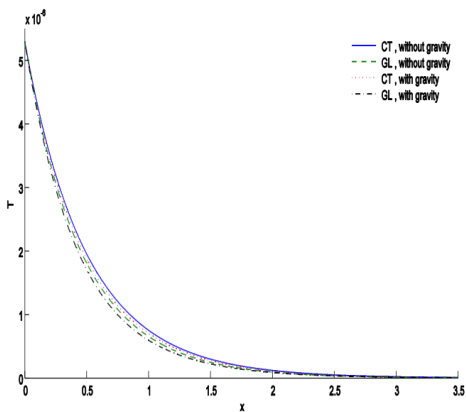


Fig. 3 The variation of CT and G-L models on T for different values of x with and without gravity

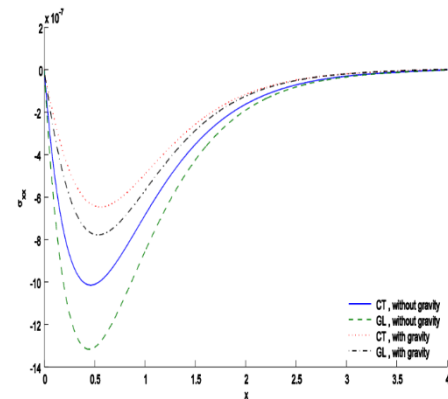


Fig. 6 The variation of CT and G-L models on σ_{xx} for different values of x with and without gravity

A comparison between CT and G-L theories on the displacement components, the temperature, the micro-rotation, the microstretch, the stress components, the distribution of m_{xy} and λ_x with different values of the earth gravity and the magnetic field respectively, with respect to the x - axis is shown in Figs. 1 -18. The grid size has been refined until the values of the displacement components, the temperature, and the micro-rotation, stabilizes. Further refinement of mesh size over 30000 elements does not change the values considerably. Thus, elements with 30000

were used for this study.

From Figs. (1, 5) and Figs. (10, 14), it appears that the distribution of the displacement u , and the micro-stretch ϕ^* , increase, then tend to zero as x - axis tends to infinity. From Figs. (2, 3, 4) and (11, 12, 13), it appears that the distribution of the displacement w , the temperature T and the micro-rotation ϕ_2 decrease tend to zero as x -axis tends to infinity. Also, it is shown from Figs. (6, 8) and (15, 17) that the stress σ_{xx} and the distribution of m_{xy} start from zero, then, decreases with an increasing of x -axis and tend to zero as x tends to infinity. Figs. (7, 10) and (17, 18) show that σ_{xz}

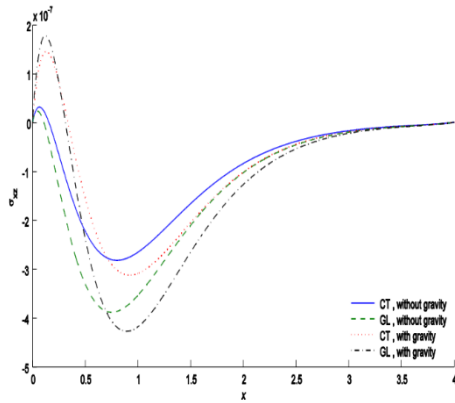


Fig. 7 The variation of CT and G-L models on σ_{xz} for different values of x with and without gravity

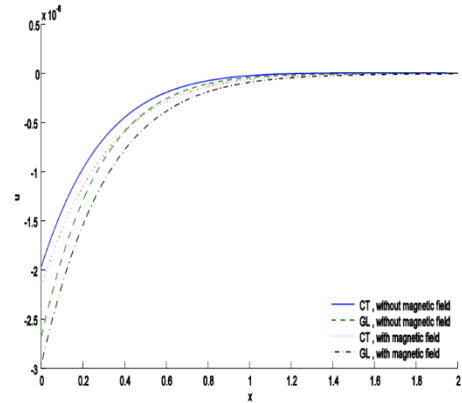


Fig. 10 The variation of CT and G-L models on u for different values of x with and without magnetic field

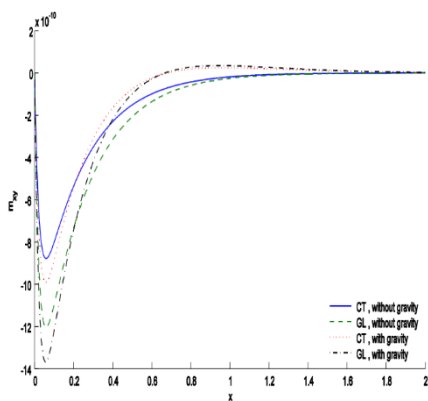


Fig. 8 The variation of CT and G-L models on m_{yz} for different values of x with and without gravity

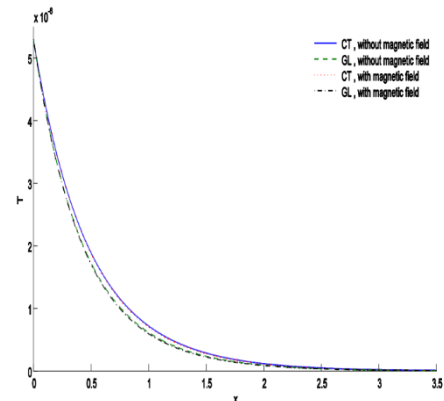


Fig. 11 The variation of CT and G-L models on w for different values of x with and without magnetic field

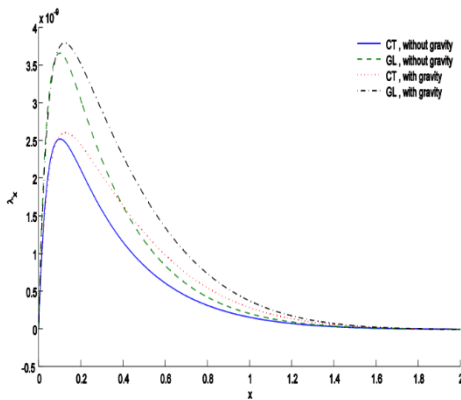


Fig. 9 The variation of CT and G-L models on λ_x for different values of x with and without gravity

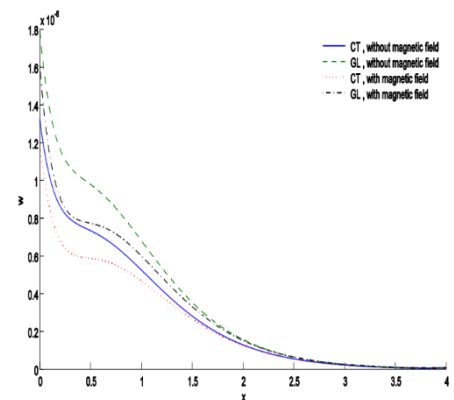


Fig. 12 The variation of CT and G-L models on T for different values of x with and without magnetic field

and λ_x increase from zero at $x=0$, decrease with an increasing of x -axis and return to increase tending to zero as x tends to infinity. It is seen from Figs. (1, 3, 5, 6, 7, 8) and (10, 12, 14, 15, 16, 17) that CT model makes a large effect compared with the G-L model on the displacement u , the temperature T , the micro-stretch ϕ^* , the stress components σ_{xx} , σ_{xz} and the distribution of m_{xy} but from Figs. (2, 4, 9) and (11, 13, 18) it makes a small effect compared with the G-L model on the distribution of displacement w , the micro-rotation ϕ_2 and λ_x . From Figs. (1, 2, 3, 4, 5, 7, 8) it is clear that the displacement components u , w , the temperature T ,

the micro-rotation ϕ_2 , the micro-stretch ϕ^* , the stress σ_{xz} and the distribution of m_{xy} decrease with the presence of the gravity field. Figs. (6, 9) display that σ_{xx} and λ_x increase with the presence of the gravity that indicates to the influence of gravity on the all parameter values increase or decrease.

Finally, with the influence of the magnetic field, it is obvious in Figs. (10, 11, 12, 14, 17, 18) that the displacement components u , w , the temperature T , the micro-stretch ϕ^* the distribution of m_{xy} and λ_x decrease with the presence of the magnetic field. Figs. (13, 15, 16) display

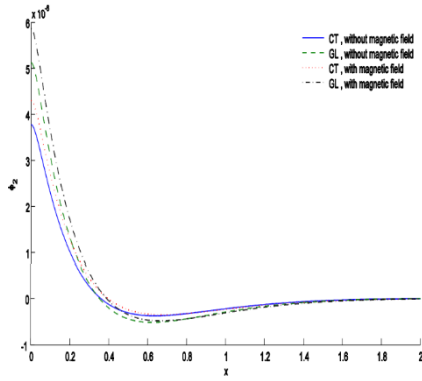


Fig. 13 The variation of CT and G-L models on ϕ_2 for different values of x with and without magnetic field

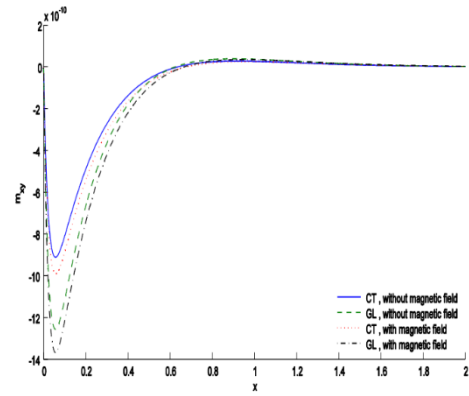


Fig. 17 The variation of CT and G-L models on m_{yz} for different values of x with and without magnetic field

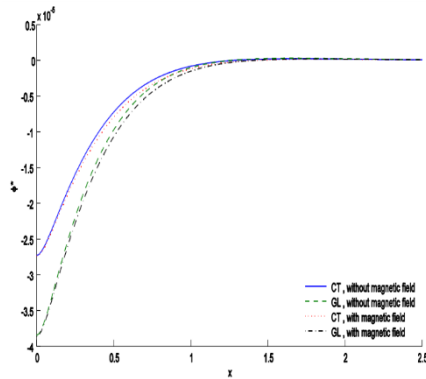


Fig. 14 The variation of CT and G-L models on ϕ^* for different values of x with and without magnetic field

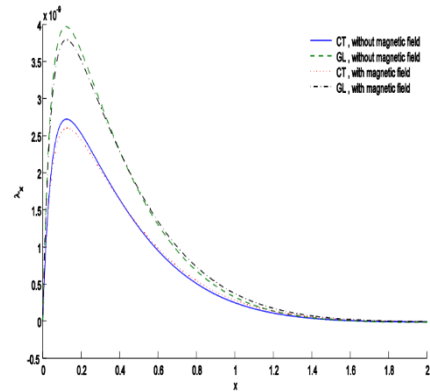


Fig. 18 The variation of CT and G-L models on λ_x for different values of x with and without magnetic field

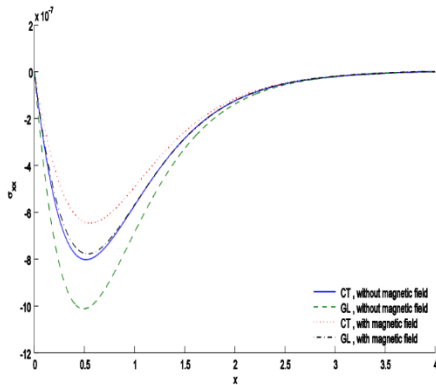


Fig. 15 The variation of CT and G-L models on σ_{xx} for different values of x with and without magnetic field

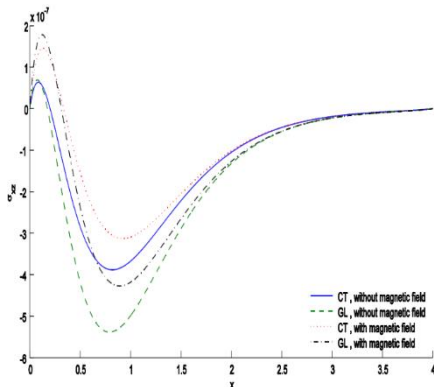


Fig. 16 The variation of CT and G-L models on σ_{xz} for different values of x with and without magnetic

that the micro-rotation ϕ_2 , the stress components σ_{xx} , σ_{xz} , increase with the presence of the magnetic field that indicate to the influence of the magnetized on the all parameter values increase or decrease.

7. Conclusions

- The presence of the micro-stretch plays a significant role in all the physical quantities with and without gravity and magnetic field.
- The curves in CT and G-L theories decrease exponentially with the increasing x , which indicates that the thermo-elastic waves are unattended and non-dispersive, while purely thermoelastic waves undergo both attenuation and dispersion.
- The solutions based upon the finite element method on the thermoelasticity problem in solids have been developed and utilized.
- The values of all the physical quantities converge to zero with an increase in the distance x , and all functions are continuous.
- The presence of magnetic field plays a significant role in all the physical quantities.
- The presence of gravity plays a significant role in all the physical quantities.
- In future study the effect of rotation will be study under multi-phase-lag.

Declaration of conflicting interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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JS

Nomenclature

- E the electric intensity
- e the dilatation
- e_{ij} the components of strain tensor
- g the earth gravity

- H the primary magnetic field vector
- J the electric current density
- j the micro-inertia moment
- $K(\geq 0)$ the thermal conductivity
- k, a, β, γ the micropolar constants
- m_{ij} the couple stress tensor
- F the Lorentz's body forces vector
- C_E the specific heat per unit mass
- t the time
- T_0 the initial temperature
- T the absolute temperature, $|(T - T_0)/T_0| \ll 1$
- u the displacement vector
- $a_0, \lambda_0, \lambda_1$ the micro-stretch elastic constant
- α_{t1}, α_{t2} the thermal expansion coefficients
- δ_{il} the Kronecker delta
- ε_{ijr} the alternate tensor
- ϕ the rotation vector
- ϕ^* the scalar microstretch
- θ the ratio of the coefficients of heat transfer
- λ, μ the Lamé constants
- μ_e the magnetic permeability
- ε_0 the electric permeability
- ρ the density of the material
- τ_1, τ_2 the two relaxation times
- τ the function of the depth
- τ_{ij} the stress components
- c_1, c_2, δ the parameters to heat conduction equation
- h the perturbed magnetic field over the constant primary magnetic field vector