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Analyzing dynamic response of nonlocal strain gradient porous beams under moving load and thermal environment

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Abstract. This research presents dynamic response analysis of a porous functionally graded (FG) nanobeam under a moving point load. The nanobeam formulation has been established with the use of a higher-order refined beam model and nonlocal strain gradient theory (NSGT) including two scale factors named nonlocal and strain gradient factors. The porous FG material has been modeled via modified power-law functions which contain porosity volume according to even or uneven porosity dispersions. Moreover, graded nonlocality has been considered in order to provide a better modeling of size effects for FG nano-size structures. The governing equations of the nanobeam have been discretized with the use of differential quadrature method (DQM) and inverse Laplace transform approach has been utilized to calculate the dynamic deflections. The main findings of the present research indicate the influences of nonlocal strain gradient factors, moving load speed, pore amount, porosity distribution and elastic medium on dynamic deflection of FG nanobeams.

Keywords: dynamic response; thermal load; moving load; porosity; strain gradient

1. Introduction

Assessment of mechanical characteristics of nano-scale structures including nano-size beams and nano-size plates according to non-classic elasticity theories has been a serious case of study in recent decade. The main reason is broad application of nano-scale structures in nano-sensors or nano-electro-mechanical systems (NEMs). The most familiar theories for modeling of nano-scale structures are nonlocal elasticity (Eringen 1983) and strain gradient (Lam *et al.* 2003) theories. In the theories, some scale factors have been introduced in order to interpolate the influences of small size (Kunbar *et al.* 2020, Akgöz and Civalek 2015, Mirjavadi *et al.* 2020a,b, Ahmed *et al.* 2020, Fenjan *et al.* 2021). Pursuant to nonlocal elasticity the stress field is necessary to be nonlocal since the relation between the stress and the strain at a point depends on the strains of that point and also surrounding points (Nami and Janghorban 2014). This nonlocality of stress field has been considered with the use of nonlocal parameter. Any value of nonlocal parameter may be determined using experiment or numerical simulation. However, the derivation of the values of nonlocal parameter based on the mentioned approaches is very difficult and time-consuming. Therefore, many studies on static and dynamic properties of nano-scale structural elements have been performed as parametric studies based on some assumed values for nonlocal parameter (Aerfi and Zenkour 2016, Li *et al.* 2015, Zhang *et al.* 2015, Lou *et al.* 2016).

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In recent years, several theoretical studies and experiments report that small size effects must be characterized via stiffness increasing mechanism or strain gradient fields (Martínez-Criado 2016). This assertion is not the same as that of nonlocal elasticity in which stiffness reduction behavior has been stated (Yahiaoui *et al.* 2018, Achouri *et al.* 2019, Berrabah *et al.* 2013). However, the influences of reduction and increment on structural stiffness at nano scales can be considered in the context of nonlocal strain gradient theory (NSGT). According to NSGT, two scale factors named nonlocal and strain gradient factors have been utilized to provide an excellent description of small size effects. The static and dynamic properties of nanobeams and other nanostructures have been broadly studied with the use of NSGT (Barati 2018, She *et al.* 2018).

The effects of different loadings on vibration behavior of nanobeams has become an important case of study in recent years. Some of these loadings are harmonic forces, impulsive loads and moving loads at top surface of the nanobeam. Forced vibrations of the nanobeam due to harmonic and impulsive loads have been investigated by several authors in the context of nonlocal elasticity theory (NET) and nonlocal strain gradient theory (NSGT). However, forced vibrations of the nanobeams due to moving loads has become very important because of the nano-sensing and nano-probing applications (Simsek 2010, Abouelregal and Zenkour 2017, Shahsavari *et al.* 2017, Zhang and Liu 2020, Liu *et al.* 2020). It has been realized that the dynamic deflections of a nanobeam due to moving loads increase by the inclusion of nonlocal parameter (Khaniki and Hosseini-Hashemi 2017).

In a FG material, all properties must be described according to the continuous gradation between the two constituents (ceramic and metal). Actually, the mechanical

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characteristics of a FG material depends on the portion or percentage of each constituent. Therefore, the effective properties of a FG material can be controlled by increasing or reducing the portion of ceramic/metal constituents. Due to excellent properties of FG materials, they have many applications in structures used in aerospace, automobile and civil engineering sections. The distribution of FG material in structures can be mathematically modeled using power-law or Mori-Tanaka models. Using power-law functions, it is possible to easily describe the continuous gradation of material properties with good accuracy, however, Mori-Tanaka scheme has provided more accurate results as reported in some studies. Both models in their traditional forms have ignored the effects of porosities inside FG materials (Atmane *et al.* 2015). The porosities may be created due to some faults during the FG material production and it is shown that they have notable impact in mechanical characteristics of FG structures (She *et al.* 2018, Ahmed *et al.* 2019, El-Hassar *et al.* 2016). In recent years, a modified power-law model has been developed and used by several authors to investigate the mechanical characteristics of FG structures including porosity effects (Mirjavadi *et al.* 2017).

Another important fact in the analysis of FG structures at nano-scale is that the nonlocal parameter cannot be constant since the FG material properties are not constant in thickness direction and they are graded based on some specific distributions. Thus, the concept of graded nonlocality has been introduced in the literature for better modeling of the nonlocal behavior of FG structures at nano-scale (Ebrahimi *et al.* 2018). In this model, the nonlocal parameter is considered to be graded in thickness direction based on power-law function. However, most researchers have assumed constant nonlocal parameter for FG nanostructures which results in unexpected findings at nano-scale. According to above discussion and literature review, it can be expressed that there is no investigation of forced vibrations of porous FG nanobeams under moving point loads by considering the effects of graded nonlocality.

The present research deals with forced vibration behavior of a nanobeam made from porous FG material which is embedded on elastic substrate and is subjected to moving point load. In this research, the moving point load is the representative of a nanoparticle moving with constant velocity at the top face of the nano-sized beam. A modified power-law model has been used to investigate the dynamic characteristics of FG nanobeams including porosity effects. The nanobeam formulation is based upon higher-order refined beam theory, whereas the size effects have been captured according to NSGT. Also, graded nonlocality has been considered in order to provide a better modeling of size effects for FG nanobeams. The established equations acquired from Hamilton's rule have been solved through DQM and the time domain part of solution has been done using inverse Laplace transform approach. It is exhibited that the movable load speed, nonlocality/strain gradient factors, pore amount, pore distribution, elastic substrate and graded nonlocality have enormous affection on dynamic response of the nano-sized beams.

2. Basic relations

2.1 Beam modeling via NSGT

In recent years, several theoretical studies and experiments report that small size effects must be characterized via stiffness increasing mechanism or strain gradient fields (Martínez-Criado 2016). This assertion is not the same as that of nonlocal elasticity in which stiffness reduction behavior has been stated. As discussed, the influences of reduction and increment on structural stiffness at nano scales can be considered in the context of NSGT. According to NSGT, two scale factors named nonlocal and strain gradient factors have been utilized to provide an excellent description of small size effects. At first step, it is essential to define the stress field components in the below form (Barati 2018):

$$S_{ij} = S_{ij}^{(0)} - \nabla S_{ij}^{(1)} \tag{1}$$

Note that the symbols $S_{ij}^{(0)}$ and $S_{ij}^{(1)}$ are used for stress components which are respectively associated with strains ε_{kl} and strain gradients $\nabla\varepsilon_{kl}$ as:

$$S_{ij}^{(0)} = \int_V Q_{ijkl}\alpha_0(x,x',e_0a)\varepsilon'_{kl}(x')dx' \tag{2a}$$

$$S_{ij}^{(1)} = l^2\int_V Q_{ijkl}\alpha_1(x,x',e_1a)\nabla\varepsilon'_{kl}(x')dx' \tag{2b}$$

Here, the symbol Q_{ijkl} is used for elastic coefficients; e_0a and e_1a have been used to define the nonlocal effects and l introduces the influences of the strain gradients. If the nonlocal functions $\alpha_0(x,x',e_0a)$ and $\alpha_1(x,x',e_1a)$ can satisfy the introduced conditions by Eringen (1983), the relationship between the stresses and strains in the context of NSGT becomes:

$$\begin{aligned}&[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]S_{ij}\\&= Q_{ijkl}[1-(e_1a)^2\nabla^2]\varepsilon_{kl}\\&-Q_{ijkl}\,l^2[1-(e_0a)^2\nabla^2]\nabla^2\varepsilon_{kl}\end{aligned} \tag{3a}$$

where ∇^2 is called Laplacian operator. The above relation can be more simplified by assuming $e_1a=e_0a=ea$ as:

$$\begin{aligned}&[1-(ea)^2\nabla^2]S_{ij} =\\&Q_{ijkl}[1-l^2\nabla^2]\varepsilon_{kl}\end{aligned} \tag{3b}$$

2.2 FG materials

The distribution of FG material in structures can be mathematically modeled using power-law or Mori-Tanaka models. Using power-law function, it is possible to easily describe the continuous gradation of material properties with good accuracy, however, Mori-Tanaka scheme has

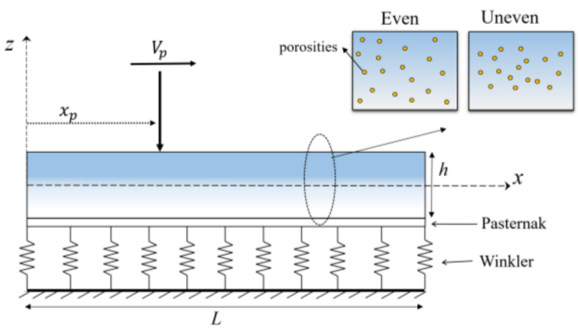


Fig. 1 Applied moving load on the FG nanobeam having pores

provided more accurate results as reported in some studies. At the first step, assume a FG nanobeam with length L and thickness h, as indicated in Fig.1. Pursuant to Mori-Tanaka FG model, the effective material properties of a FG nanobeam including the effective bulk modulus K_e, and shear modulus μ_e may be determined by:

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m(K_c - K_m)/(K_m + 4G_m/3)} \tag{4a}$$

$$\frac{G_e - G_m}{G_c - G_m} = \frac{V_c}{1 + V_m(G_c - G_m)/\breve{K}}, \quad \breve{K} = [(G_m + G_m(9K_m + 8G_m)/(6(K_m + 2G_m))] \tag{4b}$$

Here, K_m and K_c respectively denote the bulk modulus of metal and ceramic constituents; G_m and G_c respectively denote the shear modulus of metal and ceramic constituents. Also, V_c displays the volume of ceramic ingredient which has the below relation with the volume fraction of metal ingredient V_m as:

$$V_c + V_m = 1, \quad V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{5}$$

Note that the distribution of FG material in thickness direction relies on the value of material gradient index (p). According to above relations, one can express the effective Young's modulus (E), Poisson's ratio (ν) and mass density (ρ) of FG material as:

$$E(z) = \frac{9K_e G_e}{3K_e + G_e} \tag{6a}$$

$$\nu(z) = \frac{3K_e - 2G_e}{6K_e + 2G_e} \tag{6b}$$

$$\rho(z) = \rho_c V_c + \rho_m V_m \tag{6c}$$

Table 1 Material properties for the two ingredients

Property	Steel	Alumina (Al_2O_3)
E	210 (GPa)	390 (GPa)
ρ	7800 (kg/m^3)	3960 (kg/m^3)
α	13e-6	10.5e-6 (1/K)
ν	0.3	0.24

As mentioned, another approach to model a FG material is power-law function. Based upon power-law model each material property (F) can be defined using the following relation knowing that F_m and F_c respectively denote the material property of metal and ceramic constituents:

$$F(z) = (F_c - F_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + F_m \tag{7}$$

Based on above relation, it is possible to define Young's modulus (E), Poisson ratio (ν), nonlocal parameter $(ea)^2$ and strain gradient factor (l^2). In this way, the effect of graded nonlocality has been considered in this research (Ebrahimi *et al.* 2018). However, the above relations have ignored the effect of porosities inside FG materials. Using modified power-law functions, it is possible to model each material property containing porosity volume (ζ) as:

$$F(z) = (F_c - F_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + F_m - (F_c + F_m)\frac{\xi}{2} \tag{8a}$$

for even distribution

$$F(z) = (F_c - F_m)\left(\frac{z}{h} + \frac{1}{2}\right)^p + F_m - \frac{\xi}{2}(F_c + F_m)\left(1 - \frac{2|z|}{h}\right) \tag{8b}$$

for uneven distribution

It must be stated that the material properties of the metal and ceramic constituents have been presented within Table 1.

2.3 Theoretical relations

Higher-order beam theories are useful for establishing the governing equations of beams considering shear deformation effect. One of the well-known theories is refined beam theory which has the below form of displacement field (u_x, 0, u_z) as (Issad *et al.* 2018):

$$u_x(x,z) = u(x) - (z - z^*)\frac{\partial w_b}{\partial x} - [\mathrm{T}(z) - z^{**}]\frac{\partial w_s}{\partial x} \tag{9a}$$

$$u_z(x,z) = w_b(x) + w_s(x) \tag{9b}$$

The above displacement field contains axial displacement (u); bending displacement (w_b) and shear displacement (w_s). Moreover, in order to determine the neutral axis location in a FG beam, it is necessary to calculate:

$$c^* = \frac{\int_{-h/2}^{h/2} E(z)\, z dz}{\int_{-h/2}^{h/2} E(z)\, dz},\quad c^{**} = \frac{\int_{-h/2}^{h/2} E(z)\, \mathrm{T}(z) dz}{\int_{-h/2}^{h/2} E(z)\, dz} \tag{10}$$

A trigonometric shear strain function T(z) has been selected as:

$$\mathrm{T}(z) = z - \sin(\xi z)/\xi \tag{11}$$

with $\xi = \pi / h$. The derived strains based on the presented displacement field may be introduced as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - (z - c^*)\frac{\partial^2 w_b}{\partial x^2} - [\mathrm{T}(z) - c^{**}]\frac{\partial^2 w_s}{\partial x^2} \tag{12a}$$

$$\gamma_{xz} = g(z)\frac{\partial w_s}{\partial x} \tag{12b}$$

Note that $g(z) = 1 - d\mathrm{T}(z)/dz$. Next, the principle of Hamilton based on strain energy (U) and kinetic energy (K) implies that:

$$\int_0^t \delta(U + V - K)\, dt = 0 \tag{13}$$

Also, V introduces the energy of applied forces. The strain energy variation might be introduced by:

$$\delta U = \int_v S_{ij}\delta\varepsilon_{ij}\, dV = \int_v (S_{xx}\delta\varepsilon_{xx} + S_{xx}^{(1)}\delta\nabla\varepsilon_{xx} + S_{xz}\delta\gamma_{xz} + S_{xz}^{(1)}\delta\nabla\gamma_{xz})\, dV \tag{14}$$

Insertion of Eqs.(12a) - (12b) in Eq.(14) leads to:

$$\delta U = \int_0^L (N\frac{\partial\delta u}{\partial x} - M_b\frac{\partial^2\delta w_b}{\partial x^2} - M_s\frac{\partial^2\delta w_s}{\partial x^2} + Q\frac{\partial\delta w_s}{\partial x})dx \tag{15}$$

In such a way that:

$$\begin{aligned}
N &= \int_{-h/2}^{h/2}(S_{xx}^0 - \nabla S_{xx}^{(1)})dz = N^{(0)} - \nabla N^{(1)}\\
M^b &= \int_{-h/2}^{h/2} z(S_{xx}^0 - \nabla S_{xx}^{(1)})dz = M^{b(0)} - \nabla M^{b(1)}\\
M^s &= \int_{-h/2}^{h/2} \mathrm{T}(S_{xx}^0 - \nabla S_{xx}^{(1)})dz = M^{s(0)} - \nabla M^{s(1)}\\
Q &= \int_{-h/2}^{h/2} g(S_{xz}^0 - \nabla S_{xz}^{(1)})dz = Q^{(0)} - \nabla Q^{(1)}
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
N^{(0)} &= \int_{-h/2}^{h/2}(S_{xx}^{(0)})dz,\\
N^{(1)} &= \int_{-h/2}^{h/2}(S_{xx}^{(1)})dz\\
M^{b(0)} &= \int_{-h/2}^{h/2} z(S_{xx}^{b(0)})dz,\\
M^{b(1)} &= \int_{-h/2}^{h/2} z(S_{xx}^{b(1)})dz\\
M^{s(0)} &= \int_{-h/2}^{h/2} \mathrm{T}(S_{xx}^{s(0)})dz,\\
M^{s(1)} &= \int_{-h/2}^{h/2} \mathrm{T}(S_{xx}^{s(1)})dz\\
Q^{(0)} &= \int_{-h/2}^{h/2} g(S_{xz}^{(0)})dz,\\
Q^{(1)} &= \int_{-h/2}^{h/2} g(S_{xz}^{(1)})dz
\end{aligned} \tag{17}$$

The energy due to the exerted loads might be introduced as follows:

$$\delta V = \int_0^L (q\delta(w_b + w_s))dx \tag{18}$$

In such a way that:

$$q = +k_w(w_b + w_s) - (k_p - N^T)\frac{\partial^2(w_b + w_s)}{\partial x^2} + q_{dynamic} \tag{19}$$

In above relation, k_w and k_p respectively denote the Winkler and Pasternak parameters of elastic substrate; $q_{dynamic}=P_0\delta(x-V_pt)$ is the applied force due to the moving load. Here, V_p denotes the velocity of moving load. Also, $N^T = \int_{-0.5h}^{+0.5h}\alpha(z)E(z)\Delta T dz$ is the thermal load. Application of the variation in the kinetic energy leads to:

$$\begin{aligned}
\delta K = \int_0^L (I_0[&\frac{\partial u}{\partial t}\frac{\partial\delta u}{\partial t} + (\frac{\partial w_b}{\partial t} + \frac{\partial w_s}{\partial t})(\frac{\partial\delta w_b}{\partial t} + \frac{\partial\delta w_s}{\partial t})] - I_1(\frac{\partial u}{\partial t}\frac{\partial^2\delta w_b}{\partial x\partial t} + \frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial\delta u}{\partial t})\\
&+ I_2(\frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial^2\delta w_b}{\partial x\partial t}) - J_1(\frac{\partial u}{\partial t}\frac{\partial^2\delta w_s}{\partial x\partial t} + \frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial\delta u}{\partial t})\\
&+ K_2(\frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial^2\delta w_s}{\partial x\partial t}) + J_2(\frac{\partial^2 w_b}{\partial x\partial t}\frac{\partial^2\delta w_s}{\partial x\partial t} + \frac{\partial^2 w_s}{\partial x\partial t}\frac{\partial^2\delta w_b}{\partial x\partial t}))dx
\end{aligned} \tag{20}$$

where

$$(I_0, I_1, J_1, I_2, J_2, K_2) = \int_{-h/2}^{h/2}\{1, z - c^*, (z - c^*)^2, \mathrm{T} - c^{**}, (z - c^*)(\mathrm{T} - c^{**}), (\mathrm{T} - c^{**})^2\}\rho dz \tag{21}$$

The governing equations have been determined via insertion of Eqs. (15)-(20) in Eq. (13) with setting the coefficients of $\delta u, \delta w_b$ and δw_s to zero:

$$\frac{\partial N}{\partial x} = I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^3 w_b}{\partial x\partial t^2} - J_1\frac{\partial^3 w_s}{\partial x\partial t^2} \tag{22}$$

$$\frac{\partial^2 M_b}{\partial x^2} - q_{dynamic} = +I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) + I_1 \frac{\partial^3 u}{\partial x \partial t^2} - I_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} - J_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} + k_w(w_b + w_s) - (k_p - N^T)\frac{\partial^2 (w_b + w_s)}{\partial x^2} \tag{23}$$

$$\frac{\partial^2 M_s}{\partial x^2} + \frac{\partial Q}{\partial x} - q_{dynamic} = +I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) + J_1 \frac{\partial^3 u}{\partial x \partial t^2} - J_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} - K_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} + k_w(w_b + w_s) - (k_p - N^T)\frac{\partial^2 (w_b + w_s)}{\partial x^2} \tag{24}$$

Using Eq. (3), it is possible to establish the stress-strain relations of a higher-order refined FG nanobeam in the context of NSGT as:

$$S_{xx} - (ea)^2 \frac{\partial^2 S_{xx}}{\partial x^2} = (1 - l^2 \frac{\partial^2}{\partial x^2}) E(z)\varepsilon_{xx} \tag{25}$$

$$S_{xz} - (ea)^2 \frac{\partial^2 S_{xz}}{\partial x^2} = (1 - l^2 \frac{\partial^2}{\partial x^2}) G(z)\gamma_{xz} \tag{26}$$

Integration from Eqs. (25) and (26) according the nano-sized beam thickness leads to the below relations of the forces and moments as:

$$N - (ea)^2 \frac{\partial^2 N}{\partial x^2} = (A^{Local} \frac{\partial u}{\partial x} - B^{Local} \frac{\partial^2 w_b}{\partial x^2} - B_s^{Local} \frac{\partial^2 w_s}{\partial x^2}) - \frac{\partial^2}{\partial x^2}(A^{SGT} \frac{\partial u}{\partial x} - B^{SGT} \frac{\partial^2 w_b}{\partial x^2} - B_s^{SGT} \frac{\partial^2 w_s}{\partial x^2}) \tag{27}$$

$$M_b - (ea)^2 \frac{\partial^2 M_b}{\partial x^2} = (B^{Local} \frac{\partial u}{\partial x} - D^{Local} \frac{\partial^2 w_b}{\partial x^2} - D_s^{Local} \frac{\partial^2 w_s}{\partial x^2}) - \frac{\partial^2}{\partial x^2}(B^{SGT} \frac{\partial u}{\partial x} - D^{SGT} \frac{\partial^2 w_b}{\partial x^2} - D_s^{SGT} \frac{\partial^2 w_s}{\partial x^2}) \tag{28}$$

$$M_s - (ea)^2 \frac{\partial^2 M_s}{\partial x^2} = (B_s^{Local} \frac{\partial u}{\partial x} - D_s^{Local} \frac{\partial^2 w_b}{\partial x^2} - H_s^{Local} \frac{\partial^2 w_s}{\partial x^2}) - \frac{\partial^2}{\partial x^2}(B_s^{SGT} \frac{\partial u}{\partial x} - D_s^{SGT} \frac{\partial^2 w_b}{\partial x^2} - H_s^{SGT} \frac{\partial^2 w_s}{\partial x^2}) \tag{29}$$

$$Q - (ea)^2 \frac{\partial^2 Q}{\partial x^2} = (A_s^{Local} \frac{\partial w_s}{\partial x}) - \frac{\partial^2}{\partial x^2}(A_s^{SGT} \frac{\partial w_s}{\partial x}) \tag{30}$$

So that the cross-sectional rigidities can be determined by:

$$\{A^{Local}, B^{Local}, B_s^{Local}, D^{Local}, D_s^{Local}, H_s^{Local}\} = \int_{-h/2}^{h/2} E(z)\{1, (z-c^*), (\mathrm{T}-c^{**}), (z-c^*)^2, (z-z^*)(\mathrm{T}-c^{**}), (\mathrm{T}-c^{**})^2\} dz \tag{31}$$

$$A_s^{Local} = \int_{-h/2}^{h/2} g^2 G(z) dz \tag{32}$$

$$\{A^{SGT}, B^{SGT}, B_s^{SGT}, D^{SGT}, D_s^{SGT}, H_s^{SGT}\} = \int_{-0.5h}^{0.5h} E(z) l^2(z)\{1, (z-c^*), (\mathrm{T}-c^{**}), (z-c^*)^2, (z-c^*)(\mathrm{T}-c^{**}), (\mathrm{T}-c^{**})^2\} dz \tag{33}$$

$$A_s^{SGT} = \int_{-h/2}^{h/2} g^2 l^2(z) G(z) dz \tag{34}$$

A porous FG nanobeam with graded nonlocality has the below governing equations which are determined via insertion of Eqs. (27)-(30) in Eqs. (22)-(24) in such a way that:

$$(A^{Local} - A^{SGT} \frac{\partial^2}{\partial x^2}) \frac{\partial^2 u}{\partial x^2} - (B^{Local} - B^{SGT} \frac{\partial^2}{\partial x^2}) \frac{\partial^3 w_b}{\partial x^3} - (B_s^{Local} - B_s^{SGT} \frac{\partial^2}{\partial x^2}) \frac{\partial^3 w_s}{\partial x^3} - I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} + J_1 \frac{\partial^3 w_s}{\partial x \partial t^2} + (I_0^{Nonlocal} \frac{\partial^4 u}{\partial x^2 \partial t^2} - I_1^{Nonlocal} \frac{\partial^5 w_b}{\partial x^3 \partial t^2} - J_1^{Nonlocal} \frac{\partial^5 w_s}{\partial x^3 \partial t^2}) = 0 \tag{35}$$

$$\begin{aligned}&(B^{Local} - B^{SGT} \frac{\partial^2}{\partial x^2})(\frac{\partial^3 u}{\partial x^3}) - (D^{Local} - D^{SGT} \frac{\partial^2}{\partial x^2}) *(\frac{\partial^4 w_b}{\partial x^4}) - (D_s^{Local} - D_s^{SGT} \frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_s}{\partial x^4}) \\ &- I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) - I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^4 w_b}{\partial x^2 \partial t^2} + J_2 \frac{\partial^4 w_s}{\partial x^2 \partial t^2} - k_w(w_b + w_s) + (k_p - N^T) * \frac{\partial^2 (w_b + w_s)}{\partial x^2} \\ &+ (I_0^{Nonlocal}(\frac{\partial^4 w_b}{\partial x^2 \partial t^2} + \frac{\partial^4 w_s}{\partial x^2 \partial t^2}) + I_1^{Nonlocal} \frac{\partial^5 u}{\partial x^3 \partial t^2} - I_2^{Nonlocal} \frac{\partial^6 w_b}{\partial x^4 \partial t^2} - J_2^{Nonlocal} \frac{\partial^6 w_s}{\partial x^4 \partial t^2}) \\ &+ \mu_m(k_w \frac{\partial^2 (w_b + w_s)}{\partial x^2} - k_p \frac{\partial^4 (w_b + w_s)}{\partial x^4}) + N^{T,Nonlocal} = q_{dynamic} - \mu_c \frac{\partial^2 q_{dynamic}}{\partial x^2}\end{aligned} \tag{36}$$

$$\begin{aligned}&(B_s^{Local}-B_s^{SGT}\frac{\partial^2}{\partial x^2})(\frac{\partial^3 u}{\partial x^3})-(D_s^{Local}\\&-D_s^{SGT}\frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_b}{\partial x^4})-(H_s^{Local}\\&-H_s^{SGT}\frac{\partial^2}{\partial x^2})(\frac{\partial^4 w_s}{\partial x^4})+(A_s^{Local}\\&-A_s^{SGT}\frac{\partial^2}{\partial x^2})(\frac{\partial^2 w_s}{\partial x^2})-I_0(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2})\\&-J_1\frac{\partial^3 u}{\partial x\partial t^2}+J_2\frac{\partial^4 w_b}{\partial x^2\partial t^2}+K_2\frac{\partial^4 w_s}{\partial x^2\partial t^2}\\&-k_w(w_b+w_s)+(k_p-N^T)\frac{\partial^2(w_b+w_s)}{\partial x^2}\\&+(I_0^{Nonlocal}(\frac{\partial^4 w_b}{\partial x^2\partial t^2}+\frac{\partial^4 w_s}{\partial x^2\partial t^2})\\&+J_1^{Nonlocal}\frac{\partial^5 u}{\partial x^3\partial t^2}-J_2^{Nonlocal}\frac{\partial^6 w_b}{\partial x^4\partial t^2}\\&-K_2^{Nonlocal}\frac{\partial^6 w_s}{\partial x^4\partial t^2})+\mu_m(k_w\frac{\partial^2(w_b+w_s)}{\partial x^2}\\&-k_p\frac{\partial^4(w_b+w_s)}{\partial x^4})+N^{T,Nonlocal}\\&=q_{dynamic}-\mu_c\frac{\partial^2 q_{dynamic}}{\partial x^2}\end{aligned} \tag{37}$$

where

$$\begin{aligned}&(I_0^{Nonlocal},I_1^{Nonlocal},J_1^{Nonlocal},\\&I_2^{Nonlocal},J_2^{Nonlocal},K_2^{Nonlocal})\\&=\int_{-h/2}^{h/2}(1,z-c^*,(z-c^*)^2,\mathrm{T}-c^{**}\\&,(z-c^*)(\mathrm{T}-c^{**}),(\mathrm{T}-c^{**})^2)\rho(z)(ea)^2dz\end{aligned} \tag{38}$$

And $N^T=\int_{-0.5h}^{+0.5h}ea(z)\alpha(z)E(z)\Delta T dz$. Moreover, it must be clarified that the bending-extension coupling will be eliminated if the exact location of neutral axis has been incorporated. Hence, Eqs.(36) and (37) are solved to obtain the dynamic response.

3. Solution by differential quadrature method (DQM)

In the present chapter, differential quadrature method (DQM) has been utilized for solving the governing equations for NSGT porous FG nanobeam. According to DQM, at an assumed grid point (x_i, y_j) the derivatives for function F are supposed as weighted linear summation of all functional values within the computation domains as:

$$\frac{d^nF}{dx^n}|_{\;x=x_i}=\sum_{j=1}^{N}c_{ij}^{(n)}F(x_j) \tag{39}$$

where

$$C_{ij}^{(1)}=\frac{\pi(x_i)}{(x_i-x_j)\;\pi(x_j)}\qquad i,j=1,2,\dots,N,\qquad i\neq j \tag{40}$$

in which $\pi(x_i)$ is defined by

$$\pi(x_i)=\prod_{j=1}^{N}(x_i-x_j),\qquad i\neq j \tag{41}$$

And when $i=j$

$$C_{ij}^{(1)}=c_{ii}^{(1)}=-\sum_{k=1}^{N}C_{ik}^{(1)},\qquad i=1,2,\dots,N,\qquad i\neq k,\quad i=j \tag{42}$$

Then, weighting coefficients for high orders derivatives may be expressed by:

$$\begin{aligned}C_{ij}^{(2)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(1)}\\C_{ij}^{(3)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(2)}=\sum_{k=1}^{N}C_{ik}^{(2)}C_{kj}^{(1)}\\C_{ij}^{(4)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(3)}=\sum_{k=1}^{N}C_{ik}^{(3)}C_{kj}^{(1)}\qquad i,j=1,2,\dots,N.\\C_{ij}^{(5)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(4)}=\sum_{k=1}^{N}C_{ik}^{(4)}C_{kj}^{(1)}\\C_{ij}^{(6)}&=\sum_{k=1}^{N}C_{ik}^{(1)}C_{kj}^{(5)}=\sum_{k=1}^{N}C_{ik}^{(5)}C_{kj}^{(1)}\end{aligned} \tag{43}$$

According to presented approach, the dispersions of grid points based upon Gauss-Chebyshev-Lobatto assumption are expressed as:

$$x_i=\frac{L}{2}\left[1-\cos\left(\frac{i-1}{N-1}\pi\right)\right]\qquad i=1,2,\dots,N, \tag{44}$$

Next, the displacement components may be determined by

$$w_b(x,t)=W_b(x)e^{i\omega t} \tag{45}$$

$$w_s(x,t)=W_s(x)e^{i\omega t} \tag{46}$$

where W_b and W_n denote vibration amplitudes and ω defines the vibrational frequency. Then, it is possible to express obtained boundary conditions as:

$$\begin{aligned}&w_b=w_s=0,\\&\frac{\partial^2 w_b}{\partial x^2}=\frac{\partial^2 w_s}{\partial x^2}=0\;,\frac{\partial^4 w_b}{\partial x^4}=\frac{\partial^4 w_s}{\partial x^4}=0\end{aligned} \tag{47}$$

Now, one can express the modified weighting coefficients for all edges simply-supported as:

$$\begin{aligned}\bar{C}_{1,j}^{(2)}&=\bar{C}_{N,j}^{(2)}=0,\qquad i=1,2,\dots,M,\\\bar{C}_{i,1}^{(2)}&=\bar{C}_{1,M}^{(2)}=0,\qquad i=1,2,\dots,N.\end{aligned} \tag{48}$$

and

$$\bar{C}_{ij}^{(3)}=\textstyle\sum_{k=1}^{N}C_{ik}^{(1)}\bar{C}_{kj}^{(2)}\quad \bar{C}_{ij}^{(4)}=\sum_{k=1}^{N}C_{ik}^{(1)}\bar{C}_{kj}^{(3)} \tag{49}$$

Insertion of Eqs. (38) and (39) in Eqs. (30) and (31) results in:

$$\left\{[K]+\frac{\partial}{\partial t^2}[M]\right\}\begin{Bmatrix}W_{bn}\\W_{sn}\end{Bmatrix}=\begin{Bmatrix}q_{dynamic}-\mu_c\dfrac{\partial^2 q_{dynamic}}{\partial x^2}\\q_{dynamic}-\mu_c\dfrac{\partial^2 q_{dynamic}}{\partial x^2}\end{Bmatrix} \tag{50}$$

In above equation, [K] and [M] respectively display the stiffness and mass matrices. Furthermore, the normalized foundation coefficients have been selected as:

$$K_w = k_w \frac{L^4}{E_m I}, K_p = k_p \frac{L^2}{E_m I}, \tag{51}$$

Since the nanobeam has been exposed to a moving load with constant speed at its top surface, one can define the applied force as:

$$q_{dynamic} = \sum_{n=1}^{\infty} Q_n sin[\frac{n\pi}{L} x] \tag{52}$$

$$\begin{aligned} Q_n &= \frac{2}{L}\int_0^L sin[\frac{n\pi}{L} x] q(x)dx \\ &= \frac{2P_0}{L}\sin[\frac{n\pi}{L} x_p] = \frac{2P_0}{L}\sin[\frac{n\pi}{L} V_p t] \end{aligned} \tag{53}$$

Here, Q_n denotes the coefficients of Fourier series and $q(x) = P_0\delta(x - x_p)$ with P_0 as the force magnitude, x_p denotes the force location. Moreover, V_p has been called the speed of applied force. At the end, with the selection of zero initial conditions and Laplace transform method, Eq. (50) has been re-formulated as:

$$\left\{[K] + S^2[M]\right\}\begin{Bmatrix} L[W_{bn}] \\ L[W_{sn}] \end{Bmatrix} = \begin{Bmatrix} L[q_{dynamic} - \mu_c \frac{\partial^2 q_{dynamic}}{\partial x^2}] \\ L[q_{dynamic} - \mu_c \frac{\partial^2 q_{dynamic}}{\partial x^2}] \end{Bmatrix} \tag{54}$$

By solving Eq. (54) through the inverse Laplace transform approach, it is possible to derive the values of bending (W_{bn}) and shear (W_{sn}) displacements. However, the total deflection of the nanobeam is the summation of the two displacements as $W = W_{bn} + W_{sn}$. Also, the normalized deflection and the normalized velocity factor may be introduced by:

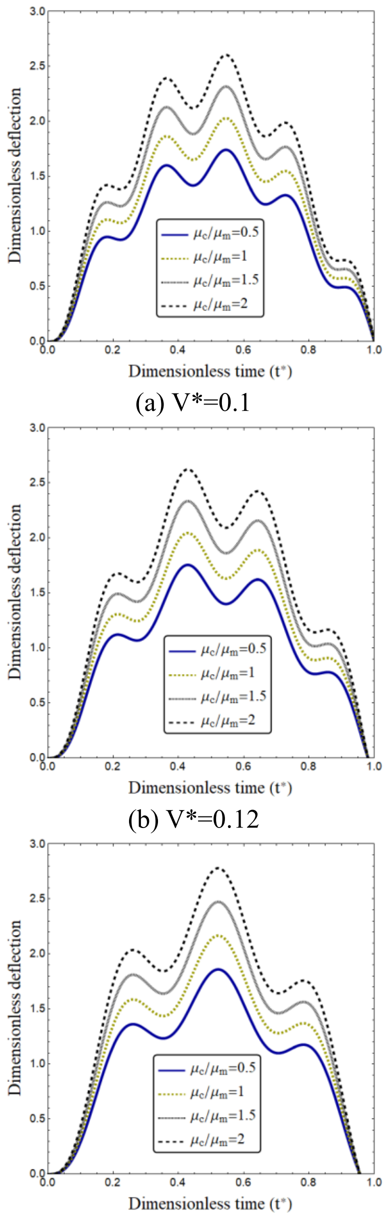
$$\bar{W} = W\frac{100E_m I}{P_0 L^3}, \quad V^* = \frac{V_p}{V_{cr}}, \quad V_{cr} = \frac{\omega_n L}{\pi} \tag{55}$$

Finally, another important factor is the normalized time in the below form:

$$t^* = \frac{V_p t}{L} \tag{56}$$

4. Obtained results and discussion

Assessment of mechanical characteristics of nano-scale structures including nano-size beams and nano-size plates according to non-classic elasticity theories has been a serious case of study in recent decade. The main reason is broad application of nano-scale structures in nano-sensors or nano-electro-mechanical systems (NEMs). The present section has been devoted to examine the dynamic responses of pore-dependent FG nano-scaled beams because of the moving load capturing both nonlocality and strain gradient influences. The effects of load velocity, material gradation, porosity distribution, graded nonlocality and two scale factors on dynamic deflection of the nanobeam have been studied in detail. Based on this research, it has been supposed that the nonlocality factor and strain gradient parameter are not fixed for FG nanobeam and they are variable in the thickness direction as $\mu(z)$ and $\lambda(z)$ according to Eq.(8). Thus, μ_m and μ_c are the nonlocal parameters of the metal and ceramic phases, respectively; λ_m and λ_c are the



(a) V*=0.1

(b) V*=0.12

(c) V*=0.15

Fig. 2 Dynamic deflection of the nano-size beam against normalized time at different load velocities according to diverse nonlocality ratios (L/h=10, p=1, K_w=K_p=0, λ_m=0.1, λ_c/ λ_m=2, μ_m=0.2)

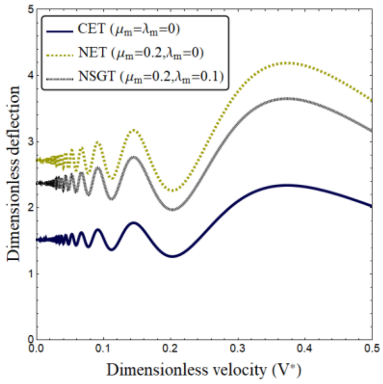


Fig. 3 Dynamic deflection of the nano-size beam against normalized velocity based on different elasticity theories (L/h=10, p=1, K_w=K_p=0, μ_c/μ_m=λ_c/ λ_m=2, t*=0.5)

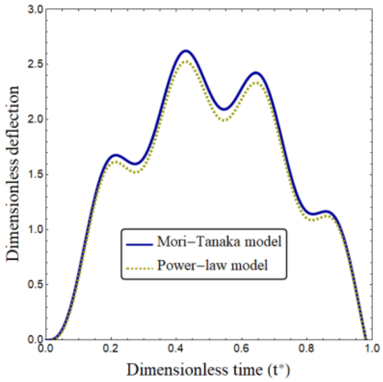


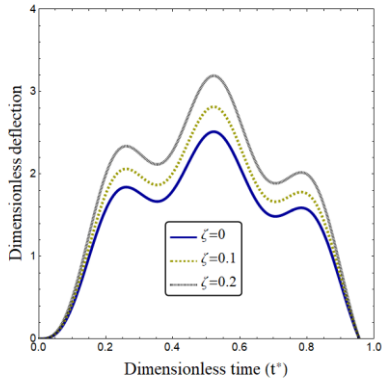
Fig. 4 Dynamical deflection of the nano-size beam against normalized velocity based on power-law and Mori-Tanaka FG models (L/h=10, p=1, K_w=K_p=0, λ_m=0.1, λ_c/λ_m=2, μ_c/μ_m=2, μ_m=0.2, V*=0.12)

strain gradient parameters of the metal and ceramic ingradients, respectively. Accordingly, μ_c / μ_m and λ_c / λ_m respectively denote the nonlocal ratio and strain gradient ratio. More discussion on this issue can be found in the following paragraphs.

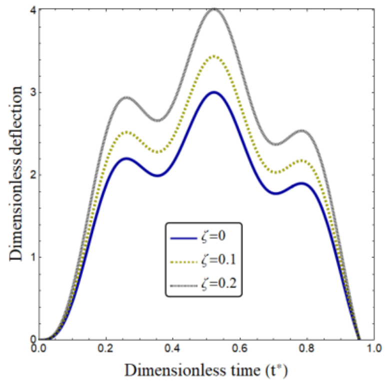
At the first step, a comparison has been provided in Table 2 with the work of Simsek (2019) to validate the vibration frequency of a FG nanobeam based on NSGT. In this regard, the validation of first dimensionless vibration frequency $\Omega = \omega L^2 \sqrt{\rho_c / E_c} / h$, at different values of nonlocal and strain gradient parameters have been carried out and an excellent agreement has been obtained between the obtained results and those of Simsek (2019).

Fig. 2. Dynamic deflection of the nano-size beam against normalized time at different load velocities according to diverse nonlocality ratios (L/h=10, p=1, K_w=K_p=0, λ_m=0.1, λ_c/ λ_m=2, μ_m=0.2).

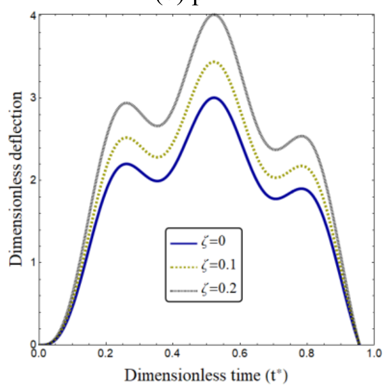
Fig. 2 plots the time history of normalized dynamic deflections at different values of load velocity factor (V* = 0.1, 0.12, 0.15) and nonlocal parameter ratio (μ_c / μ_m=0.5, 1, 1.5, 2) at the FG index p = 1. It is assumed in this research that λ_c/ λ_m=2. For every value of load velocity factor, an increment in nonlocal ratio leads to higher values of normalized dynamic deflection. This is due to reduced stiffness of FG nano-sized beam when the nonlocal ratio



(a) p=0.5



(b) p=1



(c) p=2

Fig. 5 Dynamic deflection of the nano-size beam against normalized time at different FG index according to diverse porosity factors (L/h=10, V*=0.15, K_w=K_p=0, λ_m=0.1, λ_c/λ_m=2, μ_c/μ_m=2, μ_m=0.2)

tends to higher values. Such behavior indicates that a FG nanobeam displays stiffness-softening effects when the nonlocal ratio augments. It can be concluded that the dynamics of a FG nano-scaled beam depends on the competition between the nonlocal values of metal and ceramic ingredients. In other words, considering the nonlocal parameter as a constant quantity is not a correct assumption. Thus, it must be regarded to be variable in thickness direction of a FG beam. This means that variable nonlocal parameter can be regarded as a key factor in analysis of FG beams at nano scale.

Dynamic deflection of the nano-size beam against load velocity factor based on different elasticity theories (CET, NET and NSGT) has been plotted in Fig. 3 at p=1 and

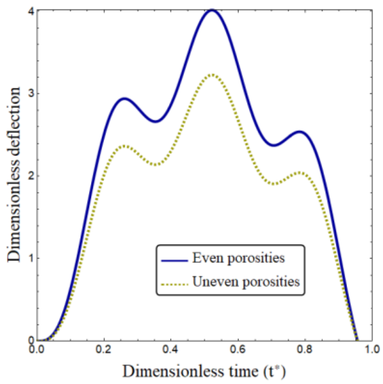
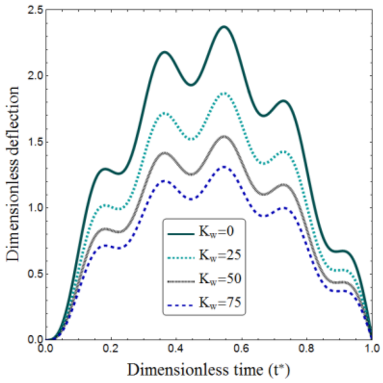
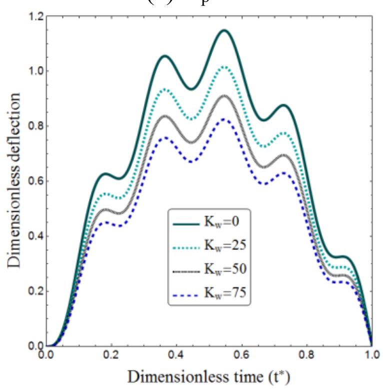


Fig. 6 Dynamic deflection of the nano-size beam against normalized time according to even and uneven porosity dispersions (L/h=10, V*=0.15, p=2, ζ=0.2, K_w=K_p=0, λ_m=0.1, λ_c/λ_m=2, μ_c/μ_m=2, μ_m=0.2)



(a) K_p=0



(b) K_p=10

Fig. 7 Dynamical deflection of nano-sized beam against normalized time for diverse foundation coefficients (L/h=10, λ_m=0.1, λ_c/λ_m=1, μ_c/μ_m=1, μ_m=0.2, p=1, ξ=0.1, V*=0.1)

t^*=0.5. For the case of classic elasticity theory (CET), it is assumed that μ_m= λ_m=0. Also, it is considered for the case of nonlocal elasticity theory that μ_m=0.2 and λ_m=0. Within this figure, the nonlocality ratio has been chosen to be μ_c/μ_m= =2. It can be understood that the dynamic deflections are appreciably impressed by the moving load rapidity. Actually, the dynamic deflection is augmented with the load velocity factor until achieving a peak point, and next it declines abruptly after that point (critical velocity). However, the dynamic deflection and the peak point are

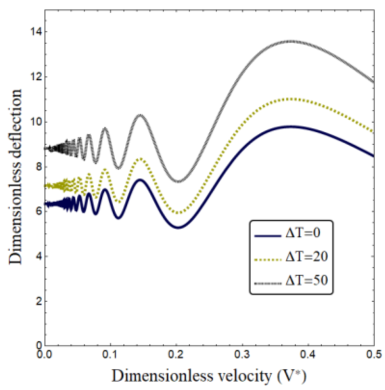


Fig. 8 Dyna Dynamic deflection of the nano-size beam against normalized velocity based on different temperatures (L/h=10, p=1, K_w=K_p=0, μ_c/μ_m=λ_c/ λ_m=2, μ_m=0.2)

dependent on the values of nonlocal and strain gradient parameters. Indeed, NET gives the maximum value of dynamic deflection due to the inclusion of nonlocal effects. However, by incorporating the strain gradient effect, NSGT renders smaller deflections than NET.

A comparison between the dynamic deflection (time history) of FG nanobeam obtained by power-law and Mori-Tanaka models has been provided in Fig.4 assuming p=1. In this figure, the load velocity factor is set to V^*=0.12. It can be seen that Mori-Tanaka model of FG materials results in higher values of dynamic deflection compared with power-law model. In fact, the FG nanobeam is more flexible by describing the material properties using Mori-Tanaka model. Actually, Mori-Tanaka model is known as a more reliable scheme for FG materials compared with power-law model.

In Fig.5, the time history of FG nanobeam has been plotted based on various porosity volume fraction (ζ=0, 0.1, 0.2) and FG gradient index (p=0.5, 1, 2) at V^*=0.15. The even porosity dispersion has been considered for this figure. It can be understood that the FG nano-sized beam becomes more pliable at a higher value of gradient index because of the greater percentage of metal constituent compared to ceramic constituent. Accordingly, the dynamic deflections of nanobeam increase by increasing the value of gradient index. However, another important factor in dynamic response of a FG nanobeam is the presence of porosities. As the porosity volume (ζ) becomes higher, the value of dynamic deflection becomes higher due to the reason that porosities inside FG material will reduce the structural stiffness.

A comparison between the dynamic deflection of porous FG nanobeam obtained by even-type and uneven-type porosities has been presented in Fig.6 assuming p=2. It is found from this figure that uneven-type porosities give smaller values of dynamic deflection compared with even-type porosities. Actually, the FG nanobeams with even-type porosities are more flexible than uneven-type porosities. The reason is uniform distribution of porosities in cross section area of the FG nanobeam in the case of even model. However, the porosities will not occur at corner of the cross section in the case of uneven pattern.

The influences of normalized elastic substrate parameters (K_w, K_p) on dynamic deflection of a porous FG nanobeam have been plotted in Fig.7 at ζ=0.1 and V^*=0.1. It should be stated that the existence of elastic substrate enhances the bending rigidity of nanobeam. However, this enhancing effect depends on the magnitudes of Winkler and Pasternak parameters. An increment in Winkler or Pasternak parameters yields lower values for dynamic deflection. Moreover, the influence of Pasternak factor on dynamic deflection is more considerable than Winkler factor.

Dynamical deflection of the nano-size beam against load velocity factor based on different temperatures (ΔT) has been plotted in Fig. 8 at p=1 Within this figure, the nonlocality ratio has been chosen to be μ_c/μ_m= =2. It can be seen that the dynamic deflection is markedly impressed by the temperature variation. Actually, the dynamic deflection is augmented with the increase of temperature. It means that at higher temperatures, the nanobeam has lower stiffness leading to greater deflections.

5. Conclusions

This article dealt with dynamical responses analysis of a porous FG nano-scaled beam in interaction with a moving point force considering the effects of graded nonlocality. A modified power-law model was used to investigate the dynamic characteristics of FG nanobeams including porosity effects. The nanobeam formulation was based upon higher-order refined beam theory, whereas the size effects have been captured according to NSGT. The governing equations were solved using DQM and inverse Laplace transform method. The primary detections have been listed as follows:

It was found that an increment in nonlocal ratio leads to higher values of normalized dynamic deflection.

It was reported that NSGT gives smaller deflections than NET.

The dynamic deflection was augmented with the load velocity until achieving a peak point, and then it dropped after this point which was called critical velocity.

The porosity volume becomes higher; the value of dynamic deflection becomes higher due to the reason that porosities inside FG material will reduce the structural stiffness.

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