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Dynamical behavior of the orthotropic elastic material using an analytical solution

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Abstract. In this work, an analytical solution is provided for the dynamical response of an orthotropic non-homogeneous elastic material. The present study has engineering applications in the fields of geophysical physics, structural elements, plasma physics, and the corresponding measurement techniques of magneto-elasticity. The analytical performances for the elastodynamic equations has been solved regarding displacements. The influences of the rotation, the magnetic field, the non-homogeneity based radial displacement and the corresponding stresses in an orthotropic material are investigated. The variations of the stresses, the displacement, and the perturbation magnetic field have been illustrated. The comparisons is performed using the previous solutions in the magnetic field absence, the non-homogeneity and the rotation.

Keywords: non-homogeneous; the rotation; orthotropic material; magnetic field; dynamical behavior

1. Introduction

Growing, attention is being enthusiastic to electromagnetic due to its various engineering applications based on the geophysical physics, magnetic structural elements, plasma physics, the elements of the magnetic storage along with the consistent measurement methods of magneto-elasticity as described in the Refs. (Kumar *et al.* 2012, Kakar and Kakar 2014, Dutta *et al.* 2016, Suman *et al.* 2017, Adda Bedia *et al.* 2019, Alimirzaei *et al.* 2019). The interaction of electromagnetic fields with the motion of a deformable solid is receiving much attention from many researchers. Among the many important problems considered in such studies, elastic wave propagation problems in the presence of a paper magnetic field were investigated. Large range of implementations of elastic materials, particularly in the industries of aerospace. The substantial practical interest to examine the elastodynamic conduct of such resources due to the impact of rapid use in the surface pressures. Furthermore, the aerospace industries, spherical constructions may also be applied in submarines, chemical based plants and nuclear reactors. Several papers have been performed to examine the static and dynamic responses of various elastic material differents types of loading as (Civalek 2009, Akgoz and Civalek 2011, Avcar 2016, Katariya *et al.* 2017, Avcar and Mohammed 2018, Javani et al 2019, Karami *et al.* 2019ab, Lata and Zakhmi 2019, Lakshmipathi and Vasudevan 2019, Lata and Kaur 2020, Rouabhia *et al.* 2020, Al-Furjan *et al.* 2020a, b, Alwabli *et al.* 2021, Mahmoud *et al.* 2020, Ramady *et al.* 2020, Al-Basyouni *et al.* 2020ab, Benmansour *et al.* 2019, Akbaş 2020, Tounsi *et al.* 2020, Asghar *et al.* 2020, Bekkaye *et al.* 2020, Cao *et al.* 2020, Dehsaraji *et al.* 2020, Khadimallah and Hussain 2020, Mehar *et al.* 2020). The elastodynamic use in anisotropic spheres is an essential issue of the transformed modern interest.

Mahmoud *et al.* (2011ab) examined the rotation impact on the plane vibrations in an isotropic transversely infinite hollow cylinder and the rotation impacts on the wave based motion over a cylindrical bore in the micropolar cubic porous crystals. Abd-Alla *et al.* (2013) investigated analytical performances for the electrostatic potential using the wave propagation systems in human wet long bones, investigated effects of inhomogeneity, gravity field and magnetic field on Rayleigh waves in an primarily stressed elastic half-space of orthotropic material subject to rotation. They examined the rotation impact and the gravity area on stonely waves in a non-homogeneous elastic orthotropic source. Abd-Alla and Mahmoud (2010 and 2013) worked on the problem of magneto-thermoelastic in non-homogeneous rotating orthotropic hollow cylindrical over the heat conduction of hyperbolic model as well as the radial vibrations problems in an isotropic non-homogeneity cylinder using the effect of the magnetic field and initial stress. The hollow spheres are normally handled in the engineering industries and the associated problems with free vibration represent one of the basic model in elastodynamics. Abd-Alla *et al.* (2011a, b) used the S-wave propagation with the non-homogeneous anisotropic initially stressed and incompressible source under the gravity field effects and the rotation impacts on an infinite non-homogeneous cylinder of orthotropic material. Moreover, they studied the effects of the non-homogeneity over wave

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propagation on an orthotropic elastic based media. Sofiyev and Karaca (2009) studied buckling and vibration of coated non-homogeneous orthotropic conical shells associated to the outward pressure. Argatov (2005) investigated the approximate outcomes of the contact axisymmetric model using an elastic sphere. The analytical performances for the vibrations of a orthotropic polarly Mindlin sectorial plate using the radial edges have been presented by Huang and Ho (2004). Mahmoud (2013) studied shear waves based on the half-space magneto-elastic over a non-homogeneous material anisotropic under the rotation impacts and considered the impacts of rotation and non-homogeneity over an infinite comprehensive medium of thermoelastic diffusion using a spherical cavity associated to the initial stress and magnetic field. Bahrami *et al.* (2013) investigated the wave propagation scheme for free vibrational study of annular circular and sectorial membranes. Kundu and Towfighi (2003) examined the elastic based wave propagation in the anisotropic curve spherical plates. Mahmoud and Abd-Alla (2012) used the analytical results of wave propagation over the non-homogeneous rotating orthotropic elastic source. Othmanand Fekry (2018) discussed the gravity and rotation effects on generalized thermo-viscoelastic medium. Theotokoglou and Stampouloglou (2008) examined the radially nonhomogeneous axisymmetric problem. Greenberg and Stavsky (2003) considered the radial laminated orthotropic hollow spheres vibrations.

This work performs an equation of an elastodynamic problem for an orthotropic non-homogeneous hollow sphere subject to the rotation, the magnetic field is solved regarding displacement potentials. Analytical performances for the elastodynamic equations is illustrated in detail for different cases with figures. The results designate the rotation effects, the magnetic field, the non-homogeneity over the radial displacement, the corresponding stresses, and the perturbation magnetic field are very pronounced.

2. Problem formulation

Consider a hollow sphere placed in an axial magnetic field in the start. For the spherically an orthotropic elastic sourcec, the spherical based coordinates $(\mathrm{r}, \theta, \varphi)$ are supportive with r radial, φ meridional and θ co-latitudinal. The basic form of spherical an orthotropic coincident with the origin. The radial displacement $\mathrm{U_r} = \mathrm{U_r}(\mathrm{r}, \mathrm{t})$ have been used is the r and t functions only, the displacement based on circumferential and the longitudinal are U_θ and U_φ that are independent of θ and φ. The dynamical equation in the $\boldsymbol{r}$ direction is given by:

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} - \frac{1}{r}\tau_{\theta\theta} - \frac{1}{r}\tau_{\varphi\varphi} + \rho(\mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{U})_r + \mu_e(\boldsymbol{J} \times \mathbf{H})_r = \rho\frac{\partial^2 U_r}{\partial t^2}, \tag{1}$$

where ρ, shows the material density of the sphere $\mathrm{f_r} = \mu_e(\boldsymbol{J} \times \mathbf{H})_r$ is defined as the component of Lorentz's force in the radial direction $(\boldsymbol{r})$, which may be written from the governing Maxwell equations:

$$\vec{J} = curl\ \boldsymbol{h}, \qquad -\mu_e\frac{\partial \vec{h}}{\partial t} = curl\ \mathbf{E}, \qquad div\ \boldsymbol{h} = 0, \tag{2a}$$

$$\begin{aligned}\boldsymbol{h} = curl\quad (\mathbf{U} \times \mathbf{H}) &= \left(0,0,-\frac{1}{r}H_0\frac{\partial(rU_r)}{\partial r}\right)\\ &= \left(0,0,-H_0\left(\frac{\partial U_r}{\partial r} + \frac{2U_r}{r}\right)\right)\end{aligned} \tag{2b}$$

$$\begin{gathered}h_\phi = -H_0\left(\frac{\partial U_r}{\partial r} + \frac{2U_r}{r}\right), \qquad f_r = \mu_e(\boldsymbol{J} \times \mathbf{H}) = \\ \mu_e H_0^2\frac{\partial}{\partial r}\left(\frac{\partial U_r}{\partial r} + \frac{2U_r}{r}\right), \quad \sigma_{rr} = \mu_e H_0^2\left(\frac{\partial U_r}{\partial r} + \frac{2U_r}{r}\right),\end{gathered} \tag{2c}$$

where σ_{rr} is the magnetic stress, $\mathbf{E}$ denotes the electric intensity, $\boldsymbol{h}$ shows the perturbed magnetic field, μ_e is the magnetic permeability, $\boldsymbol{J}$ is the electric density of current, $\mathbf{H}(0,0,\mathrm{H}_0)$ is the constant initial magnetic field, and $\mathbf{U} = (\mathrm{U_r}(\mathrm{r},\mathrm{t}),0,0)$ is the displacement vector.

$$\tau_{rr} = c_{11}\frac{\partial U_r}{\partial r} + c_{12}\frac{U_r}{r} + c_{13}\frac{U_r}{r}, \tag{3a}$$

$$\tau_{\theta\theta} = c_{12}\frac{\partial U_r}{\partial r} + c_{22}\frac{U_r}{r} + c_{23}\frac{U_r}{r}, \tag{3b}$$

$$\tau_{\varphi\varphi} = c_{13}\frac{\partial U_r}{\partial r} + c_{23}\frac{U_r}{r} + c_{33}\frac{U_r}{r}, \tag{3c}$$

$$\tau_{r\varphi} = \tau_{r\theta} = \tau_{\theta\varphi} = 0. \tag{3d}$$

Substituting from Eqs. (2) into Eq. (1), we get:

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} - \frac{1}{r}\tau_{\theta\theta} - \frac{1}{r}\tau_{\varphi\varphi} + \mu_e H_0^2\frac{\partial}{\partial r}\left(\frac{\partial U_r}{\partial r} + \frac{2U_r}{r}\right) = \rho\frac{\partial^2 U_r}{\partial t^2}. \tag{4}$$

We describe the elastic constants $\mathrm{c_{ij}}$, and the density ρ of non-homogeneous material as:

$$\begin{gathered}c_{ij} = \alpha_{ij}r^{2m}, \qquad \rho = \rho_0 r^{2m}, \quad \mu_e = \mu_e{}^*r^{2m},\\ i,j = 1,2,3,\end{gathered} \tag{5}$$

where α_{ij} and ρ_0 are the $\mathrm{c_{ij}}$ and ρ values in the case of homogeneous and m represent the the non-homogeneous term.

$$\begin{aligned}\tau_{rr} &= r^{(-1+2\ m)}\left[(\alpha_{12} + \alpha_{13})U_r + r\ (\alpha_{11})\frac{\partial U_r}{\partial r}\right],\\ \tau_{\theta\theta} &= r^{(-1+2\ m)}\left[(\alpha_{22} + \alpha_{23})U_r + r\ (\alpha_{12})\ \frac{\partial U_r}{\partial r}\right],\\ \tau_{\varphi\varphi} &= r^{(-1+2\ m)}\left[(\alpha_{23} + \alpha_{33})\ U_r + r\ (\alpha_{13})\ \frac{\partial U_r}{\partial r}\right].\end{aligned} \tag{6}$$

Substituting from Eqs. (6) and (5) into Eq. (4), then we obtain:

$$\begin{aligned}r^{-1+m}\Bigg[&[\alpha_{12} + 2m\alpha_{12} + \alpha_{13} + 2m\alpha_{13} - \alpha_{22} - 2\alpha_{23} - \alpha_{33}]U_r\\ &+ r(-r\rho_0\frac{\partial^2 U_r}{\partial t^2}\\ &+ 2[(1+m)(\alpha_{11}) + H_0^2\mu_0]\frac{\partial U_r}{\partial r} + r(\alpha_{11}\\ &+ H_0^2\mu_0)\frac{\partial^2 U_r}{\partial r^2})\Bigg] = 0.\end{aligned} \tag{7}$$

In the next part, the analytical radial vibrations results of a spherical elastic body of the orthotropic substantial is studied.

3. Solution of the problem

This section shows the analytical performances of the above model for a spherical domain of the inner and outer radious a and b , respectively using the different boundary conditions for the harmonic based vibrations. The solution of Eq. (7) is assumed in the following form:

$$U_r(r,t) = u_1(r)e^{-i\omega t}, \tag{8}$$

where ω is the natural frequency of the vibrations, t is the time.

Substituting from Eq. (8) into Eq. (7), one gets:

$$\begin{aligned} e^{-it\omega}r^{-1+m} \Bigg[& [\alpha_{12} + 2m\alpha_{12} + \alpha_{13} + 2m\alpha_{13} - \alpha_{22} - 2\alpha_{23} \\ & - \alpha_{33} + r^2\rho_0(\omega^2)]\mathrm{u}_1(r) \\ & + [2r(1+m)(\alpha_{11}) + 2rH_0^2\mathrm{\mu}_0]\frac{du_1}{dr} \\ & + r(\alpha_{11} + H_0^2\mathrm{\mu}_0)\frac{d^2u_1}{dr^2}) \Bigg] = 0, \end{aligned} \tag{9}$$

where $\mathrm{u}_1(r)$ is provided in the m terms because of the material non-homogeneity. Then one has

$$r^{-1+m}\left[\alpha_1 r^2\rho_0(\omega^2)u_1(r) + r\left(\alpha_2\frac{du_1}{dr} + r\alpha_3\frac{d^2u_1}{dr^2}\right)\right] = 0, \tag{10}$$

22here

$$\begin{aligned} &\alpha_1 = +\alpha_{12} + 2\ \mathrm{m}\ \alpha_{12} + \alpha_{13} + 2\ \mathrm{m}\ \alpha_{13} - \alpha_{22}, \\ &\alpha_2 = 2(1+\ \mathrm{m})(\alpha_{11}) + H_0^2\ \mu_0, \alpha_3 = (\alpha_{11} + H_0^2\ \mu_0), \end{aligned} \tag{11}$$

Eq. (11) shows the spherical Bessel's Eq. (9) and its generic form is given as:

$$\begin{aligned} u_1 = r^{\frac{1}{2}-\frac{\alpha_2}{2(\alpha_{11}+H_0^2\mu_0)}} \Bigg[& C_1J_{\frac{1}{2}+n}\left(\frac{\omega\sqrt{\alpha_1}\sqrt{\rho_0}}{\sqrt{\alpha_{11}+H_0^2\mu_0}}\ r\right) \\ & + C_2\ Y_{\frac{1}{2}+n}\left(\frac{\omega\sqrt{\alpha_1}\sqrt{\rho_0}}{\sqrt{\alpha_{11}+H_0^2\mu_0}}\ r\right)\Bigg]. \end{aligned} \tag{12}$$

Substituting from an above Eq. (12) into Eq. (8), one obtains the components of the displacements:

$$\begin{aligned} U_r = r^{\frac{1}{2}-\frac{\alpha_2}{2(\alpha_{11}+H_0^2\mu_0)}} \Bigg[& C_1J_{\frac{1}{2}+n}\left(\frac{\omega\sqrt{\alpha_1}\sqrt{\rho_0}}{\sqrt{\alpha_{11}+H_0^2\mu_0}}\ r\right) \\ & + C_2\ Y_{\frac{1}{2}+n}\left(\frac{\omega\sqrt{\alpha_1}\sqrt{\rho_0}}{\sqrt{\alpha_{11}+H_0^2\mu_0}}\ r\right)\Bigg] e^{-it\omega}, \end{aligned} \tag{13}$$

From Eq. (13), one get strain components in the form:

$$e_{rr} = \frac{\sqrt{\alpha_1}r^{\frac{-1}{2}-n}\sqrt{\rho_0}\omega r^{\frac{1}{2}-n}\left[C_1J_{-\frac{1}{2}+n}(k_2\ r) + C_2\ Y_{-\frac{1}{2}+n}(k_2\ r)\right]e^{-it\omega}}{\sqrt{\alpha_{11}+H_0^2\mu_0}} \tag{14}$$

$$e_{\theta\theta} = r^{\frac{-1}{2}-\mathrm{n}}\left[C_1\mathrm{J}_{\frac{1}{2}+\mathrm{n}}(k_2\ r) + C_2\ \mathrm{Y}_{\frac{1}{2}+\mathrm{n}}(k_2\ r)\right]e^{-it\omega}, \tag{15}$$

$$e_{\varphi\varphi} = r^{\frac{-1}{2}-n}\left[C_1J_{\frac{1}{2}+n}(k_2\ r) + C_2\ Y_{\frac{1}{2}+n}(k_2\ r)\right]e^{-it\omega}. \tag{16}$$

Substituting from an above Eq. (13) into Eq. (5), one obtains the stresses:

$$\begin{aligned} \tau_{rr} = \frac{1}{\sqrt{\alpha_{11}+H_0^2\mu_0}}r^{-\frac{1}{2}+2m-n}\Bigg[& \omega\sqrt{\alpha_1}r(\alpha_{11})\sqrt{\rho_0}J_{-\frac{1}{2}+n}(k_2\ r)C_1 \\ & + (\alpha_{12}+\alpha_{13})\sqrt{\alpha_{11}+H_0^2\mu_0}J_{\frac{1}{2}+n}(k_2\ r)C_1 \\ & + C_2\sqrt{\alpha_1}r(\alpha_{11})\omega\sqrt{\rho_0}Y_{-\frac{1}{2}+n}(k_2\ r) + C_2(\alpha_{12} \\ & + \alpha_{13})\sqrt{\alpha_{11}+H_0^2\mu_0}Y_{\frac{1}{2}+n}(k_2\ r)\Bigg]e^{-it\omega}, \end{aligned} \tag{17}$$

$$\begin{aligned} \tau_{\theta\theta} = \frac{1}{\sqrt{\alpha_{11}+H_0^2\mu_0}}r^{-\frac{1}{2}+2m-n}\Bigg[& \sqrt{\alpha_1}r(\alpha_{12})\omega\sqrt{\rho_0}J_{-\frac{1}{2}+n}(k_2\ r)C_1 \\ & + (\alpha_{22}+\alpha_{23})\sqrt{\alpha_{11}+H_0^2\mu_0}J_{\frac{1}{2}+n}(k_2\ r)C_1 \\ & + C_2\sqrt{\alpha_1}r(\alpha_{12})\omega\sqrt{\rho_0}Y_{-\frac{1}{2}+n}(k_2\ r) + C_2(\alpha_{22} \\ & + \alpha_{23})\sqrt{\alpha_{11}+H_0^2\mu_0}Y_{\frac{1}{2}+n}(k_2\ r)\Bigg]e^{-it\omega}, \end{aligned} \tag{18}$$

$$\begin{aligned} \tau_{\varphi\varphi} = \frac{1}{\sqrt{\alpha_{11}+H_0^2\mu_0}}r^{-\frac{1}{2}+2m-n}\Bigg[& \sqrt{\alpha_1}r(\alpha_{13})\sqrt{\rho_0}\omega J_{-\frac{1}{2}+n}(k_2\ r)C_1 \\ & + (\alpha_{23}+\alpha_{33})\sqrt{\alpha_{11}+H_0^2\mu_0}J_{\frac{1}{2}+n}(k_2\ r)C_1 \\ & + C_2\sqrt{\alpha_1}r(\alpha_{13})\sqrt{\rho_0}\omega Y_{-\frac{1}{2}+n}(k_2\ r) \\ & + C_2(\alpha_{23} \\ & + \alpha_{33})\sqrt{\alpha_{11}+H_0^2\mu_0}Y_{\frac{1}{2}+n}(k_2\ r)\Bigg]e^{-it\omega}. \end{aligned} \tag{19}$$

Substituting from an above Eq. (13) into the Eqs. (2), the component of perturbed magnetic field h_ϕ on the initial magnetic field and stress, σrr are in the form

$$\begin{aligned} h_\phi = -H_0\left[\frac{\sqrt{\alpha_1}r^{\frac{-1}{2}-n}\sqrt{\rho_0}\omega r^{\frac{1}{2}-n}\left[C_1J_{-\frac{1}{2}+n}(k_2\ r) + C_2\ Y_{-\frac{1}{2}+n}(k_2\ r)\right]e^{-it\omega}}{\sqrt{\alpha_{11}+H_0^2\mu_0}}\right] \\ - H_0r^{\frac{-1}{2}-n}\left(C_1J_{\frac{1}{2}+n}(k_2\ r) + C_2\ Y_{\frac{1}{2}+n}(k_2\ r)\right)e^{-it\omega}, \end{aligned} \tag{20}$$

where

$$n = \frac{\alpha_2}{2(\alpha_{11}+H_0^2\mathrm{\mu}_0)}, \qquad k_2 = \frac{\sqrt{\alpha_1}\sqrt{\rho_0}\omega}{\sqrt{\alpha_{11}+H_0^2\mathrm{\mu}_0}}, \tag{21}$$

where C_1, C_2 indicate the constants and $\mathrm{j_n(k_2r)}$ and $\mathrm{y_n(k_2r)}$ represent the Bessel's spherical function of the order *n* of first and second kind that is defined in Bessel's function form as:

$$\begin{aligned} j_n(k_2r) = \sqrt{\frac{\pi}{2k_2r}}J_{n+\frac{1}{2}}(k_2r), y_n(k_2r) \\ = \sqrt{\frac{\pi}{2k_2r}}Y_{n+\frac{1}{2}}(k_2r), \quad k_2 \text{ is constant.} \end{aligned} \tag{22}$$

The constants using the boundary conditions are determined as:

$$U_r(r,t) = 0, \qquad at \qquad r = a,$$
$$\tau_{rr}(r,t) + \sigma_{rr}(r,t) = -pe^{-i\omega t}, \qquad at \qquad r = b, \tag{23}$$

where p shows the constant, then from Eqs. (16), (17), and (20) we have:

$$C_1 = \frac{pb^{-m}\sqrt{p^* + \alpha_{11} + H_0^2\mu_0}Y_{n+\frac{1}{2}}(k_2a)}{J_{n+\frac{1}{2}}(k_2a)\left(Y_{n-\frac{1}{2}}(k_2b) + \left(\frac{c_1 - n + m}{b}\right)Y_{n+\frac{1}{2}}(k_2b)\right) - Y_{n+\frac{1}{2}}(k_2b)d_9}$$
$$C_2 = \frac{-pb^{-m}(p^* + \alpha_{11} + H_0^2\mu_0)J_{n-\frac{1}{2}}(k_2a)}{J_{n+\frac{1}{2}}(k_2a)\left(Y_{n-\frac{1}{2}}(k_2b) + \left(\frac{c_1 - n + m}{b}\right)Y_{n+\frac{1}{2}}(k_2b)\right) - Y_{n+\frac{1}{2}}(k_2b)d_{10}}$$
$$C_2 = \frac{-pb^{-m}(p^* + \alpha_{11} + H_0^2\mu_0)J_{n-\frac{1}{2}}(k_2a)}{J_{n+\frac{1}{2}}(k_2a)\left(Y_{n-\frac{1}{2}}(k_2b) + \left(\frac{c_1 - n + m}{b}\right)Y_{n+\frac{1}{2}}(k_2b)\right) - Y_{n+\frac{1}{2}}(k_2b)d_{10}} \tag{24}$$
$$d_9 = \left(J_{n-\frac{1}{2}}(k_2b) + \left(\frac{c_1 - n - m}{b}\right)J_{n+\frac{1}{2}}(k_2b)\right)$$
$$d_{10} = \left(J_{n-\frac{1}{2}}(k_2b) + \left(\frac{c_1 - n - m}{b}\right)J_{n+\frac{1}{2}}(k_2b)\right)$$

Substituting from Eqs. (24) into Eqs. (13), (17)-(20) then we have the corresponding radial displacement, the radial stress, the hoop stress, and the perturbation magnetic field in a radial non-homogenous material.

4. Discussion and numerical results

The numerical outcomes have been plotted graphically to display the distribution of the displacements, the stresses, and the perturbation magnetic field via the radial direction of the inhomogeneous orthotropic hollow sphere. The elastic values of the constants may be achieved from (Theotokoglou and Stampouloglou 2008) used as an example: $\alpha_{12} = 0.101$, $\alpha_{11} = 0.134$, $\alpha_{13} = 0.099$, $\alpha_{23} = 0.151$, $\alpha_{22} = 0.674$, $\alpha_{33} = 0.297$. Using the elastic constants values and b = 3 cm, a = 1 cm. The numerical simulations are obtained for the stress components and displacement in the r-direction at dissimilar rotation values in the cases of non-homogeneous substantial, an orthotropic material. Figs. 1-6 depict the variation of the displacement, the radial stress, the hoop stress, and the perturbation magnetic field together with the radial inhomogeneous hollow sphere direction with different results of the inhomogeneity exponent m, and the constant magnetic field is H_0. It is onserved in the Figs. that the radial based displacement satisfies the mechanical boundary conditions. Fig.1 shows the variation of the radial displacement, with increasing r, in case of non-homogeneity at different performances for inhomogeneity m=0.0, 0.5, 1.0, 1.5, $H_0=1.5\text{x}10^3$, Ω =1.5. The radial displacement satisfied the boundary conditions, in the case of an orthotropic inhomogeneous hollow sphere.

Fig. 2 shows the radial stress variations versus the radius r, in case of the non-homogeneity at dissimilar inhomogeneity for the values m=0.0, 0.5, 1.0, 1.5, $H_0=1.5\text{x}10^3$, Ω =1.5. It is clear from the obtained graphs that the effect of the radius r and magnetic field has a significant role in the variation of the radial stress τ_{rr}.

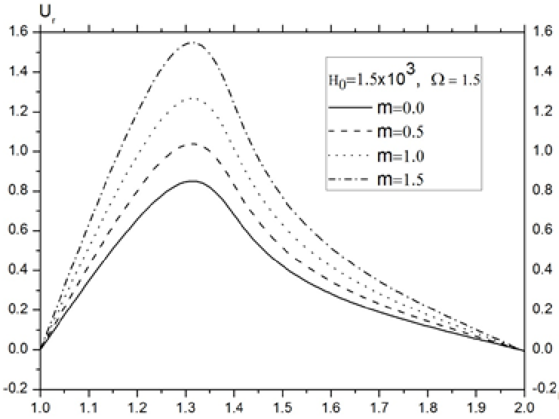

Fig. 1 The radial displacement U versus the radius r at dissimilar forms for non-homogeneity m, H_0 =1.5x10^3, Ω =1.5

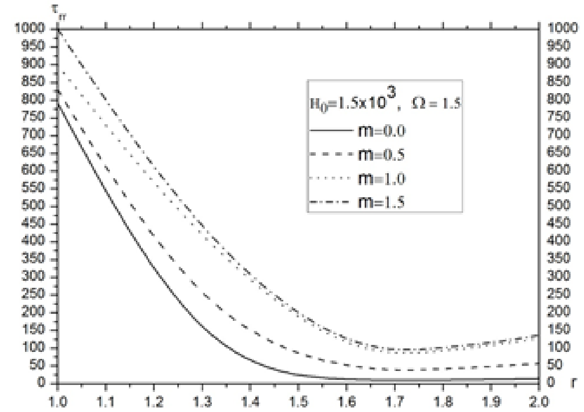

Fig. 2 Radial stress τ_{rr} variations versus the radius r at different performances for inhomogeneity m, H_0=1.5x10^3, Ω =1.5

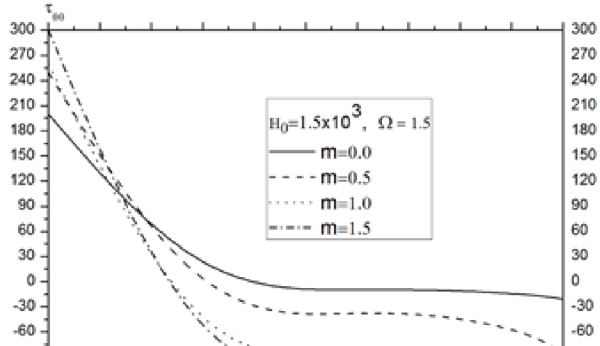

Fig. 3 Hoop stress variations $\tau_{\theta\theta}$ versus the radius r at different non-homogeneity values at m, H_0 =1.5x10^3, Ω =1.5

Fig. 3 shows the hoop stress variations versus radius r, in case of the inhomogeneity at dissimilar inhomogeneity values for m=0.0, 0.5, 1.0, 1.5, H_0 =1.5x10^3, Ω =1.5. We can conclude that the hoop stress is influenced by the values of the magnetic field and the radius r.

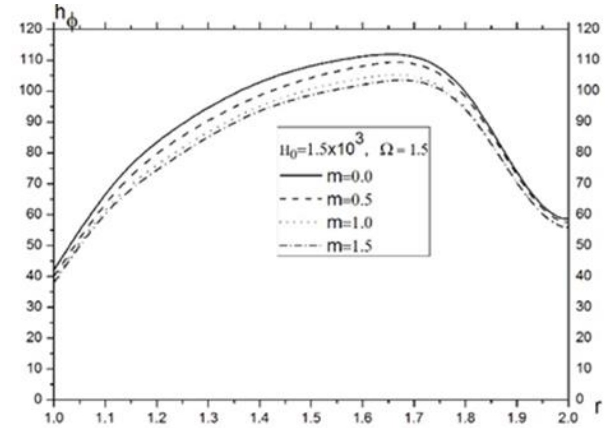

Fig. 4 Variation of the perturbation magnetic field h_ϕ versus the radius r at different values for non-homogeneity m, $H_0=1.5\text{x}10^3$, Ω =1.5

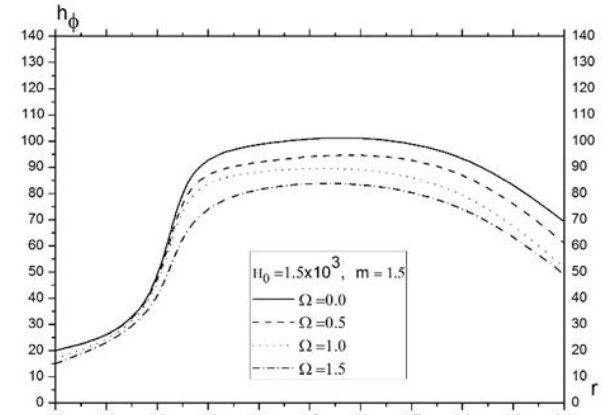

Fig. 5 Variation of the perturbation magnetic field h_ϕ versus the radius r at different values for rotation Ω, $H_0 = 1.5x10^3$, m = 1.5

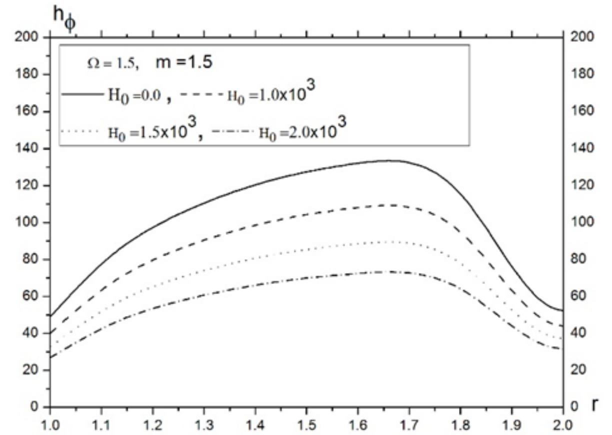

Fig. 6 Variation of the perturbation magnetic field h_ϕ versus the radius r at different values for magnetic field, Ω =1.5, m = 1.5

Fig. 4 authenticates variation of the perturbation magnetic field versus the radius r at different values for the inhomogeneity m, m=0.0, 0.5, 1.0, 1.5, $H_0=1.5\text{x}10^3$, Ω =1.5. Fig. 5 Shows the variation of the perturbation magnetic field h_ϕ versus the radius r at different values for rotation Ω =0.0, 0.5, 1.0, 1.5 , $H_0 = 1.5x10^3$, m = 1.5. Fig. 6 Shows the variation of the perturbation magnetic field h_ϕ versus the radius r at different values for magnetic field $H_0 = 0.0\ , 1.0x10^3,\ 1.5x10^3, 2.0x10^3$, Ω =1.5, m = 1.5. It is evident that the inhomogeneity , the magnetic field and the rotation have a significant influence more than the influence of the on displacement, the perturbation magnetic field, and the radial stress. Also, the influence of the magnetic field, the rotation, and the non-homogeneity based radial displacement, the stresses, and the perturbation magnetic field is pronounced. These outcomes are specific, for example, they can have different tendencies because of the dependence of the results on the mechanical of the material. Also, the influence of the non-homogeneity and the orthotropic property of the material is pronounced.

The results in this paper compared with previous results, in the absence of the magnetic field H_0=0.0, the rotation Ω =0.0, and non-homogeneity m=0.0, the results coincide with the results that have been obtained. These results are specific, for example, considered; one more case may have different trends because of the dependence of the results on the mechanical properties of the material as is displayed in Refs.(Civalek and Acar 2007, Mahmoud 2016, Islam *et al.* 2017, Panjehpour *et al.* 2018, Shinde *et al.* 2018, Abualnour *et al.* 2019, Ahmed *et al.* 2019, Azmi *et al.* 2019, Zouatnia and Hadji 2019, Barati 2019, Mehar *et al.* 2019, Hussain and Naeem 2019, Lata and Kaur 2019, Avcar, 2019, Yaylaci *et al.* 2019, Lago *et al.* 2020, Perera *et al.* 2020, Selmi 2020, Timesli 2020abc, Ahmed, *et al.* 2020, Merzoug *et al.* 2020, Yaylaci and Avcar 2020, Mishra *et al.* 2020, Rachedi *et al.* 2020, Karami and Janghorban 2020, Kaddari *et al.* 2020, Al-Furjan *et al.* 2020cd, Tahir *et al.* 2021, Bellifa *et al.* 2021, Arshid *et al.* 2021, Heidari *et al.* 2021, Al-Furjan *et al.* 2021ab, Hirane *et al.* 2021) that have more applications in the scientific and materials science and technical disciplines.

5. Conclusions

In the present study, the exact solutions for radial displacements stress of an orthotropic hollow sphere subjected to the magnetic field perturbation. The rotation and the effect of the non-homogeneity are also examined. The distribution of the displacements, stresses and perturbation of the magnetic field are plotted and discussed in detail under various effects. The present technique applies to other homogeneous materials. The numerical results are obtained and represented graphically. The results indicate that the effects of the magnetic field, the rotation, and the non-homogeneity on radial displacements, the stresses, and the perturbation magnetic field pronounced. The results have engineering applications in the fields of geophysical physics, structural elements, plasma physics, and the corresponding measurement techniques of magneto-elasticity.

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