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Quantitative risk assessment for wellbore stability analysis using different failure criteria

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Abstract. Uncertainties in geomechanical input parameters which mainly related to inappropriate data acquisition and estimation due to lack of sufficient calibration information, have led wellbore instability not yet to be fully understood or addressed. This paper demonstrates a workflow of employing Quantitative Risk Assessment technique, considering these uncertainties in terms of rock properties, pore pressure and in-situ stresses to makes it possible to survey not just the likelihood of accomplishing a desired level of wellbore stability at a specific mud pressure, but also the influence of the uncertainty in each input parameter on the wellbore stability. This probabilistic methodology in conjunction with Monte Carlo numerical modeling techniques was applied to a case study of a well. The response surfaces analysis provides a measure of the effects of uncertainties in each input parameter on the predicted mud pressure from three widely used failure criteria, thereby provides a key measurement for data acquisition in the future wells to reduce the uncertainty. The results pointed out that the mud pressure is tremendously sensitive to UCS and SHmax which emphasize the significance of reliable determinations of these two parameters for safe drilling. On the other hand, the predicted safe mud window from Mogi-Coulomb is the widest while the Hoek-Brown is the narrowest and comparing the anticipated collapse failures from the failure criteria and breakouts observations from caliper data, indicates that Hoek-Brown overestimate the minimum mud weight to avoid breakouts while Mogi-Coulomb criterion give better forecast according to real observations.

Keywords: geomechanical key parameters; wellbore instability; quantitative risk assessment; response surface; failure criteria; likelihood of success

1. Introduction

Todays, wellbore instability is counted amongst the most critical issues for the oil and gas industry. Kick, tight hole, packing off, stuck pipe, mud loss, wellbore breakout and well tensile fracture are the most common challenging problems that consecutively occur during a drilling operation and threaten the well construction and field developing projects. Accordingly, obtaining a proper mud weight window i.e., precise and reliable determination of minimum and maximum permissive mud pressure positively affects the safety of operation, non-productive time (NPT), efficiency and total cost of the project (Plumb *et al.* 2000).

Developing a proper pre-drilling geomechanical scheme is the first step to guarantee a safe and successful drilling project. Building a mechanical earth model (MEM) can be presented as a practical solution to minimize the risk of wellbore instability and impending consequences. Mechanical earth model is a numerical representation of stress state and rock mechanical properties for a particular stratigraphic section in an oil field or a basin (Noeth *et al.* 2015). Pore pressure and fracture gradient predictions/calculations, real time decision-making, mud weight window prediction and drilling performance optimization are the most significant applications of building a MEM.

As any drilling or excavation operation changes the equilibrium regime of underground strata stress state, therefore the adjacent rocks must take over the carried load of removed column of rock to re-establish this regime. This phenomenon leads to stress concentration on the boundary of the drilled well. It is obvious that without a suitable supporting introduced pressure into the borehole called “drilling mud”, failure in the formation and well collapse is unavoidable. Therefore, selecting a suitable failure criterion plays an essential role to approach to the best possible geomechanical model in a well planning and drilling project.

A few studies on the application of different failure criteria in determination of safe mud pressure under uncertain situations have been carried out. For instance, Wiprut and Zoback (2000), Zoback *et al.* (2003) and Fjaer *et al.* (2008), used Mohr-Coulomb criterion to obtain the permitted limits of maximum and minimum mud pressure to avoid any well collapse or mud loss to design a safe and optimized well plan Based on the results of true-triaxial tests, Al-Ajmi and Zimmerman (2005) developed Mogi-

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Coulomb failure criterion that the effect of intermediate principal stress (σ2) had been considered. Elyasi and Goshtasbi (2015) calculated allowable mud pressure utilizing Mohr-Coulomb, Mogi-Coulomb and Hoek-brown failure criteria. However, they did not consider uncertainties associated with estimations of the input parameters such as stresses and mechanical rock properties and rather used some predetermined values.

To build a trustworthy mechanical earth model and choose the best failure criterion, wide range of data need to be applied. But due to lack of data sources, variability of information as a function of the depth, limit amount of calibration and poor interpretation of stress state, all the geomechanical data are not available, accurate or fully dependable which expose the deterministic methods to the high degree of uncertainties that may put the project in the irreparable damages. Thus, probabilistic methods must be principally used to quantify obtained mud weight windows especially if they are issued high risk of uncertainties.

A probabilistic approach based on quantitative risk assessment (QRA) for oil and gas drilling projects was introduced by Ottesen *et al.* (1999), which was further developed by Moos *et al.* (2003), to diagnose uncertainties related to data, distinguish the risk of wellbore collapse and finally minimize operational damages and consequent costs.

To evaluate and quantify the errors, choosing an appropriate distribution function is the first step in this system. For the next step, in order to demonstrate rock properties, deformation behavior and link input parameters to purpose output, a suitable constitutive model (failure criterion) must be identified. After that, determination of the thresholds between success and failure should be taken a place via Limit State Function (LSF). To decrease the likelihood of any possible instability related to the well drilling projects, the limit state function makes a connection between conventional models versus operational failures through quantifying drilling operation failures and allocating fit range of mud densities. Then, to create a Response Surface and determining the Likelihood of Success (LS) as a function of drilling fluid pressure, plenty of wellbore instability simulations should be carried out to ensure a valid and secure well drilling operation by quantifying uncertainties and faults associated with key parameters of wellbore, estimated for input and output data utilizing probabilistic distribution functions (Aadnoy and Looyeh 2010, Plazas 2016, Guan and Sheng 2017).

In this study, QRA technique was applied on the log-derived rock elastic/strength parameters and stress state at a drilled well case to determine the disparities, errors and uncertainties associated with the obtained values, build limit state functions and response surfaces using three widely used failure criteria, recognize most influential parameters and their sensitivities in each failure criterion, and finally evaluate how the likelihood of success that prevents the wellbore form both collapse and fracture can be maximized.

2. Available petrophysical and geomechanics data

The first step in the geomechanical study is data collection. Building a MEM requires integrating data from various sources to precisely describe the formations in terms of geomechanical characteristics. In reality not every single essential data can be gotten, so we need to realize how to draw the most information from accessible data. This geomechanical study is aimed to accomplish the objectives by maximizing the utilization of available data by selecting appropriate techniques for calculations.

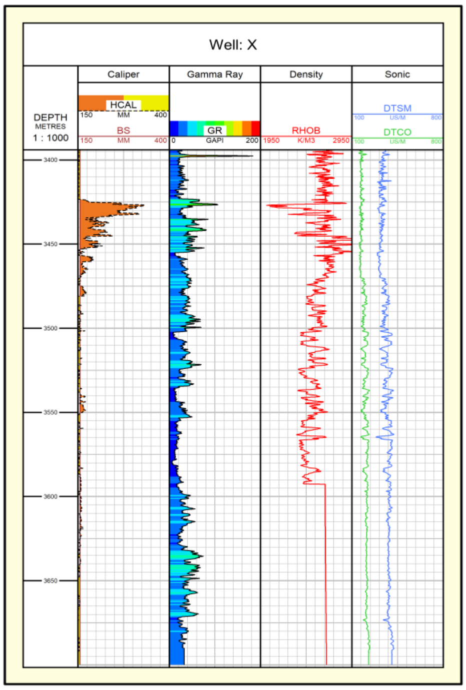


Fig. 1 Wireline logs of Well X

Wireline logs which are extensively used to extract geomechanical parameters are the key source of geomechanical characterization in this research study. The logs data are utilized to determine the petrophysical properties, the in-situ stress state, rock elastic and rock mechanical strength properties. The conventional and advanced well logs used in this study to determine the mud pressure window (MPW) include caliper, gamma ray, density, porosity together with compressional and shear sonic slowness logs.

A number of static formation pressure data from MDT tests of different intervals and two leak-off test data (LOT) are available. The results of these two different kinds of tests were used to calibrate the mechanical earth model (i.e., mechanical and stress parameters estimated based on developed relations).

3. Assessment of key physical parameters and their probability distribution functions

To set up the QRA for wellbore stability investigation, key input parameters are required to be estimated and quantified by fitting a distribution function to their variations. The key input parameters, ordinarily considered by quantitative risk assessment method for wellbore stability study, are elastic properties (Young's Modulus and Poisson's ratio), in-situ stresses (overburden stress, minimum and maximum horizontal stress), pore pressure, unconfined compressive strength and internal friction angle (Zhang 2013, Han *et al.* 2014).

Available petrophysical logs are shown in Fig. 1. The estimation of rock mechanical properties, pore pressure, in-situ stresses as well as the well logging interpretation results are discussed in sections 3.1 to 3.3. Subsequently, the fitting results of different parameters are introduced.

3.1 Estimation of rock mechanical properties

Wireline log data were utilized to derive dynamic elastic properties and then established correlation was applied to derive the static properties.

3.1.1 Elastic properties

Supposing elastic isotropy, the dynamic elastic rock properties are estimated from bulk density and sonic travel time (i.e., compressional slowness Δt_C and shear slowness Δt_S) (Fjaer *et al.* 2008). The static rock properties can be calculated using empirical relations. Log-derived dynamic rock properties whenever are possible should be calibrated with core-derived static rock properties. However, core samples are usually only taken over the reservoir section and are biased towards the stronger rock types. Therefore, over intervals without cores for laboratory tests empirical relations are often the only way to estimate the static rock properties. Numerous empirical relations to convert dynamic Young's modulus to static Young's modulus have been published for different rock types. To find the best fitting correlation, various published relationships were considered. For example, Eissa and Kazi (1988) correlation results in approximately identical static and dynamic Young Modulus. Horsrus (2001) and Ohen (2003) give much lower value of static young Modulus than the dynamic value for a wide range which doesn't sound logical and should not be applied in this field. Accordingly, correlation proposed by Ameen *et al.* (2009) was found to be the best to transform dynamic Young's modulus to its static value as given below:

$$E_s = 0.541 \times E_d + 12.852 \tag{1}$$

3.1.2 Uniaxial compressive strength (UCS)

Ideally a relation between UCS and log data should be established from core laboratory tests. However, since core is usually not available for all rocks encountered in a wellbore, empirical relations developed from similar rock types have to be utilized. Numerous relations have been proposed by many authors that relate UCS to wireline log data. Most of these equations utilize compressional slowness or velocity, Young's Modulus and porosity. The obtained UCS can only be utilized in specific fields with similarities in geological settings.

To estimate the UCS of the formations intersecting Well X, correlation proposed by Militzer (1973) was used.

$$E_s = 0.541 \times E_d + 12.852 \tag{2}$$

where DT is compressional slowness.

3.1.3 Internal friction angle

There have been relatively few endeavors to find relationships between the angle of internal friction angle (φ) and geophysical measurements in light of the way that even week rocks have generally high φ, and there are comparatively complicated relations between φ and mechanical characteristics of rock such as rock stiffness (Gholami *et al.* 2014).

In this study, to estimate the friction angle, the correlation proposed by Ameen *et al.* (2009) was applied as given below:

$$\varphi = 49.03 - 1.26 \times (\mathrm{PHIE} \times 100) \tag{3}$$

Where PHIE is effective porosity (v/v) and can be obtained using NPHI log.

3.2 Pore pressure prediction

Pore pressure is mostly used for determination of in-situ stress, effective stress philosophy and design of safe mud pressure window. It is because of the importance of pore pressure that several studies have only focused on the role of pore pressure on well stability (Wei and Yan 2014, Zhu *et al.* 2016). Formation fluid pressure is measurable utilizing well testing methods such as XPT or DST. Furthermore, empirical relations dependent on petrophysical information have been introduced for prediction of formation pore pressure along the well (Das and Chatterjee 2017).

In this study, Eaton most widely publicized pore pressure estimation technique (1975) that estimate pore pressure gradient from sonic transit time (Eq. (4)) was used to obtain the pore spaces pressure in the intervals of overburden and reservoir section those contain clay rich sedimentary rocks and calibrated using utilized mud pressure during drilling and wellbore instability events such as mud loss and well kick (influx of reservoir fluid into the well).

$$P_{pg} = OBG - (OBG - P_{ng})\left(\frac{\Delta t_n}{\Delta t}\right)^3 \tag{4}$$

where OBG is the overburden pressure (vertical stress) gradient, P_{ng} is normal pressure gradient, $\Delta \mathrm{t_n}$ is the sonic transit time gotten from well log in shale at the normal pressure and Δt is sonic transit time from well logging.
For reservoir section, pore pressure gradient obtained from XPT and MDT tools is used to calibrate pore pressure.

3.3 In-situ stresses magnitude

The magnitude of vertical stress (also called lithostatic pressure) is produced by the weight of the overlaying materials and is generally determined using wireline logs or from core density ascribing to integration of formation bulk density from surface to the interesting depth according to Eq. (5) (Zang and Stephansson 2010).

$$E_s = 0.541 \times E_d + 12.852 \tag{5}$$

where g is gravity (gravitational acceleration).

The determination of horizontal stresses magnitude encompasses technical challenges in geomechanical simulation. A number of laboratory and field techniques have been presented for the estimation of the horizontal in-

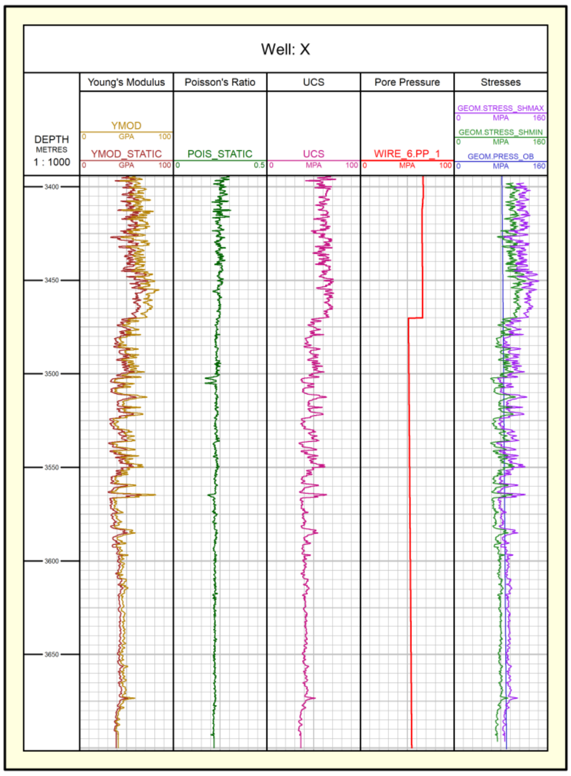


Fig. 2 Continuous profile of the rock elastic and strength parameters, pore pressure and in-situ stresses

situ stresses magnitude as focal mechanism, hydraulic fracturing, Leak-off test or extended leak off test, mini-frac tests, stress relief and strain recovery (Zang and Stephansson 2010). Comparing methods for determination of two horizontal stresses, the magnitude of S_{hmin} is more straightforward to determine if it happens to be the minimum in-situ stress (σ_3) and is less than the vertical stress. With the above-mentioned tests, indirect measurements of σ_3 (and therefore S_{hmin}) can be obtained with reasonable accuracy. However, the magnitude of the maximum horizontal stress (S_{Hmax}) is more difficult to determine, and it is best quantified by an integrated approach that employs some appropriate modeling and involves rigorous calibration to observed features such as wellbore breakout or drilling-induced tensile fractures.

Poroelastic method is a well proven developed approach which successfully utilized in this study for determination of horizontal in-situ stresses along the well in this tectonically active field by Eqs. (6) and (7) (Ostadhassan *et al.* 2012, Maleki *et al.* 2014).

$$S_{Hmax} = \frac{\vartheta}{1-\vartheta}(\sigma_V - \alpha P_P) + \alpha P_P + \frac{E_s}{1-\vartheta^2}(\varepsilon_y + \vartheta\varepsilon_x) \quad (6)$$

$$S_{hmin} = \frac{\vartheta}{1-\vartheta}(\sigma_V - \alpha P_P) + \alpha P_P + \frac{E_s}{1-\vartheta^2}(\varepsilon_x + \vartheta\varepsilon_y) \quad (7)$$

where ϑ is Poisson's ratio, E_s is static Young's modulus, α is Biot's coefficient, P_P is pore pressure and ε_x and ε_y are strains in S_{Hmax} and S_{hmin} directions, respectively which are defined by Eqs. (8) and (9) ((Kidambi and Kumar 2016).

$$\varepsilon_x = \frac{\sigma_V \times \vartheta}{E}\left[\frac{1}{1-\vartheta} - 1\right] \quad (8)$$

$$\varepsilon_y = \frac{\sigma_V \times \vartheta}{E}\left[1 - \frac{\vartheta^2}{1-\vartheta}\right] \quad (9)$$

Evaluation results of rock mechanical properties, pore pressure and stress determination are demonstrated in Fig. 2.

The stress model indicated that the stress regime in interval between 3395 m to 3470 m is primarily reverse faulting ($S_{Hmax} > S_{hmin} > S_V$) and the interval between 3470m and 3700m have mainly a strike-slip ($S_{Hmax} > S_v > S_{hmin}$) stress regime as it can be seen in Fig. 2. It's well proven that the necessary mud weight for wellbore stability is significantly affected by formation stress regime. Hence, we performed QRA for both sections independently.

3.4 Distribution functions for geomechanical properties

This section details how the probability distribution functions for the used input parameters in the wellbore stability analysis (i.e., rock strength, pore pressure and in-situ stresses) were defined.

Quantification is important since log data, which are utilized as the sources of data for estimation of input parameters, are exposed to different uncertainties because of common mechanical or electrical functioning in a defective way. Moreover, the assessed properties from log data should be calibrated against field or core data which are subjected to their own uncertainties during measurements. The effect of inconsistency in geological setting of the filed may similarly escalate the level of uncertainty when necessary calibration points are not sufficient. Aside from these, the correlations utilized for dynamic to static elastic properties conversion are field specific and, in many cases, extracted within adequate representative data. The lab data that are utilized to calibrate the log based extracted properties are normally constrained to few samples which may not be well representative of the whole log area.

Several distribution functions, including Log Logistic, Inverse Gaussian, Weibull, Beta, Cauchy, Logistic and Chi-Squared were utilized for describing the probable changes in the logs-derived data. Clear definitions and properties, plus methods of inference, applications, algorithms, characterizations, and reference to other related distributions of the above-mentioned statistical distributions can be found in Johnson *et al.* (1994).

For each individual parameter the best function was chosen and utilized for quantitative risk assessment. Taking a gander at the distribution functions of key parameters, it is seen that none of them can foresee the whole variety of the log information. For every one of the fitted distributions, goodness of fit (GOF) tests were conducted using Kolmogorov Smirnov, Anderson-Darling and Chi-Squared tests.

3.4.1 Goodness of fit tests

The goodness of fit (GOF) tests measure the compatibility of a random sample with a theoretical probability distribution function. In other words, these tests show how well the distribution you selected fits to your data.

Kolmogorov-Smirnov

This test is used to decide if a sample comes from a hypothesized continuous distribution. It is based on the empirical cumulative distribution function (ECDF). Assume that we have a random sample $x_1, ..., x_n$ from some distribution with CDF F(x). The empirical CDF is denoted by:

$$F_n(x) = \frac{1}{n}.[Number\ of\ observations \leq x] \tag{10}$$

The Kolmogorov-Smirnov statistic (D) is based on the largest vertical difference between the theoretical and the empirical cumulative distribution function:

$$D = {}_{1\leq i\leq n}^{max}\left(F(x_i) - \frac{i-1}{n}, \frac{i}{n} - F(x_i)\right) \tag{11}$$

For more information on Kolmogorov-Smirnov test see Chakravarti *et al.* (1967).

Anderson-Darling

The Anderson-Darling procedure is a general test to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function. This test gives more weight to the tails than the Kolmogorov-Smirnov test. The Anderson-Darling statistic (A^2) is defined as:

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i - 1).[\ln F(X_i) + \ln(1 - F(X_{n-1+1}))] \tag{12}$$

For more information about Anderson-Darling test see Stephens (1974).

Chi-Squared

The Chi-Squared test is used to determine if a sample comes from a population with a specific distribution. This test is applied to binned data, so the value of the test statistic depends on how the data is binned. Please note that this test is available for continuous sample data only.

Although there is no optimal choice for the number of bins (k), there are several formulas which can be used to calculate this number based on the sample size (N). For example, EasyFit employs the following empirical formula:

$$k = 1 + log_2 N \tag{13}$$

The data can be grouped into intervals of equal probability or equal width. The first approach is generally more acceptable since it handles peaked data much better. Each bin should contain at least five or more data points, so certain adjacent bins sometimes need to be joined together for this condition to be satisfied. For more information see Snedecor and Cochran 1989).

Comparing the above mentioned three tests show that The Anderson Darling does a good job at fitting the tails of the distribution and Kolmogorov-Smirnov is good at fitting the mid-range of the distribution. According to the tests, both the tails and mid-range of distribution is fitted good by Beta. In addition, Chi-square test is a nonparametric test. It is based upon the assumption that the test statistics will tend to follow a chi-square distribution. Therefore, it is less reliable as parametric functions.

Table 1 Goodness of fit for σ_h distribution functions

Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
	Statistic	Rank	Statistic	Rank	Statistic	Rank
Weibull	0.03703	1	1.0302	1	8.724	1
Weibull (3P)	0.04341	2	1.2078	2	12.544	2
Beta	0.04686	3	1.294	3	16.943	3
Log-Logistic (3P)	0.05422	4	2.1517	4	29.787	4
Logistic	0.07903	7	2.9701	5	29.868	5
Inv. Gaussian (3P)	0.06982	5	3.011	6	37.53	7
Inv. Gaussian	0.07112	6	4.1103	7	44.881	8
Log-Logistic	0.09382	9	4.782	8	37.429	6
Cauchy	0.08563	8	8.824	9	64.103	9
Chi-Squared (2P)	0.09776	10	9.2749	10	79.07	10
Chi-Squared	0.17497	11	36.015	11	246.99	11

The fitted distributions were ranked and the best one was picked for the subsequent investigations. An exemplary of ranking each distribution function for minimum horizontal stress is presented in Table 1, which demonstrates that Weibull is the best likelihood distribution function for minimum horizontal stress data in this case.

Beta-General was the best function chosen for shear wave transit time (Δt_S), internal friction angle and vertical stress logs of depth 3395 m to 3470 m and pore pressure and vertical stress for depth from 3470 m to 3700 m as it could capture most of information variety. This distribution specifies a beta distribution with a characterized least and most value utilizing shape parameters α_1, α_2 which are constantly greater than zero.

On the other hand, Weibull function which was fitted to porosity, Young's modulus, uniaxial compressive strength, minimum and maximum horizontal stresses from depth 3395 m to 3470 and both compressional and shear waves transit time for depth from 3470 m to 3700 m is described by a shape (λ) and a mean (μ) parameter.

Then again, log logistic function was the best function for compressional wave transit time (Δt_P), Poisson's ratio and minimum horizontal stress parameters belonging to interval 3395 m to 3470 m and density, porosity, Young's modulus, Poisson's ratio, internal friction angle, uniaxial compressive strength and minimum and maximum horizontal stresses of interval 3470 m to 3700 m. This distribution function is described by two parameters. The parameter α is a scale parameter and is additionally the middle of the distribution. The parameter $b>0$ is a shape parameter. These two parameters must have a positive value. As it can be seen, this function has a poor presentation in covering the adjustments in the variation of density log of interval 3470-3700 m.

Cushy distribution function which is the best fitted for

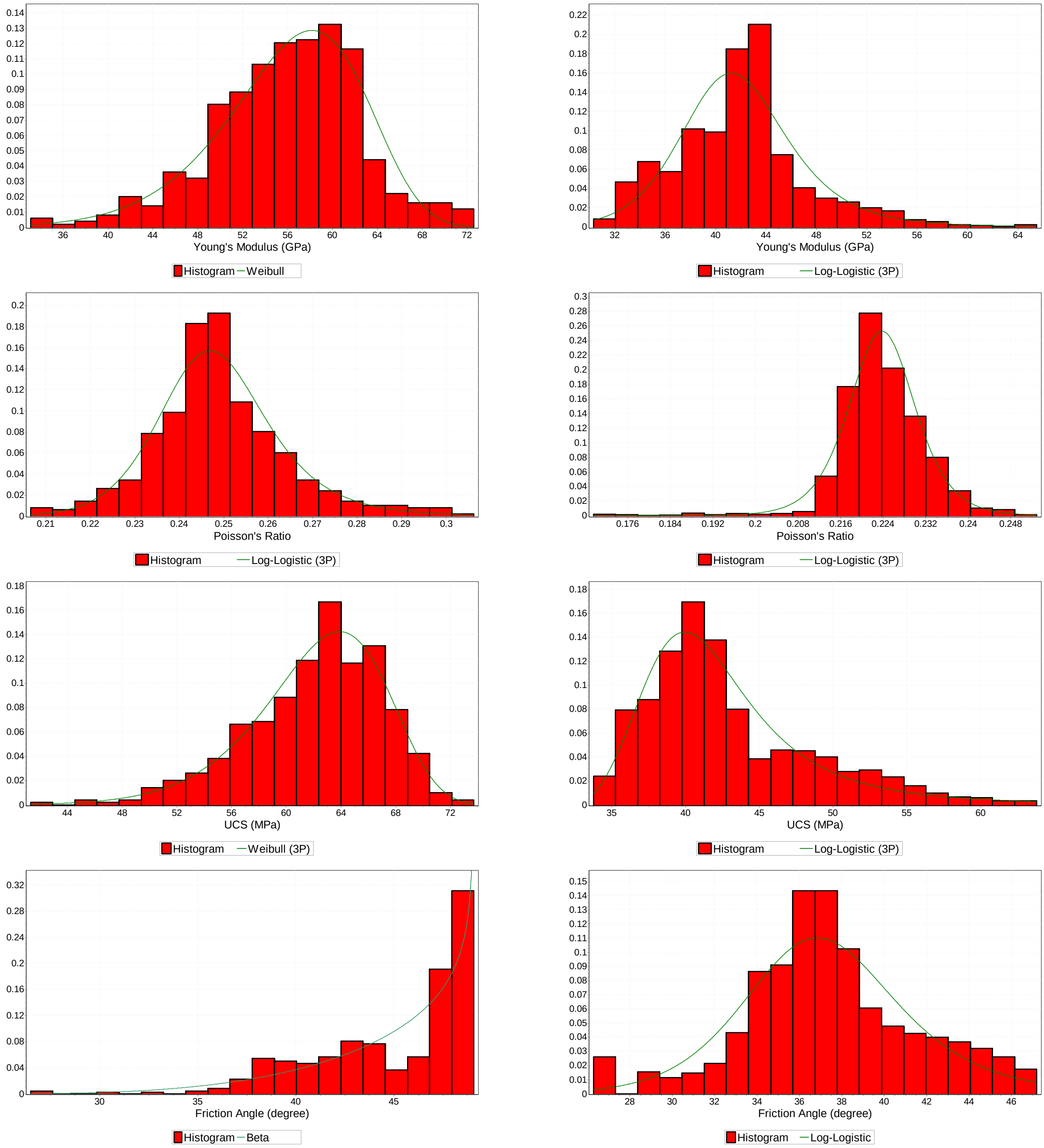


Fig. 3 Probability distribution functions fitted and used for quantifying the deviation of rock elastic and strength parameters for RF (left) and SS (right) stress regimes

density log of interval 3395m to 3470m has only one parameter: k, a positive integer that determined the quantity of degrees of freedom.

From the above discussion, it tends to be inferred that the sources of uncertainty would be very high in estimation of geomechanical parameters since the wireline logs which are displayed in Fig. 1 ought to be used. This uncertainty will even be enlarged when the stated geomechanical parameters are utilized for determination of safe mud weight window.

Figs. 3 and 4 demonstrate the uncertainties of the geomechanical input parameters which are given by probability density functions that are determined by means of the minimum, the maximum, and the most likely values of each parameter. These quantify the uncertainties in the input parameters required to compute the mud weight limits necessary to avoid wellbore instabilities.

Probability density function of the Weibull distribution was utilized to monitor the data variation associated with minimum and maximum horizontal stresses corresponding to Well X at the section with RF Stress regime. This function generates a distribution with the shape parameter

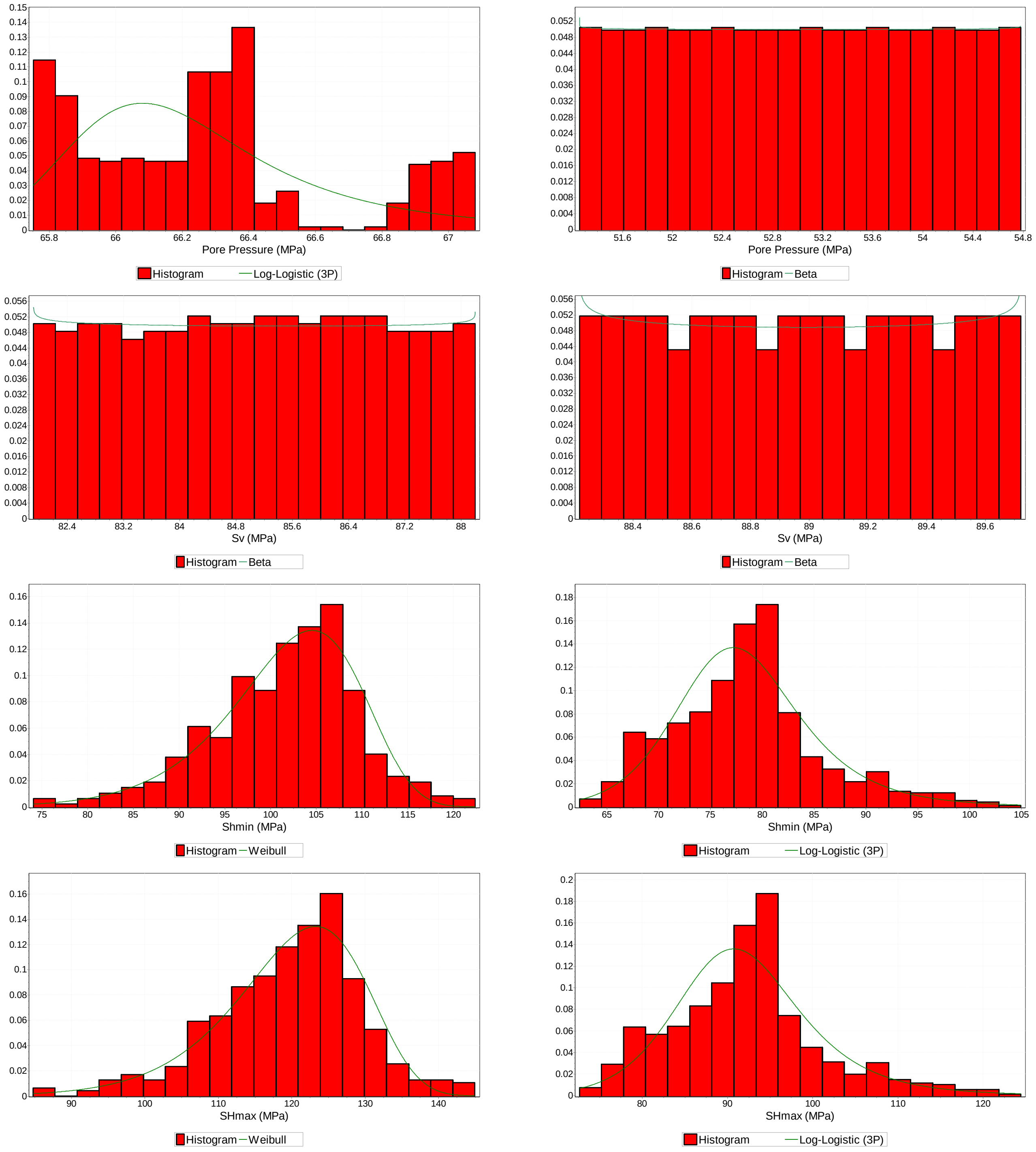


Fig. 4 Histogram and probability distribution functions fitted and used for quantifying the variation pore pressure and in-situ stresses for RF (left) and SS (right) stress regimes

alpha and a scale parameter beta. The results are shown in Fig. 4. Although, there might be a little uncertainty in using this distribution function, the fitted function seems to be very sophisticated. The only problem is the fact that nobody can be sure about the accuracy of the values obtained for the S_{Hmax} and S_{hmin} stresses in the first place. On the other hand, Log-logistic probability distribution function was used to capture most variation of horizontal stresses corresponding to Well X at the section with mainly SS Stress regime (3470-3700m) as shown in Fig. 4.

4. Quantitative risk assessment and mud weight determination

Determination of the appropriate drilling fluid density by rock failure analysis is an essential step to control wellbore instability. To determine wellbore failure stresses, the rock strength has to be known, an appropriate constitutive model has to be selected, and finally an accurate rock failure criterion must be chosen. Poro-elastoplastic modeling of the mechanical wellbore failure is the most common approach to investigate wellbore instability. This section presents an application of a probabilistic approach to investigate wellbore stability using three widely used failure criteria as a tool for planning well drilling operations in the future field developments.

4.1 Rock failure criteria

A failure criterion specifies at which stress condition failure occurs. The application of failure criteria would be limited to the soil, rock or metal based on their features. In wellbore stability analysis, the rock failure criterion is used to determine borehole breakout/fracture pressure due to tensile, shear or compressive failure. Numerous rock failure criteria have been proposed with different characteristics. In this paper, the application of QRA for three most common rock failure criteria are compared and analyzed.

4.1.1 Mohr-Coulomb

The Mohr-Coulomb is the most commonly used failure criterion in geomechanics and has a simple linear form. Mohr-Coulomb criterion is based on the two-dimensional Mohr's stress circle which is a useful theory in analyzing rock failure. Coulomb concluded that rock failure will occur alongside a plane due to acting shear stress on that plane and rock failure in compression takes place when the shear stress (τ) that is developed on a specific plane reaches a value that is sufficient to overcome the natural cohesion of the rock as well as the frictional force that opposes motion along the failure plane. The criterion can be written as:

$$\tau = c + \sigma_n tan\varphi \tag{14}$$

where the parameter c is known as cohesion, σ_n is the normal stress acting on the failure plane and parameter φ is the angle of internal friction. The Mohr-Coulomb failure criterion can be expressed in term of maximum (σ_1) and minimum principal stresses (σ_3):

$$\sigma_1 = C_0 + q\sigma_3 \tag{15}$$

where C_0 is uniaxial compressive strength which is related to the cohesion and the angle of internal friction by Eq. (16) and q is the slope of the line relating maximum and minimum principal stresses which is given by Eq. (17):

$$C_0 = 2c\frac{cos\varphi}{1 - sin\varphi} \tag{16}$$

$$q = \frac{1 + sin\varphi}{1 - sin\varphi} \tag{17}$$

It is worthy to note that these two major rock mechanical properties (C_0 and q) have been considered as basic input data for the evaluation of material parameters for different failure criteria.

4.1.2 Hoek-Brown

The most representative and commonly used non-linear criterion is the Hoek-Brown criterion (Hoek and Brown 1980, Sheorey 1997). This failure criterion present that at failure of rock mass the relation between maximum (σ_1) and minimum principal stresses (σ_3) is given by:

Table 2 Minimum, most likely and maximum values of the input data used in the QRA

Parameter	Minimum Value	Most likely Value	Maximum Value
UCS (MPa)	33.7	41.5	63.8
Friction Angle (degree)	26.3	37	47.2
Pore Pressure (MPa)	51.2	53	54.8
Sv (MPa)	88.2	88.9	89.7
SHmax (MPa)	72.6	94.5	124.5
Shmin (MPa)	62.4	81.5	105

$$\sigma_1 = \sigma_3 + \sqrt{mC_0\sigma_3 + sC_0^2} \tag{18}$$

where m and s are material constants (depending on the both rock properties and fracture characteristics). The values for s and m are various from rock to rock which are presented in Sheorey (1997). The main challenge in this criterion is determination of rock mass' parameters from the intact rock properties. However, several studies have been performed in this regard and to derive the strength of anisotropic rock from strength of the corresponding truly intact rock (Bagheripour *et al.* 2011).

4.1.3 Mogi-Coulomb

Based on the results from polyaxial compressive tests, Mogi (1971) concluded that intermediate principal stress (σ_2) influences the rock strength and the fracture occurs along a plane in the σ_2 direction. Mogi pointed out that mean normal stress which opposes the creation of a fracture is $\sigma_{m,2} = \frac{\sigma_2+\sigma_3}{2}$, rather than the octahedral normal stress (σ_{oct}). Al-Ajmi and Zimmerman (2005) simplified nonlinear Mogi criterion and defined linear Mogi criterion which is known as "Mogi-Coulomb":

$$\tau_{oct} = a + b\sigma_{m,2} \tag{19}$$

where a and b are constant given by:

$$a = \frac{2\sqrt{2}}{3}c\ cos\varphi \tag{20}$$

$$b = \frac{2\sqrt{2}}{3}sin\varphi \tag{21}$$

QRA was applied to examine the effect of uncertainties in the key parameters on the required mud weight determined from three widely used failure criteria. Key parameters for the analysis can be divided in two main groups including rock mechanical and stress data. Uniaxial compressive strength and internal friction angle are the rock mechanical variables. Overburden, the two horizontal stresses and pore pressure are the effective stress variables. The minimum, most likely, and maximum amounts of each parameter of each interval are given by probability density functions. The mentioned values for interval 3470 m to 3700 m are tabulated in Table 2.

Succeeding determination of each parameter distribution function, mud pressure to avoid wellbore failure was

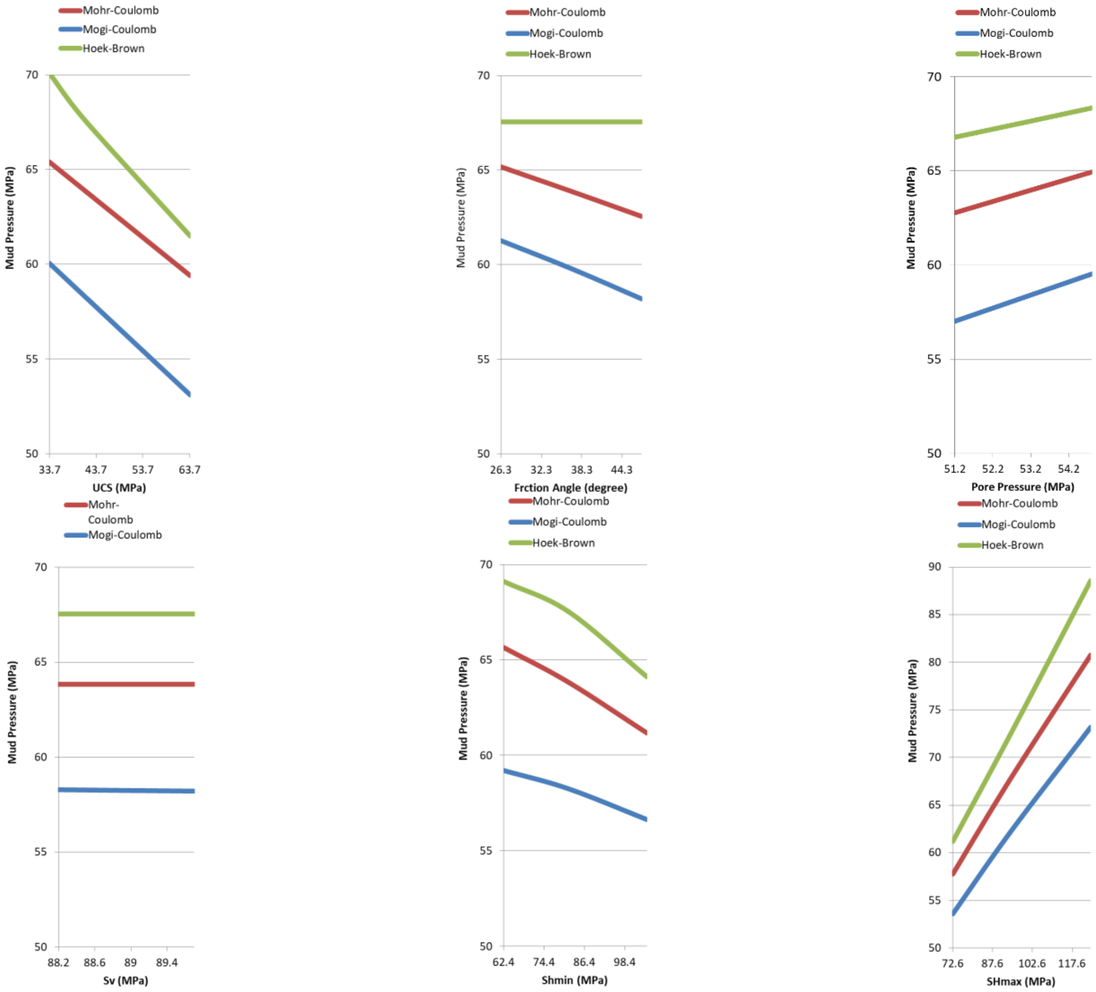


Fig. 5 Response surfaces for the key drilling parameters illustrating the sensitivity of mud pressure for well collapse

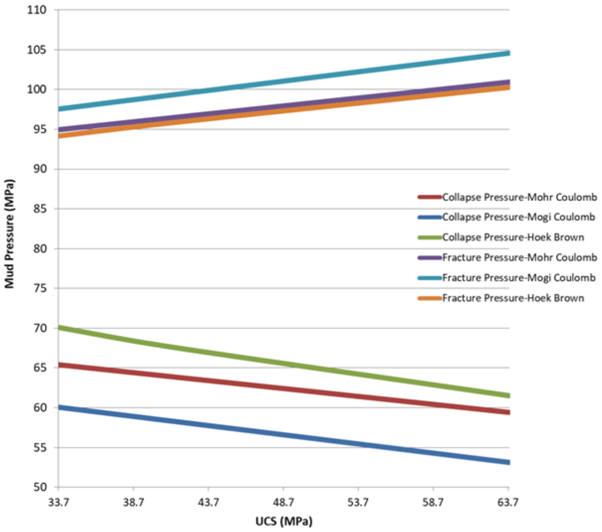
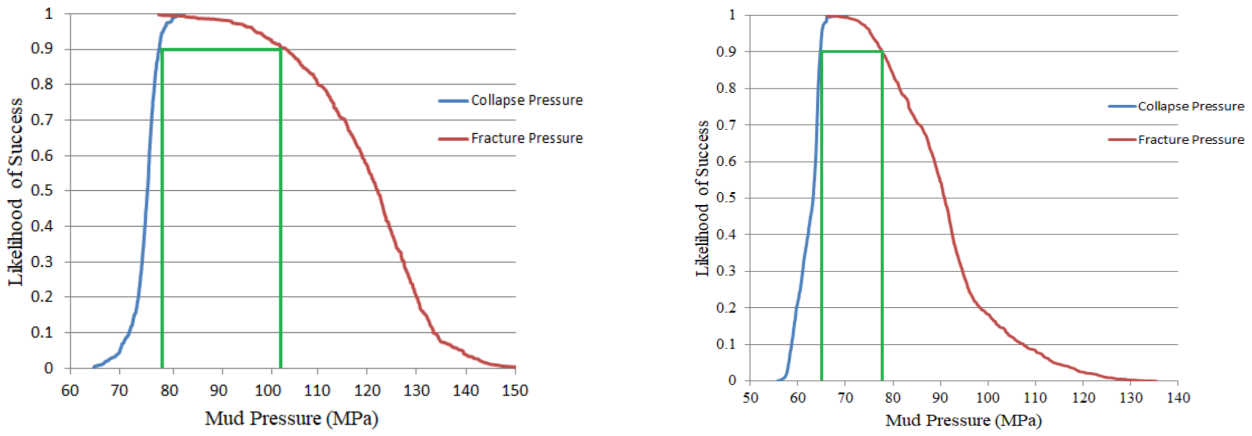


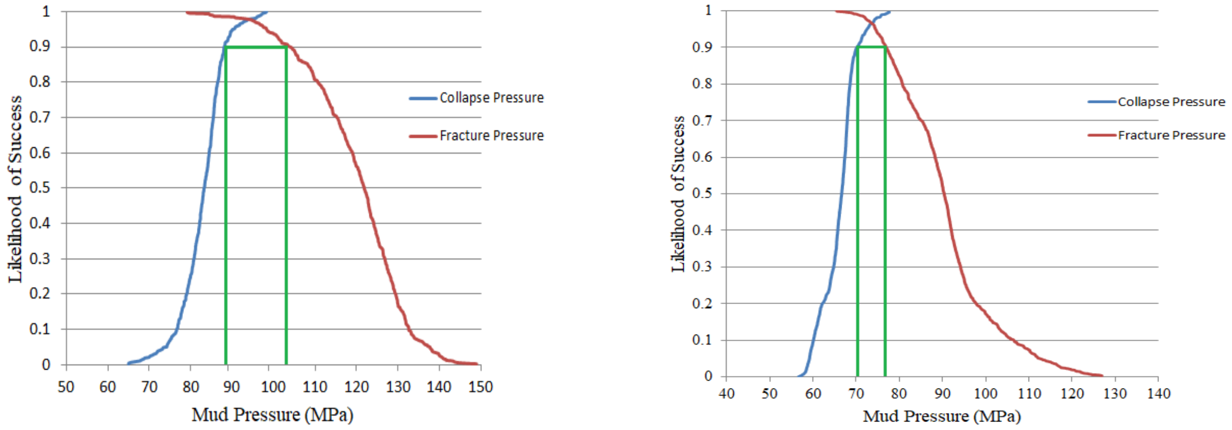
Fig. 6 Response surfaces of well collapse and fracture pressures for uniaxial compressive strength

estimated using each rock failure criteria. In other words, once the input uncertainties have been quantified, response surfaces for the well collapse (Fig. 5) and fracture (shown here for UCS in Fig. 6) can be defined. Response surfaces reveal the sensitivity of the anticipated mud pressure to the uncertainty of each input parameter.

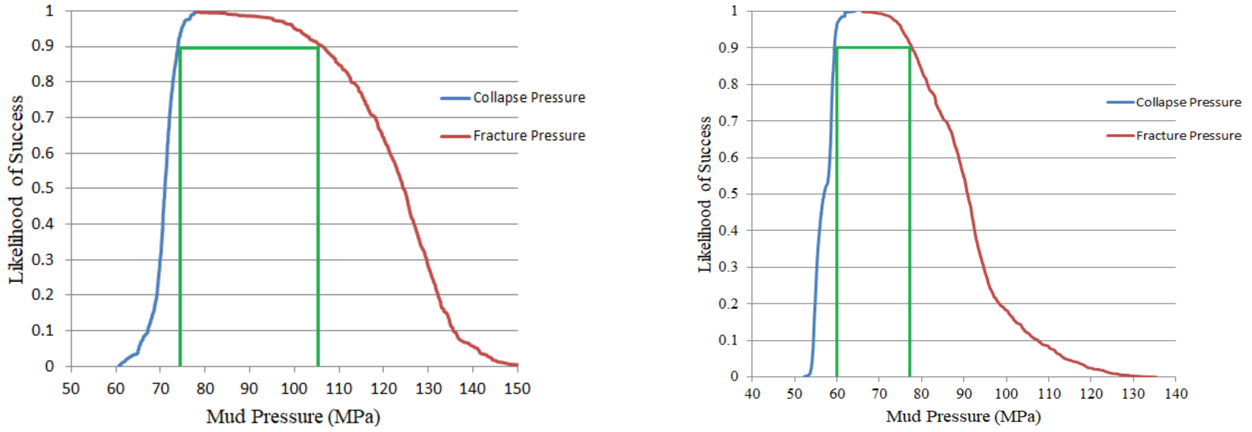
The response surfaces are quite flat for S_V magnitudes suggesting that this parameter is known with adequate accuracy not to require further analysis and uncertainty of S_V results in negligible change in mud pressure required to stabilize the well. However, the predictions of mud pressure are tremendously sensitive to the uniaxial compressive



(a) Mohr-Coulomb



(b) Hoek-Brown



(c) Mogi-Coulomb

Fig. 7 Cumulative likelihood of success (avoiding wellbore failure) for RF (left) and SS (right) stress regimes

strength and maximum horizontal stress. For the weakest rocks likely to be encountered (i.e., UCS of 33.7 MPa) a mud pressure of 60.1 MPa, 65.4 MPa and 70.1 MPa is required obtained from Mogi-Coulomb, Mohr-Coulomb and Hoek-Brown, respectively. Based on the Mogi-Coulomb failure criterion, the strongest rock (i.e., UCS of 63.8 MPa), in contrast, will be stable even if the fluid pressure in the well is lower than the pore pressure in the reservoir (i.e., mud pressure of 53.1 MPa). On the other hand, the change in S_{Hmax} value from 72.6 MPa to 124.5MPa alters the Mogi-Coulomb required mud pressure from 50.6 MPa to 75.2 MPa. Therefore, the critical parameters required to improve the predictions of the safe mud pressure window are rock strength and maximum horizontal stress.

Also, it can be seen from Fig. 5 that Hoek-Brown criterion does not consider the friction angle into account; and in contrast to S_{Hmax} and pore pressure, increasing in S_{hmin}, UCS and internal friction angle decreases the amount of the required mud weight

Response surfaces of uniaxial compressive strength for the both the well collapse and fracture are shown in Fig. 6.

Based on the response surfaces in Fig. 6, it can be pointed out that minimum required pressure from Mogi-Coulomb criterion is lower than the both Mohr-Coulomb

Table 3 Allowable mud pressure window (MPW) provides at least 90% chance of drilling success

Criterion	MPW (3395-3470 m), MPa		MPW (3470-3700 m), MPa	
	From	To	From	To
Mohr-Coulomb	77.9	103.6	64.7	77.7
Hoek-Brown	88.1	103.8	70.1	76.9
Mogi-Coulomb	73.9	106.2	59.4	77.7

and Hoek-Brown criteria and Hoek-Brown criterion is more conservative than the other two. Also, the maximum allowable mud pressure obtained from Mogi-Coulomb is higher than the other two. Consequently, the drilling safe mud pressure window resulted from Mogi-Coulomb is the broadest and Hoek-Brown is the most limited.

4.2 Mud weight determination under uncertain condition

Once these response surfaces of key parameters and corresponding failure criteria are established, Monte-Carlo simulations were run using the trends (inclinations) of response surfaces, to generate cumulative probability density curves for wellbore instabilities related pressures such as collapse and fracture. Based on these cumulative curves, the probability of drilling success for both intervals is defined by mud pressure window, considering all the uncertainties related to input parameters. Higher the probability of success indicates more narrow safe mud pressure/weight window which trends towards deterministic answer.

To conduct wellbore stability analysis and determine safe mud weight window, a scientific workflow to build a comprehensive integrated QRA-MEM with rigorous calibration to the observed features was applied as given below:

-To calibrate pore pressure in reservoir sections, pore pressure gradient obtained from XPT and MDT tools alongside with the applied drilling mud weights were used.

-Maximum horizontal stress was calibrated using particular modes of formation breakout seen on caliper log.

Drilling mud weights obtained from the calculations were compared with the actual drilling data, geomechanics related aspects of drilling events (such as loss, gain, stuck pipe, kick and etc.) and downhole measurement/integrity tests. For this purpose, DDRs as well as end of well report were utilized.

Fig. 7 shows the cumulative likelihood of preventing wellbore collapse and fracture for both intervals as a function of the mud pressure obtained from different failure criteria. The left-hand and right-hand curves display the cumulative probability of avoiding wellbore collapse and wellbore fracture for intervals with RF and SS stress regimes, respectively the green horizontal line between the curves of both intervals indicate the range of mud pressures that will simultaneously provide at least a 90% chance of avoiding both wellbore collapse and fracture (the operational mud window). It is worthy to note that the mud pressures used to drill Well X in the first interval (3395-3470 m) and the second interval (3470-3700 m) were 71 MPa and 62.5 MPa, respectively

As it can be seen in Fig. 7, according to the implemented failure criteria, the mud pressure shall be kept in the range tabulated in Table 3.

It is worthy to note that the mud weight used to drill Well X in the first interval (RF stress regime) and the second interval (SS stress regime) were 2.25 gr/cc (mud pressure of 66.4-67.8 MPa depending on the well depth) and 1.95 g/cc (mud pressure of 74.1-76.5 MPa), respectively. As can be seen on the caliper log, some collapses were observed while drilling the first interval. On the other hand, according to the well daily drilling reports, no significant problem happened while drilling the second interval. Therefore, it seems that the mud weight needs to be increased in the first section which fairly matches the results obtained from QRA.

According to Table 3, the safe mud window resulted from Mogi-Coulomb is the widest and Hoek–Brown is the narrowest. On the other hand, comparing the anticipated collapse failures from the failure criteria and breakouts observations from caliper data, it can be realized that Hoek-Brown overestimate the minimum mud weight to avoid breakouts while Mogi-Coulomb criterion give better forecast according to real observations. However, Mohr-Coulomb gives results between these two criteria. Therefore, Mogi-Coulomb provide better description of stress condition at failure. This can be associated to the fact that this criterion is a polyaxial failure criteria where the role of the intermediate principal stress is considered in failure analysis while the other two implicitly ignore that effect.

5. Conclusions

This paper outlines a complete methodology to execute wellbore stability analysis by three widely used failure criteria along with a probabilistic approach that consists of four different steps: uncertainty quantification in input parameters, identify most influential parameters and their sensitivities utilizing response surfaces, Monte Carlo simulations and finally determine likelihood of success in each failure criterion that prevents the wellbore form both collapse and tensile fracture.

The response surfaces analysis provides a measure of the effects of uncertainties in each input parameters on the mud pressure predictions by each failure criterion, thereby revealing the key measurements needed to lessen uncertainties in the most cost-effective way. It showed that uncertainty of S_V results in negligible change in mud pressure required to stabilize the well while mud pressure is tremendously sensitive to UCS and S_{Hmax}. Also, in contrast to S_{Hmax} and pore pressure, increasing in S_{hmin}, UCS and internal friction angle magnitude decreases the amount of the required mud weight.

The results pointed out that the predicted safe mud window from Mogi-Coulomb is the widest while the Hoek-Brown is the narrowest. On the other hand, comparing the anticipated collapse failures from the failure criteria and breakouts observations from caliper data, it is understood that Hoek-Brown overestimate the minimum mud weight to avoid breakouts while Mogi–Coulomb criterion give better forecast according to real observations. It is noteworthy that since empirical Hoek-Brown failure criterion follow the Mohr's stress circle, it does not consider the effect of intermediate principal stress as well as Mohr-Coulomb criteria. Therefore, Mogi-Coulomb provides better description of stress condition at failure. This can be associated with the fact that this criterion is a polyaxial failure criterion where the role of the intermediate principal stress is considered in failure analysis while the other two implicitly ignore that effect.

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