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Estimation of tensile strength and moduli of a tension-compression bi-modular rock

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Abstract. The Brazilian test has been widely used to determine the indirect tensile strength of rock, concrete and other brittle materials. The basic assumption for the calculation formula of Brazilian tensile strength is that the elastic moduli of rock are the same both in tension and compression. However, the fact is that the elastic moduli in tension and compression of most rocks are different. Thus, the formula of Brazilian tensile strength under the assumption of isotropy is unreasonable. In the present study, we conducted Brazilian tests on flat disk-shaped rock specimens and attached strain gauges at the center of the disc to measure the strains of rock. A tension-compression bi-modular model is proposed to interpret the data of the Brazilian test. The relations between the principal strains, principal stresses and the ratio of the compressive modulus to tensile modulus at the disc center are established. Thus, the tensile and compressive moduli as well as the correct tensile strength can be estimated simultaneously by the new formulas. It is found that the tensile and compressive moduli obtained using these formulas were in well agreement with the values obtained from the direct tension and compression tests. The formulas deduced from the Brazilian test based on the assumption of isotropy overestimated the tensile strength and tensile modulus and underestimated the compressive modulus. This work provides a new methodology to estimate tensile strength and moduli of rock simultaneously considering tension-compression bi-modularity.

Keywords: Brazilian test; tensile modulus; compressive modulus; tension-compression bi-modular rock; indirect tensile strength

1. Introduction

The tensile strength of rock is among one of the most important parameters influencing rock excavation, rock stability and rock support. The difficulties in performing a direct uniaxial tension test on rock have led to the development of a number of indirect methods for determining the tensile strength of rock (Chen *et al.* 2019, Jaeger *et al.* 2007, Liu *et al.* 2018, Perras and Diederichs 2014, Wei *et al.* 2017).

The most popular of these tests is the Brazilian test. The test was first introduced by the Brazilian engineer Carneiro in 1943 (Carneiro 1943). Since then, studies on the Brazilian test have been developed greatly and various key points have been investigated concerning the test's validity. Hondros (1959) revised the analytical solution of the Brazilian test assuming that the load is distributed along the symmetric finite arcs of the disc periphery. Fairhurst (1964) and Hudson *et al.* (1972) concluded that the results of the Brazilian test always underestimate rock tensile strength because the failure often initiate at the loading points. Thus, Wang *et al.* (2004) suggested using the Brazilian disc with flattened ends to ensure that a crack would initiate at the center of the disc. Wijk (1978) and Yu *et al.* (2006) stated that the formula for the indirect tensile strength of rock is inaccurate when the Brazilian disc has a significant thickness. Although there are debates regarding the Brazilian test interpretations, the International Society for Rock Mechanics (ISRM) suggested the Brazilian test as an effective method in 1978 owing to the simple operation and easy data processing (Ulusay and Hudson 2007). At present,

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the test is used worldwide to measure the tensile strength of rock, concrete and other brittle materials (Cai and Kaiser 2004, Chen and Irfan 2018, Cho *et al.* 2012, Liang *et al.* 2012, Ning *et al.* 2011, Park *et al.* 2018, Roy and Singh 2016, Yu *et al.* 2014, Yuan and Shen 2017, Zhou *et al.* 2020).

To calculate the tensile strength from the Brazilian test, the stress distribution of the disc must be known, particularly at the rock disc center, where a crack initiates when the tensile criterion is met. This stress can be obtained by an analytical solution, which assumes that the rock is isotropic. The implicit assumption for this solution is that the elastic moduli of rock are the same both in tension and compression. This is also a common assumption when dealing with other rock mechanics issues. In general, the compressive elastic modulus and Poisson's ratio determined from the uniaxial compressive test are used to evaluate the displacement, strain and stress distributions of rock, regardless of the rock is in tension or compression. It is reasonable for metal materials, but unreasonable for rock materials (Wei *et al.* 2019). The compressive and tensile moduli of rock may be significantly different even if there are no joints, fissures or beddings in the rock (Stimpson and Chen 1993, Sundaram and Corrales 1980). Thus, rock is characterized by the tension-compression bi-modularity. The tensile strength calculation formula of the Brazilian disc based on the isotropic hypothesis is worth discussing. However, less attention is paid to this problem.

In addition, some researchers have studied the deformation characteristics of rock using the Brazilian test. Ye *et al.* (2009) estimated the tensile modulus of rock in the Brazilian test by attaching a strain gauge at the center of the disc along the horizontal direction to measure the local strain. Gong *et al.* (2010) developed a method to determine the elastic tensile modulus by measuring the total horizontal displacement. Liu (2010) adopted the optical digital image correlation (DIC) technique to measure the displacement field of a Brazilian disc surface and obtained its elastic constants from the measured data. However, none of these methods considered the difference of tensile and compressive moduli of the material. Afterwards, Ye *et al.* (2012) proposed a method to determine the rock tensile and compressive moduli simultaneously by measuring the displacement field on the Brazilian disc. However, the method assumed that the Poisson's ratio in compression and tension as the same. To remove this limitation, Patel and Martin (2018) derived new equations for the displacements in the Brazilian test considering the bi-modularity in the constitutive relations. Unfortunately, there is an intrinsic inconsistency in this work. They assumed that the modulus of rock in tension and compression is different; however, the stress field equations used in the solution procedure were still based on the isotropic hypothesis.

During the Brazilian test, the rock is subjected to compressive stress along the vertical direction and tensile stress along the horizontal direction. Thus, the tension-compression bi-modular model is equivalent to the transversely isotropic model in the Brazilian test. Chen *et al.* (1998) developed an analytical solution for a transversely isotropic disc subjected to the Brazilian test. Claesson and Bohloli (2002) improved the solution method and derived approximate expressions for the principal stresses at the disc center. Their solution is also a fit for calculating the tensile strength of rocks having bi-moduli in tension and compression.

On the basis of their work, a further research is conducted in the present study. The objective of this research is to estimate the tensile strength and modulus of rock simultaneously by considering tension-compression bi-modularity via Brazilian test. The influence of different elastic properties in tension and compression on the principal stresses of rock at the center of Brazilian disc will be investigated, and the calculation formula of indirect tensile strength will be modified. First, it is proven that the tensile and compressive moduli of no bedding intact rock are different. Afterwards, the analytical solution for a Brazilian disc considering tension-compression bi-modularity (transversely isotropy) is presented. The results of stresses and strains under different ratios of compressive to tensile moduli are provided. Finally, these results are fitted by cubic polynomial functions. The relation between the principal strains, principal stresses and ratio of compressive to tensile moduli at the disc center is also established. Thus, a convenient and efficient method is proposed to determine the elastic tensile modulus, compressive modulus and tensile strength of rock simultaneously using the Brazilian test.

2. Validation of tension-compression bi-modularity of rock

For a homogeneous, isotropic and linearly elastic Brazilian disc, the stresses at the central point of the disc are (Timoshenko and Goodier 2013)

$$\sigma_x = -\frac{2P}{\pi DL} \tag{1}$$

$$\sigma_y = \frac{6P}{\pi DL} \tag{2}$$

where σ_i ($i = x, y$) is the stress in i-direction, P is the applied load, D is the disc diameter, and L is the disc thickness. Substituting P with P_t in Eq. (1), the tensile strength of rocks σ_t is determined by

$$\sigma_t = -\frac{2P_t}{\pi DL} \tag{3}$$

where P_t is the failure load.

According to the generalized Hooke's law, the strains at the disc center satisfies the following equations:

$$\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E} = -\frac{2P}{\pi DL}\frac{(1+3\nu)}{E} \tag{4}$$

$$\varepsilon_y = \frac{\sigma_y - \nu\sigma_x}{E} = \frac{2P}{\pi DL}\frac{(3+\nu)}{E} \tag{5}$$

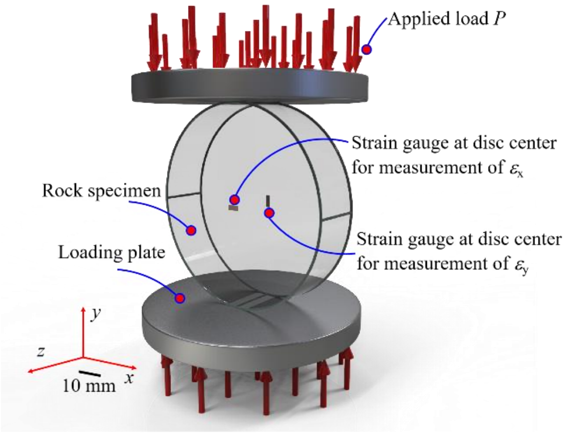
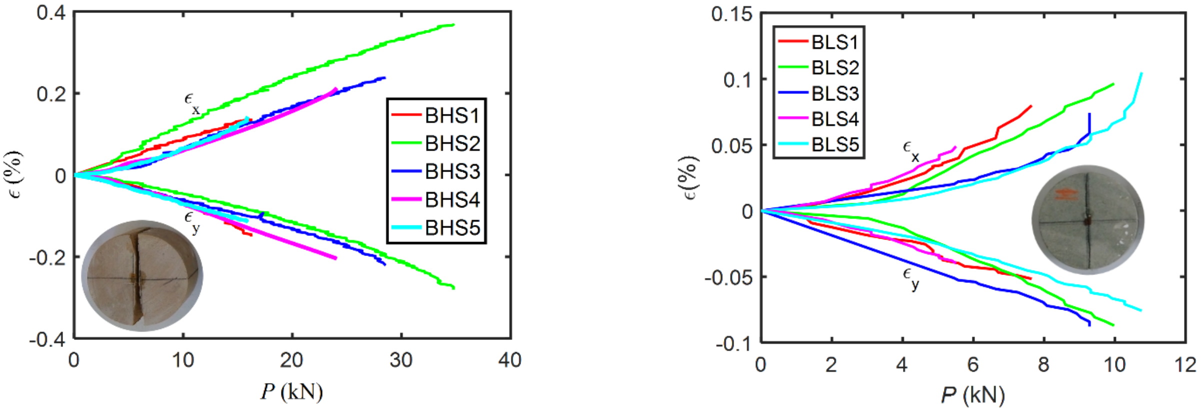
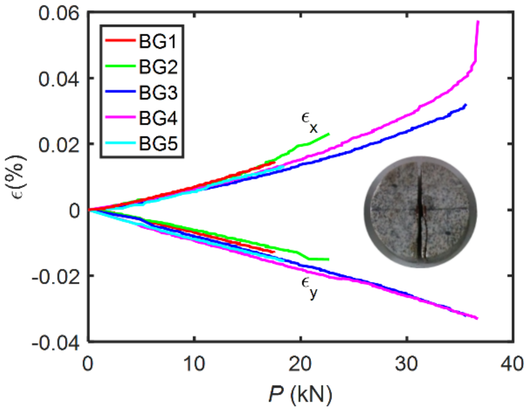


Fig. 1 Schematic diagram of strain gauge layout in Brazilian test



(a) Yellow sandstone (b) Green sandstone



(c) Granite

Fig. 2 Curves of strains at Brazilian disc center versus applied load

where ε_i ($i = x, y$) is the strain in i-direction, ν is Poisson's ratio, and E is Young's modulus. Therefore,

$$E_x = -\frac{2P}{\pi DL}\frac{(1+3\nu)}{\varepsilon_x} \tag{6}$$

$$E_y = \frac{2P}{\pi DL}\frac{(3+\nu)}{\varepsilon_y} \tag{7}$$

In the present study, all the calculations follow the sign convention of positive tension and negative compression. Three types of intact rock (i.e., yellow sandstone, green sandstone and granite) are chosen in the test. We check it carefully by eyes to ensure that there are no joints, fissures or beddings in the rocks. The Brazilian tests were conducted on flat disk-shaped rock specimens. During the experimental process, the applied load was measured by the electronic force transducer and the lateral strain ε_x and vertical strain ε_y at the disc center were measured by the strain gauges. As shown in Fig. 1, a horizontal strain gauge and a vertical strain gauge were attached at the center of the

Table 1 Results of Brazilian tests

Rock type	No.	D (mm)	L (mm)	σ_t (MPa)	E_y (GPa)	E_x (GPa)	E_y/E_x
Yellow sandstone	BHS1	80.97	40.41	3.16	7.18	5.91	1.21
	BHS2	80.94	41.10	6.65	6.41	4.23	1.52
	BHS3	80.95	41.44	5.42	6.97	4.29	1.62
	BHS4	80.89	41.11	4.61	6.37	4.18	1.52
	BHS5	80.94	40.31	3.11	7.15	3.53	2.03
	Average			4.59	6.85	4.43	1.54
Green sandstone	BLS1	81.02	39.20	1.52	7.62	3.98	1.91
	BLS2	81.08	39.31	1.99	6.11	3.12	1.95
	BLS3	80.97	39.58	1.85	9.44	5.49	1.72
	BLS4	81.02	39.43	1.10	8.12	4.35	1.87
	BLS5	80.99	39.70	2.14	7.80	4.12	1.90
	Average			1.72	7.82	4.21	1.86
Granite	BG1	81.10	39.81	3.48	83.94	47.83	1.75
	BG2	80.89	39.43	4.52	93.51	47.71	1.96
	BG3	81.11	39.31	7.08	77.68	47.03	1.65
	BG4	80.95	39.70	7.30	77.25	41.24	1.87
	BG5	80.97	39.76	3.67	83.74	43.99	1.90
	Average			5.21	83.22	45.56	1.83

disc on the opposite sides of the specimen to measure ε_x and ε_y, respectively. The curves of strains of Brazilian disc center versus applied load are presented in Fig. 2 for five different samples for each rock type. The tensile modulus E_x and compressive modulus E_y of the Brazilian disc can be determined by Eqs. (6) and (7), respectively. It should be noted that substituting the monitoring data into Eqs. (6) and (7) directly is not appropriate because the data curves are not completely linear. Thus, it is more reasonable to use the data of linear section ΔP, $\Delta\varepsilon_x$ and $\Delta\varepsilon_y$ to calculate tensile modulus E_x and compressive modulus E_y of rock.

Table 1 lists the results of the Brazilian tests. The mean values of the tensile strength σ_t, compressive modulus E_y and tensile modulus E_x of yellow sandstone are 4.59 MPa, 6.85 GPa and 4.43 GPa, respectively. The mean values of the tensile strength σ_t, compressive modulus E_y and tensile modulus E_x of green sandstone are 1.72 MPa, 7.82 GPa and 4.21 GPa, respectively. The mean values of the tensile strength σ_t, compressive modulus E_y and tensile modulus E_x of granite are 5.21 MPa, 83.22 GPa and 45.56 GPa, respectively. Obviously, the compressive and tensile moduli of rock from the Brazilian disc are different. Therefore, rock materials do not conform to the isotropic assumption. The application of the tension-compression bi-modular model is more reasonable for rock.

3. Analytical solution for a Brazilian disc considering tension-compression bi-modularity

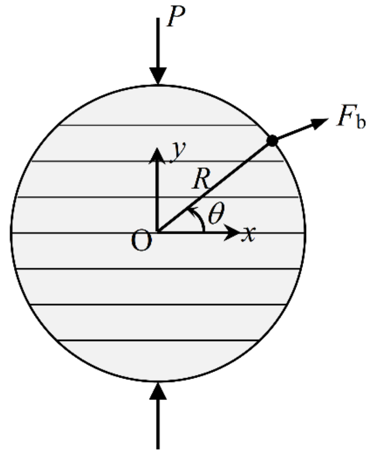


Fig. 3 Brazilian test for the tension-compression bi-modular rock

The rock material is assumed to be tension-compression bi-modularity here. In the Brazilian test, the tensile stresses are in horizontal direction and the compression stresses are in vertical direction. Therefore, the tension-compression bi-modular model is equivalent to a transversely isotropic model. The rock disc has radius R and thickness L. The disc is compressed between two parallel platens. Actually, the opposing normal forces are applied over a strip involving a small angle. However, the differences between the stresses in the disc due to a point or a narrow strip load are negligible, particularly around the disc center (Claesson and Bohloli 2002). Therefore, a point load is used in the solution, as shown in Fig. 3.

The stress system acting in the disc is simplified as a plane stress problem. The governing equations for the disc can be listed as follows (Jaeger *et al.* 2007):

(1) Equilibrium equations:

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \end{aligned} \tag{8}$$

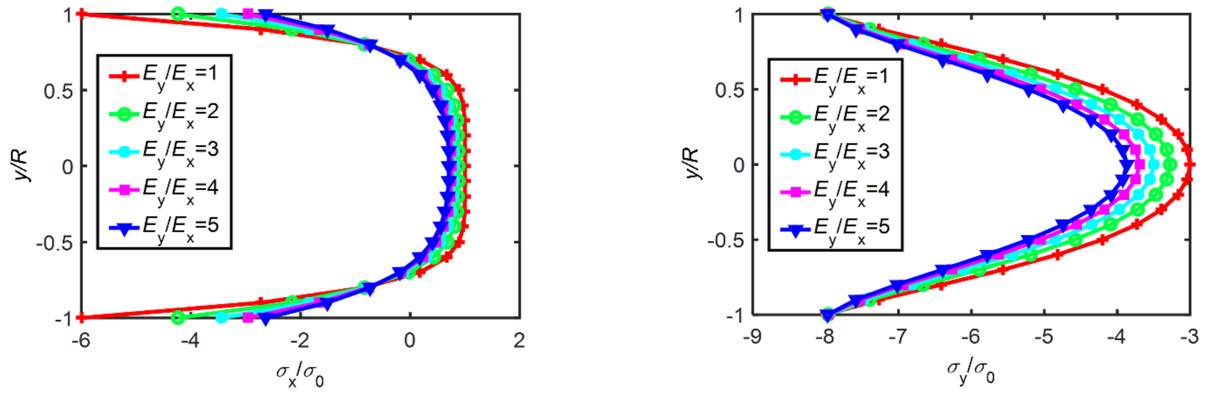
(2) Strain-displacement equations:

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{9}$$

(3) Constitutive equations:

$$\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E_x} - \nu_{yx}\frac{\sigma_y}{E_y} \\ \varepsilon_y &= \frac{\sigma_y}{E_y} - \nu_{xy}\frac{\sigma_x}{E_x} \\ \gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}} \end{aligned} \tag{10}$$

where ε_x, ε_y and γ_{xy} are the strain components, σ_x, σ_y and τ_{xy} are the stress components, and E_x and E_y are the Young's moduli in tension and compression, respectively. ν_{yx} is the



(a) s_x/s_0 (b) s_y/s_0

Fig. 4 Stress distributions of the Brazilian disc on the vertical centerline *Oy* for different ratios of E_y/E_x (E_y = 10 GPa, ν_{yx} = 0.35 and $G_{xy} = E_y/(1+\nu_{yx})$)

Poisson's ratio characterizing the lateral extension strain response due to the compressive stress in the y direction and ν_{xy} is the inverse. G_{xy} is the shear modulus on the *x*-*y* plane. In addition, $\nu_{yx}/E_y = \nu_{xy}/E_x$.

Considering the elastic theory, the expression for the stress distribution according to Claesson and Bohloli (2002) is

$$\begin{aligned} \sigma_x(x,y) &= \sigma_{x1} + 2\mathrm{Re}\left[\mu_1^2\phi_1'(x+\mu_1 y) + \mu_2^2\phi_2'(x+\mu_2 y)\right] \\ \sigma_y(x,y) &= \sigma_{y1} + 2\mathrm{Re}\left[\phi_1'(x+\mu_1 y) + \phi_2'(x+\mu_2 y)\right] \\ \tau_{xy}(x,y) &= \tau_{xy1} - 2\mathrm{Re}\left[\mu_1\phi_1'(x+\mu_1 y) + \mu_2\phi_2'(x+\mu_2 y)\right] \end{aligned} \tag{11}$$

where μ_1 and μ_2 are complex-valued constants. ϕ'_1 and ϕ'_2 are determined by

$$\varphi_1'(z) = \sum_{m=2}^{\infty} A_m P'_{1,m}(z) \quad \text{and} \quad \varphi_2'(z) = \sum_{m=2}^{\infty} B_m P'_{2,m}(z) \tag{12}$$

where

$$\begin{aligned} P_{j,m}(z) &= \left(W_j^+(z)\right)^m + \left(W_j^-(z)\right)^m \qquad m = 1,2,3,\ldots \\ W_j^+(z) &= \frac{z/R + \tilde{R}_j(z)}{1 - i\mu_j} \\ W_j^-(z) &= \frac{z/R - \tilde{R}_j(z)}{1 - i\mu_j} \\ \tilde{R}_j(z) &= \sqrt{(z/R)^2 - 1 - \mu_j^2} \qquad j = 1,2 \end{aligned} \tag{13}$$

The above equations lay the theoretical foundation for later studies, but the derivation process is too complicated. Therefore, the derivation process is given in Appendix A.

4. Strains and stresses of a Brazilian disc considering tension-compression bi-modularity

There are four independent elastic constants (i.e., compressive modulus E_y and corresponding Poisson's ratio ν_{yx}, and tensile modulus E_x and shear modulus G_{xy}) in Eqs. (8)-(13). Based on the experimental results, it is assumed that E_y = 10 GPa, ν_{yx} = 0.35, $G_{xy} = E_y/(1+\nu_{yx})$ and E_y/E_x is from 1 to 10. The solution procedure to the set of equations given above is implemented in MATLAB.

Table 2 Stresses and strains at the disc center for different E_y and E_y/E_x from analytical solution

E_y/E_x	E_y = 10 GPa			E_y = 100 GPa		
	σ_x/σ_0	σ_y/σ_0	$\varepsilon_y/\varepsilon_x$	σ_x/σ_0	σ_y/σ_0	$\varepsilon_y/\varepsilon_x$
1	1.00	-3.00	-1.63	1.00	-3.00	-1.63
2	0.88	-3.27	-1.23	0.88	-3.27	-1.23
3	0.81	-3.50	-1.04	0.81	-3.50	-1.04
4	0.75	-3.69	-0.92	0.75	-3.69	-0.92
5	0.71	-3.87	-0.84	0.71	-3.87	-0.84
6	0.67	-4.03	-0.78	0.67	-4.03	-0.78
7	0.64	-4.17	-0.74	0.64	-4.17	-0.74
8	0.61	-4.31	-0.71	0.61	-4.31	-0.71
9	0.59	-4.44	-0.68	0.59	-4.44	-0.68
10	0.57	-4.56	-0.66	0.57	-4.56	-0.66

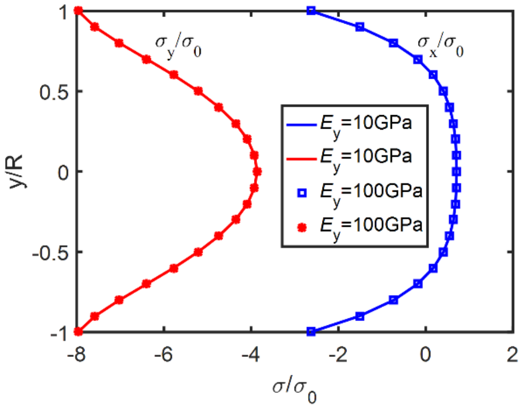
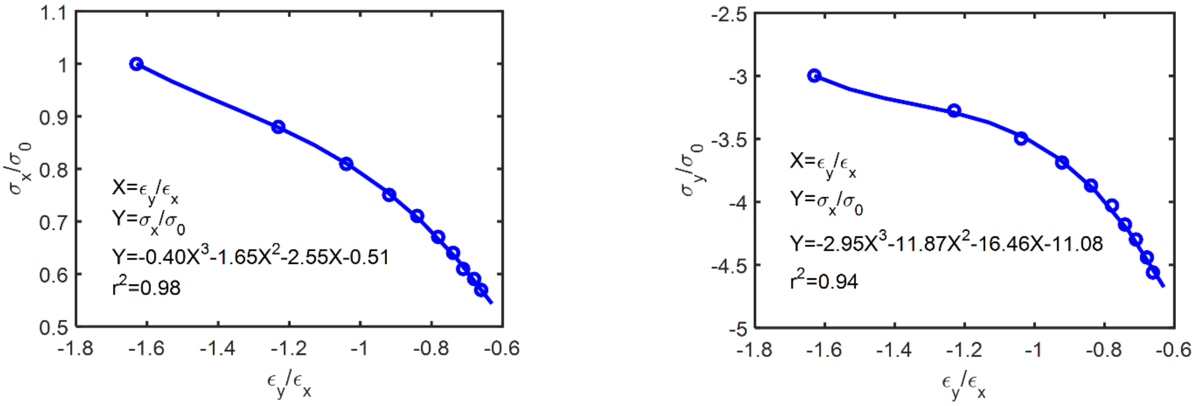
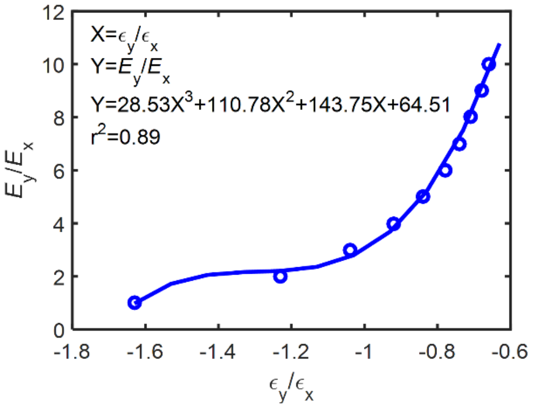


Fig. 5 Stress distributions of Brazilian disc on the vertical centerline *Oy* for different E_y (E_y/E_x = 5, ν_{yx} = 0.35 and $G_{xy} = E_y/(1+\nu_{yx})$

Fig. 4 presents the stress distributions of the Brazilian disc on the vertical centerline *Oy* for different ratios of the compressive modulus E_y to the tensile modulus E_x. The



(a) $\varepsilon_y/\varepsilon_x$ versus s_x/s_0 (b) $\varepsilon_y/\varepsilon_x$versus s_y/s_0



(c) $\varepsilon_y/\varepsilon_x$ versus E_y/E_x

Fig. 6 Fitted curves of strain versus stress and strain versus elastic modulus at the Brazilian disc center

principal stresses are normalized by $\sigma_0 = 2P/\pi DL$ and the y-coordinate is normalized by the disc radius R. As can be seen from Fig. 4, as the ratio of E_y and E_x increases, the tensile stress σ_x on the vertical centerline Oy decreases, while the compressive stress σ_y increases. Table 2 lists the stresses and strains at the disc center for different E_y/E_x , which also indicates the same phenomenon. Moreover, the ratio of ε_y and ε_x decreases with an increase of the ratio of E_y to E_x. Meanwhile, the influence of compressive modulus change on the solution is studied. The stress distributions of Brazilian disc on the vertical centerline Oy for different E_y are shown in Fig. 5. It was found that the stress and strain solutions are the same regardless of the compressive modulus E_y is 10 GPa or 100 GPa. It indicates that the stress and strain solutions are applicable to any type of rock.

5. Estimation of moduli and tensile strength of rock

In this section, a method is proposed to develop an estimation for the tensile modulus, compressive modulus and tensile strength of rock simultaneously. According to the results of Table 2, the stresses and elastic moduli at the Brazilian disc center are related to the strains. The cubic polynomial function is used to fit the data:

$$\frac{\sigma_x}{\sigma_0} = a_1\left(\frac{\varepsilon_y}{\varepsilon_x}\right)^3 + b_1\left(\frac{\varepsilon_y}{\varepsilon_x}\right)^2 + c_1\left(\frac{\varepsilon_y}{\varepsilon_x}\right) + d_1 \tag{14}$$

$$\frac{\sigma_y}{\sigma_0} = a_2\left(\frac{\varepsilon_y}{\varepsilon_x}\right)^3 + b_2\left(\frac{\varepsilon_y}{\varepsilon_x}\right)^2 + c_2\left(\frac{\varepsilon_y}{\varepsilon_x}\right) + d_2 \tag{15}$$

$$\frac{E_y}{E_x} = a_3\left(\frac{\varepsilon_y}{\varepsilon_x}\right)^3 + b_3\left(\frac{\varepsilon_y}{\varepsilon_x}\right)^2 + c_3\left(\frac{\varepsilon_y}{\varepsilon_x}\right) + d_3 \tag{16}$$

In Fig. 6, the discrete data in Table 2 are plotted as empty circles and the solid lines are their best-fitting curves. Based on the discrete data, the parameters, a_i, b_i, c_i and d_i, and the corresponding coefficients of correlation are estimated and given in Fig. 6. It was found that the cubic polynomial function provides a good fit to the data sets with very high coefficients of correlation (r^2).

Combining Eqs. (14), (15) and (10), a series of mechanical parameters (e.g., indirect tensile strength, compressive modulus and tensile modulus) of the Brazilian specimen can be determined using the experimental data ε_y, ε_x and P. Table 3 lists the modified mechanical parameters of Brazilian specimens. It can be seen that the values of E_y/E_x from constitutive equations (Eq. (10)) and fitting formula (Eq. (16)) are almost equal, which indicates that the model in the present study is valid.

The results of Table 2 show that when E_y/E_x is 1, $\varepsilon_y/\varepsilon_x$, σ_x/σ_0 and σ_y/σ_0 at the disc center are -1.63, 1 and -3, respectively. However, the experimental results of Table 3 show that all $|\varepsilon_y/\varepsilon_x|$ values are less than 1.63, all $|\sigma_x/\sigma_0|$

Table 3 Modified results of Brazilian tests

Rock type	No.	$\varepsilon_y/\varepsilon_x$	σ_x/σ_0	σ_y/σ_0	σ_t (MPa)	E_y	E_x	E_y/E_x	
								Eq.(10)	Eq.(16)
Yellow sandstone	BHS1	-1.35	0.92	-3.17	2.90	7.50	4.63	1.62	1.67
	BHS2	-1.08	0.83	-3.44	5.48	7.13	2.61	2.73	2.72
	BHS3	-1.01	0.79	-3.55	4.30	7.96	2.47	3.23	3.23
	BHS4	-1.07	0.82	-3.45	3.79	7.10	2.57	2.76	2.75
	BHS5	-0.81	0.69	-3.95	2.14	8.95	1.62	5.53	5.46
	Average				3.72	7.73	2.78		
Green sandstone	BLS1	-0.85	0.72	-3.83	1.09	9.28	1.94	4.79	4.79
	BLS2	-0.83	0.71	-3.88	1.40	7.53	1.48	5.07	5.05
	BLS3	-0.95	0.77	-3.64	1.42	11.02	2.98	3.70	3.72
	BLS4	-0.87	0.73	-3.79	0.80	9.81	2.17	4.52	4.54
	BLS5	-0.86	0.72	-3.82	1.54	9.48	2.02	4.69	4.70
	Average				1.25	9.42	2.12		
Granite	BG1	-0.93	0.76	-3.68	2.64	98.75	25.41	3.89	3.91
	BG2	-0.83	0.71	-3.88	3.19	115.24	22.63	5.09	5.07
	BG3	-0.99	0.79	-3.57	5.56	89.23	26.57	3.36	3.37
	BG4	-0.87	0.73	-3.79	5.31	93.38	20.49	4.56	4.57
	BG5	-0.86	0.72	-3.82	2.64	101.90	21.51	4.74	4.74
	Average				3.87	99.70	23.32		

Table 4 Results of the uniaxial compression and tension tests

Rock type	No.	Compression		Tension	
		σ_c (MPa)	E_c (GPa)	σ_t (MPa)	E_t (GPa)
Yellow sandstone	UHS1	71.35	9.24	3.86	2.32
	UHS2	57.03	7.22	3.74	2.58
	UHS3	79.17	6.88	4.39	2.62
	UHS4	70.57	8.36	3.79	2.51
	UHS5	58.53	7.28	3.96	2.92
	Average	67.33	7.79	3.95	2.59
Green sandstone	ULS1	30.11	8.38	1.02	2.93
	ULS2	29.98	9.59	1.10	2.47
	ULS3	30.34	8.46	0.82	1.99
	ULS4	28.53	9.98	0.84	1.18
	ULS5	31.60	8.99	1.13	1.01
	Average	30.11	9.08	0.98	1.92
Granite	UG1	99.82	94.56	5.61	25.32
	UG2	88.11	96.28	3.39	31.16
	UG3	99.95	94.44	2.54	26.28
	UG4	128.67	97.87	2.41	20.82
	UG5	111.93	99.32	5.64	30.27
	Average	105.69	96.49	3.92	26.77

values are less than 1, all $|\sigma_y/\sigma_0|$ values are greater than 3, and all $|E_y/E_x|$ values are greater than 1. That means, the compressive modulus E_y of rock should be greater than its tensile modulus E_x.

Based on Table 3, the mean values of the tensile strength σ_t, compressive modulus E_y and tensile modulus E_x for the yellow sandstone are 3.72 MPa, 7.73 GPa and 2.78 GPa, respectively. The mean values of the tensile strength σ_t, compressive modulus E_y and tensile modulus E_x for the green sandstone are 1.25 MPa, 9.42 GPa and 2.12 GPa, respectively. The mean values of the tensile strength σ_t, compressive modulus E_y and tensile modulus E_x for the granite are 3.87 MPa, 99.70 GPa and 23.32 GPa, respectively. In comparison to Table 1, the tensile strength σ_t and tensile modulus E_x of rock are overestimated and the compressive modulus E_y is underestimated when the common calculation formula is used without consideration of the tension-compression bi-modularity of rock.

In addition, a series of unconfined compressive and tensile direct tests were conducted. Table 4 lists the results of the uniaxial compressive and tensile tests. It can be seen that the elastic moduli of rock during the unconfined compressive and tensile direct tests are different. However, in comparison to Table 3, the compressive and tensile moduli from the direct tests are nearly identical to those from the Brazilian tests. Although there are some differences due to the discreteness of rock properties, the results of unconfined compressive and tensile direct tests generally agree well with the Brazilian test results. Thus, for yellow sandstone, green sandstone and granite, Eqs. (10) and (14)-(16) can be used together to estimate the tensile modulus, compressive modulus and tensile strength of rock simultaneously.

6. Conclusions

The characteristics of tensile strength and deformation of rock are studied using the Brazilian test and its results are validated by the results of direct tension and compression tests. The tensile and compressive moduli of intact rocks are different. Thus, the formula of Brazilian tensile strength under the assumption of isotropy is unreasonable. In the Brazilian test, the rock is subjected to compressive stress along the vertical direction and tensile stress along the horizontal direction. Therefore, the tension-compression bi-modular model is equivalent to the transversely isotropic model in the Brazilian test, which can be used to interpret the data of the Brazilian test. The following conclusions can be drawn:

- The relations between the principal strains, principal stresses and the ratio of compressive modulus to tensile modulus at the disc center are given out. As the ratio of E_y/E_x increases, the tensile stress σ_x decreases, while the compressive stress σ_y increases. The ratio of $\varepsilon_y/\varepsilon_x$ decreases with the increase of the ratio of E_y/E_x.
- Combining Eqs. (10) and (14)-(16), a series of mechanical parameters (e.g., indirect tensile strength, compressive modulus and tensile modulus) of the Brazilian specimen can be determined using the experimental data ε_y, ε_x and P. Thus, a new method is proposed to estimate the tensile strength, tensile and compressive moduli simultaneously considering tension-compression bi-modularity of rock using the Brazilian test.
- The tensile strength, tensile and compressive moduli obtained using the new method were found to be in agreement with the values obtained from the direct tension and compression tests. However, the formulas deduced from the Brazilian test based on the assumption of isotropy overestimates the tensile strength and tensile modulus and underestimates the compressive modulus.

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GC

Appendix

Assuming the boundary force $F_b=(X_b(\theta),Y_b(\theta))$ is a function of angle θ, see Fig. 3, $X_b(\theta)$ and $Y_b(\theta)$ can be expressed as a Fourier series (Claesson and Bohloli 2002):

$$\begin{aligned} X_b(\theta) &= \sum_{m=1}^{\infty}\left[X_{cm}\cos(m\theta)+X_{sm}\sin(m\theta)\right] \\ Y_b(\theta) &= \sum_{m=1}^{\infty}\left[Y_{cm}\cos(m\theta)+Y_{sm}\sin(m\theta)\right] \end{aligned} \tag{A.1}$$

where X_{cm} and Y_{cm} are cosine coefficients, and X_{sm} and Y_{sm} are sine coefficients. The moment of the applied forces is null. Thus, the solution for the first Fourier component (m=1) is

$$\sigma_{x1}=X_{x1} \quad \tau_{xy1}=X_{s1}=Y_{c1} \quad \sigma_{y1}=Y_{s1} \tag{A.2}$$

For the boundary conditions, there are

$$\begin{aligned} \sum_{m=2}^{\infty}\left[X_{em}e^{im\theta}+\overline{X}_{em}e^{-im\theta}\right] &= \frac{2}{R}\mathrm{Re}\left[\mu_1\phi_1(z_{b1})+\mu_2\phi_2(z_{b2})\right] \\ \sum_{m=2}^{\infty}\left[Y_{em}e^{im\theta}+\overline{Y}_{em}e^{-im\theta}\right] &= -\frac{2}{R}\mathrm{Re}\left[\phi_1(z_{b1})+\phi_2(z_{b2})\right] \end{aligned} \tag{A.3}$$

where $z_{b1}(\theta)$ and $z_{b2}(\theta)$ are the arguments of ϕ_1 and ϕ_2:

$$z_{bj}(\theta)=Rcos(\theta)+\mu_j Rsin(\theta) \quad j=1,2 \tag{A.4}$$

and

$$X_{\mathrm{em}}=\frac{X_{\mathrm{cm}}-\mathrm{i}X_{\mathrm{sm}}}{2\mathrm{mi}} \quad \text{and} \quad Y_{\mathrm{em}}=\frac{Y_{\mathrm{cm}}-\mathrm{i}Y_{\mathrm{sm}}}{2\mathrm{mi}} \tag{A.5}$$

After some derivations, the values of $P_{\mathrm{j,m}}$ at the boundary of the disc become

$$P_{\mathrm{j},m}\left(z_{\mathrm{bj}}\right)=\mathrm{e}^{\mathrm{im}\theta}+\mathrm{t}_{\mathrm{j}}^{\mathrm{m}}\mathrm{e}^{-\mathrm{im}\theta} \quad \text{and} \quad \mathrm{t}_{\mathrm{j}}=\frac{1+i\mu_j}{1-i\mu_j} \quad j=1,2 \tag{A.6}$$

The equations determining the expansion coefficients A_{m} and B_{m} can be written in the following way:

$$\begin{aligned} &\begin{pmatrix} (\mu_1-\overline{\mu}_2)\left(1-(t_1\overline{t}_1)^m\right) & (\mu_2-\overline{\mu}_2)\left(1-(t_1\overline{t}_2)^m\right) \\ (\overline{\mu}_1-\mu_1)\left(1-(t_1\overline{t}_1)^m\right) & (\overline{\mu}_1-\mu_2)\left(1-(t_2\overline{t}_2)^m\right) \end{pmatrix}\begin{pmatrix} A_m \\ B_m \end{pmatrix} \\ &= R\begin{pmatrix} X_{em}+\overline{\mu}_2Y_{em}-\overline{t}_1^{\,m}\left(\overline{X}_{em}+\overline{\mu}_2\overline{Y}_{em}\right) \\ -X_{em}-\overline{\mu}_1Y_{em}-\overline{t}_2^{\,m}\left(\overline{X}_{em}+\overline{\mu}_1\overline{Y}_{em}\right) \end{pmatrix} \end{aligned} \quad m=2,3,\ldots \tag{A.7}$$

The Fourier coefficients for opposing normal forces applying at the angle $\theta=\pi/2$ is

$$\begin{aligned} &X_{\mathrm{em}}=\mathrm{p}_{\mathrm{em}}\sin\left(\frac{\pi}{2}-\theta\right)e^{im\left(\frac{\pi}{2}-\theta\right)} \quad \text{and} \quad Y_{\mathrm{em}}=\mathrm{p}_{\mathrm{em}}\cos\left(\frac{\pi}{2}-\theta\right)e^{im\left(\frac{\pi}{2}-\theta\right)} \\ &\mathrm{p}_{\mathrm{em}}=\frac{P}{\pi RL}\frac{\sin(0.5\pi m)}{m} \quad m=1,2,\ldots \end{aligned} \tag{A.8}$$