

# Development of orthotropic Winkler-like model for rotating cylindrical shell: Stability analysis

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**Abstract.** Vibration investigation of rotating functionally graded cylindrical shells with fraction laws is studied here. Shell motion equations are framed according to the orthotropic Winkler-like model. For isotropic materials, the physical properties are same everywhere where the laminated and functionally graded materials, they vary from point to point. The influence of the polynomial, exponential and trigonometric fraction laws is investigated with simply supported condition. Also the variations have been plotted against the circumferential wave mode, length-to-radius and height-to-radius ratio. Moreover, backward and forward frequency pattern is observed increasing and decreasing for the various position of angular speed. The frequency first increases and gain maximum value for circumferential wave number. It is also exhibited that the effect of frequencies is investigated by varying the surfaces with stainless steel and nickel as a constituent material. The frequencies of trigonometric law is less than remaining laws.

**Keywords:** circumferential wave mode; FGM; length-to-radius ratio; simply supported; Winkler-like model

## 1. Introduction

As shells can be designed in various geometrical shapes like cylindrical, spherical, parboiled etc. But a cylindrical shell is very simple owing to its geometrical dimensions and is extensively studied for its vibrations. Vibration investigation of static and rotating cylindrical shells is a significant discipline in theoretical and applied mechanics. These shells have wide applications in engineering science and technology. For example their uses are observed in civil, mechanical, electrical, nuclear engineering, aerodynamics, missile technology etc.

More than one type of materials is used to structure the functionally graded materials and their physical properties vary from one surface to the other surface. In these surfaces, one has highly heat resistance property while other may preserve great dynamical perseverance and differs mechanically and physically in regular manner from one surface to other surface, making them of dual physical appearance. All these materials have changeable outer and inner sides and their physical properties greatly differ from

each other (Suresh and Mortensen 1997, Koizumi 1997). These materials are organized by various techniques and their applications are seen in dynamical elements such as plates, beams and shells. Moreover, they are also observed in space crafts, nuclear reactors and missiles technology etc. Bryan (1890) is considered to be the primer research worker who examined studied vibrations of rotating cylindrical shells. The free vibrations of a rotating ring were related with those of these shells.

Sharma *et al.* (1998) determined frequencies of composite cylindrical shells containing fluid. They estimated the axial modal deformations by trigonometric functions. Di Taranto and Lessen (1964) investigated the vibrations of thin isotropic and infinite long rotating cylindrical shells. Darvishgohari presented approach considers hybrid control strategy to reduce the amount of transmitted sound through a multilayered doubly curved sandwich shell equipped with piezoelectric layer and shunt circuit. The construction is composed of some piezoelectric actuator and sensor layers as an active controller as well as resistance-inductance shunt circuit as passive controller. Sharma (1974) analyzed vibration frequencies circular cylinder with using the Rayleigh - Ritz formulation and made comparisons of his results with some experimental ones. Talebitooti *et al.* (2019) designed a robust controller against the uncertainties in piezoelectric patches including sensor and actuator based on sliding mode

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method to control the radiated sound from cylindrical shells. Accordingly, in order to extract and discretize the dynamic equations of a smart cylinder equipped with piezoelectric patches, the Hamilton's principle and the Rayleigh-Ritz method are, respectively, used. Srinivasan and Lauterbach (1971) conducted the research on isotropic long rotating cylindrical shells including influence of coriolis actions on their travelling modes. Chung *et al.* (1981) studied the frequency response of fluid-filled CSs and presented an analysis of experimental and analytical investigation. Gohari *et al.* (2020) employed a robust controller on the basis of sliding mode control to propose a novel strategy nominated as self-adjusting boundary layer in order to prevent occurrence of chattering phenomenon. Since the boundary layers and the controller parameters are adjusted just for the special conditions, it is possible that the system losses its desirable performance and leads to this event. Padovan (1975) did analysis of pre-stress influence on buckling and vibration aspects of rotating cylindrical shells. Goncalves and Batista (1987) gave an analytical investigation of submerged CSs with fluid. Jweeg *et al.* (2010) investigated the free vibration solution will be developed for laminated simply supported closed cylindrical shells. This solution is obtained using General Third Shell Theory (GTT). Also the critical in-plane fatigue load is studied and the required equilibrium equations are developed, the effects of tension or compression in-plane load on the natural frequencies are discussed also. The natural frequencies and in-plane fatigue load results are very close to those obtained by other researchers. Talebitooti *et al.* (2016) investigated the acoustic behavior of the laminated composite cylindrical shell, excited by an oblique plane sound wave, is investigated. The cylindrical shell is assumed to be infinitely long with uniform airflow in the external fluid medium. To provide an analytical solution of Sound Transmission Loss (STL) based on Third-order Shear Deformation Theory (TSDT), the displacements are developed as the cubic order of the thickness coordinate. Sewall and Naumann (1968) considered the vibration analysis of CSs based on analytical and experimental methods. The shells were strengthened with longitudinal stiffeners. Jweeg and Alazzawy (2007) developed the transient solutions for laminated simply supported closed cylindrical shells subjected to a uniform dynamic pressure at the outer surface of the cylinder. These solutions are obtained by using General Third Shell Theory (GTT). Rectangular pulse, triangular pulse, sinusoidal pulse and (ramp-constant) load-time varying functions are studied and the required equilibrium equations are developed. The central deformation and principle stresses are investigated for different cross-ply laminates. Talebitooti *et al.* (2018a,b) investigated a diffuse acoustic field to analyze the wave propagation on infinite doubly curved laminated composite shell sandwiching a porous material which is extensively used in aerospace structures. In fact, the main goal is to study the influence of embedded porous core on Sound Transmission Loss (STL) of the structure. Accordingly, the displacements are extended up to cubic order of thickness coordinate based on Third-order Shear Deformation Theory (TSDT) known as HSDT

including no effect of shear correction factor. Zohar and Aboudi (1973) studied vibrations of rotating cylindrical shells having finite length and matrix approach was used to derive the shell vibration. Zarastvand *et al.* (2019) carried out the reviews of the shell papers of power transmission during the last 60 years. For this purpose, a series of categories are first highlighted in order to classify the issues that should be searched. The review is then directed with emphasis on the kinds of shells with different geometries containing cylindrical and doubly curved shells. Najafizadeh and Isvandzibaei (2007) applied ring supports to CSs for vibration analysis of along the tangential direction and founded their research on angular deformation theory of higher order. The angular deformation was used for shell equations and determined the effects of constituent volume fractions and shell configurations on the shell vibrations. FG material parameters were changed step by step. Wang and Chen (1974) performed frequencies of rotating cylindrical shells based on energy variational approach. Fox and Hardie (1985) examined vibrations of rotating cylindrical shells. They used shell theory due to Flugge for shell motion equations. Talebitooti *et al.* (2018) analysed of the four-sides simply supported doubly curved composite shell interlayered with porous material used in aerospace applications is considered based on Third order Shear Deformation Theory (TSDT). The focus is specifically placed on presenting the effect of boundaries on Sound Transmission Loss (STL) of the poroelastic structure. Then, the results are compared with those of infinite shell.

Pankaj *et al.* (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Li and Lam (1998) studied influence of edge conditions vibration frequencies and modes of rotating composite CSs. Talebitooti *et al.* (2019) performed the multi-objective vibroacoustic optimization of the double-walled doubly curved composite shells having poroelastic lining in its core in a diffuse field based on Non-dominated sorting Genetic Algorithm-II. Amabili *et al.* (1999) used Donnell's shallow-shell model with the quiescent, dense, inviscid and incompressible fluid. Also the dense fluid is studied for the influence of both the internal and external side of the shell. In the external side of the shell, the fluid was considered as an unbounded domain in the radial direction, while internally, the shell was considered as filled completely. The shell motion equations were used for rotating cylindrical shell by different researchers (Saito and Endo 1996, Wang and Sivadas and Ganesan, 1994, Chen *et al.* 1993). Darvish *et al.* (2020) presented an analytical model to embed porous materials in a finite cylindrical shell in order to obtain the sound transmission loss coefficient. Although the circumferential modes are considered only for calculating the amount of the transmitted noise through an infinitely long cylinder, the present study employs the longitudinal modes in addition to circumferential ones to analyze the vibroacoustic performance of a simply supported cylinder subjected to the porous core based on the first order shear deformation theory. Lam and Loy (1994) investigated the

vibrations of rotating composite and sandwich cylindrical shells. They performed comparisons of vibration frequencies of composited rotating cylindrical shells and evaluated the results applying different shell theories. Asadijafari *et al.* (2021) focused the acoustic characteristic of a simply supported doubly curved composite shell subjected to the Pasternak-type elastic foundation. To carry out the sound analysis of a finite shell, displacements and rotation terms and acoustic pressures are developed based on the infinite longitudinal and transversal modes.

Penzes and Kraus (1972) applied generalized end conditions to analyze vibrations of rotating cylindrical shells. The analysis of rotating shells was confined to some special cases owing to need of approximate approach and calculation process. With powerful numerical methodologies, shell vibration analysis has completely revolutionized by advanced computers. Zarastvand *et al.* (2021) proposed a strategy based on modeling poroelastic doubly curved composite shells on a Pasternak-type Elastic Foundation (PEF). Likewise, a general formulation is developed considering Hamilton's principle to extract dynamic equations based on the shear deformation shallow shell theory (SDSST). Ergin and Temarel (2002) did a vibration study of cylindrical shells. The shells lied in a horizontal direction and contained fluid and submerged in it. Recently some researcher used different methods for nonlinear modeling (Alzabeebee 2020, Lata, and Kaur 2019, Uyar and Aksoy 2019, Bouazza *et al.* 2019, Boulefrakh *et al.* 2019, Bouanati *et al.* 2019).

In this analytical study, vibrations of rotating functionally grade cylindrical shells have been investigated for the distribution of material composition of material with two categories of material. Here the orthotropic Winkler-like model has been applied to derive the shell frequency equation with various volume fraction laws. Dynamical behavior of a cylindrical shell is described with regard to the reference surface, length, radius and thickness quantities and boundary conditions applied its ends. For motion of a static cylindrical shell, a stationary wave is generated due to vibration. Moreover, on increasing the rotating speed, the backward frequencies increases and forward frequencies decreases. The frequencies are higher for trigonometric fraction law.

## 2. Volume fraction laws

Vibrations of rotating FG circular cylindrical shells are inspected for three volume fraction laws viz.: polynomial, exponential and trigonometric. These laws control functionally graded material composition in the shell radial direction.

Polynomial Law (Law-I)

$$V_f = \left( \frac{z}{h} + \frac{1}{2} \right)^x \quad (1)$$

Exponential Law (Law-II)

$$V_f = 1 - e^{-\left( \frac{z}{h} + \frac{1}{2} \right)^x} \quad (2)$$

Trigonometric Law (Law-III)

$$V_{f1} = \cos^2 \left[ \left( \frac{z}{h} + \frac{1}{2} \right)^x \right] \quad (3)$$

$$V_{f2} = \sin^2 \left[ \left( \frac{z}{h} + \frac{1}{2} \right)^x \right] \quad (4)$$

### 2.1 Effective material properties

The best use of functionally graded materials is found in highly heated systems. For these conditions their fabric properties are temperature-reliant. If  $\varpi$  designates a fabric property and depends on the absolute temperature  $T$  (K), Touloukian (1973) defined the following law:

$$\varpi = \varpi_0 (\varpi_{-1} \varpi^{-1} + \varpi_1 T + \varpi_2 T^2 + \varpi_3 T^3) \quad (5)$$

Here  $\varpi_0$ ,  $\varpi_{-1}$ ,  $\varpi_1$ ,  $\varpi_2$  and  $\varpi_3$  define the thermal coefficients and the temperature  $T$  (K) is measured in the Kelvin degree. The ensuing fabric properties of a FGM is stated as:

$$\varpi = \sum_{i=1}^{\gamma} \varpi_i V_{fi} \quad (6)$$

where  $\varpi_i$  designates the fabric property and  $V_{fi}$ , the volume fraction of the  $i^{\text{th}}$  FGM in that order.  $\gamma$  is the number of functionally graded ingredient fabric. When volume fractions of these constituent materials are added, their result is one that is

$$\sum_{i=1}^{\gamma} V_{fi} = 1 \quad (7)$$

The term  $V_f$  is designated as total volume fraction of FG-CS, respectively. The power exponent is denoted as  $X$  and  $h$  for thickness and  $z$  is the coordinate which varies from zero to infinity.

On mixing two or more than two materials like nickel and stainless steel, functionally graded materials are obtained. On changing the constituent materials, the shell is divided into two types (Type-I and Type-II). Their arrangement has profound influence on the formation of FG-CSs. At temperature 300K, the material properties for FG-CS are:  $E$ ,  $\nu$ ,  $\rho$  for Stainless steel are, 0.317756 and 8166 Kg/m<sup>3</sup> and Nickel are  $2.05098 \times 10^{11}$  N/m<sup>2</sup>, 0.3100, and 8900 Kg/m<sup>3</sup>.

So the Young's modulus  $E_{fgm}$ , for three different laws are defined as:

FGM Polynomial Law (Law-I)

$$E = (E_1 - E_2) \left( \frac{z}{h} + 0.5 \right)^X + E_2 \quad (8)$$

FGM Exponential Law (Law-II)

$$E = (E_1 - E_2) \left( 1 - a \left( \frac{z+1}{h} \right)^x \right) + E_2 \tag{9}$$

FGM Trigonometric Law (Law-III)

$$E = (E_1 - E_2) \sin^2 \left[ \left( \frac{z}{h} + \frac{1}{2} \right)^x \right] + E_2 \tag{10}$$

Now for Poisson ratio  $\nu_{fgm}$  and mass density  $\rho_{fgm}$  is same as Eqs. (8)-(10). When  $z = -h/2$  is substituted in the expressions (8) to (10),  $E=E_2$ ,  $\nu=\nu_2$  and  $\rho=\rho_2$  which are material properties of Type-I and when substitution  $z = h/2$  is made in the above expressions,  $E=E_1$ ,  $\nu=\nu_1$  and  $\rho=\rho_1$  for a Type-II. These substitutions show that there is an incessant change of fabric properties of the fabric Type-I at the shell internal surface to those of the fabric Type-II, at the shell external surface where  $z=0$  is associated to its mid surface. A FG-CS comprised of two materials is heterogeneous shell and its vibration characteristics are can be studied by applying any appropriate thin shell theory provided its radius to thickness ratio is greater than twenty.

### 3. Orthotropic Winkler-like model

The Winkler-like model is to study the vibration of FG-CSs in their natural conditions is,

$$(c^2 + 1)k_2 \frac{\partial^2 u}{\partial \beta^2} - \left[ -1 + N \frac{1 - \nabla^2}{K} \right] \frac{\partial^2 u}{\partial \alpha^2} + (\mu_1 + k_2) \frac{\partial^2 v}{\partial \alpha \partial \beta} + \left[ \mu_1 \frac{\partial}{\partial \alpha} + c^2 \left( k_2 \frac{\partial^3}{\partial \alpha \partial \beta^2} - \frac{\partial^3}{\partial \alpha^3} \right) \right] w = \frac{\partial^2 u}{\partial t^2} \tag{11}$$

$$\left[ (\mu_1 + k_2) \frac{\partial^2}{\partial \alpha \partial \beta} \right] u + \left[ k_2(1 + 3c^2) \frac{\partial^2}{\partial \alpha^2} - \frac{N}{K}(1 - \nabla^2) \frac{\partial^2}{\partial \alpha^2} + k_1 \frac{\partial^2}{\partial \beta^2} \right] v + \left[ k_1 \frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2) \frac{\partial^3}{\partial \alpha^2 \partial \beta} \right] w = \left( \frac{\partial^2 v}{\partial t^2} + \Omega \frac{\partial w}{\partial t} - \Omega^2 v \right) \tag{12}$$

$$\left[ \mu_1 \frac{\partial}{\partial \alpha} - c^2 \left( \frac{\partial^3}{\partial \alpha^3} - k_2 \frac{\partial^3}{\partial \alpha \partial \beta^2} \right) \right] u + \left[ k_1 \frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2) \frac{\partial^3}{\partial \alpha^2 \partial \beta} \right] v + \left[ \left( 1 + \frac{1}{c^2} \right) k_1 + \frac{\partial^4}{\partial \alpha^4} + k_1 \frac{\partial^4}{\partial \beta^4} + 2k_1 \frac{\partial^2}{\partial \beta^2} + (2\mu_1 + 4k_2) \frac{\partial^4}{\partial \alpha^2 \partial \beta^2} \right] c^2 w + \frac{N}{K}(1 - \nabla^2) \frac{\partial^2 w}{\partial \alpha^2} + (1 - \nabla^2) \frac{R^2 P}{K} = \left( \frac{\partial^2 w}{\partial t^2} - \Omega \frac{\partial v}{\partial t} - \Omega^2 w \right) \tag{13}$$

where  $P = -K_m w$  by Winkler model. The negative mark demonstrates that the direction of pressure  $P$  is reverse to for FG-CSs,  $K_m$  denotes elastic constant of medium. Here the boundary conditions are simply supported.

The solution for simply supported boundary conditions

can be written as (Flügge 1962, Forsberg 1964, Warburton 1965).

$$\begin{cases} u(\alpha, \beta, t) = A \cos n\beta \cos k\alpha \\ v(\alpha, \beta, t) = B \sin n\beta \sin k\alpha \\ w(\alpha, \beta, t) = C \cos n\beta \sin k\alpha \end{cases} \tag{14}$$

After putting Eq. (14) into Eqs. (11)-(13), this is homogeneous system of linear equations, therefore in matrix form the above system can be written as,

$$[E^{(2)}(n, L/Rm)]_{3 \times 3} \begin{bmatrix} U \\ V \\ W \end{bmatrix} = [0 \ 0 \ 0]^T \tag{15}$$

### 4. Simulation results and discussion

In this section, the versatile numerical method has been used in current study for the frequency analysis of rotating cylindrical shells. For the convergence rate of cylindrical shell, the present results are compared with (Lam and Loy, 1998) as shown in Table 1. Table 2 shows the comparison of present results with finite element method (Chen *et al.* 1993). The results are well-matched with earlier investigations. The proposed model based on orthotropic Winkler-like model can incorporate in order to accurately predict the acquired results of material data point. There is seen minute difference among the two sets of frequencies. Hence consequently it is concluded that on the base of these comparisons of analytical frequencies, the present

Table 1 Non rotating frequency comparison with (Lam and Loy 1998)

m	Method	n			
		3	4	5	6
1	Lam and Loy (1998)	759.9	1459.3	2360.9	3463.9
	Present	746.3	1448.2	2298.1	3422.4

Table 2 Rotating frequency comparison with (Chen *et al.* 1993)

		Finite element method (Chen <i>et al.</i> 1993)	Present model
		0.00167	0.00122
		0.00447	0.00418
		0.00847	0.00828
		0.01364	0.01346

Table 3 Rotating frequency variation of FGM Type-I and-II versus, m (X= 0.7, L = 5 m, h = 0.003 m, R = 1 m, Ω = 0.5 rps)

n	m	m				
		1	2	3	4	5
Type-I	1	Backward 28.7028	32.5056	41.3138	63.7345	89.7865
	Forward 28.6802	32.4256	41.2765	63.6420	89.542	
Type-II	1	Backward 32.468	34.2579	45.2334	66.5532	91.458
	Forward 32.256	34.0256	45.1357	66.4570	91.358	

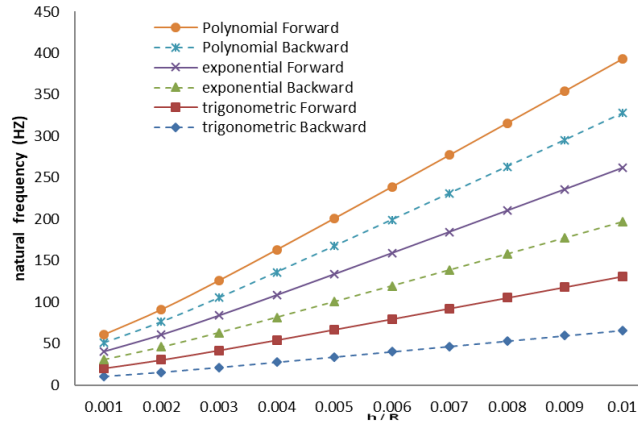


Fig. 1 Type-I S-S frequencies of rotating FG-CSs versus  $L/R$  ( $m=1, L/R=15, X=0.5, n=2, \Omega=3$ )

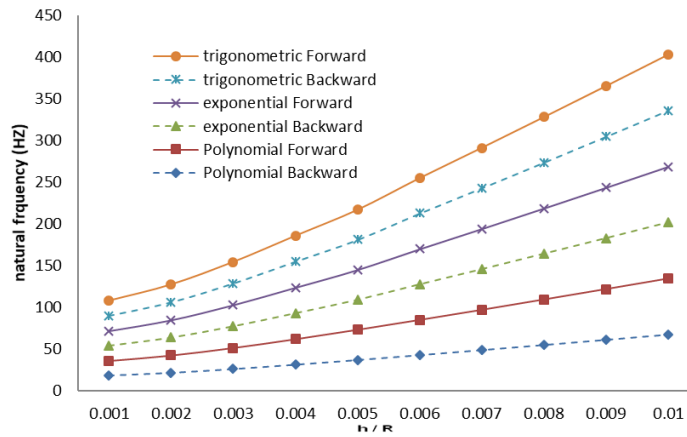


Fig. 2 Type-II S-S frequencies of rotating FG-CSs versus  $L/R$ : ( $m=1, L/R=10, X=0.7, n=2, \Omega=3$ )

Table 4 Rotating Type-I S-S frequency of FG-CSs against circumferential wave number  $n$  ( $m=1, h/R=0.02, L/R=10, \Omega=1, X=0.7$ )

$n$	Polynomial		Exponential		Trigonometric	
	Backward	Forward	Backward	Forward	Backward	Forward
1	48.523	48.224	48.431	48.152	48.326	48.118
2	18.758	18.504	18.515	18.360	18.154	18.099
3	20.633	20.441	18.467	20.276	20.125	19.994
4	35.501	35.351	35.301	35.251	35.189	35.039
5	56.204	56.082	55.941	54.718	55.586	55.364
6	81.758	81.654	81.500	81.396	81.233	81.129
7	112.01	111.93	111.98	111.47	111.32	111.11
8	146.95	146.87	146.84	148.72	146.66	146.09
9	186.56	186.49	186.37	186.25	186.19	186.03
10	230.83	230.77	230.71	230.65	230.51	230.38

Table 5 Rotating Type-II S-S frequency of FG-CSs against circumferential wave number  $n$  ( $m=1, h/R=0.03, L/R=10, \Omega=1, X=30$ )

$n$	Polynomial		Exponential		Trigonometric	
	Backward	Forward	Backward	Forward	Backward	Forward
1	49.524	49.225	49.432	49.153	49.327	48.119
2	18.759	19.505	19.516	19.361	19.155	19.010
3	22.634	21.442	19.468	21.277	21.126	20.995
4	36.502	36.352	36.302	36.252	36.190	36.040
5	57.205	57.083	56.941	55.719	56.587	56.365
6	82.759	82.655	82.500	82.397	82.234	82.130
7	113.02	112.94	112.98	112.48	112.33	112.12
8	147.96	147.88	148.84	149.73	147.67	147.10
9	187.57	187.50	187.37	187.26	187.20	187.04
10	231.84	231.78	231.71	231.66	231.51	231.39

numerical techniques selected here has good efficiency, validity, and robustness.

Table 3 shows the rotating frequencies versus  $n$  (wave number) and  $m$  (axial wave mode) for both Types (I and II). The frequencies for backward and forward waves increase indefinitely as  $n$  and  $m$  grows for FG-CSs. Moreover the order of constituent material of the shell impresses the frequency values. Figs. 1 and 2 show the frequency value

w.r.t  $h/R$ . The frequency values increases on increasing  $h/R$  respectively for Types (I and II). The influence of the various volume fraction laws for height-to-radius ratios is investigated with simply supported boundary condition. The frequency outcomes of polynomial law are higher than that of other two laws. The frequency of Type-II is greater than Type-I. It is due to the material used to form the functionally graded material.

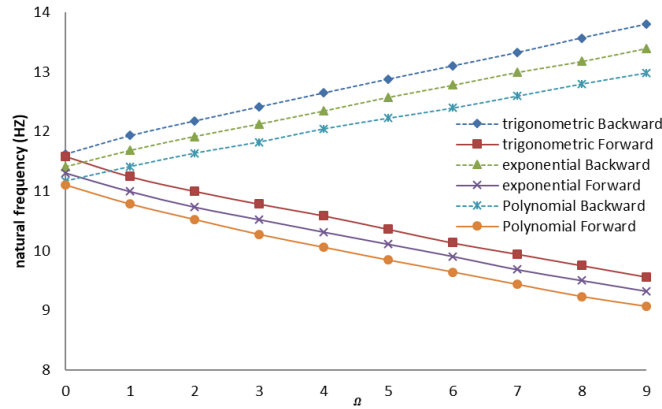


Fig. 3 Type-I S-S frequencies of rotating FG-CSs versus angular speed ( $m=1, h/R=0.05, n=2, L/R=10, X=30$ )

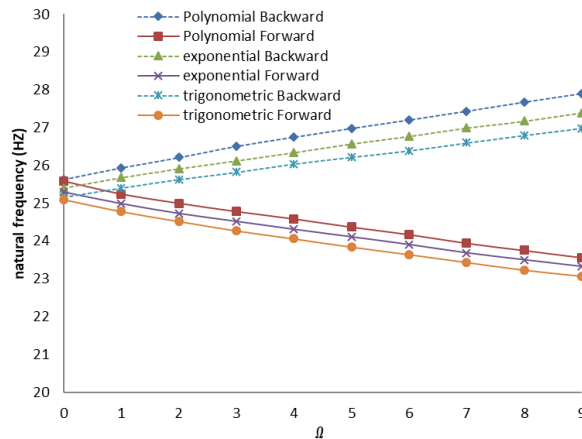


Fig. 4 Type-II S-S frequencies of rotating FG-CSs versus angular speed ( $m=1, h/R=0.05, n=2, L/R=10, X=30$ )

Table 6 Rotating Type-I S-S frequency of FG-CSs against  $L/R$ : ( $m=1, h/R=0.01, X=10, n=2, \Omega=3$ )

$L/R$	Polynomial		Exponential		Trigonometric	
	Backward	Forward	Backward	Forward	Backward	Forward
3	157.52	156.76	157.41	156.66	157.30	156.56
4	104.55	103.78	104.47	103.71	104.36	103.61
5	78.355	77.591	78.289	77.524	78.278	77.514
6	64.560	63.796	64.502	63.738	64.491	63.728
7	56.947	56.182	56.895	56.130	56.884	56.120
8	52.560	51.796	52.513	51.749	52.502	51.739
9	49.926	49.161	49.882	49.882	49.871	49.872
10	48.278	47.513	48.237	47.472	48.235	47.462
11	47.207	46.442	47.168	46.403	47.166	46.392
12	46.486	45.721	46.449	45.684	46.448	45.673

Table 7 Rotating Type-I S-S frequency of FG-CSs against circumferential wave number  $L/R$  ( $m=1, h/R=0.01, X=0.7, n=2, \Omega=3$ )

$L/R$	Polynomial		Exponential		Trigonometric	
	Backward	Forward	Backward	Forward	Backward	Forward
2	267.72	267.03	268.21	267.48	268.71	267.93
3	148.92	148.16	149.73	148.41	150.23	148.86
4	92.701	91.939	92.556	92.094	92.606	92.139
5	62.760	61.997	62.866	62.103	62.916	62.147
6	45.242	44.479	45.319	44.556	45.369	44.601
7	34.239	33.475	34.298	33.534	34.348	33.579
8	26.945	26.182	26.992	26.229	27.492	26.274
9	21.910	21.146	21.948	21.185	21.998	21.130
10	18.322	17.558	18.354	17.591	18.401	17.636
11	15.703	14.940	15.731	14.968	15.781	15.418

Tables 4 and 5 shows the variations of Type-I and -II frequencies with simply supported-simply supported (S-S) versus  $h/R$ . The angular speed is taken as  $\Omega = 1$ rps and volume fraction exponent ( $X$ ) is 0.7. The behavior of the Tables shows that frequencies first decreases, reach its maximum values on increasing  $n$ . The reason of said frequency behavior the flexural and membrane of the shell strain energy. The material work of the shell is restricted by three volume fraction laws: (I), (II), (III). It shows that

frequency determined by the polynomial law is the higher than that calculated from other two laws. The frequency first increases and gain maximum value on increasing the wave number. It shows that frequency determined by the trigonometric law is less than that evaluated from other two laws.

Figs. 3 and 4 demonstrate the natural frequencies (Hz) of a rotating functionally graded cylindrical shell drawn against the angular speeds. These results have been

obtained for the circumferential wave,  $n = 2$  with the polynomial, exponential and trigonometric volume fraction laws. Although by increasing the angular speed all backward frequencies enhance, an opposite manner can be observed for forward frequencies in this case (Chen et al, 1993a,b). For the polynomial volume law, the forward and backward frequencies are the highest than those for other two laws.

Tables 6 and 7 show the rotating frequencies against  $L/R$  and having with different physical parameters. The shell rotation speed is  $\Omega=1$  (rps). Due to the rotation, the bifurcation frequencies (backward and forward) gets higher on decreasing the ratio  $L/R$ , these frequencies slower down for types (I and II). The frequencies of Type-II is greater than Type-I. It is due to the material values used in the formation of shell.

## 5. Conclusions

Effect of volume fraction laws for rotating shell is presented using the orthotropic Winkler-like model with simply supported condition. For isotropic materials, the physical properties are same everywhere where the laminated and functionally graded materials, they vary from point to point. The frequency behavior is investigated with fraction laws versus circumferential wave number, length-to-radius and height-to-radius ratio. The frequency first increases and gain maximum value with the increase of circumferential wave mode. Moreover, the effect of height-and length-to-radius ratio is investigated. It is examined that the backward and forward frequencies increases and decreases on increasing the ratio of height- and length-to-radius ratio. It has been observed that the influence of fraction laws on shell frequencies is very prominent i.e., it increases their values. It is observed that the backward and forward frequencies monotonically increases and decreases respectively, on increasing the rotating speed. Stability of a cylindrical shell depends highly on these aspects of material. More the shell material sustains a load due to physical situations, the more the shell is stable. An extension of present study can be done for investigating the rotating FG-shells with ring supports.

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## Declaration of conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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