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Modified Lysmer's analog model for two dimensional mat settlements under vertically uniform load

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Abstract. A two dimensional model of linearly elastic soil spring used for the settlement analysis of the flexible mat foundation is suggested in this study. The spring constants of the soils underneath the foundation were modeled assuming uniformly vertical load applied onto the foundation. The soil spring constants were back calculated using the three-dimensional finite element analysis with Midas GTS NX program. Variation of the soil spring constants was modeled as a two-dimensional polynomial function in terms of the normalized spatial distances between the center of foundation and the analytical points. The Lysmer's analog spring for soils underneath the rigid foundation was adopted and calibrated for the flexible foundation. For validations, the newly proposed soil spring model was incorporated into a two dimensional finite difference analysis for a square mat foundation at the surface of an elastic half-space consisting of soft clays. Comparative study was made for elastic soils where the shear wave velocity is 120~180 m/s and the Poisson's ratio varies at 0.3~0.5. The resulting foundation settlements from the two dimensional finite difference analysis with the proposed soil springs were found in good agreement with those obtained directly from three dimensional finite element analyses. Details of the applications and limitations of the modified Lysmer's analog springs were discussed in this study.

Keywords: soil stiffness, mat foundation, flexural displacement, modified Lysmer's analog model

1. Introduction

The settlements of a mat foundation under vertical loads can be estimated by a number of analyses suggested in the past years. In general, the settlements of a mat foundation can be simulated using either analytical formulas or numerical analyses. The analytical formula is relatively simple but it has the limit to model the variabilities of the soil profile and the foundation settlements. As to the numerical analysis, the available models can be one-dimensional (1D) beam on elastic foundation (i.e., Winkler foundation), or two-dimensional (2D) mat underlain by a set of soil springs. Of course, the three-dimensional (3D) structure-foundation-soil system can also be modeled as a continuum. In order to monitor 2D variations of the foundation displacements, the two-dimensional mat on soil springs is the one that has been adopted mostly for mat foundation and piled raft foundation analyses. Comparisons of various analyses can be found in Omer and Arbabi (2015). Discussions of the soil spring models used for the mat foundation or piled raft foundation have been reported by Civalek and Ozturk (2010), Suchart *et al*. (2012), Bouzid *et al*. (2013), Lee *et al*. (2015), Zhang *et al*. (2016), Jeong *et al*. (2017) and Hazzar *et al*. (2019).

It was pointed out that the Pasternak model (1954) is a well-used one out of the two-parameter foundation models. If a mat foundation has smaller dimensions of length and width but relatively large thickness or the foundation is quite rigid resting on soft soils, the foundation can be treated as a rigid structure. No flexural deflections will occur in such case. If a mat has large dimensions on length and width, and it is very thin; the rigidity of the foundation would be relatively small, the foundation will exert flexural deformations, differential settlements of the foundation will occur accordingly. Note that the rigidity of the mat foundation, K_r can be computed using following equation (ACI Committee 336 1988).

$$K_r = \frac{EI_b}{E_s B^3} \tag{1}$$

where E is the modulus of elasticity of the foundation, B is the foundation width, I_b is the moment of inertia of the foundation per unit length perpendicular to B, E_s is the modulus of elasticity of the soil. According to ACI Committee 336 (1988), if the foundation has $K_r \geq 0.5$, it can be treated as rigid foundation. For foundation with $K_r < 0.5$, it is a flexible foundation.

Fig. 1(a) depicts a rigid mat in clays where the vertically

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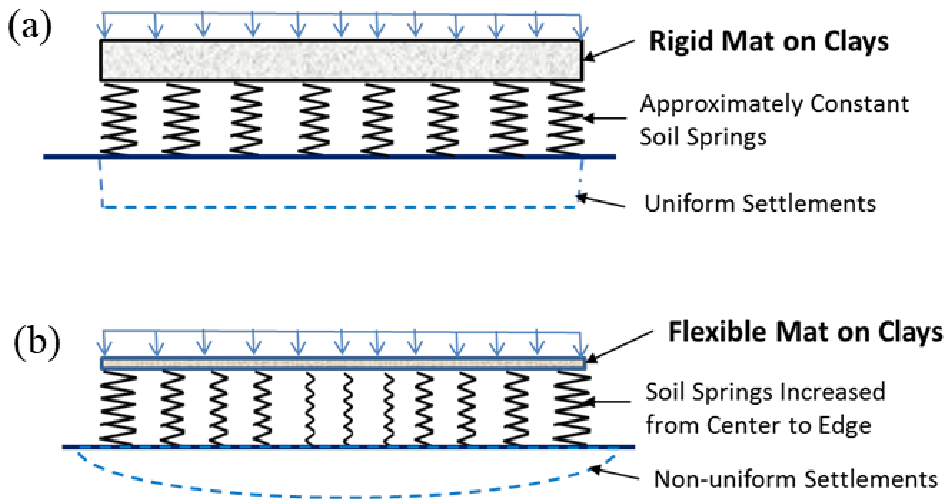

Fig. 1 1D Beam model on a set of soil springs for: (a) rigid and (b) flexible foundations on clayey soils

uniform load is applied on top of the foundation. As it has been known, uniform settlements will result in such foundation. The contact pressures near the edge would be found larger than that at the center (Barnes, 1995). If the Winkler foundation was considered (see Fig. 1(a)), the total spring constant of the soils can be approximated by the Lysmer's analog equation (Lysmer and Richart, 1966). On the other hand, if a flexible mat foundation in clays was applied with the same load, non-uniform settlements of the foundation would be expected. The contact pressures of the soils became more complicated. Distributions of the contact pressures will vary with the soil stiffness. The author (Chang *et al.* 2021) found that the contact pressures are smaller at the center when the clays are relatively soft, however as the soil stiffness increases, the contact pressures will become larger at the center. The contact pressures of a flexible foundation in stiff clays were typically noticed in the past. Although the contact pressures can be varied, the corresponding soil spring constants were found always larger at the edge of flexible foundation. Therefore, variations of the interactive soil springs of a flexible foundation in clays can be shown in Fig. 1(b).

In such case, various soil spring models have been suggested to model the foundation displacements (Teodoru and Musat 2010, Worku and Seid 2020). A recent discussion on spatial variations of the soil springs of a rectangular mat can be found in Loukidis and Tamiolakis (2017). Three-dimensional finite element analysis was adopted to analyze the foundation displacements due to a set of column loads. Polynomial function was then suggested to model the spring stiffness. Similarly, Chang *et al.* (2019) also suggested that the soil springs underneath a square mat can be back-calculated and modeled through three-dimensional finite element analysis on elastic soils. Jeong *et al.* (2017) has conducted three dimensional FEM analyses to show that the shape, width and thickness of the mat will affect the foundation behaviors located in the ground of different soils. They suggested that non-uniform subgrade reaction modulus should be developed for the task problem. Because the soil reaction pressures and the soil stiffness could be very different for rigid and flexible foundations, to develop a feasible soil model for the reactions underneath the flexible foundation is important to the design engineers.

Determination of the subgrade reaction modulus of the foundation is not an easy task, since it is not a unique fundamental property (Becker and Moore 2006). Many researchers (Vesic 1961, Meyerhof and Baikie 1963, Horvath 1989, Daloglu and Vallabhan 2000, Elachachi *et al.* 2004, Ziaie *et al.* 2006, Farouk and Farouk 2016, Dungca *et al.* 2018) have suggested different but suitable expressions. For example, one can refer to Richart *et al.* (1970) on soil spring constants used for rigid foundation at the surface of an elastic half-space. The one proposed for soil spring constant, K_z by Lysmer and Richart (1966) for the rigid foundation under the vertical load can be written as,

$$K_z = \frac{4G_s r_0}{1-\upsilon} \tag{2}$$

where G_s is the shear modulus of the soil, υ is the Poisson's ratio of the soil, and r_o is the equivalent radius of the foundation. This equation has been well known as the Lysmer's analog model.

Alternate stiffness model of the foundation soils can be found in the study of Novak and Beredugo (1972). The dynamic stiffness of the soils asides the foundation was also suggested rather than those presented for soils underneath the foundation. It was learnt that for the cases of foundation vibrations, if the dimensionless frequency a_o (where $a_0 = \omega r_0 / V_s$; ω is operating circular frequency, V_s is the shear wave velocity of the soil) was in between 0~2, the soil stiffness could be approximated as constant. If the dynamics of the foundation became more important, one can refer to the frequency-dependent soil impedances (Gazetas 1991). If the operating frequency ω is zero, the one associated with the dynamic impedance functions will be the same as the Lysmer's analog model. For the effects of embedment, the finite thickness of the soils, and the variability of soils along

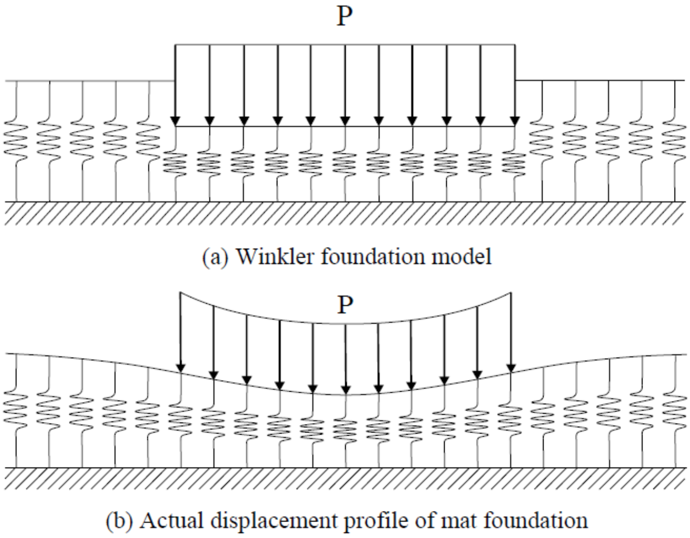

Fig. 2 Foundation under uniformly distributed load (from Lee *et al*. 2015)

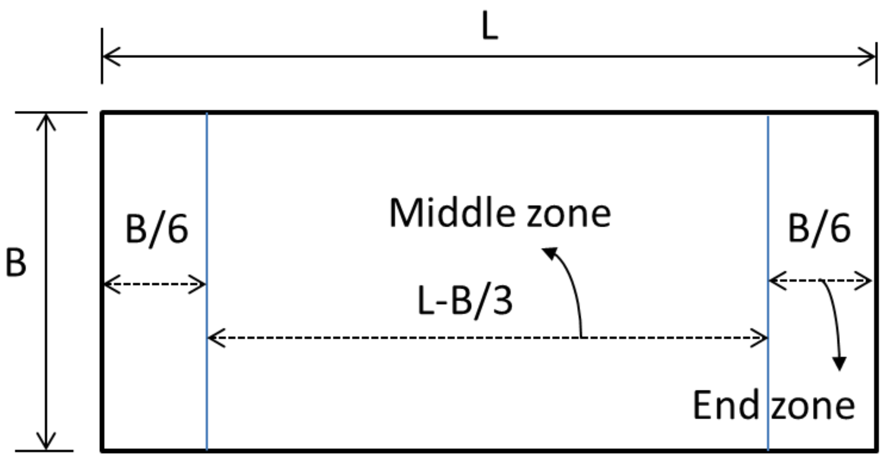

Fig. 3 Layout of foundation with middle zone and end zone (after FEMA 356, 2000)

the depth, one can find corresponding models. It should be pointed out that all the above models were suggested only for rigid foundation.

For the flexible foundations, one can find comprehensive discussions in Lee *et al*. (2015). Fig. 2 depicts the modelling of the foundation springs without and with the soil interactions. It was pointed out that there are many studies that had considered the interactions between soil springs (i.e., the shears in the subgrade). It was suggested that the Pasternak's subgrade model (1954) has been adopted frequently to monitor the shallow foundation settlements. The Pasternak's model was suggested based on force equilibriums with the use of Laplacian operator on foundation displacements, such as follows

$$p(x,y)=k_s\omega(x,y)-k_g\nabla^2\omega(x,y) \tag{3}$$

In Eq. (3), p is soil reactions underneath the mat (units in F/L^2), k_s is the coefficient of subgrade reaction (units in F/L^3) and k_g is the constant to consider the shear influence in the subgrade (units in F/L). According to Selvadurai (1979), k_s and k_g can be denoted as follows,

$$k_s=\frac{E_s}{H(1+\upsilon)(1-2\upsilon)} \tag{4}$$

$$k_g=\frac{E_sH}{6(1+\upsilon)} \tag{5}$$

where E_s is the Young's modulus of soils and H is the thickness of the ground soils (or can be regarded as the effective depth of stress influence of the foundation). The above model has been named as the two-parameter model. The thickness of the soils can be taken as 2B for square foundation where B is the width of the foundation. For a strip foundation, H can be taken as 4B. Notice that in such modelling, the complexity arises in the solutions since not only the single displacement is of interest but also the adjacent nodal displacements will affect the soil reactions. In addition, the nonlinearities of the subgrade springs would be difficult to analyze if such model was used to simulate

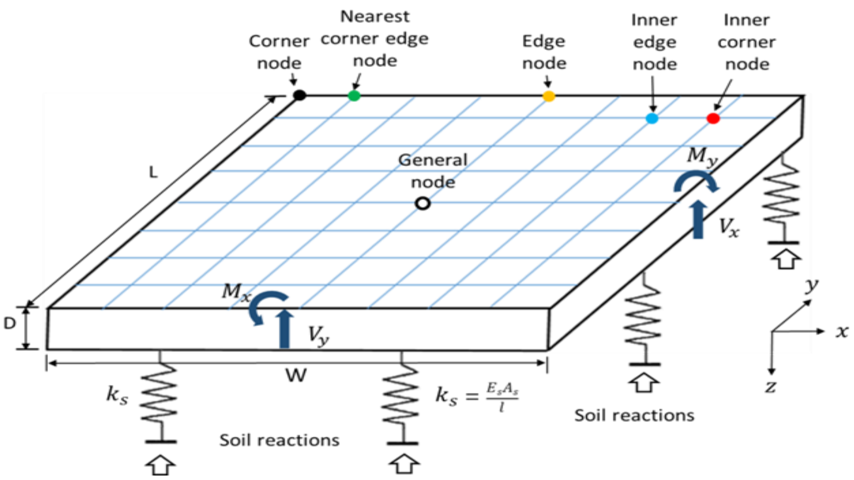

Fig. 4 2D model of the mat on ground surface (from Chang *et al*. 2018)

the nonlinear problems.

Alternate two soil spring model can be found in FEMA 356 (2000) and Adhikary *et al*. (2014). Different soil springs were suggested directly on various zones the foundation. Eq. (6) and Eq. (7) were suggested respectively for soils at the middle zone and the end zone of the foundation. Fig. 3 reveals the definition of middle and end zones in the longer dimension (i.e., L) of the rectangular foundation. The stiffness per unit length, k_z at these zones were suggested as,

$$k_z = \frac{0.73G_s}{(1-\upsilon)} \quad \text{at middle zone} \tag{6}$$

$$k_z = \frac{6.83G_s}{(1-\upsilon)} \quad \text{at end zone} \tag{7}$$

Obviously the discontinuity of soil spring constants is resulted in such model. Similar suggestion of different springs at various zones of the foundation can be also found in Coduto (2001), Loukidis and Tamiolakis (2017) where the soil springs were suggested at both longer and shorter sides of the foundation.

2. Mat foundation analysis

Theory of Plate can be categorized as thin plate and thick plate. In general, if the thickness of the plate (D) is less than a tenth of the width (B) of plate, it can be treated as thin-plate. The Kirchhoff- Love classical plate theory was suggested for thin plate. The Thick Plate theory considers the in-plane shear strains whereas the Thin Plate theory does not.

2.1 WERAFT analysis

According to Timoshenko and Woinowsky-Krieger (1959), governing equation of the vertical displacements of a thin plate subjected to vertically uniform load (q) and point load (P) can be written as follows,

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{12(1-\mu^2)}{ED^2} + \frac{12P(1-\mu^2)}{ED^3(\partial x \partial y)} \tag{8}$$

where w is the vertical displacement of the mat, μ and E are the Poisson's ratio and Young's modulus of mat, D is thickness of the mat, and x and y are the spatial variables. Bowles (1977) has demonstrated the finite difference formula of the above equation for the infinite plate. It should be noted that the soil resistances underneath the mat need to be considered if such analysis was adopted.

For a mat foundation located at the ground surface as shown in Fig. 4, the moments and shear forces are assumed zero at edge of the foundation. Assuming that at top and bottom edges of the mat where y=constant, M_x (bending moment rotating at the x-direction) and V_y (vertical shear force at the surface normal to y-direction) can be vanish. Similarly, at the left and right edges of the mat where x=constant, the boundary forces M_y and V_x are becoming zero. Chang *et al*. (2018) suggested the 2D finite difference formulations to compute the foundation displacements under vertically uniform loads. Computer program WERAFT-S was suggested (Lien, 2018) accordingly for the mat foundation settlement analysis.

2.2 Analysis using Lysmer's analog model

For the soil resistance underneath the mat, various spring models can be used. For simplicity, the Lysmer's Analog model was first adopted to verify the WERAFT-S analysis. Eq. (7) can be modified by replacing q with q*; where $q^* = q - \sum k_s w_k / A_r$; w_k is foundation settlement at the kth node, A_r is the total area of the mat which is equal to ΣA_{rk} where A_{rk} stands for the area of mat at the kth node. The soil stiffness k_s at each node can be computed as an averaged spring constant from the Lysmer's Analog model, i.e., $k_s = K_z/m$, where m is total number of nodes at the foundation. With the observations that the stiffness of the foundation soils seems to be larger than the average value for the nodes along the edges of foundation, the so-called effective area ratio, n at the nodes along the edges as well as

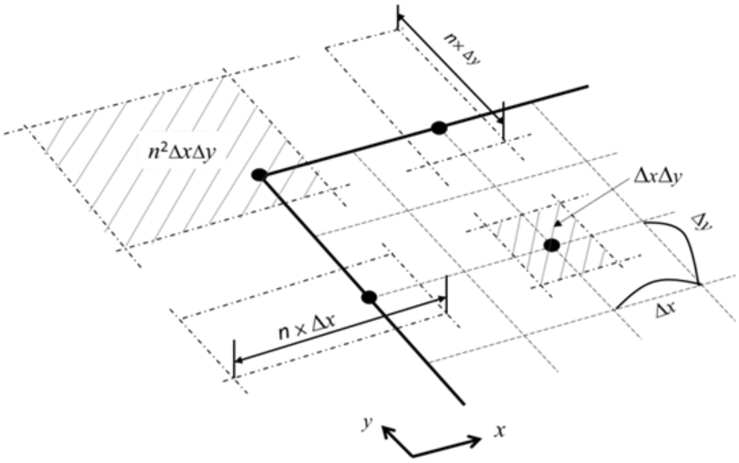

Fig. 5 Effective area of the soils at nodes along foundation edge (from Chang *et al.* 2018)

Table 1 Numerical model used in WERAFT-S analysis

Soils	Shear wave velocity (V_s) = 120 m/s, 150 m/s, 180 m/s; υ =0.3, 0.4, 0.5; γ_s = 20 kN/m^3
Foundation	Raft : length×width×thickness =26 m×26 m×1 m E = 3×10^4 Mpa, γ = 24 kN/m^3, υ =0.15
Load	Uniform load q with intensity of 100 kPa

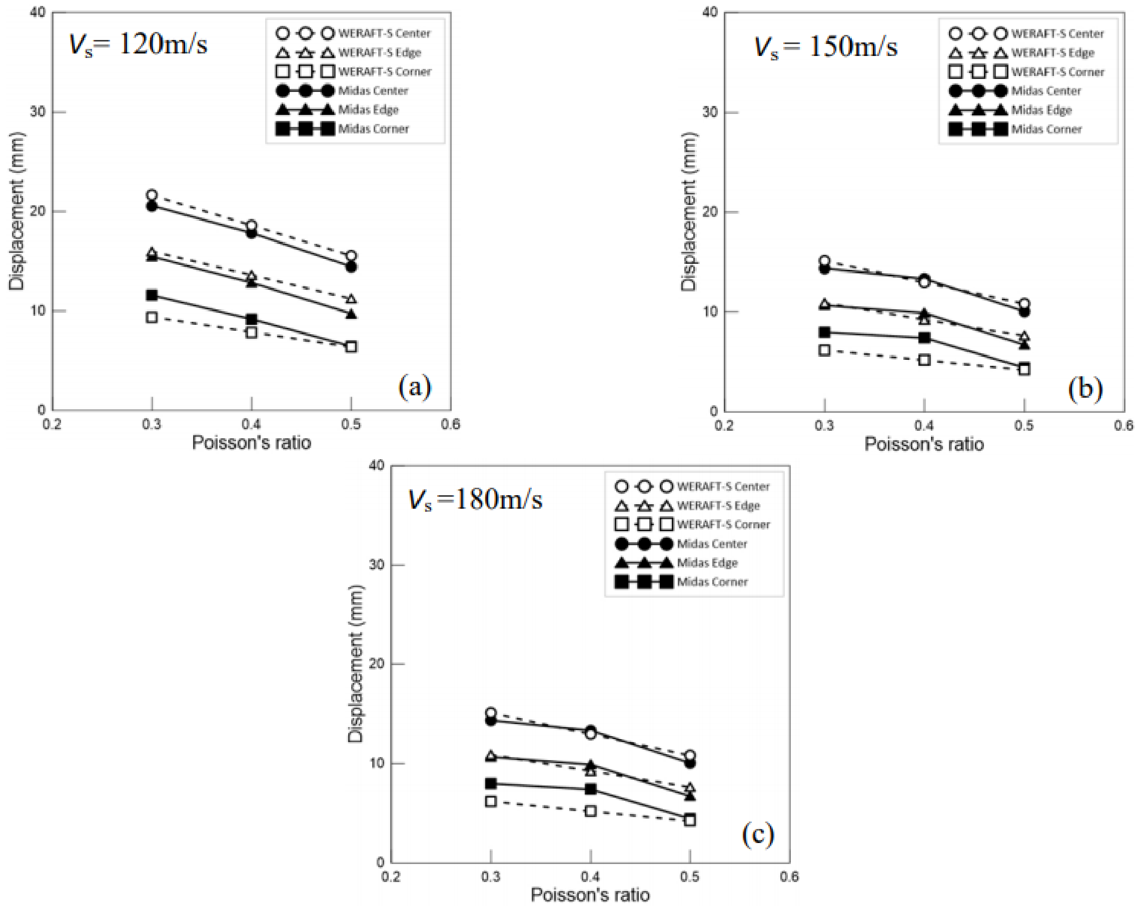

Fig. 6 Comparisons of settlements of mat foundation varying υ of the soils at 0.3, 0.4 and 0.5, (a) V_s =120 m/s (b) V_s =150 m/s and (c) V_s =180 m/s

at the corners was suggested to yield compatible solutions to the three dimensional FEM analysis using Midas GTS-NX program (Chang *et al.* 2018). Fig. 5 illustrates the effective area ratio, n used at different nodal points of the mat. It has been reported that the area ratio n of 2.5~3 would be appropriate in the studies conducted by Chang *et*

Table 2 Comparisons of the foundation settlements for original WERAFT-S analysis

Shear wave velocity, V_s	Analysis	Midas-GTS	WERAFT-S w/ Lysmer's spring
	Location	Settlement (mm)	
120 m/s	Center	27.7	28.7
	Edge	20.3	21.7
	Corner	14.5	13.2
150 m/s	Center	17.8	18.6
	Edge	12.8	13.6
	Corner	9.2	7.9
180 m/s	Center	13.3	13.0
	Edge	9.9	9.2
	Corner	7.4	5.2

al. (2018).

Table 1 presents the numerical model adopted in this study. Note that the study is aimed on linearly elastic soil behavior, therefore the shear wave velocity of the soil is used. Corresponding Young's modulus of the soil in this study would be in the range of 76~198 MPa. If the empirical formula of Vs and the undrained shear strength (S_u) of the clays (Dickenson 1994, Ashford *et al.* 1997) was used, the corresponding Su will be around 32~76 kPa. The results of WERAFT-S analysis using Lysmer's Analog spring model and those from 3D Midas analysis are shown in Table 2 for various soils with Poisson's ratio of 0.4.

Fig. 6 presented the comparisons of the foundation settlements when the Poisson's ratio of the soil is changed with shear wave velocities of the soils. It should be noted that with the use of effective area ratio n of 3, the settlements were found slightly larger than those from the FEM solutions at the center and the middle edge for soils with V_s =120 m/s, 150 m/s and 180 m/s. The settlements obtained from WERAFT-S were found much smaller than FEM solutions at the corners. Despite that the numerical tools are very different, it is believed that the major deviations are caused by following reasons, 1. The Lysmer's spring is suitable only for rigid foundation. The predictions become inadequate when the foundation exerts more flexibility, and 2. The averaged spring constant was installed in the WERAFT-S analysis while the shear in the subgrade and the interactions between the springs were completely ignored in the modelling. In addition, the effective area ratio is not a fixed value which can satisfy all the cases. In order to make the WERAFT-S analysis more accessible, following soil spring model modified from the Lysmer's Analog model is proposed.

3. Modified Lysmer's analog model

To modify the soil spring model used for flexible foundation, a study was proposed based on 3D FEM analysis using Midas program. The variations of the soil resistances underneath the numerical model of the foundation were interpreted with a normalized function *f(x)* from the centre to the edge of the foundation along the middle axis (i.e., y=0), where *f(x)* was computed by taking soil stiffness at the centre as the reference. Similarly, a normalized function *g(y)* can be obtained from the centre to the edge when x=0 (see Fig. 7). For symmetry, *g(y)* cam be expressed in the same form of *f(x)*. The soil stiffness along the centroid line of the foundation can be computed by dividing the soil reaction forces underneath foundation with

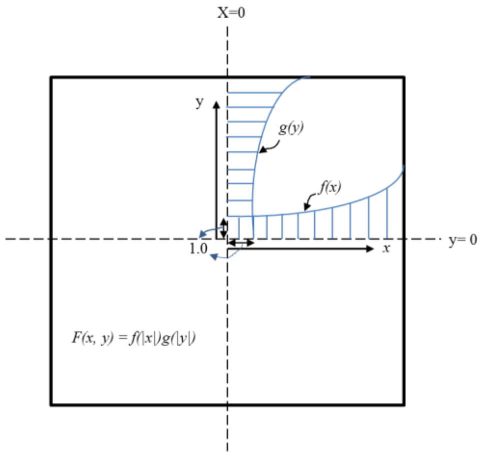

Fig. 7 Schematic layout of the normalized functions for soil stiffness underneath the mat

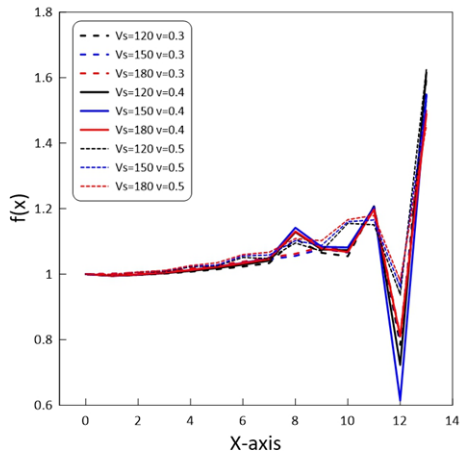

Fig. 8 Normalized curves for soil stiffness underneath the numerical model of a mat foundation from FEM analysis

Table 3 Normalized function *f*(x) of soil stiffness in terms of normalized spatial distance x from the center to the points of computation

Shear wave velocity, V_s(m/s)	Poisson's ratio, υ	*f*(x) from regression analysis ($0 \le x \le 1.0$))	r^2
150	0.3	$f(x) = 2.6612x^5 - 2.8259x^4 + 0.2343x^3 + 0.5606x^2 - 0.08024x + 1.00156$	0.991
	0.4	$f(x) = 8.6613x^5 - 16.9555x^4 + 11.2677x^3 - 2.6108x^2 - 0.1867x + 0.997909$	0.976
	0.5	$f(x) = 8.1555x^5 - 16.3070x^4 + 11.2931x^3 - 2.9018x^2 + 0.2897x + 0.996683$	0.991

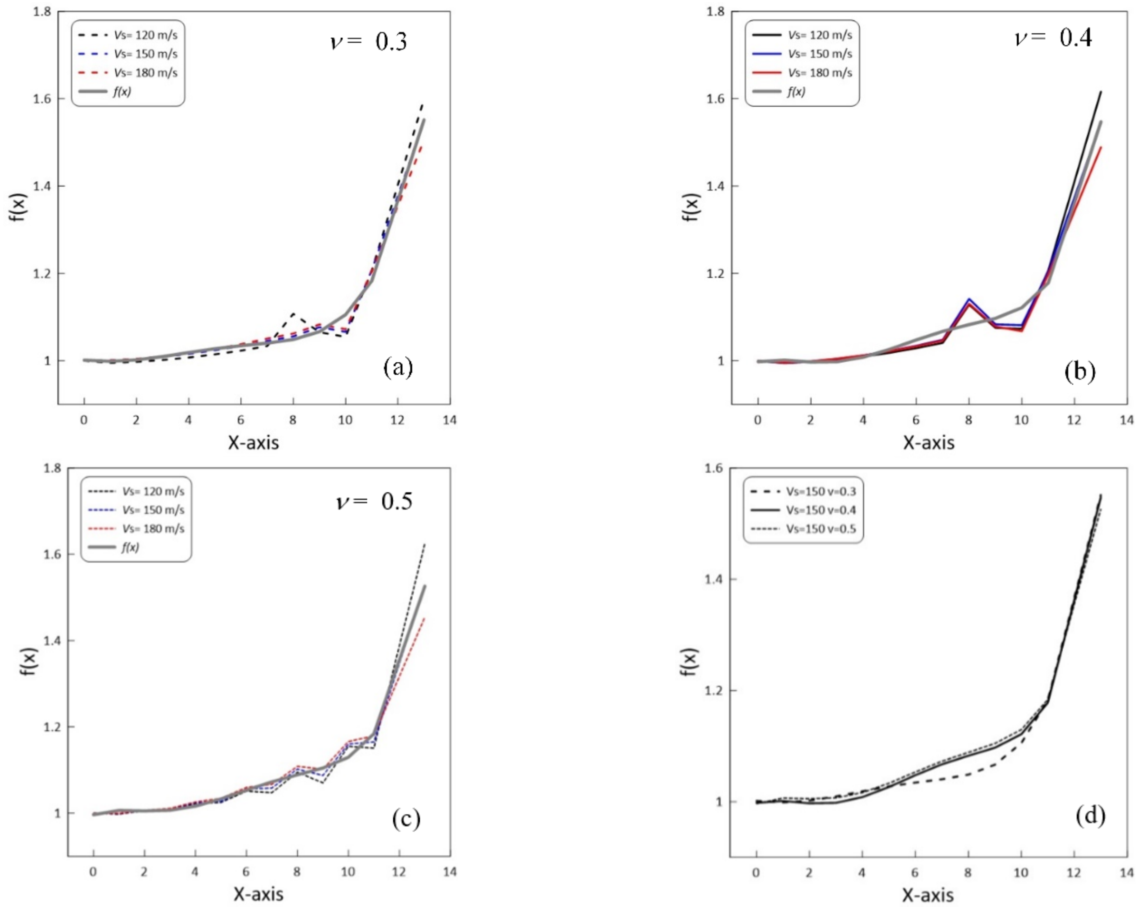

Fig. 9 Normalized functions *f(x)* and their data of the soil stiffness with respect to the distance from the center of foundation in x-direction for mat foundations on soils of various velocities (a) υ = 0.3, (b) υ = 0.4, (c) υ = 0.5 and (d) comparison of the regression curves based on V_s = 150 m/s

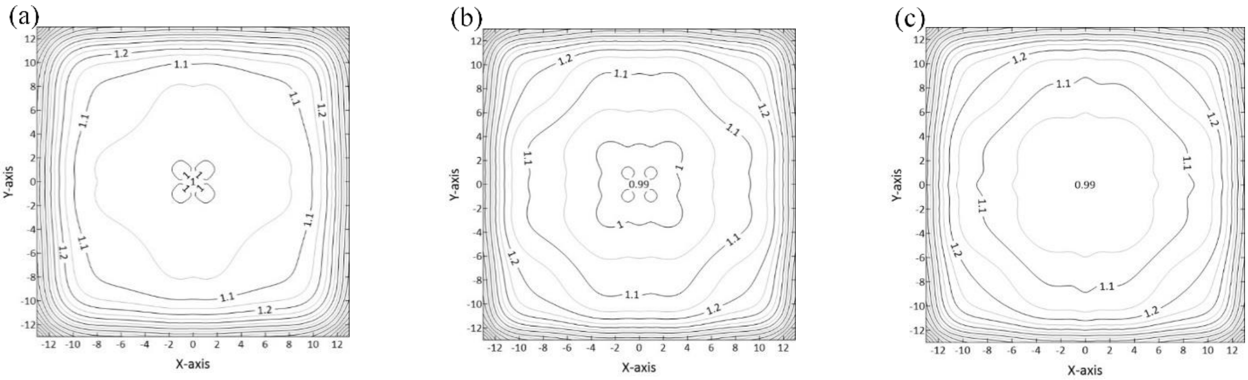

Fig. 10 Contour plots for 2D functional of *F(x,y)* of mat foundation on soils with V_s=150 m/s, (a) υ = 0.3, (b) υ = 0.4 and (c) υ = 0.5

the foundation displacements. As a result, two-dimensional normalized function *F(x,y)* can be written as $f(|x|)\times g(|y|)$ assuming that the soil spring constants are distributed symmetrically with respect to the centre of the foundation (Chang *et al.* 2019).

For the numerical model parameters discussed in this study, the corresponding *f(x)* can be found by regression analysis on the back-calculated soil stiffness as shown in Fig. 8. Note that the soils stiffness will have significant changes at the foundation edge. However, for the simplicity of modelling, the significant changes of the soil stiffness at the edges were neglected. In addition, the effects of the wave velocity seem to be only significant at the edge. Therefore, the soil stiffness was modelled by smooth mathematic functions of the Poisson's ratio of soils only. The influences of shear wave velocity were ignored. Table 3 shows the normalized polynomial functions for the soil stiffness at V_s=150 m/s. Note that in the functions, the spatial variable x is normalized with respect to half of the foundation width (*B/2*) in order to yield a more generalized equation for any size of the foundation. Fig. 9 depicts the comparisons of these functions with the curves based on the

Table 4 Calibration factor of soil stiffness for flexible foundations

Shear wave velocity, V_s (m/s)	Poisson's ratio, υ	η by Eq. (9)	η by Eq. (10)
	0.3	0.975	1.038
120	0.4	0.965	1.061
	0.5	0.985	1.161
	0.3	0.983	1.030
150	0.4	0.973	1.056
	0.5	0.993	1.158
	0.3	0.988	1.023
180	0.4	0.912	0.937
	0.5	0.999	1.155

data from FEM analysis excluding the significant changes at the foundation edge.

In Fig. 9, it can be found that the regression curves of the normalized soil stiffness for the soils with Poisson's ratio of 0.4 and 0.5 are very similar while the result of the soil stiffness would be slightly different for soils with Poisson's ratio of 0.3. Note that the increase of shear wave velocity of the soils would affect more the normalized soil stiffness at the foundation edge.

For simplicity, the corresponding functions for soils with shear wave velocity of 150 m/s will be used for V_s=120 m/s and V_s=180 m/s. The contour plots of the two dimensional function *F(x,y)* can be shown in Fig. 10 for a mat foundation on soils whose V_s is 150m/s with various Poisson's ratios. Applying *F(x,y)* to modify k_s (where $k_s=K_z/m$) from the Lysmer's analog spring, i.e., $k_s^* = k_s \times F(x,y)$, the results obtained for WERAFT-S were studied and it was found that the solutions can be modified further with the calibration factor η on Lysmer's analog spring. The calibration factor η is necessary because the Lysmer's model is suitable only for rigid foundation. The calibration factor, η can be computed by two methods, 1. Using the maximum displacement occurred at the foundation centre, 2. Using the overall displacements occurred at the centre, the middle edge and the corner. The calibration factor, η was calculated by dividing the displacement(s) from WERAFT-S analysis (applied with the normalized function *F(x,y)* with those from Midas analysis. Eq. (9) and Eq. (10) show the alternate computations. Note that Eq. (9) was proposed in order to calibrate the soil stiffness from the Lysmer's model, while Eq. (10) was proposed to compensate the solution differences caused by ignoring the sharp changes near at the foundation edges. Table 4 depicts the calibration factors obtained using these two equations. As a result, the modified soil springs k^*_s for flexible mat foundation was suggested as the expression in Eq. (11).

$$\eta = \frac{w_{WERAFT-S}}{w_{Midas\ GTS}} \quad \text{(from center node)} \tag{9}$$

$$\eta = \frac{\Sigma\left[\frac{w_{WERAFT-S}}{w_{Midas\ GTS}}\right]i}{3} \quad \text{(from center, middle edge and comer nodes)} \tag{10}$$

$$k_s^* = \left(\frac{K_z}{m}\right) \times F(x,y) \times \eta = \left(\frac{K_z}{m}\right) \times f(|x|) \times g(|x|) \times \eta \tag{11}$$

where K_z can be computed using Lysmer's model (i.e., Eq. (2)), m is the total number of the nodes of the mat, calibration factor η can be found in Table 4, where x and y in above equation can be defined as the ratio of the distances from the centre of foundation to the points of calculation in *x*-direction or *y*-direction to half of the foundation width (i.e., *B/2*) or half of the foundation length (i.e., *L/2*), respectively. With such model of the soil springs, the effective area ratio n is no longer required for nodes along the edge.

4. Comparisons using the proposed model

The results of the WERAFT-S analysis with the proposed soil stiffness were resolved and compared to those from the Midas analysis. Again, Table 5 shows the comparisons on model foundation settlements for the revised analysis using the newly proposed soil model while the Poisson's ratio of the soil is 0.4. It can be found that the foundation settlements are becoming more consistent with the modified Lysmer's analog spring model. If Eq. (9) was used, the solutions from WERAFT-S at the centre of the foundation were found much closer to the ones from Midas, however those at the foundation edge and corner would have relatively larger deviations. In order to yield better estimations for displacements at edge and corners, Eq. (10) can be used. In that case, the foundation displacement at the centre will have larger deviations.

Corresponding plots of the model foundation settlements can be found in Figs. 11 and 12 for mats on soils with various shear wave velocities. It can be found that the displacements at centre of the foundation can be captured using Eq. (9). Larger deviations of the foundation displacements at edge and corner would be found. If Eq. (10) was used, better approximations of overall foundation displacements can be achieved, however such compensation would result in smaller displacements at centre of the foundation especially when the ground soils became

Table 5 Comparisons of foundation settlements

Shear wave velocity, V_s	Analysis	Midas-GTS	WERAFT-S w/ proposed spring using Eqs. (9) and (11)	WERAFT-S w/ proposed spring using Eqs. (10) and (11)
	Location	Settlement (mm)		
	Center	27.7	27.6	25.2
120 m/s	Edge	20.3	22.6	20.5
	Corner	14.5	17.3	15.6
	Center	17.8	17.8	16.5
150 m/s	Edge	12.8	14.2	13.1
	Corner	9.2	10.6	9.7
	Center	13.3	13.3	13.0
180 m/s	Edge	9.9	10.5	10.2
	Corner	7.4	7.6	7.4

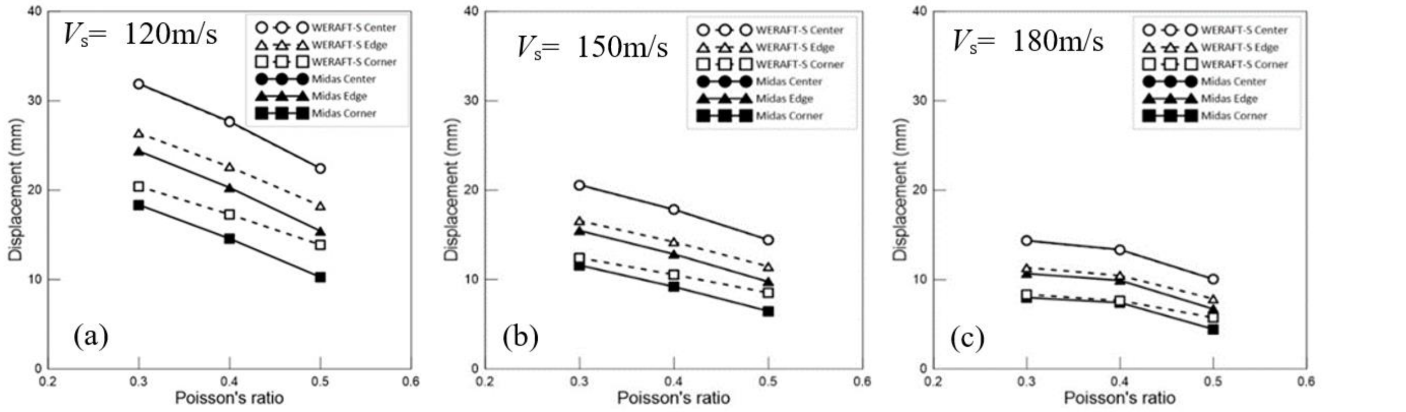

Fig. 11 Comparisons of settlements of mat foundation from Midas and WERAFT-S (using Eq. (9) and Eq. (11)) varying υ of the soils at 0.3, 0.4 and 0.5, (a) *Vs*=120 m/s (b) *Vs*=150 m/s and (c) =180 m/s

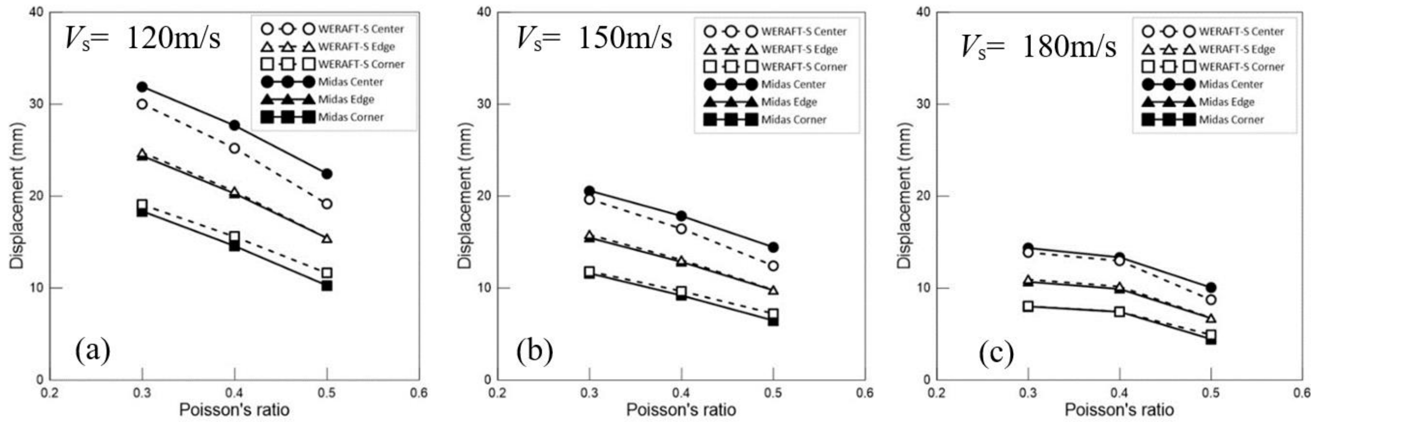

Fig. 12 Comparisons of settlements of mat foundation from Midas and WERAFT-S (using Eq. (10) and Eq. (11)) varying υ of the soils at 0.3, 0.4 and 0.5, (a) *Vs*=120 m/s (b) *Vs*=150 m/s and (c) *Vs*=180 m/s

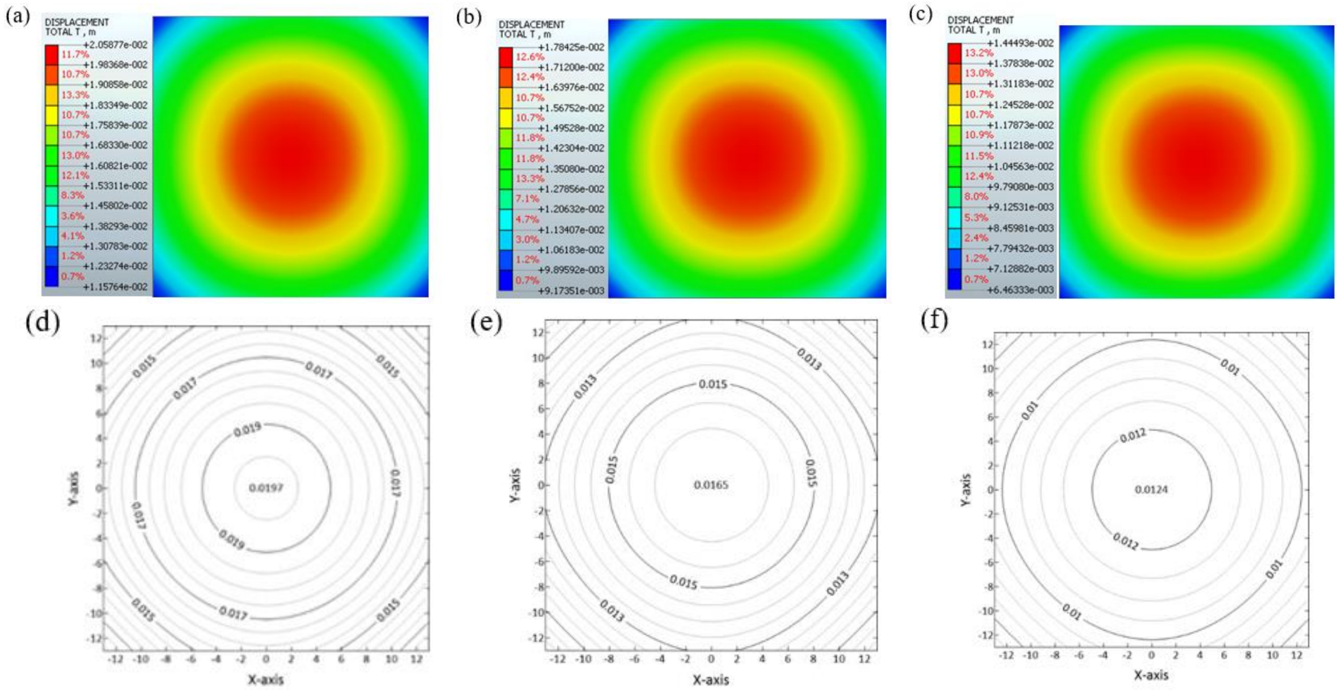

Fig. 13 Contour plots of 2D settlements of mat foundation from Midas and WERAFT-S using (Eq. (10) and Eq. (11)) varying υ of the soils at 0.3, 0.4 and 0.5, (a)-(c) Midas analysis and (d)-(f) WERAFT-S analysis

softer. The engineers may use Eq. (9) in design practice to compute the maximum foundation displacement occurred at the centre of the foundation, and use Eq. (10) to compute the displacements at the edge and the corner to find the differential settlements with respect to the maximum one at the centre from Eq. (9). Fig. 13 reveals the comparisons of solutions from Midas and WERAFT-S using Eq. (10) for the 2D foundation displacements on soils with shear wave velocity of 150 m/s and various Poisson's ratios.

Further studies on mat foundations under vertically uniform loads in nonlinear clays and sands have been conducted by Chang *et al.* (2021). Not only the foundation displacements but also the soil reaction pressures and the spring constant of the soils were studied using Mohr-Coulomb model of the soils. It is learned that the soil reaction pressures under the mat are varied according to the stiffness and strength of the soils, however the distributions of the resulted spring constants are very similar regardless of the soils and their properties. The largest spring constants were always occurred at the edge of the foundation whereas the smallest one was found at the centre.

5. Conclusions

• This paper presents a modified Lysmer's analog spring model for a vertically uniform loaded flexible mat foundation on ground surface where the thickness of foundation (D) is less than 0.1 times the width of foundation (B). Linearly elastic half-space was assumed for the ground soils. The soils were considered relatively soft, where the shear wave velocity of soils was varied at 120, 150 and 180m/s and the Poisson's ratio of the soils was assumed at 0.3, 0.4 and 0.5. Corresponding Young's modulus are around 76~198 MPa.

• A two-dimensional finite difference analysis WERAFT-S suggested by the authors was adopted to estimate the 2D settlements of a square mat foundation ($L \times B \times D$=26 m×26 m×1 m) resting on the ground surface under vertically uniform loads with intensity of 100kPa. The analysis was compared with the assured solutions from 3D FEM analysis using Midas GTS-NX program.

• In order to achieve appropriate soil stiffness under the foundation, soil reactions from the 3D FEM analysis were analyzed. The corresponding soil spring constants were computed dividing the soil reaction forces by the foundation displacements along the center line of foundation. The soil spring constant was then normalized with the smallest one found at the center. Polynomial functions along the x-direction and the y-direction of the foundation were suggested by conducting the regression analysis on these spring constants. For more applicability regardless of the foundation size, the spatial distances (i.e., x and y) between the center of foundation and the point of interest were divided by half of the foundation width and/or length, and the normalized spatial distances (0~1) were used in the polynomial functions. A 2D function $F(x,y)$ expressed as $f(x) \times g(y)$, is suggested to model the variations of soil stiffness underneath the foundation.

• Lysmer's analog spring which can be simulated as a number of mini springs was adopted to calculate the soil spring constant at center of the foundation. To calibrate the Lysmer's analog model for flexible foundation, not only the 2D function $F(x,y)$ was suggested, but also a calibration factor, η was proposed. This factor can be computed in alternate forms in order to achieve the maximum foundation displacement and the foundation differential settlements.

• The proposed model is thus termed as modified Lysmer's analog model that can be used for flexible mat foundation. The model was found effective to provide agreeable elastic solutions for finite difference analysis WERAFT-S with 3D FEM computations.

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