

Dynamic stability analysis of a rotary GPLRC disk surrounded by viscoelastic foundation

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Abstract. The research presented in this paper deals with dynamic stability analysis of the graphene nanoplatelets (GPLs) reinforced composite spinning disk. The presented small-scaled structure is simulated as a disk covered by viscoelastic substrate which is two-parametric. The centrifugal and Coriolis impacts due to the spinning are taken into account. The stresses and strains would be obtained using the first-order-shear-deformable-theory (FSDT). For Poisson ratio, as well as various amounts of mass densities, the mixture rule is employed, while a modified Halpin-Tsai model is inserted for achieving the elasticity module. The structure's boundary conditions (BCs) are obtained employing GPLs reinforced composite (GPLRC) spinning disk's governing equations applying principle of Hamilton which is based on minimum energy and ultimately have been solved employing numerical approach called generalized-differential quadrature-method (GDQM). Spinning disk's dynamic properties with different boundary conditions (BCs) are explained due to the curves drawn by Matlab software. Also, the simply-supported boundary conditions is applied to edges $\theta = \pi/2$, and $\theta = 3\pi/2$, while, cantilever, respectively, is analyzed in $R=R_i$, and R_0 . The final results reveal that the GPLs' weight fraction, viscoelastic substrate, various GPLs' pattern, and rotational velocity have a dramatic influence on the amplitude, and vibration behavior of a GPLRC rotating cantilevered disk. As an applicable result in related industries, the spinning velocity impact on the frequency is more effective in the higher radius ratio's amounts.

Keywords: GPLRC 2-D cantilevered disk; viscoelastic foundation; rotation; FSDT; finite element approach; numerical model; dynamic stability

1. Introduction

Nowadays, reinforcements of GPLs has been attracted by a lot of researchers due to improvement in the mechanical characterizations (Jafari Fesharaki and Roghani 2019), owing to its boosted the thermal conductivity, along with flame retardant. Dynamic stability analysis of structure has been attracted a lot of attention (Zuo *et al.* 2013, Liu *et al.* 2015, Pang *et al.* 2018, Zhu *et al.* 2018, Abedini *et al.* 2019, Zhang *et al.* 2019a, Alam *et al.* 2020a, Liu *et al.* 2020a, b, c, d, Wang *et al.* 2020, Yang *et al.* 2020, Zhang *et al.* 2020a, b, Zhu *et al.* 2020). Gholami *et al.* (Gholami and Ansari 2019) reported GPLRC plate's nonlinear frequency analysis applying higher-order shear deformation theory (HSDT). They, however, solved the governing motion equations through multistep computational model (Abedini and Zhang 2020, Cao 2020, Liu *et al.* 2020e, Mousavi *et al.* 2020, Zhang *et al.* 2020). The sandwich plate's forced and Free vibration considering core of viscoelasticity which is combined by layers of GPLRC have been investigated by Ref. (Mohseni and Shakouri 2020). Employing classical theory and energy approach, the researchers obtained the governing motion equations and ultimately Navier approach

has been used to solve them precisely. GPLRC microplate's frequency investigation employing a small-scaled model and considering analytical approach has been firstly presented by Mohammad-Rezaei Bidgoli *et al.* (2019). A considerable outcome was that arrange of factors of size-dependent elements has dramatically contributed in the GPLRC microplate's frequency characteristics. The low velocity influence on the GPLRC plate's deflection considering imperfect configuration was reported by Ref. (Song *et al.* 2019). They applied mixture approach as well as Halpin-Tsai for computing the elasticity module along with other material properties. GPLRC plate's resonance behavior employing Navier method (Abedini *et al.* 2020b, Kordestani *et al.* 2020, Xu *et al.* 2020, Zhang *et al.* 2020d) has been scrutinized by Ref (Karami *et al.* 2019). They have simulated the GPLRC plate through using Kirchhoff plate model and associated BCs and governing equations have been determined by principle of minimum energy. In the mentioned research they have demonstrated that the impact of layers' number on their structure's stability has not that depended on the types of GPLs. Moreover, GPLRC plate's natural frequency investigation using computational solution approach has been reported by Gunasekaran *et al.* (2020). Using FEM, GPLRC curved plate's forced vibration (Zhang *et al.* 2020e) performances subjected to high-temperature situation (Cui *et al.*) has been introduced by Re. (Tran *et al.* 2020). A prominent outcome that they disclosed in their research was that GPLs' configurational characters have a dramatic impact on the curved GPLRC

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plate's resonance. In addition, GPLRC curved sector plate's critical temperature considering a nonlinearity known as von Kármán model and first-order-shear-deformation model has been reported by Javani *et al.* (2020). They have employed GDQM and eventually disclosed that by using GPLs' small amount into the matrix, crucial temperature tends to be raised. This kind of structure can be used in many systems (Sun *et al.* 2019, Wang *et al.* 2020b) such as concrete (Zhang *et al.* 2019b, Abedini and Zhang 2020c, Alam *et al.* 2020b, Ju *et al.* 2020, Sun *et al.* 2020, Wu *et al.* 2021) in various conditions via experimental date (Alam *et al.* 2020c, Huang *et al.* 2020, Li *et al.* 2020, Zheng *et al.* 2020f). It is uncovered that circular plates would be applied in multifarious engineering and industrial applications (Bai *et al.* 2020). Due to different dynamic and static forces that would drop the system's stability (Long *et al.* 2015, Xiong *et al.* 2016, Wu *et al.* 2019, Zhang and Wang 2019c, d, Dong *et al.* 2020, Zhang and Wang 2020g, Dong and Cui 2021), the presented structure is usually is subjected to a wide range of environment situations. As well as this, undesirable performances would happen for the mentioned structures including cracks, buckling, resonance and deformations. Then, it would be highly important for designers to understand the mechanical performances of this system type for analyzing the structure considering a significant reliability. Due to the GRLs reinforcements' benefits, this class of material would be applied as a composite filler (Zhu *et al.* 2018) in the circular plates. Employing 3D-elasticity model, Yang *et al.* (2017) scrutinized circular/annular plate's bending investigation. They. However, disclosed that distribution pattern, BCs, and other configurational and physical factors of the GPLs have vital contributions in the structure's strain and stress fields. GPLRC annular plate's bending and Frequency characteristics employing 3D-elasticity model have been evaluated by Liu *et al.* (2019). They have solved the system's governing motion equations via state-space based DQM. They uncovered that a range of BCs, and other structure's physical and geometrical factors and GPLs have been playing a prominent role in the GPLRC annular plate's bending and Frequency characteristics. GPLRC circular plate's nonlinear forced vibration in an extreme temperature situation has been reported by Wu *et al.* (Wu, Zhu *et al.* 2020). Using iterative approach and GDQM, they solved the nonlinear governing motion equations considering related BCs. Eventually, they demonstrated that the ratio of nonlinear frequency drops when GPL fillers increases within the GPLRC circular plate's outer layers. Spinning disks, plates and have a broad range of applications including, compressors, computer disks, pump, and rotors., In the area of spinning circular/annular plate's instability/stability investigation, Hu *et al.* (2016) analyzed rotating circular plate's nonlinear performance subjected to magnetic field. The structure presented by them was simulated via thin plate model and ultimately they have solved the nonlinear governing motion equations through using multiple scales and Galerkin approaches. They disclosed that, rotational speed, and magnetic excitation have an important contribution in the rotating circular plate's instability/stability. Ref. (Bagheri *et al.* 2019) dealt

with stability study of a rotating homogeneous isotropic annular plate. By applying von Kármán nonlinearity and FSDT, the model was derived. Mahinzare *et al.* (2018) evaluated functionally graded (FG) spinning circular plate's frequency characteristics using a classical model, while governing motion equations have been solved via DQM. They, however, demonstrated that related factors to FG, angular speed, and external voltage have a significant contribution in the smart (FG) circular plate's frequency behavior. Furthermore, Qin *et al.* (2018) studied cylindrical shell's vibrational performances coupled with circular disk within framework of Rayleigh–Ritz approach. Recently, hygro-thermal environment's impact of the magneto-electro-elastic (MEE) imperfect circular plate's multi-field responses has been reported by Ref. (Dai *et al.* 2019). the thermal situation has been created using a steady-state heat conduction. Ultimately, the governing equation has been solved via DQM, then the outcomes revealed the external conditions' impacts (extreme temperature and moisture) on the imperfected annular MEE plate's multi-field responses. Refs. (El-Hassar *et al.* 2016, Fahsi *et al.* 2017, Issad *et al.* 2018, Sadoun *et al.* 2018, Younsi *et al.* 2018, Ahmed *et al.* 2019, Boulefrakh *et al.* 2019, Chikr, Kaci *et al.* 2020, Rabhi *et al.* 2020), however, reported structures' instability/stability investigation considering a wide range of solution approaches (Ghabussi *et al.* 2019, Habibi *et al.* 2019, Al-Furjan *et al.* 2020a, b, c, d, e, f, g, h, i, j, k, l, m, Ebrahimi *et al.* 2020, Habib *et al.* 2020, Lori *et al.* 2020, Moayedi *et al.* 2020, Safarpour *et al.* 2020, Shokrgozar *et al.* 2020). The bi-axial micro-scanner's instability investigation which is subjected to electromagnetic actuation such as size dependency and damping impacts has been studied by Ref. (Atabak *et al.* 2020). stress-based non-local elasticity for the fluid-conveying C-BN hybrid-nanotube's instability located in a magneto-thermal situation has been scrutinized by Sedighi *et al.* (Sedighi *et al.* 2020a). In their research, the size dependency had been considered employing Eringen's strain-based non-local differential theory. hetero-nanotube's dynamics made of boron nitride (BN) and carbon (C) nanotubes in thermal and magnetic situation applying a FE model has been reported by Ref. (Sedighi *et al.* 2020b). Lately, Safarpour *et al.* (2019) have studied frequency, and bending information of cylindrical shell, truncated GPLRC conical shell, and circular plate through using DQM and three-dimensional model of elasticity. Then, it found that GPLs' pattern types, GPLs', weight fraction, configurational factors including ratio of length to mid-radius and semi-vertex angle have vital contributions in the GPLRC structures' bending, and frequency. They (Bisheh *et al.* 2019), moreover, analyzed FG- annular and circular GPLRC plate's bending and natural frequencies considering a range of BCs. They derived and solved, respectively, the annular and circular FG-GPLRC plate's governing equations applying three-dimensional elasticity model, and DQM. To the best of aforementioned scientists' knowledge, spinning cantilevered GPLRC disk's frequency and amplitude information which is covered by viscoelastic substrate have not been reported in the aforementioned literature. In the present study, modified Halpin-Tsai's micromechanics is

employed for approximating effective elastic properties. Furthermore, a computational approach is applied to solve governing motion equations and derived via principle of minimum energy. Despite semi-computational solution, an FE method has been introduced applying the FE package for simulating the spinning cantilevered GPLRC disk's response. The outcomes derived by the FE simulation demonstrates a close agreement with the results of semi-computational approach. A significant attention is given for analyzing some physical and configurational factors' impacts on the frequency, and amplitude behaviors of a spinning cantilevered GPLRC disk covered by viscoelastic substrate.

2. The mathematical formulations for spinning GPLRC cantilevered disk

A rotating GPLRC cantilevered microdisk surrounded by viscoelastic foundation is shown in Fig. 1.

Four patterns have been considered for modelling GPLRC reinforcements, in the current manuscript. The functions of volume fraction are defined for each figure as follows (Song *et al.* 2017):

$$V_{GPL}(k) = V_{GPL}^* U - GPLRC \quad (1)$$

$$V_{GPL}(k) = 2|2k - N_L - 1|V_{GPL}^* / N_L \quad X - GPLRC \quad (2)$$

$$V_{GPL}(k) = 2(2k - 1)V_{GPL}^* / N_L \quad A - GPLRC \quad (3)$$

$$V_{GPL}(k) = 2\left[1 - \left(|2k - N_L - 1| / N_L\right)\right]V_{GPL}^* \quad O - GPLRC \quad (4)$$

Ref (Song *et al.* 2017) provides us with details of factors applied in Eqs. (1)-(4). The dependence of V_{GPL}^* and its weight fraction g_{GPL} may be explained by:

$$V_{GPL}^* = \frac{g_{GPL}}{g_{GPL} + (\rho_{GPL} / \rho_m)(1 - g_{GPL})} \quad (5)$$

Here the mass density of GPL and polymer matrix are depicted as ρ_{GPL} and ρ_m respectively. The GPLRC shell's elasticity modulus has been supposed using modified Halpin-Tsai micro-sized mechanics (De Villoria and Miravete 2007)

$$E = \left(\frac{3}{8} \left(\frac{1 + \xi_L \eta_L V_{GPL}}{1 - \eta_L V_{GPL}} \right) + \frac{5}{8} \left(\frac{1 + \xi_W \eta_W V_{GPL}}{1 - \eta_W V_{GPL}} \right) \right) \times E_M \quad (6)$$

$$\xi_L = 4L_{GPL} / 2t_{GPL} \quad \xi_W = (4w_{GPL} / 2t_{GPL})$$

$$\eta_L = (-E_M + E_{GPL}) / (\xi_L E_M + E_{GPL})$$

$$\eta_W = (-I + \frac{E_{GPL}}{E_M}) / (\xi_W + \frac{E_{GPL}}{E_M})$$

Eventually, Poisson's ratio ν_c are defined as below by applying the rule of mixture and polymer micro GNP composite's density ρ_c :

$$\begin{aligned} \nu &= \nu_{GPL} V_{GPL} + \nu_M V_M, \\ \rho &= \rho_{GPL} V_{GPL} + \rho_M V_M. \end{aligned} \quad (7)$$

2.1 Disk's displacement fields

The fields of displacement would be expressed by the below equations, according to FSDT:

$$\begin{aligned} u(R, \beta, z, t) &= u_0(R, \beta, t) + zu_1(R, \beta, t) \\ v(R, \beta, z, t) &= v_0(R, \beta, t) + zv_1(R, \beta, t) \\ w(R, \beta, z, t) &= w_0(R, \beta, t) \end{aligned} \quad (8)$$

2.2 Strain-stress of the structure

The strain- stress relations would be explained as below, according to the FSDT:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \\ \sigma_{xz} \\ \sigma_{\theta z} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{x\theta} \\ \varepsilon_{xz} \\ \varepsilon_{\theta z} \end{bmatrix} \quad (9)$$

$$Q_{11} = Q_{22} = \frac{\bar{E}}{1 - \bar{\nu}^2}, \quad Q_{12} = Q_{21} = \frac{\bar{E}\bar{\nu}}{1 - \bar{\nu}^2}, \quad Q_{44} = Q_{55} = Q_{66} = \frac{\bar{E}}{2(1 + \bar{\nu})}$$

Then, the components of strain may be calculated as (Hosseini-Hashemi *et al.* 2010, Ghiasian *et al.* 2014):

$$\left. \begin{aligned} \varepsilon_{rr} &= \left(\frac{\partial u}{\partial R} + z \frac{\partial u_1}{\partial R} \right) \\ \varepsilon_{\beta\beta} &= \frac{\partial v}{R \partial \beta} + \frac{u}{R} = \left(\frac{\partial v_0}{R \partial \beta} + z \frac{\partial v_1}{R \partial \beta} + \frac{u_0}{R} + z \frac{u_1}{R} \right) \\ \gamma_{r\beta} &= \frac{\partial v}{\partial R} + \frac{1}{R} \frac{\partial u}{\partial \beta} - \frac{1}{R} v = \left(\frac{\partial v_0}{\partial R} + z \frac{\partial v_1}{\partial R} + \frac{1}{R} \frac{\partial u_0}{\partial \beta} + \frac{z}{R} \frac{\partial u_1}{\partial \beta} - \frac{1}{R} v_0 - z \frac{v_1}{R} \right) \\ \gamma_{rz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial R} = \left(u_1 + \frac{\partial w_0}{\partial R} \right) \\ \gamma_{\beta z} &= \frac{\partial v}{\partial z} + \frac{1}{R} \frac{\partial w}{\partial \beta} = \left(v_1 + \frac{1}{R} \frac{\partial w_0}{\partial \beta} \right) \end{aligned} \right\} \quad (10)$$

2.3 Extended Hamilton's principle

Relations between motion equations and BCs would be defined as the following equations, according to the principle of minimum energy (Sedighi and Daneshmand 2014, Ouakad and Sedighi 2019, Al-Furjan *et al.* 2020n, o, p, q, Moraveji Tabasi *et al.* 2020, Wang *et al.* 2020c, Al-Furjan *et al.* 2021):

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W) dt = 0 \quad (11)$$

Spinning disk's Speed vectors would be defined as

below (Hu and Wang 2016, Hu and Li 2019):

$$\left(\frac{\partial u}{\partial t} - \Omega v\right) i + \left(\frac{\partial u}{\partial t} + \Omega(R+u)\right) j + \left(\frac{\partial w}{\partial t}\right) k \quad (12)$$

It would be important to mention that j , k , and i are unit vectors in the θ , z , and R orientations, respectively. Eventually, the pertained rotating system's kinetic energy would be obtained as:

$$T = T_1 + T_2 = \int_V \frac{1}{2} \rho \left[\left(\frac{\partial u}{\partial t} - \Omega v\right)^2 + \left(\frac{\partial v}{\partial t} + \Omega(R+u)\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right] dV \quad (13)$$

The kinetic energy could be expressed as below, if spinning terms are not considered:

$$\delta T_1 = \int_V \rho \left(\frac{\partial u}{\partial t} \frac{\partial \delta u}{\partial t} + \frac{\partial v}{\partial t} \frac{\partial \delta v}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial \delta w}{\partial t} \right) dV : \quad (14)$$

$$\left[\begin{array}{l} \left\{ -I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 \frac{\partial^2 u_1}{\partial t^2} \right\} \delta u_0 + \left\{ -I_1 \frac{\partial^2 u_0}{\partial t^2} - I_2 \frac{\partial^2 u_1}{\partial t^2} \right\} \delta u_1 \\ \left\{ -I_0 \frac{\partial^2 v_0}{\partial t^2} - I_1 \frac{\partial^2 v_1}{\partial t^2} + I_3 c_1 \left(\frac{\partial^2 v_1}{\partial t^2} + \frac{\partial^3 w_0}{R \partial \beta \partial t^2} \right) \right\} \delta v_0 \\ \left\{ -I_1 \frac{\partial^2 v_0}{\partial t^2} - I_2 \frac{\partial^2 v_1}{\partial t^2} \right\} \delta v_1 + \left\{ -I_0 \frac{\partial^2 w_0}{\partial t^2} \right\} \delta w_0 \end{array} \right] R dR d\beta$$

The kinetic energy could be defined as below, if spinning terms are considered:

$$\delta T_2 = \int_V \rho \left(\Omega \left(-v \frac{\partial \delta u}{\partial t} - \frac{\partial u}{\partial t} \delta v + (R+u) \frac{\partial \delta v}{\partial t} + \frac{\partial v}{\partial t} \delta u \right) + [(R+u)\delta u + v\delta v] \Omega^2 \right) dV : \quad (15)$$

$$\left[\begin{array}{l} \left(\Omega \left(-\left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) - \left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) \right) \right) \delta u_0 \\ \left(-\Omega^2 \left(R + I_0 u_0 + I_1 u_1 - c_1 I_3 \left[u_1 + \frac{\partial w_0}{\partial R} \right] \right) \right) \delta u_0 \\ \left(\Omega \left(-\left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) + \left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) \right) \right) \delta v_0 \\ \left(-\Omega^2 \left(I_0 v_0 + I_1 v_1 - c_1 I_3 \left[v_1 + \frac{\partial w_0}{R \partial \beta} \right] \right) \right) \delta v_0 \\ \left(\Omega \left(\left(I_0 \frac{\partial^2}{\partial t \partial R} v_0 + I_1 \frac{\partial^2}{\partial t \partial R} v_1 \right) - \left(I_0 \frac{1}{R} \frac{\partial^2}{\partial t \partial \beta} u_0 + I_1 \frac{1}{R} \frac{\partial^2}{\partial t \partial \beta} u_1 \right) \right) \right) \delta w_0 \\ \left(-\left(I_0 \frac{1}{R} \frac{\partial^2}{\partial t \partial \beta} u_0 + I_1 \frac{1}{R} \frac{\partial^2}{\partial t \partial \beta} u_1 \right) + \left(I_0 \frac{\partial^2}{\partial t \partial R} v_0 + I_1 \frac{\partial^2}{\partial t \partial R} v_1 \right) \right) \delta w_0 \\ \left(+\Omega^2 \left(\left(I_0 \frac{1}{R} \frac{\partial}{\partial \beta} v_0 + I_1 \frac{1}{R} \frac{\partial}{\partial \beta} v_1 \right) + \left(I_0 \frac{\partial}{\partial R} u_0 + I_1 \frac{\partial}{\partial R} u_1 \right) \right) \right) \delta w_0 \\ \left(-\Omega(I_1) \left(\left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) - \left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) \right) \right) \delta u_1 \\ \left(-\Omega^2(I_1)(R + I_0 u_0 + I_1 u_1) \right) \delta u_1 \\ \left(\Omega(I_1 - c_1 I_3) \left(-\left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) + \left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) \right) \right) \delta v_1 \\ \left(-\Omega^2(I_1)(I_0 v_0 + I_1 v_1) \right) \delta v_1 \end{array} \right] dA$$

where (Safarpour *et al.* 2017, Safarpour *et al.* 2018, Al-Furjan *et al.* 2020m, n, o, p, q, r):

$$\{I_0, I_1, I_2, I_3\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \{I, z, z^2, z^3\} dZ \quad (16)$$

here Ω denotes spinning velocity. Also, the current composite structure's strain energy would be derived as:

$$\delta U = \frac{1}{2} \iiint_V \sigma_{ij} \delta \varepsilon_{ij} dV \quad (17)$$

$$= \int_A \left[\begin{array}{l} \left(N_{RR} \frac{\partial \delta u_0}{\partial R} + M_{RR} \frac{\partial \delta u_1}{\partial R} \right) \\ + \left(N_{\beta\beta} \frac{\partial \delta v_0}{R \partial \beta} + M_{\beta\beta} \frac{\partial \delta v_1}{R \partial \beta} + N_{\beta\beta} \frac{\delta u_0}{R} + M_{\beta\beta} \frac{\delta u_1}{R} \right) \\ \left(N_{R\beta} \frac{\partial \delta v_0}{\partial R} + M_{R\beta} \frac{\partial \delta v_1}{\partial R} + N_{R\beta} \frac{\partial \delta u_0}{R \partial \beta} + M_{R\beta} \frac{\partial \delta u_1}{R \partial \beta} \right) \\ + \left(-c_1 P_{R\beta} \left[\frac{\partial \delta u_1}{R \partial \beta} + \frac{\partial^2 \delta w_0}{R \partial \beta \partial R} \right] - N_{R\beta} \frac{\delta v_0}{R} - M_{R\beta} \frac{\delta v_1}{R} \right) \\ + \left((N_{Rz}) \left(\delta u_1 + \frac{\partial \delta w_0}{\partial R} \right) \right) + \left((N_{\beta z}) \left(\delta v_1 + \frac{\partial \delta w_0}{R \partial \beta} \right) \right) \end{array} \right] dA$$

which:

$$\{N_{RR}, M_{RR}\} = \int_z \{ \sigma_{RR}, z \sigma_{RR} \} dz;$$

$$\{N_{\beta\beta}, M_{\beta\beta}\} = \int_z \{ \sigma_{\beta\beta}, z \sigma_{\beta\beta} \} dz;$$

$$\{N_{Rz}, M_{Rz}, Q_{Rz}\} = \int_z \{ \sigma_{Rz}, z \sigma_{Rz}, z^2 \sigma_{Rz} \} dz; . \quad (18)$$

$$\{N_{R\beta}, M_{R\beta}, Q_{R\beta}\} = \int_z \{ \sigma_{R\beta}, z \sigma_{R\beta}, z^2 \sigma_{R\beta} \} dz;$$

$$\{N_{\beta z}, M_{\beta z}, Q_{\beta z}\} = \int_z \{ \sigma_{\beta z}, z \sigma_{\beta z}, z^2 \sigma_{\beta z} \} dz.$$

It should be mentioned that the centrifugal force which is in result of the structure's spinning feature in the may be written as (Mahinzare, Ranjbarpur *et al.* 2018; Hu and Li 2019):

$$\delta W_1 = \int_A \left(N^{Rotation} \right) \left(\frac{\partial w_0}{\partial R} \frac{\partial \delta w_0}{\partial R} + \frac{\partial w_0}{R^2 \partial \beta} \frac{\partial \delta w_0}{\partial \beta} \right) dA \quad (19)$$

where

$$N^{Rotation} = \int_{-h/2}^{h/2} \left(\frac{\rho \Omega^2}{8} (2R)(3+v) \right) dz \quad (20)$$

Since it is obvious from figure 1, the internal and upper composite layers are covered by viscoelastic substrate. Then, the viscoelastic foundation's work, would be obtained as:

$$\delta W_2 = \int_A \left[K_w w \delta w + C_d \dot{w} \delta \dot{w} \right] dA \quad (21)$$

Ultimately, pertaining BCs and governing motion equations would be extracted using inserting Eqs. (19), (17), and (13) in the principle of minimum energy (Eq. (11)) which would be defined as:

$$\delta u_0 : \quad (22)$$

$$\frac{\partial}{\partial R} N_{RR} - \frac{N_{\beta\beta} - N_{RR}}{R} + \frac{\partial}{R \partial \beta} N_{R\beta} = I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 u_1}{\partial t^2}$$

$$+ \left(\Omega \left(-\left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) - \left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) \right) - \Omega^2 (R + I_0 u_0 + I_1 u_1) \right)$$

$$\delta v_0 : \frac{\partial}{R\partial\beta} N_{\beta\beta} + \frac{2N_{R\beta}}{R} + \frac{\partial}{\partial R} N_{R\beta} = I_0 \frac{\partial^2 v_0}{\partial t^2} + I_1 \frac{\partial^2 v_1}{\partial t^2} + \left(\Omega \left(- \left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) + \left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) \right) - \Omega^2 (I_0 v_0 + I_1 v_1) \right) \quad (23)$$

$$\delta w_0 : \frac{\partial}{\partial R} (N_{Rz}) + \frac{\partial}{R\partial\beta} (N_{\beta z}) + \frac{\partial}{\partial R} N_{Rz} + \frac{\partial}{R\partial\beta} N_{\beta z} - N^{Rotation} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{R^2 \partial \beta^2} \right) - K_w w - C_d \dot{w} = \left(I_0 \frac{\partial^2 w_0}{\partial t^2} \right) + \left(\Omega \left(\left(I_0 \frac{\partial^2}{\partial R \partial t} v_0 + I_1 \frac{\partial^2}{\partial R \partial t} v_1 \right) - \left(I_0 \frac{1}{R} \frac{\partial^2}{\partial t \partial \beta} u_0 + I_1 \frac{1}{R} \frac{\partial^2}{\partial t \partial \beta} u_1 \right) \right) - \left(I_0 \frac{1}{R} \frac{\partial^2}{\partial R \partial \beta} u_0 + I_1 \frac{1}{R} \frac{\partial^2}{\partial R \partial \beta} u_1 \right) + \left(I_0 \frac{\partial^2}{\partial t \partial R} v_0 + I_1 \frac{\partial^2}{\partial t \partial R} v_1 \right) \right) + \Omega^2 \left(\left(I_0 \frac{\partial}{R\partial\beta} v_0 + I_1 \frac{\partial}{R\partial\beta} v_1 \right) + \left(I_0 \frac{\partial}{\partial R} u_0 + I_1 \frac{\partial}{\partial R} u_1 \right) \right) \quad (24)$$

$$\delta u_1 : \frac{\partial}{\partial R} M_{RR} - \frac{M_{\beta\beta} - M_{RR}}{R} + \frac{\partial}{R\partial\beta} M_{R\beta} - N_{Rz} = I_1 \frac{\partial^2 u_0}{\partial t^2} + I_2 \frac{\partial^2 u_1}{\partial t^2} + \left(-\Omega (I_1 - c_1 I_3) \left(\left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) - \left(I_0 \frac{\partial}{\partial t} v_0 + I_1 \frac{\partial}{\partial t} v_1 \right) \right) - \Omega^2 (I_1) (R + I_0 u_0 + I_1 u_1) \right) \quad (25)$$

$$\delta v_1 : \frac{\partial}{R\partial\beta} M_{\beta\beta} + \frac{2}{R} M_{R\beta} + \frac{\partial}{\partial R} M_{R\beta} - M_{\beta z} = I_1 \frac{\partial^2 v_0}{\partial t^2} + I_2 \frac{\partial^2 v_1}{\partial t^2} + \left(\Omega (I_1) \left(- \left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) + \left(I_0 \frac{\partial}{\partial t} u_0 + I_1 \frac{\partial}{\partial t} u_1 \right) \right) - \Omega^2 (I_1) (I_0 v_0 + I_1 v_1) \right) \quad (26)$$

Moreover, general pertained BCs may be written as:

$$\begin{aligned} \delta u_0 = 0 & \quad or \quad N_{RR} n_R + \frac{1}{R} N_{R\beta} n_\beta = 0 \\ \delta v_0 = 0 & \quad or \quad N_{R\beta} n_R + \frac{1}{R} N_{\beta\beta} n_\beta = 0 \\ \delta w_0 = 0 & \quad or \quad \left(\left((N_{Rz}) + N_{Rz} \right) n_R + \left(\left(\frac{N_{\beta z}}{R} + \frac{N_{\beta z}}{R} \right) n_\beta \right) \right) = 0 \\ \delta u_1 = 0 & \quad or \quad (M_{RR}) n_R + \left(\frac{M_{R\beta}}{R} \right) n_\beta = 0 \\ \delta v_1 = 0 & \quad or \quad (M_{R\beta}) n_R + \left(\frac{M_{\beta\beta}}{R} \right) n_\beta = 0 \end{aligned} \quad (27)$$

3. Solution procedure

This section deals with a solution approach (Huang *et al.* 2019, Gholipour *et al.* 2020a, b, Pang *et al.* 2020) known as GDQM to solve the current problem's equations. In this approach the n^{th} -order derivatives of a adequately smooth

function f respect to related detached points in the whole domain's range may be assumed as a linear weighted sum function of the values at each points in the all allowable input range as follows (Shu 2012):

$$\frac{\partial^n f}{\partial r^n} = \sum_{m=1}^M C^{(n)}_{j,m} f_{m,k} \quad (28)$$

where, $C^{(n)}$ illustrates weighting parameters for the derivative of n^{th} -order in the orientation of its radius, which would be obtained as:

$$\begin{aligned} C_{ij}^{(1)} &= - \sum_{j=1, i \neq j}^n C_{ij}^{(1)} & i = j \\ C_{ij}^{(1)} &= \frac{M(x_i)}{(x_i - x_j) M(x_j)} & i, j = 1, 2, \dots, n \text{ and } i \neq j \end{aligned} \quad (29)$$

here,

$$M(x_i) = \prod_{j=1, j \neq i}^n (x_i - x_j) \quad (30)$$

Eq. (27)'s derivatives would be defined as the relations given below:

$$\begin{aligned} C_{ii}^{(n)} &= - \sum_{j=1, i \neq j}^n C_{ij}^{(n)} \\ 1 \leq n \leq N-1 & \text{ while } j, i = 1, 2, \dots, N \\ C_{ij}^{(n)} &= r \left[C_{ij}^{(n-1)} C_{ij}^{(1)} - \frac{C_{ij}^{(n-1)}}{(x_i - x_j)} \right] \\ i \neq j, 2 \leq n \leq N-1 & \text{ while } j, i = 1, 2, \dots, N. \end{aligned} \quad (31)$$

Then by employing Chebyshev greed polynomials points, the seed through r-axes would be distributed as:

$$r_i = \frac{R_0 - R_i}{2} \left(1 - \cos \left(\frac{(i-1)}{(N_i-1)} \pi \right) \right) + R_i \quad i = 1, 2, 3, \dots, N_i \quad (32)$$

However, disk's displacement fields may be written as:

$$\begin{Bmatrix} u_0(R, \beta, t) \\ v_0(R, \beta, t) \\ w_0(R, \beta, t) \\ u_1(R, \beta, t) \\ v_1(R, \beta, t) \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} u_{0n}(R) \times \sin(n\beta) \\ v_{0n}(R) \times \cos(n\beta) \\ w_{0n}(R) \times \sin(n\beta) \\ u_{1n}(R) \times \sin(n\beta) \\ v_{1n}(R) \times \cos(n\beta) \end{Bmatrix} e^{i\omega t} \quad (33)$$

Ultimately, the formulation of the GDQ considering BC relations may be defined as:

$$\left\{ \begin{Bmatrix} [M_{dd}] & [M_{db}] \\ [M_{bd}] & [M_{bb}] \end{Bmatrix} \omega_n^2 + \begin{Bmatrix} [C_{dd}] & [C_{db}] \\ [C_{bd}] & [C_{bb}] \end{Bmatrix} \omega_n + \begin{Bmatrix} [K_{dd}] & [K_{db}] \\ [K_{bd}] & [K_{bb}] \end{Bmatrix} \right\} \begin{Bmatrix} \delta_d \\ \delta_b \end{Bmatrix} = 0 \quad (34)$$

Here, domain and boundary grid-points are explained by b and d , respectively. Furthermore, displacement vector are denoted by δ index. M , K , and C would be, respectively, the mass, stiffness, and damping. Then, by using Eq. (33) the novel system would be as:

$$[K_{db}] \delta_b + [K_{dd}] \delta_d = 0 \quad (35a)$$

Table 1 Material properties and efficiency parameter of GPLRC structure

| Graphene Platlets | E^{GPL} [Gpa] | ν^{GPL} | ρ^{GPL} [kg/m ³] | α^{GPL} [$\times 10^{-6}/k$] | t^{GPL} [μm] | t^{GPL} [nm] | w^{GPL} [μm] |
|----------------------|---------------------------|--------------------|---|---|---|--------------------------|---------------------------------------|
| | 1010 | 0.186 | 1062.5 | 5 | 2.5 | 1.5 | 1.5 |
| Copper matrix | E^{m} [Gpa] | ν^{m} | ρ^{m} [kg/m ³] | | α^{m} [$\times 10^{-6}/k$] | | |
| | 130 | 0.34 | 8960 | | 17e-6 | | |

Table 2 Comparison of the non-dimensional natural frequency of the current structure with the published article in the literature

| | R_o/R_i | S-S | | | | C-C | | | |
|--------------------------|-----------|-------------|--------|--------|--------|-------------|--------|--------|--------|
| | | Mode number | | | | Mode number | | | |
| | | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Ref. (Han and Liew 1999) | 0.001 | 14.585 | 51.681 | 112.89 | 198.34 | 27.180 | 75.264 | 148.01 | 245.37 |
| Current study | 0.001 | 13.524 | 50.202 | 111.76 | 198.51 | 28.414 | 77.263 | 151.05 | 249.15 |
| Ref. (Han and Liew 1999) | 0.050 | 14.424 | 50.309 | 107.15 | 182.45 | 26.434 | 71.128 | 135.14 | 215.18 |
| Current study | 0.050 | 13.428 | 49.009 | 106.19 | 182.18 | 27.579 | 72.666 | 136.26 | 214.36 |
| Ref. (Han and Liew 1999) | 0.100 | 13.774 | 46.847 | 94.570 | 151.81 | 24.529 | 62.040 | 111.02 | 167.06 |
| Current study | 0.100 | 13.154 | 46.161 | 93.694 | 151.26 | 25.458 | 62.734 | 110.29 | 163.31 |
| Ref. (Han and Liew 1999) | 0.150 | 13.118 | 42.530 | 81.419 | 124.82 | 22.130 | 52.662 | 90.186 | 131.25 |
| Current study | 0.150 | 12.738 | 42.082 | 81.136 | 124.14 | 22.809 | 52.735 | 88.862 | 127.41 |
| Ref. (Han and Liew 1999) | 0.200 | 12.350 | 38.237 | 70.124 | 104.10 | 19.743 | 44.813 | 74.760 | 106.71 |
| Current study | 0.200 | 12.226 | 38.143 | 70.175 | 104.14 | 20.194 | 44.583 | 73.528 | 103.65 |

$$[K_{bb}]\delta_b + [K_{bd}]\delta_d = 0 \quad (35b)$$

here, the freedom degrees' vector may explain as:

$$\delta_b = -\frac{[K_{dd}]}{[K_{db}]} \delta_d \quad (36)$$

By inserting of Eq. (35) in Eq. (34b):

$$\left([K_{bd}] - [K_{bb}][K_{db}]^{-1}[K_{dd}]\right)\delta_d = 0 \quad (37)$$

Then

$$K^* = [K_{bd}] - [K_{bb}][K_{db}]^{-1}[K_{dd}] \quad (38)$$

and

$$[C_{bd}]\delta_d + [C_{bb}]\delta_b = 0 \quad (39)$$

By inserting Eq. (35) in Eq. (38), have:

$$\left([C_{bd}] - [C_{bb}][K_{db}]^{-1}[K_{dd}]\right)\delta_d = 0 \quad (40)$$

Then

$$C^* = [C_{bd}] - [C_{bb}][K_{db}]^{-1}[K_{dd}] \quad (41)$$

and

$$[M_{bd}]\delta_d + [M_{bb}]\delta_b = 0 \quad (42)$$

Also, by inserting Eq. (35) in Eq. (41), have:

$$\left([M_{bd}] - [M_{bb}][K_{db}]^{-1}[K_{dd}]\right)\delta_d = 0 \quad (43)$$

Then

$$M^* = [M_{bd}] - [M_{bb}][K_{db}]^{-1}[K_{dd}] \quad (44)$$

Eventually, by solving the next relation, structure's displacement fields and frequency information may be derived through using GDQM.

$$K^* + M^* \omega^2 + C^* \omega = 0 \quad (45)$$

It could be mentioned that the non-dimensional frequency may be calculated as:

$$\bar{\omega} = \omega R_0^2 \sqrt{\rho h / D}, \quad D = Eh^3 / (12(1-\nu^2)) \quad (46)$$

The final goal of the current paper which is boosting disks' dynamics, would lead to introducing a famous filler, known as hybrid multi-scale nano-sized composites, although these reinforcements' thermo-mechanical properties along with epoxy's properties employed in this research are demonstrated in Table 1.

4. Validation

For examination of the current approach, the annular plate's frequency factors extracted in this scrutinization

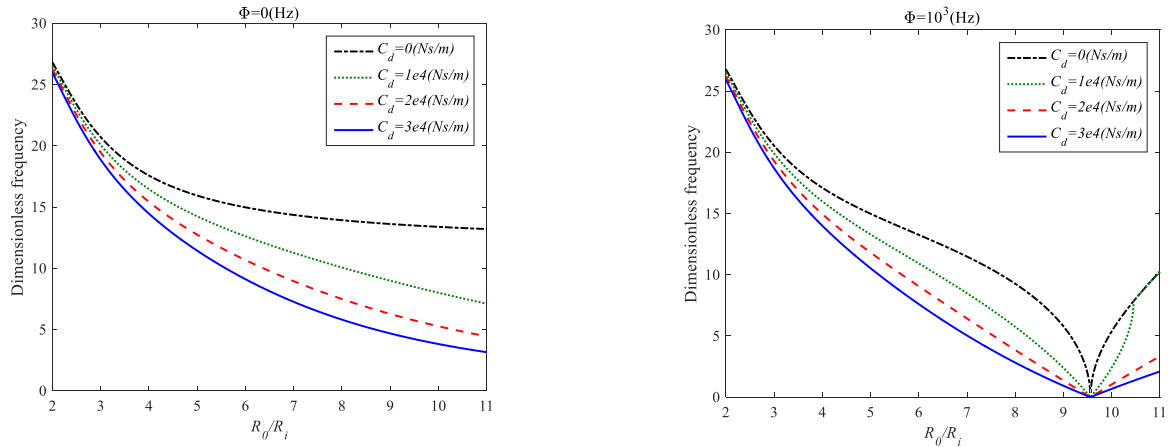


Fig. 2 The influences of C_d , Φ , R_o/R_i parameters on the frequency information of a rotating GPLs reinforced disk

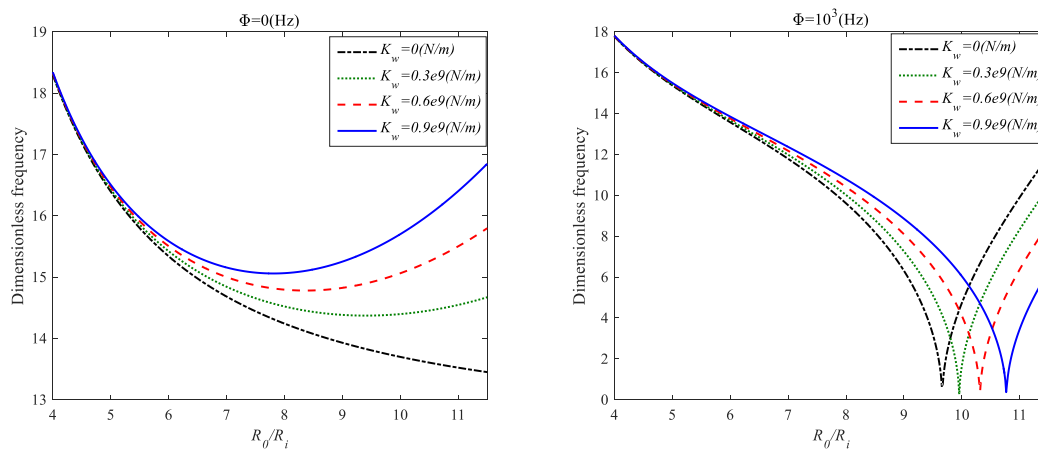


Fig. 3 The influences of K_w , Φ , R_o/R_i parameters on the frequency information of a rotating GPLs reinforced disk

are validated with those reported in Han and Liew (1999) for a wide range of axisymmetric cases of modes of vibration through the ratio of outer radius to inner radius R_o/R_i considering three different BCs as depicted in Table 2. Due to the tables reported in this paper, the present study approximates the annular plate’s dynamics very much closer than those presented in Ref. (Han and Liew 1999). The system’s kinematics would be explained through FSDT in Ref. (Han and Liew 1999). Moreover, it would be mentioned that the discrepancies in a couple of outcomes drop dramatically when we want to analyze higher modes of vibrations and larger values in ratios of R_o/R_i .

5. Numerical results

We present Fig. 2 for investigation of the influences of the damping parameter of the foundation (C_d), rotating speed (Φ), and radius ratio (R_o/R_i) on the frequency information of a rotating GPLs reinforced disk.

If we have attention to Fig. 2 can see that when the rotation speed is zero, the radius ratio will be a reason for ever decreasing the frequency of the structure while when $\Phi > 0$, the frequency of the disk is decreased as long as having a critical radius ratio. For better understanding, at

the value of radius ratio that the frequency of the system will be zero, the critical R_o/R_i is appeared. According to above figure when the damping parameter of the foundation increases the frequency of the rotary system can improve but this matter will be seen at the higher value of radius ratio. Also, Fig. 2 prove that in the condition that $\Phi > 0$, increase in the damping parameter cannot make any change in the critical radius ratio.

Fig. 3 is candidate for having a study about the impacts of the Winkler or elastic parameter of the foundation (K_w), rotating speed (Φ), and radius ratio (R_o/R_i) on the frequency information of a rotating GPLs reinforced disk. We should attention to Fig. 3 because there is shown that if we have a rotary disk without doubt we will have critical radius ratio while a static disk does not have limitation on the R_o/R_i parameter. Generally, there is an indirect relation between the radius ratio of the composite disk and dynamic stability of the structure. As a main full result, increasing the K_w parameter cannot play an effect on the frequency of the disk at the lower value of the radius ratio while the Winkler parameter will provide an enhancement on the critical radius ratio and frequency of the GPLs reinforced disk at the higher outer radius. When the system is static or the rotational speed is zero and at the higher value of K_w , the effect of elastic foundation is overcome by

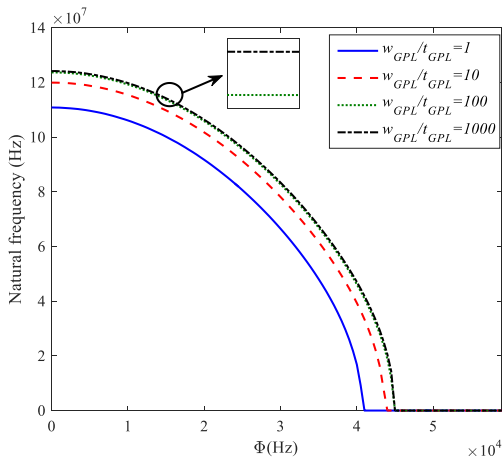


Fig. 4 Geometrical property of the GPLs and rotating speed effects on the real part of frequency of the rotary cantilevered disk

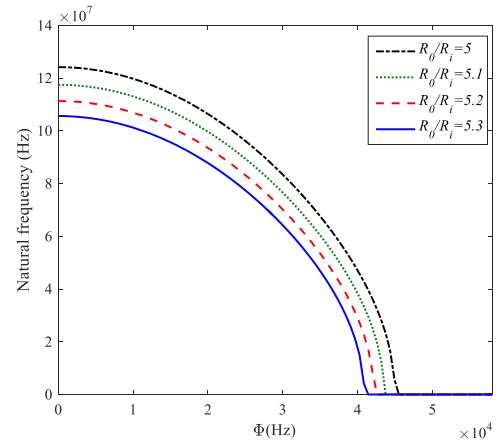


Fig. 6 Applied rotational speed effect on the real part of frequency of the rotary cantilevered disk for four values of inner to outer radius ratio

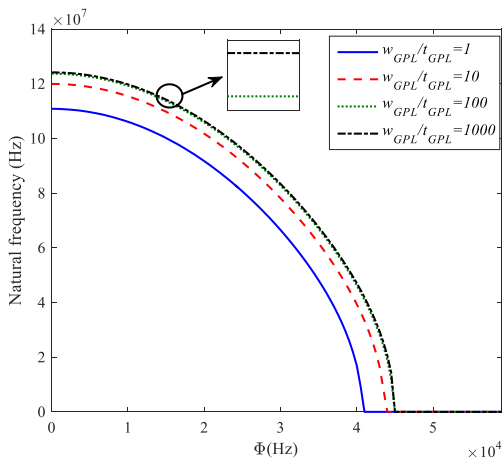


Fig. 4 Geometrical property of the GPLs and rotating speed effects on the real part of frequency of the rotary cantilevered disk

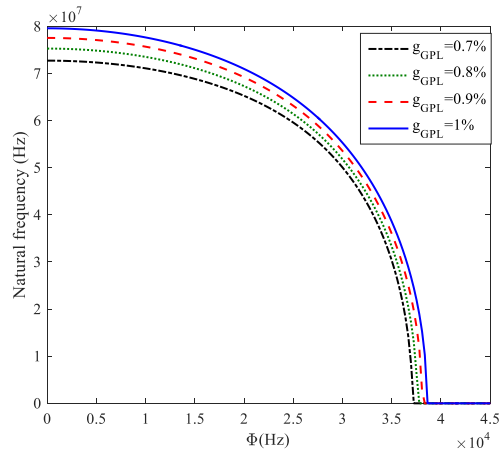


Fig. 7 Applied rotational speed influence on the real part of frequency of the rotary cantilever disk for four values of GPLs weight fraction

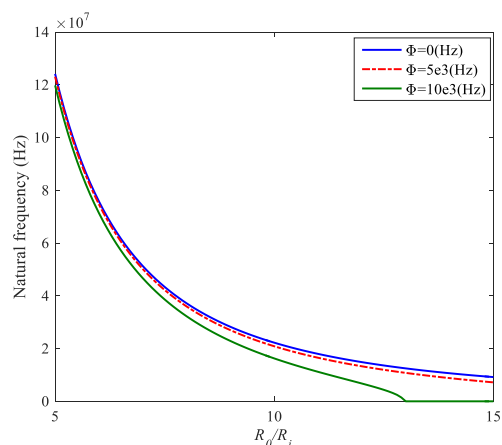


Fig. 5 Inner to outer radius ratio effect on the real part of frequency of the rotary cantilever disk for three values of the rotating speed

The given diagram in Fig. 4 evaluates the width to thickness ratio of GPLs and rotating speed effects on the frequency of the rotary cantilevered disk.

In view of Fig. 4 adding the wide GPLs in the epoxy matrix of the disk and increasing the rotating speed have direct and indirect effects on the frequency of the rotary cantilevered disk, respectively. Also, change in the high values of the geometrical information of the GPLs does not make any effect on the critical speed and the extent of the stable or unstable area. Fig. 5 takes part to study the frequency of the rotary cantilever disk by considering the inner to outer radius ratio effect for three values of the rotating speed.

Fig. 5 illustrates that as long as the internal to external radius ratio and spinning velocity continue to grow, the real part frequency of the structure continues to decrease. In addition, the rotating speed effect on the real part of frequency is more effective in the higher values of the radius ratio.

Fig. 6 carries out research into the effects of inner to outer radius ratio and applied rotational speed on the frequency of the cantilevered disk. Based on the finding of

the radius ratio and the relation between the R_o/R_i and frequency of the system will change from indirect to direct.

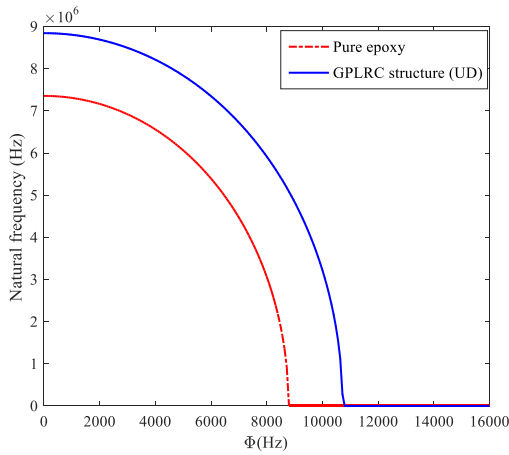


Fig. 8 Applied rotational speed effect on the real part of the frequency of the rotary cantilever disk for three GPLRC patterns

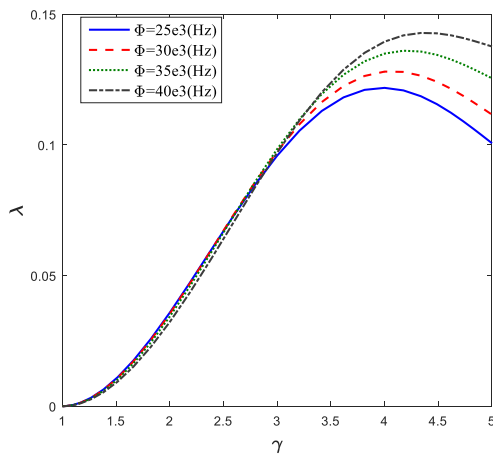


Fig. 9 x/R_i versus dimensionless amplitude of the rotary cantilever disk for four value of the external rotating speed and Pattern 4, $g_{GPL}=1\%$, $R_0/R_i=5$

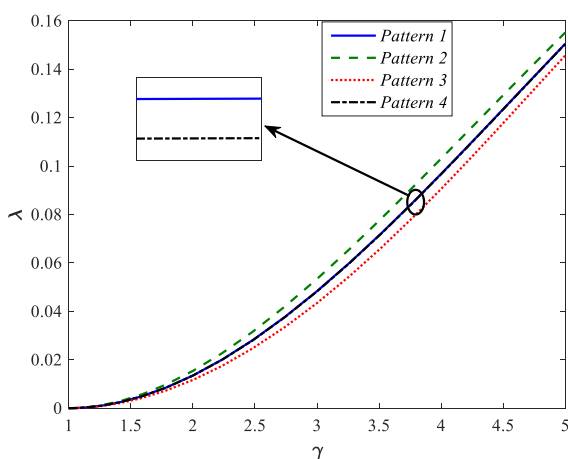


Fig. 10 γ parameter versus dimensionless amplitude of the rotary cantilever disk for four GPLRC patterns, $h=0.1R_i$ $g_{GPL}=0.05\%$, and $R_0/R_i=5$

Fig. 6 can conclude that the rotational speed to buckle the structure and stable area of the system are rising as a result of increasing the outer radius relative to the inner

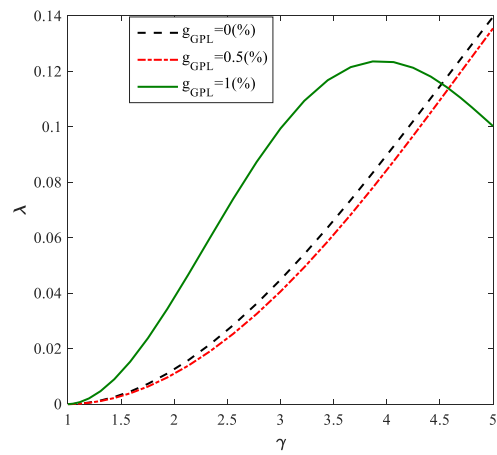


Fig. 11 x/R_i versus non-dimensional amplitude of the spinning cantilever disk for three value of g_{GPL} , pattern 2, $h=0.1R_i$ $g_{GPL}=1\%$, and $R_0/R_i=5$

radius.

Fig. 7 is intended to provide information for studying the reinforcement material percentage effect on the frequency of the cantilever disk.

The presence of Fig. 7 is essential to prove that ever-increasing the value fraction of the reinforcement provides improvements in the rotational speed to buckle the structure. Also, the stable area is expanded by having more GPLs in the matrix of the rotary cantilever disk. The good reason for this enhancement is that the structure is being harder with adding the GPLs and provides the higher critical rotating speed. As an alternative explanation, the higher the value fraction of GPLs, the better dynamic responses and the lower instable area in the system.

For the purpose of a research on the frequency of the cantilever disk with taking into account various GPLs patterns, Fig. 8 is provided.

The principal objective of figure 8 is that the best structure is GPLRC one for having a structure with the highest critical rotating speed and the widest stable area. When the structure made by pure epoxy material, the damping of the system is happened at the lowest rotating speed and the system is more ready for changing its condition from stable to instable in comparison with composite structure.

Fig. 9 conducts an analysis for providing amplitude data with attention to the inserted external spinning velocity influence on the matter. Through Fig. 9 can find out that the picked at the dimensionless amplitude ($\lambda = w/R_0$) of the structure is being far from the free edge of the disk thanks to have a decline in the value of the applied rotating speed. In other word when the applied speed reduces, the maximum amplitude can be close to inner reduce of the disk. Furthermore, for $1 \leq \gamma \leq 3$ and $3 < \gamma \leq 5$, there is an indirect and direct effect from applied rotating speed on the dimensionless amplitude of the cantilever disk.

For the purpose of having a research on the dimensionless amplitude of the cantilever disk with taking into account various GPLRC patterns, Fig. 10 is presented.

Fig. 10 shows that when the pattern 2 and pattern 3 are employed to reinforced the structure, being close to the

outer edge of the disk leads to increase and decrease in the amplitude of a cantilever disk, respectively. By considering pattern 1 and pattern 4 for reinforcing the rotary disk, the relation between the γ parameter and amplitude is equal and similar.

The given diagram in Fig. 11 evaluates the value fraction of GPLs (g_{GPL}) effect on the dimensionless amplitude of the rotary cantilever disk.

The principal objective of figure 11 is that not only there is a decline in the pick of the amplitude owing to using the more GPLs, but also an ever-increasing in the value of g_{GPL} coefficient is a reason for having the maximum amplitude far from the edge of the cantilever rotary disk.

6. Conclusions

In this paper, amplitude and vibrational behavior of a spinning cantilevered GPLRC disk covered by viscoelastic substrate have been introduced. Energy approaches have been applied for deriving the differential governing motion equations. The outcomes have been verified when it comes to contrasting them with published outcomes presented in the literature review. Despite from computational solution, an FE approach has been applied using the FE package for simulating the smart cantilevered GPLRC disk's response. The outcomes demonstrated from the FE simulation shows an excellent agreement with the semi-computational approach. The outcomes reported in this research depicted that viscoelastic substrate, angular velocity speed, GPL weight, and multifarious GPLs' patterns have a dramatic influence on the characteristics pertained to the frequency and amplitude of the spinning cantilevered GPLRC disk. The computational outcomes disclosed that:

- when the rotation speed is zero, the radius ratio will be a reason for ever decreasing the frequency of the structure while when $\Phi > 0$, the frequency of the disk is decreased as long as having a critical radius ratio.
- in the condition that $\Phi > 0$, increase in the damping parameter cannot make any change in the critical radius ratio.
- if we have a rotary disk without doubt we will have critical radius ratio while a static disk does not have limitation on the R_o/R_i parameter.
- owing to consideration of the shear correction factor via FSDT, the peaked at the rotating speed in the stable area reduces.
- change in the high values of the geometrical information of the GPLs does not make any influence on the critical velocity and the extent of the instable or stable area.
- the spinning velocity impact on the frequency is much more efficient in the higher amounts of the radius ratio.
- when the structure made by pure epoxy material, the damping of the system is happened at the lowest rotating speed and the system is more ready for changing its condition from stable to instable in comparison with composite structure.
- lowest displacement at the radial mode shape is seen for the structure when is made by pattern 2.
- As long as the radius ratio drops, the concentration of

stress at the disk's internal edge would intensify.

- the picked at the dimensionless amplitude ($\lambda = w/R_o$) of the structure is being far from the free edge of the disk thanks to have a decline in the value of the inserted spinning velocity.

Acknowledgments

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