

# Physical modelling of bending moments in single piles under combined loads in layered soil

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**Abstract.** Pile foundations are very important elements in the design of railways and bridges. In many projects, piles are under the combination of vertical and horizontal loads. In this regard, the maximum moment which is applied to the piles is a key factor for designers. Therefore, in this study, experimental investigations on loaded single piles in sandy soils with different energies in various conditions of loading and relative densities were performed. Nine physical modeling tests were created to evaluate the load-deflection response, bending moment and strain along with the driven aluminum model pile. The single piles were subjected under both horizontal and combination of horizontal and vertical loads. The dimensions of soil tank and model pile were designed by considering chamber size effects and internal scale effects. The results showed a significant difference in magnitude and shapes of bending when the model pile was subjected to combined loads with a relative density of 30% (loose) and 75% (dense). Additionally, the combined load test results demonstrated that the bending moments and lateral deflection of the pile head increase substantially in the presence of vertical loads. Finally, two computational models were proposed to estimate the maximum moment on the piles.

**Keywords:** physical modeling; combined load; lateral load; single pile; bending moment; layered sandy soil

## 1. Introduction

Piles (i.e., deep foundations) are widely used to support loads from various superstructures. These types of foundations are subjected to a combination of lateral (derived from seismic, wind and earth pressure), vertical loadings and moments due to the eccentricity of the vertical force, the fixity of the superstructure to the piles and the lateral loads' location on the pile (Khari *et al.* 2013a). However, piles are usually subjected to lateral and vertical loads; in fact, they are required to design for the bending moments (Garala and Madabhushi 2021). The bending moment of an along pile plays a very important role in pile designing.

This depends on pile installation, flexibility, and pile head conditions to attach to the superstructures (Nguyen 2016). On the other hand, passive soil resistance is very important in resisting the piles laterally loaded and the bending moments (Bellezza 2020). The length of pile shaft beyond which rotates and deflects under the lateral loads as active pile length, plays a significant role in determining the bending moments. The active length of pile (i.e., effective length of pile) is defined as the pile length below which the pile deflection can be neglected for practical purposes (Wang and Liao 1987). The active length

of pile depends on the pile's diameter, the Young's modulus of pile and soil. However, it is remarkable that the pile active length differs for the different lateral loadings e.g., dynamic and static (Karatzia and Mylonakis 2016, Stacul and Squeglia 2020). Therefore, the soil reaction around the piles can be highlighted as an effective parameter on the piles moment. Hence, soil-pile interaction phenomenon should be considered to analyze of the piles under the vertical and horizontal loadings. It is noticeable that soil-pile displacements and interaction between them play an essential role in the behavior of piles and it may induce deflections and bending moments which cause serviceability problems and even damage to the passive piles (Finno *et al.* 1991, Song *et al.* 2012, Nguyen *et al.* 2016, 2020, Alzabeebee and Chapman 202, Moayedi *et al.* 2020).

A number of approaches have been implemented in the analysis of pile behavior under pure lateral loads (Maheshwari *et al.* 2004, Nguyen *et al.* 2020, Souri *et al.* 2020) and a combination of vertical and horizontal loads (Hazzar 2017, Lu *et al.* 2018, Deb 2019). In addition, the boundary element and finite element methods (FEMs) (Bentley and El Naggar 2000) and tensile behavior of piles (Amiri *et al.* 2020, Zhou *et al.* 2020, Wang *et al.* 2020) were also investigated by various scholars in the area of piling and its possible issues. Similarly, the methods for determining the ultimate bearing capacity of columns like elements subjected to pure vertical loads have been evaluated/investigated by several researchers (Di Laora *et al.* 2019, Dehghanbandaki *et al.* 2020, Hoang *et al.* 2020,

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Chen *et al.* 2020, Li *et al.* 2020). Because of the soil nonlinearity behavior, the related response between the soil-pile is not linear so that each point of the pile length exits a nonlinear relationship (Wu and Finn 1997). To the design the laterally loaded piles, Amiri *et al.* (2020) estimated the geotechnical characteristics of kaolinite improved by the crushed glass and the pile's behavior floated and embedded in the mixed kaolinite. Their study introduced a new friction coefficient that affects the interaction between the pile and soft soil. Dehghanbandaki *et al.* (2020) investigated an experimental and numerical modeling of piles constructed with the mixing of cement and peat soil. They carried out the bearing capacity pile with the FEM and the empirical design methods. They failed the piles group via a rigid rectangular foundation by applying the stress rate of 1 kPa per minute. The ultimate bearing capacity of the group piles was the target of their study. The effects of driving and drilling on the piles' installation were studied by Kim *et al.* (2004). Their outcomes showed that the driving energy in the driven piles plays an important role in the bending moment when compared to the drilled piles. In another similar study, White *et al.* (2008) studied the bending moments in the composited piles subjected to a large scale loads. The results indicated that the maximum moments occur at depth ranges from 2.7 to 5.4 times pile diameters. In addition, the authors in the same study observed that the piles bending moment are time-dependent due to the soil movement under the combined loads. Karkush and Jafar (2015) investigated the pile's behavior adjacent to surcharge loads. Different intensities of surcharge were used in their study on both flexible and rigid piles. Their results showed that the maximum bending moments were observed in different locations with the increasing of the distance between the pile and the surcharge.

In light of the above discussion, many studies have been performed for determining the pile behavior subjected to lateral and/or vertical loadings (Khari *et al.* 2019b). However, the influence of combined loads has not been investigated by the numerical methods. In essence, the pile response subjected to combined loads can be significantly different from that under lateral or vertical loads owing to the vertical and lateral load interaction. In addition, the mentioned approaches indicate that the analysis of piles involving nonlinear properties of soil is more desirable because the surrounding soil in real fields behave nonlinearly and are more significant near the top part of the piles. In other words, the piles are embedded in layered soils in real conditions (Harandizadeh *et al.* 2019). To date, little information is available in the literature about the bending moments to model piles embedded in layered cohesion-less soil under combined loads. Furthermore, investigating the effects of some parameters such as soil density, the driving energy and vertical loads on the behavior of combined loading pile is uncommon compared to that of horizontally loaded piles in the single-layer soil.

In this study, a series of tests were planned and investigated on the single piles to fill this gap. The mentioned tests were performed under various conditions of

vertical loads on the laterally loaded pile. Furthermore, the emphasis was focused on the pile head deflection and bending moment due to the combined loads (i.e., vertical and lateral) in layered cohesion-less soil in both loose and dense sand. The remaining parts of this study are organized as follows. Section 2 describes the settings of the test conducted, properties of the used material and the procedure of tests. Then, in Section 3, the results obtained by the tests on the bending moments in the piles will be discussed in detail. Eventually, the conclusion remarks and study limitations will be given in the last section (i.e., Section 4).

## 2. Test set-up and procedures

### Test Set-up

The schematic diagram and real experimental of the tests are shown in Fig 1. The soil tank used was a box with dimensions of 900 mm, 700 mm and 650 mm in length, width and height, respectively. The box boundary effects were minimized with the extension of the size of the soil tank up to  $8-12D$  and  $3-4D$  ( $D$  = pile diameter) in the direction and perpendicular to lateral load, respectively. In addition, similar to the study of Khari *et al.* (2014b), the soil thickness was chosen and kept at least 6 times of pile diameter below the tip of the pile. The lateral load was applied to the model pile using disc weights placed on a loading pan connected to a flexible steel wire, strung over a pulley supported by a loading platform. As shown in Fig 1, the pile head was connected to the guide rod and hammer which was driven into the prepared sample by hammer blows released from a certain height. The hammer weight and drop height were adjustable to achieve the desired driving energy. The extra details regarding the physical modeling tests can be found from Fig. 1.

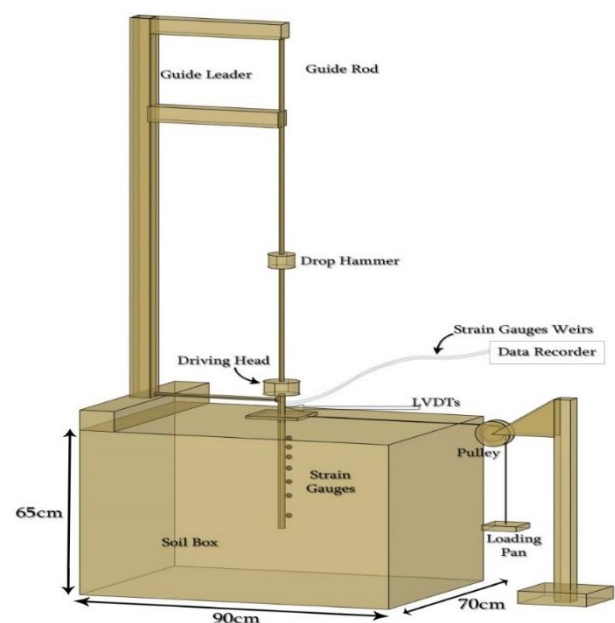


Fig. 1 Schematic diagram of experimental set up in this study

Table 1 Basic soil properties used in physical modeling tests

Items	Results
Medium diameter ( $D_{50}$ )	0.53(mm)
Uniformity coefficient ( $C_u$ ),	0.17
Minimum unit weights	16.83 (kN/m <sup>3</sup> )
Maximum unit weights	13.74 (kN/m <sup>3</sup> )

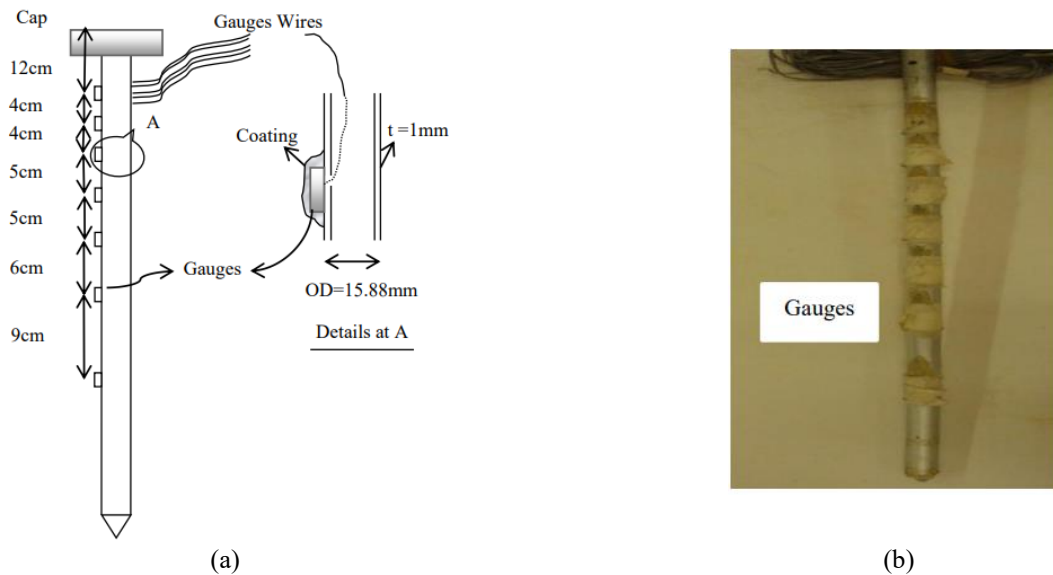


Fig. 2 Schematic (a) and actual (b) model pile instrumented

### Properties of used soil

The tests were conducted on dried river sand at the laboratory temperature. The clean and fine to medium sand was classified as *SP*, according to the Unified Soil Classification System (*USCS*). Soil properties including relative density, classification and unit weight were carried out based on ASTM. The soil properties are shown in Table 1. For all tests, the sand was placed with relative densities ( $D_r$ ) of 30% and 75% for loose and dense sand. The sand samples were reconstructed by using the raining technique using the newly designed mobile pluviator was utilized in this research. Falling height and rate of pouring had the opposite effect on the relative density. To achieve dense and loose sand samples, two selected patterns consisted of 11 holes (diameter = 18 mm) and 16 holes (diameter = 10 mm) distributed evenly in the shutter plate were used. The height of free fall corresponding to the  $D_r$  of 30% and 75% was found to be 700 mm for all tests. Detailed descriptions of the newly designed mobile pluviator can be found in Khari *et al.* (2019a, b).

### Model pile and instrumentation

A model pile with a hollow circular section was made of aluminum alloy (Young's modulus of 69.8GPa), with an outer diameter ( $D$ ) of 15.88 mm, wall thickness of 1 mm (Fig 2). The dimensions of pile were found based on the dimension analysis (Khari *et al.* 2013b). The model pile was subjected to combined loads embedded ( $l$ ) 580 mm deep with the head located about 40 mm above the ground surface model. It is worth noting that the model piles had  $l/D > 35$  which was known as a flexible pile at all

consistencies of sand considered (Khari *et al.* 2013a). Regarding the sand selected, the pile diameter to soil particle size ( $D_{max}$ ) ratio was chosen as 16.38. This ratio was greater than the suggested values in the literature; therefore, the particle size effect was not expected.

A typical test included nine instruments, seven electrical strain gauges (having a resistance of 350  $\Omega$ ) attached to pile in seven levels to measure the strain along the model pile and two linear variable differential transducers (*LVDTs*) to measure the head pile deflection. As shown in Fig. 2, the strain gauges were fixed along the outer surface of the model pile so that the distances of the gauges were closer with larger spacing near the ground surface and towards the pile tip, respectively. To prevent damage from direct contact between the gauges and sand during pile driving and loading, the gauges were coated with adhesive tape. In the pilot tests, the gauge constants were calculated for every gauge separately. The constants were calculated based on the relationship between moment and measured strain obtained by conducting a bending test on the model pile. All of the instruments were connected to the data logger and reading of 0.00 was set to the indicators.

### Tests carried out

The soil tank was filled using the mobile pluviator based on the desirable relative density in each depth. The surface of the model ground was leveled so that the final height of the reconstructed samples was 650 mm. At least 24 hours elapsed before applying any load to the model pile. After finishing the sample preparation, the model pile was driven in the center of the soil tank by means of the driving

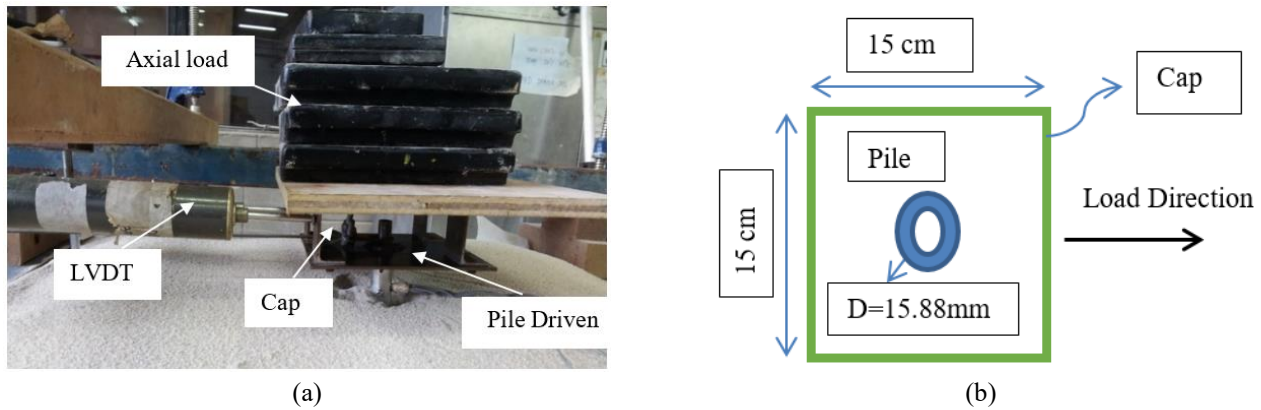


Fig. 3 Axial loads on pile cap (a) and schematic of cap dimensions (b)

Table 2 Test conditions and relevant parameters

Test	ED (J)	Layered Soil	Relative density ( $D_r$ )	V- load (Kg)
TS-1	2	1	Dense; $D_r=75\%$	$V_h$
TS-2	4	1	Dense; $D_r=75\%$	$V_h$
TS-3	2	1	Loose; $D_r=30\%$	$V_h$
TS-4	4	1	Loose; $D_r=30\%$	$V_h$
TS-5	2	2	<ul style="list-style-type: none"> <li>• Top layer (15 cm) # Loose</li> <li>• Bottom Layer (50 cm) # Dense</li> </ul>	$V_h$
TS-6	4	2	<ul style="list-style-type: none"> <li>• Top layer (15 cm) # Loose</li> <li>• Bottom Layer (50 cm) # Dense</li> </ul>	$V_h$
TS-7	4	2	<ul style="list-style-type: none"> <li>• Top layer (15cm) # Loose</li> <li>• Bottom Layer (50 cm) # Dense</li> </ul>	$V_f=20\% V_{ult}$
TS-8	2	2	<ul style="list-style-type: none"> <li>• Top layer (15 cm) # Loose</li> <li>• Bottom layer (50 cm) # Dense</li> </ul>	$V_f=40\% V_{ult}$
TS-9	4	2	<ul style="list-style-type: none"> <li>• Top layer (15 cm) # Loose</li> <li>• Bottom layer (50 cm) # Dense</li> </ul>	$V_f=40\% V_{ult}$

**Note:**

$V_h$  = Applied horizontal load

$V_f$  = Applied vertical load

$V_{ult}$  = Ultimate load (Ultimate bearing capacity of the pile)

ED = Driving energy

system. The tests were conducted on the model pile with two different driving energies and layered sandy soil with different relative densities ( $D_r$ ). To achieve the two driving energies ( $E_D$ ) of 2 J and 4 J, the drop hammer (weight=0.825 kg) was dropped from 0.25 m and 0.50 m, respectively. The combined load tests were performed with vertical loads ( $V_f$ ) equal to 0% (pure lateral load), 20% and 40% of the ultimate vertical load ( $V_{ult}$ ) and each test when the entire vertical loads applied then lateral loads applied by gradually increasing the dead weight in the pan. A series of tests were performed on the single pile connected by a rigid cap with fixed heads to investigate the influence of layered soil, driving energy, soil density and combined loads (Table 2). The recorded strain values were converted to moment by multiplying strain by the previously estimated gauge constants. Fig. 3 shows the details of loading conditions.

### 3. Results and discussion

#### 3.1 Effects of relative density on strains

In order to evaluate the influence of the relative density

of sand samples prepared on the strains along the pile shaft, the tests have been conducted with the different levels of the loads applied at the pile cap. In addition to achieve the considered densities, the tests have been conducted on the two densities of 30% and 75% as loose and dense sand, respectively. The measured strains by each gauge attached on the model pile under lateral loadings are shown in Fig. 4. The results obtained from the test TS1 (one layer dense of soil) were compared with the test TS-5 carried out on the two-layered sample prepared. According to Fig. 5, the strains showed the same almost trend regarding the position of the gauges. Furthermore, the highest strain values were measured in the TS-1 and TS-5 tests in G2 and G4, respectively. In fact, when the sample was prepared in two layers of loose and dense, the maximum strain was obtained at depth of  $14D$  in TS-5 and  $10D$  for TS-1. It may be happened due to the loose layer placed at the top of the ground surface. While the shape of the curve in TS-1 was almost the same, the trend curves were not the same in the TS-5 test because of the different soil resistance in the two layers of the sample. In addition, it can be seen that the strains in the gauges G7 and G6 in comparing with the

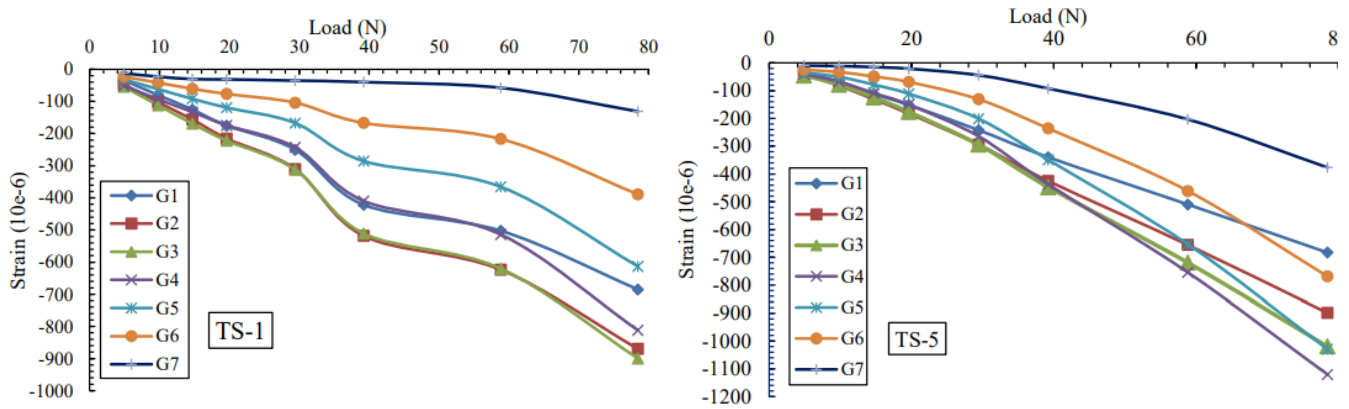


Fig. 4 Strain versus load in the TS-1 and TS-5 tests

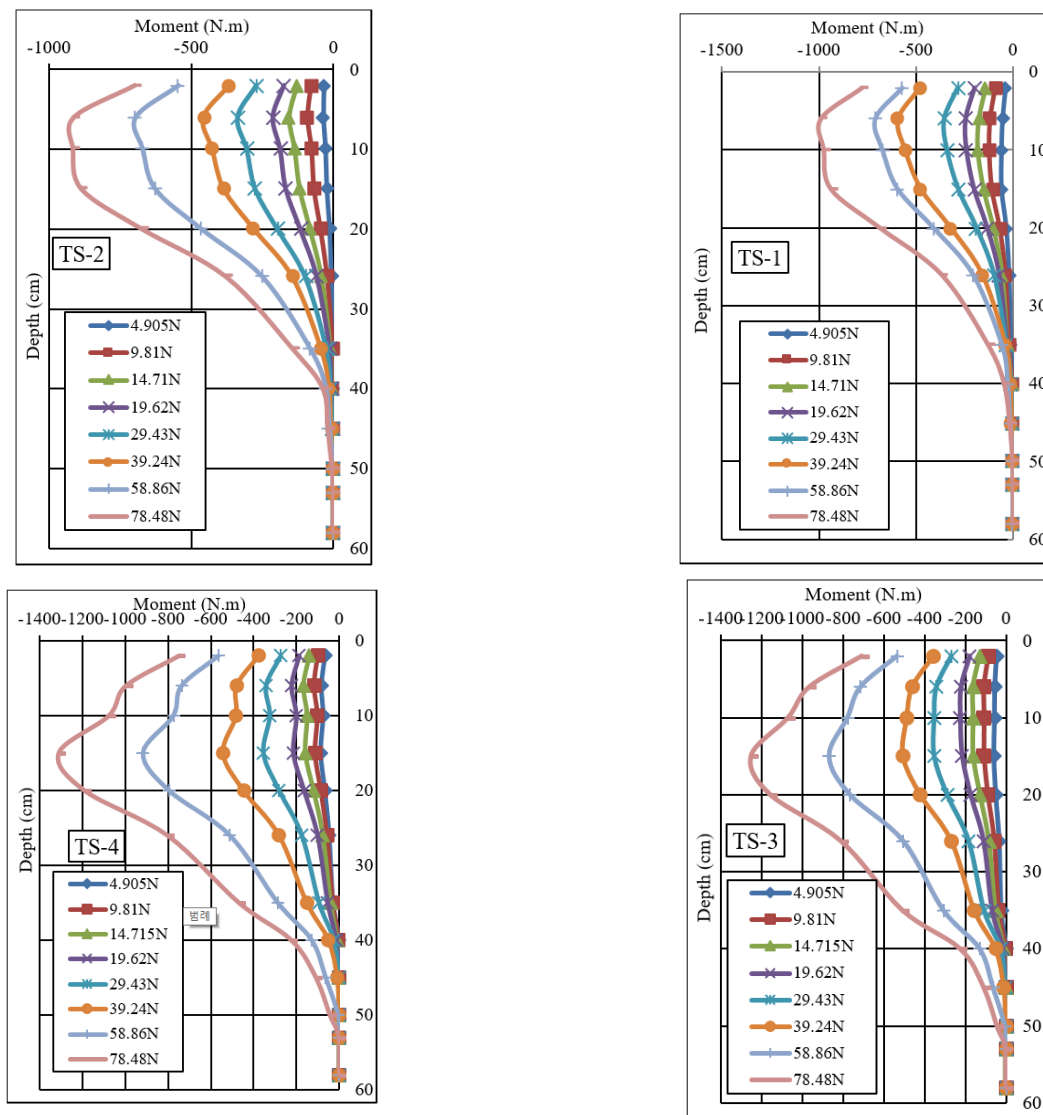


Fig. 5 Bending moment profiles in the one-layer sand; TS-1, TS-2, TS-3, TS-4

gauges closer to the sample surface were decreased as the vertical load's intensity increased. It is an expected finding that the strains in the G6 and G7 of the TS-5 were not following the previous trend in the TS-1. The outcomes show that reducing the soil strength caused a reduction in the bending moments in the single pile under the different

driving energies.

### 3.2 Effects of driving energy on the bending moments

Fig. 5 illustrates the variation of bending moment along with a pile in the TS-1, TS-2, TS-3 and TS-4 tests under the

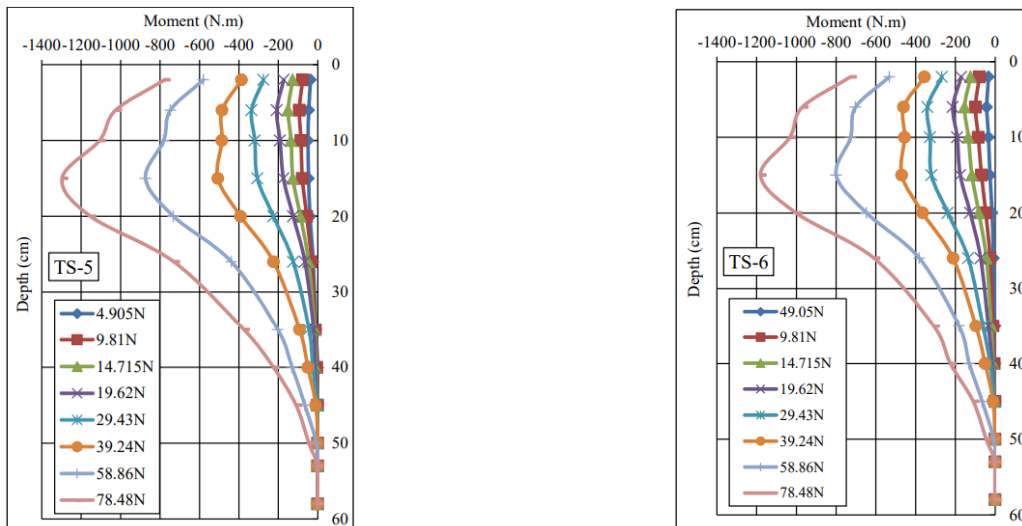
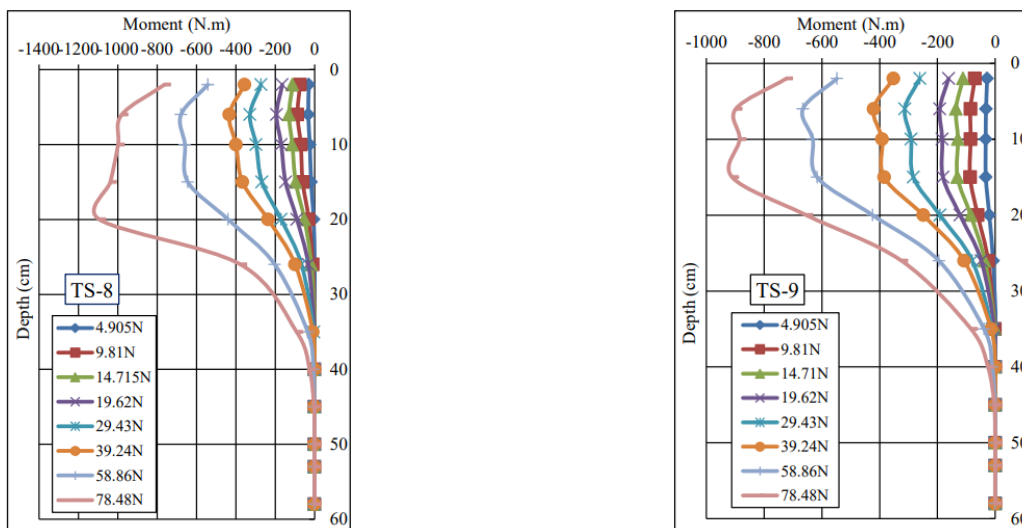


Fig. 6 Bending moment profiles in the TS-5 and TS-6 tests

Fig. 7 Bending moment profiles in the two layer sand and  $V_f = 40\% V_{ulti}$ ; TS-8, TS-9

pure lateral loadings in one-layer sandy soil with different driving energies of 2J and 4 J. As shown in the Fig. 5, the maximum values of bending moments in the TS-1 and TS-2 test were equal to 720 N.m and 690 N.m, respectively, at a depth of  $3.8D$  under the lateral load of 58.86N due to the densification of the ground surrounding the pile and the increase of the effective horizontal soil stress with pile driving. However, the maximum bending moment 870 N.m and 1520N.m occurred at a depth of  $9.5D$  in the TS-3 and TS-4 test at the same lateral load, respectively. Thus, the maximum bending moments were increased by about 25% when the relative density of sand was decreased from 75% to 30%. While the maximum bending moments were occurred at the same depth with increasing of the driving energy of 2J to 4J, but this depth was increased with the change of the relative density. According to the results, it is mentionable that the influence of relative density on the moments is more highlight than the  $E_D$ . Due to the increasing relative density in soil prepared and the surrounded soil pressure around the pile raised that which changes the active length of pile. This rise in relative

density caused an increase in total interaction between soil and pile which resulted in higher bending moments. In additional, a comparison of the TS-1 and TS-3 results suggest that the active length pile was increased in the loose sample under the same  $E_D$ .

### 3.3 Effects of layered soil on the bending moments

The interaction of the pile and the surrounded soil may involve soil-pile interaction when a pile is subjected to the driving energy and combined loads. According to the soil-pile phenomenon, Fig. 6 presents the bending moment profiles of the model piles driven in the two-layer sand. The piles were driven into the soil under the two different  $E_D$  of 2J and 4J. The moments increased with intensity increasing of the lateral loads with the disc weights. In addition, for both of the tests, the maximum bending moment occurred at almost the same depths, which was about  $9.5D$  below the ground surface. However, the values of bending moment in the TS-5 test were higher than the TS-6 test when the driving energy increased from 2J to 4J. The increasing of

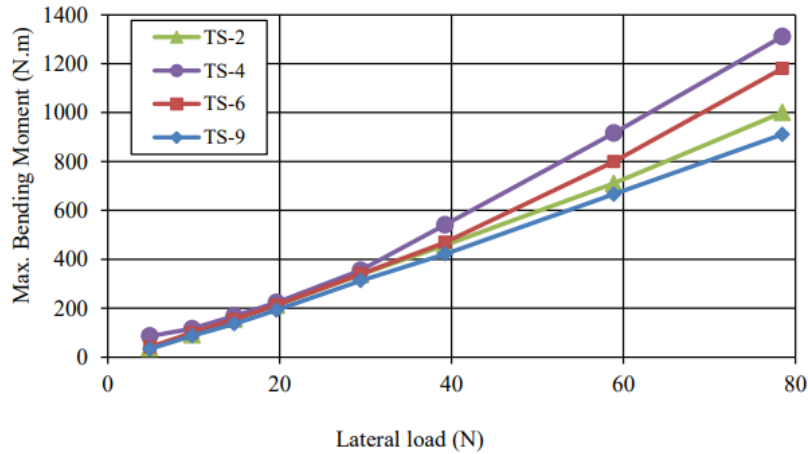


Fig. 8 Maximum bending moment of pile with  $E_D=2J$ ; TS-2, TS-4, TS-6, TS-9

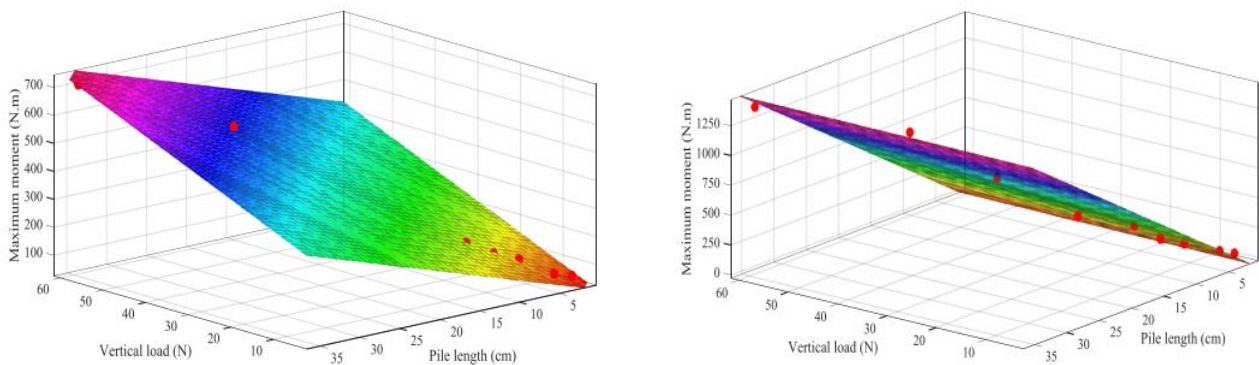


Fig. 9 Decision surfaces for maximum moments for the single piles (TS-1 & TS-6)

moments was more significant in higher lateral loads. This difference can be caused due to the increase of the surrounding stress around the model pile. The obtained results in the TS-1 and TS-5 indicates that maximum moments along the pile were occurred in 3.8D and 9.5D, respectively. In other words, the layered soil and the higher density caused the maximum moment was at the far ground surface with the same horizontal loads.

### 3.4 Effects of combined loads on the bending moments

As mentioned previously, available experimental results on single piles subjected to lateral loads are focused on vertical or axial loads. Base on the lack of the pile's behavior under the two vertically and horizontally loads, TS-7, TS-8, and TS9 were conducted with the combined loads and the bending moments recorded. The vertical load was intensity increased in the 8 steps and the axial load was constant during each of the tests based on the ultimate load's pile. The moment profiles of the model pile in TS-8 and TS-9 is illustrated in Fig. 7. Since the tests were conducted in the same layered sandy soil (Dense and Loose) and subjected to the same vertical load. Therefore, it is expected that the test with higher ED shows more bending moments. As can be seen from Fig. 7, the maximum values of the moment were decreased when the driving energy was increased from 2J to 4J. While the

moments increased with the increasing of the lateral loads in both of the tests, the location of the maximum values was decreased toward the below ground surface, especially in TS-8. The rate of the measured moments reduced had the same beyond of the ground surface at all of the vertical loads in each test but the maximum moments' depth differed in the TS-8 and TS-9 owing to the axial loads applied to the pile cap.

### 3.5 Maximum bending moment

The maximum bending moment values-lateral load for the model pile-driven with  $E_D=2J$  is presented in Fig. 8. The Figure shows that for any given lateral load, the maximum bending moments had the highest values in the one loose layer of sandy soil without any vertical load (TS-4). However, in the two-layer sample of sand with a vertical load of  $40\%V_{ult}$ , the maximum bending moments decreased by about 20-50%. In addition, the obtained curve of TS-9 shows a linear criterion in comparison with the TS-6 curve which may be due to the increase of the vertical load. The general trend of bending moment-depth of this study is consistent with the trends obtained by Lu and Zhang (2018). Several researchers have proposed computational estimation models in geotechnical engineering, especially in behaviour of the piles (Chu *et al.* 2008, Jebur *et al.* 2018, Eslami *et al.* 2011, Medina *et al.* 2019, Zhang and Goh 2016, Nejad *et al.* 2009, Nagao *et al.* 2020, Khari *et al.*

2019a, Harandizadeh *et al.* 2019). In this study, two decision surfaces for the estimation of the maximum moment of piles are proposed. The results of the experimental tests as TS-1 (pile in 1 layered soil under just horizontal loading) and TS-7 (pile in 2 layered soil under the combination of vertical and horizontal loading) were used for each vertical load for decision surfaces as shown in Fig 9. Due to different behaviour of pile in one layer and two layers, in this study we have developed two computational estimation models. Therefore, Model-1 for pile in 1 layer soil and Model-2 for pile in two layers were developed. In this figure, the pile lengths and the applied vertical loads to the piles are considered as the input parameters of the computational model while the maximum moment is considered as the output of the model. Curve fitting APP in MATLAB software was utilized for model developments. Based on the Fig. 9, an increase in both pile length and vertical load (TS-1 and TS-7) caused an increase in the maximum moment of the piles. To be mentioned that, the estimation of maximum bending moment using the developed models is limited to geotechnical properties of used sand. Eqs. (1) and (2) show the details of the developed models.

Model-1:

$$\text{Maximum moment} = p00 + p10 * (\text{pile length}) + p01 * (\text{Vertical load})$$

Eq. (1)

Coefficients (with 95% confidence bounds):

$$\begin{aligned} p00 &= -7.056 (-62.41, 48.3) \\ p10 &= 9.622 (-15.09, 34.34) \\ p01 &= 6.881 (-8.344, 22.11) \end{aligned}$$

RSME = 38

R-square = 0.97

Model-2:

$$\text{Maximum moment} = p00 + p10 * (\text{pile length}) + p01 * (\text{Vertical load})$$

Eq (2)

Coefficients (with 95% confidence bounds):

$$\begin{aligned} p00 &= -99.57 (-213.5, 14.34) \\ p10 &= 44.12 (-6.739, 94.98) \\ p01 &= 0.2624 (-31.07, 31.59) \end{aligned}$$

Goodness of fit:

R-square: 0.9806

Adjusted R-square: 0.9741

RMSE: 42.22

#### 4. Conclusions

To the author's knowledge, the determination of bending moments in model piles embedded in layered cohesionless soil under combined loads have not investigated in previous researches. Thus, in the present study, a series of model tests were performed on single piles to determine the pile response subjected to lateral and vertical loads in layered dry sandy soil with a relative density of 30% (loose) and 75% (dense). The dimensions of the soil tank and model pile were designed by considering chamber size effects and internal scale effects. The following conclusions are drawn from the experimental works.

- The lateral behavior of piles under combined loads is

influenced by the vertical loads in the layered sandy soils.

- An increase in the vertical loads decreased the lateral deflection which may be because of an increase of fixity pile head in the presence of vertical loads.

- The location of the maximum bending moment was not changed when the model pile was subjected to the lateral load with the different driving energy, however, the moment values increased.

- The presence of the vertical load significantly decreases the lateral resistance of the driven pile.

- The lateral deflection decreases with the increasing of driving energy in one layer of sandy soil owing to the densification of the surrounding ground and the increase of the effective horizontal soil stress with the pile driving.

- Finally, two computational estimation models were proposed to estimate the maximum moment of the piles.

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