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Optimum design of cantilever retaining walls under seismic loads using a hybrid TLBO algorithm

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Abstract. The main purpose of this study is to investigate the performance of the proposed hybrid teaching-learning based optimization algorithm on the optimum design of reinforced concrete (RC) cantilever retaining walls. For this purpose, three different design examples are optimized with 100 independent runs considering continuous and discrete variables. In order to determine the algorithm performance, the optimization results were compared with the outcomes of the nine powerful meta-heuristic algorithms applied to this problem, previously: the big bang-big crunch (BB-BC), the biogeography based optimization (BBO), the flower pollination (FPA), the grey wolf optimization (GWO), the harmony search (HS), the particle swarm optimization (PSO), the teaching-learning based optimization (TLBO), the jaya (JA), and Rao-3 algorithms. Moreover, Rao-1 and Rao-2 algorithms are applied to this design problem for the first time. The objective function is defined as minimizing the total material and labor costs including concrete, steel, and formwork per unit length of the cantilever retaining walls subjected to the requirements of the American Concrete Institute (ACI 318-05). Furthermore, the effects of peak ground acceleration value on minimum total cost is investigated using various stem height, surcharge loads, and backfill slope angle. Finally, the most robust results were obtained by HTLBO with 50 populations. Consequently the optimization results show that, depending on the increase in PGA value, the optimum cost of RC cantilever retaining walls increases smoothly with the stem height but increases rapidly with the surcharge loads and backfill slope angle.

Keywords: optimization; retaining wall; structural design; seismic loads; teaching-learning based optimization

1. Introduction

Engineering structures are designed by considering the design objectives such as structural safety, cost, and functionality. It is expected that an engineering design should not only meet the minimum requirements of the both functionality and the safety but also the costs of it should be the least as possible. Unlike the other criteria for engineering design, achieving the minimum cost target involves more uncertainty. Although there are strict specification and codes on structural safety and functionality, there are no rules, restrictions, or equations to achieve the minimum cost design. Therefore, the construction cost of the engineering design varies depending on the experience of the structural engineer, and it is generally overdesigned in terms of cost.

Meta-heuristic optimization algorithms are one of the proposed approaches to obtain low-cost designs regardless of the engineer's experience. Researches on cost optimization of the some structural elements and systems such as frames (Kaveh and Sabzi 2012), slab (El Semelawy *et al.* 2012), footings (Chaudhuri and Maity 2020), piles (Chan *et al.* 2009), bridge frames (Perea *et al.* 2008), retaining walls are available in the literature. This study focuses on the cost optimization of reinforced concrete cantilever retaining wall, which is one of the most common types of structures used in the sites consists the slopes in order to prevent the both sliding and overturning.

Reinforced concrete retaining walls are structures that prevent slope stability problems that may occur due to differences in ground surface level due to filling or excavation. It is reasonably convenient to use meta-heuristic algorithms because they have limited and few numbers of design variables to be optimized. In the literature, a significant number of meta-heuristic algorithms have been employed to achieve the minimal cost design of reinforced concrete cantilever retaining walls under static loads, e.g., genetic algorithm (GA) (Pei and Xia 2012, Kaveh *et al.* 2013), big bang-big crunch algorithm (BB-BC) (Camp and Akin 2012), particle swarm optimization (PSO) algorithm (Pei and Xia 2012, Gandomi *et al.* 2015, Kayhan and Demir 2016), harmony search (HS) algorithm (Kaveh and Abadi 2011, Akin and Saka 2015), teaching-learning based optimization (TLBO) algorithm (Temur and Bekdaş 2016), flower pollination algorithm (FPA) (Mergos and Mantoglou 2019), charged system search algorithm (Kaveh and Behnam 2013, Talatahari and Sheikholeslami 2014), ant colony optimization algorithm (Ghazavi and Bonab 2011), simulated annealing algorithm (Ceranic *et al.* 2001, Yepes *et al.* 2008, Pei and Xia 2012), firefly algorithm (Sheikholeslami *et al.* 2015, Gandomi *et al.* 2017), differential evaluation algorithm (Nandha Kumar and Suribabu 2017), biogeography based optimization (BBO) algorithm (Gandomi *et al.* 2017), grey wolf optimization algorithm (Bekdas and Temur 2018, Kalemci *et al.* 2020),

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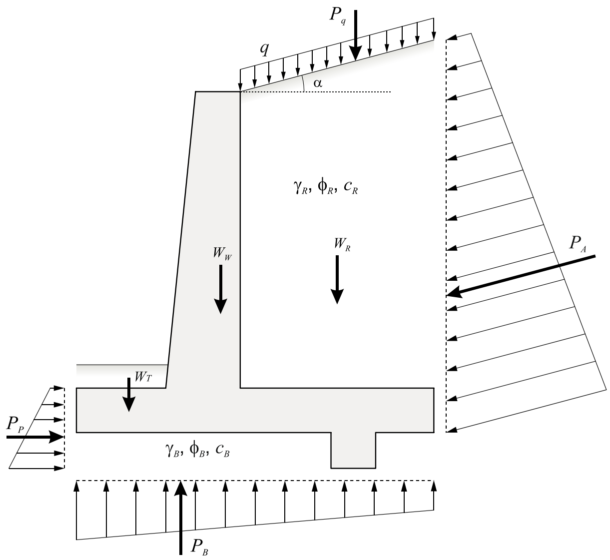


Fig. 1 Static forces acting on a cantilever retaining wall

interior search algorithm (Gandomi *et al.* 2017), dolphin echolocation optimization algorithm (Kaveh and Farhoudi 2016), bacterial foraging optimization algorithm (Ghazavi and Salavati 2011), gravitational search algorithm (Khajehzadeh *et al.* 2014), chaotic imperialist competitive algorithm (Pourbaba *et al.* 2013), jaya algorithm (JA) (Öztürk *et al.* 2020), Rao-3 algorithm (Kalemci and Ikizler 2020), black hole algorithm (Yepes *et al.* 2020), shuffled shepherd optimization algorithm (Kaveh *et al.* 2020).

On the other hand, in the case of seismic loads, a limited number of studies have been published. The optimum design of retaining walls under seismic loads were investigated by genetic algorithm (Rahbari *et al.* 2017a, b), ray optimization algorithm (Kaveh and Khayatazad 2014), and BBO with Lévy flights (Aydogdu 2017), previously.

Population-based heuristic algorithms require controlling parameters. The number of the generations and the population size are common control parameters for all heuristic algorithms. In addition to this, many algorithms use specific control parameters to their own. For example, GA requires the mutation probability, BB-BC requires the influence parameter of global best, BBO requires the maximum mutation probability, HS requires the pitch adjusting rate, PSO requires the inertia weight. The robustness of these algorithms and the calculation time could vary regarding to adjustment of the unique controlling parameters of the algorithms. In the previous studies on optimum design of cantilever retaining walls under seismic loads, algorithms which require specific parameters were applied. Heuristic algorithms that only use common control parameters and do not require specific parameters have been developed in recent years. TLBO, JA, and Rao Algorithms (Rao-1, Rao-2, and Rao-3) (Rao 2020) are examples of these specific parameter-less algorithms using only common control parameters. All the algorithms applied in the optimum design of retaining walls require algorithm-specific parameters, except TLBO, JA, and Rao-3. It was determined that the results obtained with TLBO under static loads have high convergence and robustness performance in relevant parameters, e.g., standard deviation, minimum cost, average cost (Temur and Bekdaş 2016). The optimization results obtained from the Rao-3 algorithm were not compared with another algorithm (Kalemci and Ikizler 2020). Unlike previous studies, this study focused on algorithms without special parameters. Furthermore, the original TLBO is modified in scope of this study. The modified algorithm is named as Hybrid Teaching-Learning Based Optimization Algorithm (HTLBO).

The aim of this study is two-fold: The first one is to investigate the performance of the proposed algorithm named Hybrid Teaching-Learning Based Optimization on the cost optimization of reinforced concrete cantilever retaining walls under seismic loads. For this purpose, eleven different meta-heuristic algorithms have been applied for comparison of results: BB-BC (Camp and Akin 2012), BBO (Aydogdu 2017), FPA (Mergos and Mantoglou 2019), GWO (Kalemci *et al.* 2020), HS (Kaveh and Abadi 2011), PSO (Gandomi *et al.* 2015), TLBO (Temur and Bekdaş 2016), JA (Öztürk *et al.* 2020), and Rao Algorithms (Rao-1, Rao-2, and Rao-3 (Kalemci and Ikizler 2020)). The second purpose is to investigate the effect of peak ground acceleration on the minimum cost of the reinforced concrete cantilever retaining walls. For this purpose, 21 different PGA values increase from 0 to 0.4 g in intervals of 0.02 g, 6 different stem heights from 4.5 m to 7 m in intervals of 0.5 m, 7 different surcharge loads from 0 to 30 kN/m² in intervals of 5 kN/m², and 21 different backfill slope angles from 0 to 20° in intervals of 1° were considered.

The paper is organized as follows: Section 2 presents the design rules of the reinforced concrete cantilever retaining walls. Design variables, constraints and objective function are explained in Section 3. Then, Rao algorithms, JA, and the proposed algorithm are presented in Section 4. Design examples are described in Section 5 and the optimization results are compared. Finally, the conclusions drawn from the benchmark problems are summarized in Section 6.

2. Design of reinforced concrete cantilever retaining walls

Cantilever retaining walls are a type of structures that are in equilibrium state under horizontal and vertical loads applied by their own weight, surcharge loads, backfill, and base soil. Forces acting on a cantilever retaining wall under static loads are illustrated in Fig. 1, where W_W is the total weight of the reinforced concrete wall, W_R is the weight of retained soil on the heel, and W_T is the weight of the soil on the toe. P_A, P_P, P_B, and P_q are the forces resulting from active, passive earth pressure, bearing capacity of the sub-base soil, and surcharge load, respectively; α denotes the inclination of the backfill with horizontal, q is the distributed surcharge load, γ is the unit weight of the soils, ϕ is the internal friction angle of the soils, c is the cohesion of the soils.

The retaining systems built in the earthquake zones should meet the requirements of the seismic excitation with a minimum factor of safety as 1.1, conventionally Common geotechnical failure modes considered in the stability analysis of retaining walls are overturning, sliding, and bearing stress. Moreover, the external forces such as the shear force and the bending moment demands for all elements of cantilever retaining walls must be satisfied. The details of stability and design calculations are presented in the following subsections.

2.1 Earth pressure conditions

In this study, a simplified approach developed by Mononobe, Matsuo (Mononobe and Matsuo 1929), and Okabe (Okabe 1924), adapted by Kramer (Kramer 1996) for dry and cohesionless soils, was used to calculate seismic loads. Mononobe-Okabe (M-O) theory is based on converting seismic loads to equivalent static loads. The resultant maximum lateral active earth force acting on the retaining walls under seismic loads is calculated as follows:

$$P_{ae} = \frac{1}{2}\cdot\gamma\cdot H^2\cdot(1-k_v)\cdot K_{ae} \tag{1}$$

In Eq. (1); γ, H, k_v, and K_{ae} represent the unit weight of the backfill material or soil stands behind the wall, the height of the wall, the vertical acceleration coefficient, and the dynamic active earth pressure coefficient, respectively. Although both horizontal and vertical accelerations occur during the earthquake, it is assumed that these two forces are unlikely to peak simultaneously (Christopher *et al.* 1990). Therefore, k_v value is taken as zero.

Active earth pressure coefficient for seismic loads is calculated using Eq. (2).

$$K_{ae} = \frac{sin^2(\phi+\beta-\theta)}{cos\,\theta\cdot sin^2\beta\cdot sin(\beta-\theta-\delta)\cdot\left[1+\sqrt{\dfrac{sin(\phi+\delta)\cdot sin(\phi-\theta-\alpha)}{sin(\beta-\delta-\theta)\cdot sin(\alpha+\beta)}}\right]^2} \tag{2}$$

where, ϕ is the internal friction angle of the retained soil; β is the slope of the backfill to the horizontal; δ is the angle of interface friction between the wall and the soil; α is the backfill angle. θ is the seismic inertia angle and calculated as follows:

$$\theta = tan^{-1}\left(\frac{k_h}{1-k_V}\right) \tag{3}$$

The k_h used in the equation of the seismic inertial angle is vertical acceleration coefficient, which is calculated using Eq. (4):

$$k_h = \begin{cases} \dfrac{PGA}{2\cdot g} & if \quad d\le 1 \\ \dfrac{0.74\cdot PGA}{g}\cdot\left(\dfrac{PGA\cdot 25.4mm}{d\cdot g}\right)^{0.25} & if \quad d>1 \end{cases} \tag{4}$$

where PGA, g, and d are the peak ground acceleration, the gravitational acceleration, and the typical allowable deflection, respectively. According to the M-O theory, the total maximum active earth pressure consists of two components, the static active earth pressure (P_a) and the dynamic active earth pressure (ΔP_{ae}), and is formulated as in the Eq. (5).

$$P_{ae} = P_{ae} + \Delta P_{ae} \tag{5}$$

where

$$P_a = \frac{1}{2}\cdot\gamma\cdot H^2\cdot K_a \tag{6}$$

The M-O theory is based on the Coulomb's active earth pressure theory (Coulomb 1776). Therefore, the active earth pressure coefficient for the static load is calculated using Coulomb's equation presented below:

$$K_a = \frac{sin^2(\beta+\phi)}{sin^2\beta\cdot sin(\beta-\delta)\cdot\left[1+\sqrt{\dfrac{sin(\phi+\delta)\cdot sin(\phi-\alpha)}{sin(\beta-\delta)\cdot sin(\alpha+\beta)}}\right]^2} \tag{7}$$

The total maximum passive earth force is calculated by the following equation:

$$P_{pe} = \frac{1}{2}\cdot\gamma\cdot D_1^2\cdot(1-k_v)\cdot K_{pe} \tag{8}$$

where D_1 is the depth of the passive soil. K_{pe} is the dynamic active earth pressure coefficient, formulated as follows:

$$K_{pe} = \frac{sin^2(\phi+\beta-\theta)}{cos\,\theta\cdot sin^2\beta\cdot sin(\beta+\theta)\cdot\left[1-\sqrt{\dfrac{sin\,\phi\cdot sin(\phi-\theta+\alpha)}{sin(\beta+\theta)\cdot sin(\alpha+\beta)}}\right]^2} \tag{9}$$

The total maximum passive earth pressure consists of two components, similar to the active force, static passive earth pressure (P_p) and dynamic passive earth pressure (ΔP_{pe}). P_p can be calculated using the following expression under the static load conditions:

$$P_p = \frac{1}{2}\cdot\gamma\cdot D_1^2\cdot K_p + 2\cdot c\cdot D_1\cdot\sqrt{K_p} \tag{10}$$

where c is the cohesion of the passive soil. The passive earth pressure coefficient for the static load is calculated using Coulomb's equation presented below:

$$K_P = \frac{sin^2(\beta-\phi)}{sin^2\beta \cdot sin(\beta+\delta)\cdot\left[1+\sqrt{\frac{sin(\phi+\delta)\cdot sin(\phi+\alpha)}{sin(\beta+\delta)\cdot sin(\alpha+\beta)}}\right]^2} \tag{11}$$

2.2 Stability of retaining walls

Retaining walls are designed considering three common failure modes: overturning, sliding, and bearing capacity. The factor of safety against overturning failure SF_O is defined as

$$SF_O = \frac{\sum M_R}{\sum M_O} \tag{12}$$

where ΣM_R is the sum of the resisting moments, and ΣM_O is the sum of the overturning moments. These moments are calculated about the toe. The factor of safety for sliding stability (SF_S) is defined as the ratio between the sum of resisting ($\sum F_R$) forces and sum of sliding ($\sum F_S$) forces (Eq. (13)).

$$SF_S = \frac{\sum F_R}{\sum F_S} \tag{13}$$

The sliding force consists of the sum of the horizontal components of the active earth pressure. Total resisting force is calculated as follows:

$$\sum F_R = \sum V \cdot \tan\left(\frac{2\cdot\phi_B}{3}\right) + \frac{2\cdot L_F \cdot c_B}{3} + P_P \tag{14}$$

in which $\sum V$ is sum of the vertical loads, L_F is the friction length against sliding between the soil underneath wall and the base. The factor of safety is required to prevent bearing capacity failure (SF_B) is defined as the ratio between ultimate bearing capacity (q_u) of the base soil and maximum bearing stress (q_{max}) on the base of the wall (Eq. (15)).

$$SF_B = \frac{q_u}{q_{\max}} \tag{15}$$

In this study, the ultimate bearing capacity of the basement soil of the wall, q_u is calculated determined by using the Meyerhoff's method proposed by Meyerhof (1953). The base of the retaining walls is calculated as a shallow foundation in bearing analysis. Hence, the maximum (q_{max}) and minimum (q_{min}) bearing stresses on the base are formulated as below:

$$q_{\substack{min \\ max}} = \frac{\sum V}{B}\cdot\left(1 \pm \frac{6\cdot e}{B}\right) \tag{16}$$

where B is the width of the wall base, e is the eccentricity between the sum of moments about the toe and the sum of vertical loads ($\sum V$) expressed as Eq. (17).

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V} \tag{17}$$

2.3 Reinforced concrete design

In this study, the requirements of American Concrete Institute (ACI 318-05) are considered in reinforced concrete design. Accordingly, the moment capacity for all the sections of the reinforced concrete wall (stem, toe, heel, and key) is calculated as follows:

$$M_n = \phi \cdot A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right) \tag{18}$$

where ϕ is the strength reduction factor; A_s is the area of longitudinal tension reinforcement, f_y is the characteristic yield strength of the steel; d is the distance from extreme compression fiber to the centroid of longitudinal tension reinforcement; a is the depth of equivalent rectangular stress block.

The shear strength of retaining wall sections is calculated as follows:

$$V_n = \phi \cdot 0.17 \cdot \sqrt{f_c{}'} \cdot b \cdot d \tag{19}$$

in which f_c is the characteristic compressive strength of the concrete, b is the width of the section. The shear forces and the bending moments for all sections are calculated at the joint. The reinforcement ratios used to calculate the maximum and minimum reinforcement area are defined as follows:

$$\rho_{min} = 0.25 \cdot \frac{\sqrt{f_c}}{f_y} \geq \frac{1.4}{f_y} \tag{20}$$

$$\rho_{\max} = 0.85 \cdot \beta_1 \cdot \frac{f_c}{f_y} \cdot \left(\frac{600}{600 + f_y}\right) \tag{21}$$

in which β_1 is the factor relating depth of equivalent rectangular stress block to the neutral axis. The development length of the reinforcement is calculated as in Eq. (22).

$$l_d = \begin{cases} \left(\dfrac{f_y \cdot \psi_t \cdot \psi_e \cdot \lambda}{2.1 \cdot f_c}\right) \cdot d_b \geq 300\ mm & for \quad d_b < 19\ mm \\ \left(\dfrac{f_y \cdot \psi_t \cdot \psi_e \cdot \lambda}{1.7 \cdot f_c}\right) \cdot d_b \geq 300\ mm & for \quad d_b \geq 19\ mm \end{cases} \tag{22}$$

where ψ_t, ψ_e, and λ are the factors related to reinforcement location, reinforcement coating, and unit weight of concrete, respectively. In this study, these factors are equal to 1.0.

3. Formulation of optimum design problem

3.1 Design variables

In the design of reinforced concrete cantilever retaining walls with shear key, there are two different types of design variables: cross-sectional dimensions and reinforcement design. In this study, 24 design variables are considered in the design. Eight of these variables are related to cross-sectional dimensions, 16 are related to reinforcement

Table 1 Design variables

Description		Design variable
Variables related to cross-section dimension	Total base width	x_1
	Width of the toe	x_2
	Thickness at the bottom of the stem	x_3
	Thickness at the top of the stem	x_4
	Thickness of the base slab	x_5
	Distance of the shear key from the toe	x_6
	Thickness of the shear key	x_7
	Height of the shear key	x_8
Variables related to reinforcing steel area per unit length of the wall	Tension bars of stem	R_1
	Shrinkage and temperature bars of stem	$R_{2\text{-}4}$
	Tension bars of the toe	R_5
	Shrinkage and temperature bars of toe	$R_{6\text{-}8}$
	Tension bars of the heel	R_9
	Shrinkage and temperature bars of heel	$R_{10\text{-}12}$
	Tension bars of the key	R_{13}
	Shrinkage and temperature bars of key	$R_{14\text{-}16}$

Table 2 Reinforcement design variable combinations

Index Number	Bar diameter (mm)	Bar Spacing (mm)	Cross-Sectional Area* (mm^2)
1	10	300	262
2	10	295	266
3	10	290	271
.	.	.	.
.	.	.	.
.	.	.	.
751	40	75	16755
752	40	70	17952

* in Ascending Order

design. Design variables and their descriptions are given in Table 1.

The variables depending on cross-sectional dimensions are generated by meta-heuristic algorithms considering dimension limits. The reinforcement requirements of the retaining wall are calculated for the unit length of the wall. The forces acting for each part of the wall (toe, stem, heel, and shear key) and the steel bar areas required to satisfy these forces are determined. The reinforcement design is completed by selecting the reinforcement diameter and appropriate spacing for the calculated reinforcement area from the reinforcement design variable combinations in Table 2.

3.2 Design constraints

The constraints used in designing of the reinforced concrete retaining walls may be classified under four headings: stability, the limitation of the dimensions, capacity of the force, and reinforcement arrangement (Table 3). The stability constraints include overturning, sliding, and bearing capacity failure modes. Also, the occurrence of the tensile stresses in the sub-base soil can be considered as a constraint of this category. Cross-sectional limits are defined to prevent infeasible dimensions. The constraints related to force capacity and reinforcement arrangement are defined conforming to the ACI 318-05 requirements.

All the design constraints must be satisfied at the end of the optimization process. For each constraint not satisfied, the penalty value is implemented to the objective function value so as to obtain a feasible design.

3.3 Objective function

In this study, minimizing the costs of the reinforced concrete cantilever retaining walls is defined as an objective function. The concrete, the reinforcing steel, and formwork costs are taken into consideration account in the for determining the total cost. Thus, the cost function (f_{cost}) can be formulated as below:

$$f_{cost} = C_c \cdot V_c + C_s \cdot W_s + C_f \cdot A_f \tag{23}$$

In Eq. (23); C_c, C_s, and C_f are the unit cost of the concrete, the reinforcing steel, and formwork; V_c, W_s, and A_f are the volume of the concrete, the weight of steel, and the area of formwork per unit length of the wall, respectively. (Unit costs include the cost of the material, labor, and the installation)

A penalty function is defined to satisfy the design constraints as following:

$$\Phi = \left(1 + \sum_{i=1}^{34} g_i\right)^4 \tag{24}$$

where, Φ is the penalty value, g is the constraint function, i is the constraint number. The objective function value (F) is calculated by multiplying the penalty value by the total cost value (Eq. (25)).

$$F = f_{cost} \cdot \Phi \tag{25}$$

The penalty value equals "1" if all constraints are satisfied, in contrast it is greater than "1" if any constraint is exceeded. Therefore, the objective function value of the candidate solution is increased if the constraints are not satisfied.

4. The proposed hybrid approach

In this study, a hybrid algorithm is proposed using six different optimization algorithms, named Teaching-Learning Based Optimization, Flower Pollination Algorithm, Jaya Algorithms, and Rao Algorithms (Rao-1, Rao-2, and Rao-3). The process of generating a new population for each algorithm is unique, but the initialization procedures are the same. The initial matrix is created randomly within the range that set by the lower and upper bounds. If the fitness value of the candidate

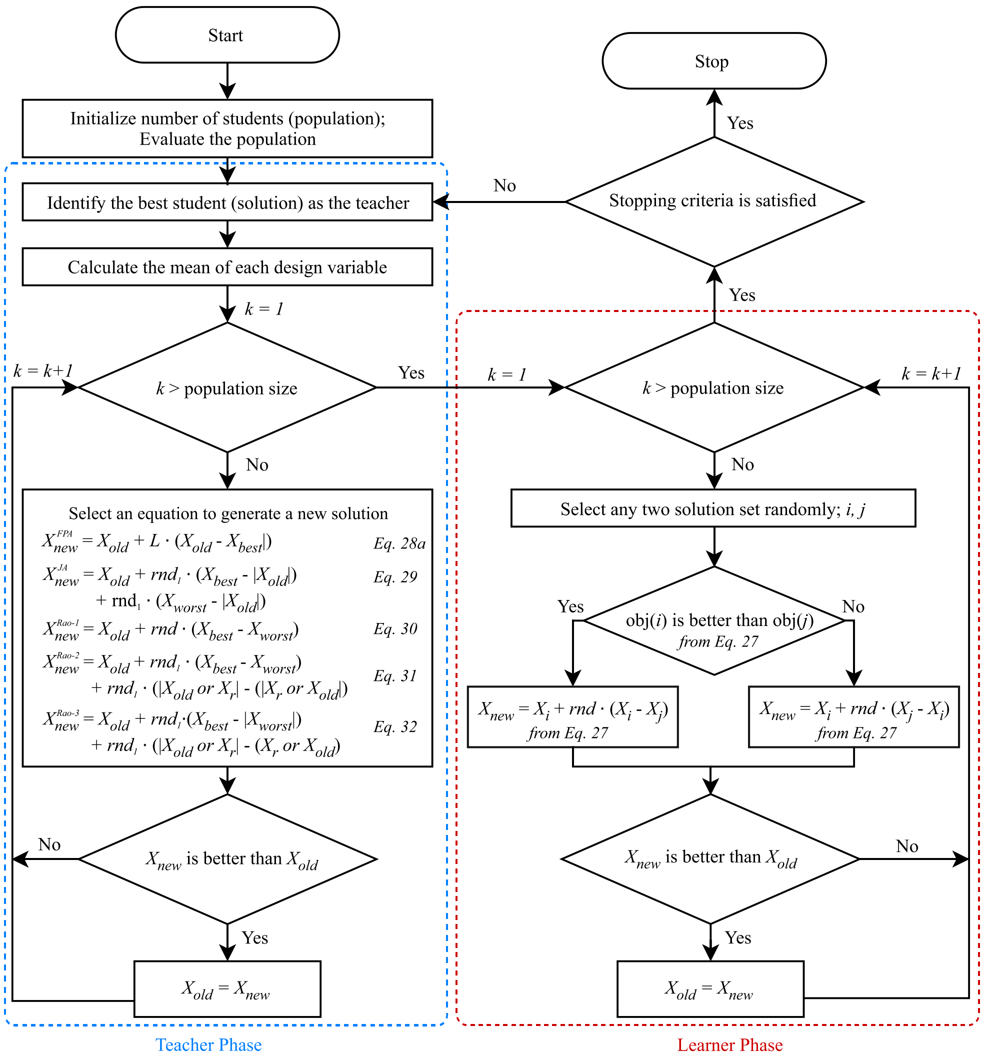


Fig. 2 Flow chart of the hybrid teaching-learning based algorithm

population is better than the old ones, these populations are interchanged. Furthermore, the best and the worst populations are re-defined using fitness values of all populations.

4.1 Teaching-learning based optimization algorithm

Teaching-Learning Based Optimization (TLBO) algorithm, which simulates the learning process of students in the classroom, is an algorithm without specific parameters developed by Rao et. al. (2011). TLBO consists of two phases: In the first one, new populations (X_{new}) are generated by Eq. (26) using the previous value (X_{old}), the best value (X_{best}), and the average value (X_{mean}) of the variables.

$$X_{new} = X_{old} + rnd \cdot \left(X_{best} - T_F \cdot X_{mean} \right) \tag{26}$$

where; *rnd* is the uniformly distributed random number within the range [0, 1] and T_F is the teaching factor that takes the value 1 or 2,. In the second phase, the new solutions are generated using two different populations (i, j) randomly selected among the existing populations and formulated as follows:

$$X_{new} = \begin{cases} X_{old} + rnd \cdot \left(X_i - X_j \right) & if \quad f\left(X_i\right) > f\left(X_j\right) \\ X_{old} + rnd \cdot \left(X_j - X_i \right) & if \quad f\left(X_i\right) < f\left(X_j\right) \end{cases} \tag{27}$$

Numerous variations have been developed in order to increase the performance of the TLBO algorithm in different optimization problems (Shukla *et al.* 2018a, b, 2019, 2020).

4.2 Flower pollination algorithm

The Flower Pollination Algorithm (FPA) using the Lévy flight distribution was developed by Yang (2012). In order to generate new populations, The one of two different population generating approaches is selected using a specific parameter called key probability (p). The first approach uses the Lévy flight distribution (L) and the best value of the variables. The second one uses a uniformly distributed random number (rnd_2) and two selected populations (i, j) from existing ones similar to the approach in the second phase of TLBO. The process of generating new populations is formulated in the Eq. (28).

$$X_{new} = X_{old} + L \cdot \left(X_{old} - X_{best} \right) \quad if \quad rnd_1 > p \tag{28}$$

$$X_{new} = X_{old} + rnd_2 \cdot \left(X_i - X_j \right) \quad if \quad rnd_1 < p \tag{29}$$

4.3 Jaya algorithm

Jaya Algorithm (JA) is a simple yet powerful meta-heuristic optimization algorithm for solving both the constrained and the unconstrained optimization problems, developed by Rao (2016). The simplicity of JA is in the step of generating value of the variables for the candidate populations. The new values of the variables are generated as below:

$$X_{new} = X_{old} + rnd_1 \cdot \left(X_{best} - |X_{old}| \right) - rnd_2 \cdot \left(X_{worst} - |X_{old}| \right) \tag{30}$$

where; X_{worst} is the worst solution of the variables.

4.4 Rao algorithms

Three new specific parameter-less algorithms are recently developed by Rao (2020). The only difference among these algorithms is the equations used for generating the new populations. The equations of these algorithms called Rao-1, Rao-2 and Rao-3 are presented in the Eqs. (31)-(33), respectively.

$$X_{new} = X_{old} + rnd \cdot \left(X_{best} - X_{worst} \right) \tag{31}$$

$$X_{new} = X_{old} + rnd_1 \cdot \left(X_{best} - X_{worst} \right) + rnd_2 \cdot \left(|X_{old} \ or \ X_r| - |X_r \ or \ X_{old}| \right) \tag{32}$$

$$X_{new} = X_{old} + rnd_1 \cdot \left(X_{best} - |X_{worst}| \right) + rnd_2 \cdot \left(|X_{old} \ or \ X_r| - \left(X_r \ or \ X_{old} \right) \right) \tag{33}$$

in which X_r is the randomly picked candidate solution. If the fitness value of the old candidate solution is better than the fitness value of the randomly picked candidate solution, the term of "X_{old} or X_r" equals X_{old}. Otherwise, the term of "X_{old} or X_r" becomes X_r. Similarly, if the fitness value of the old candidate solution is better than the fitness value of the randomly picked candidate solution, then the term of "X_r or X_{old}" equals X_r. Otherwise, the term of "X_r or X_{old}" becomes X_{old}.

Among these algorithms, only Rao-3 was applied in the optimization of RC cantilever retaining walls (Kalemci and Ikizler 2020), but the results of the study were not compared with different methods.

4.5 The proposed hybrid teaching-learning based optimization algorithm

Teaching-Learning Based Optimization Algorithm (TLBO) was applied to the optimum design problem of retaining walls and obtained lower standard deviation values than the compared other powerful algorithms such as PSO, BB-BC, HS, and JA earlier in the literature. Although it produces relatively successful results in comparisons, TLBO is far from perfection and needs improvements. For this purpose, a hybrid approach based on TLBO is proposed in this study. In this algorithm called Hybrid Teaching-Learning Based Optimization Algorithm (HTLBO), the new population generating procedures of FPA (Eq. (28)), JA (Eq. (29)), Rao-1 (Eq. (30)), Rao-2 (Eq. (31)), and Rao-3 (Eq. (32)) algorithms have been applied to the first phase of TLBO to improve both exploration and exploitation features. Accordingly, one of these algorithms is randomly selected in the first phase and the new population is generated by the procedures of the selected algorithm. In the second stage, the learner phase equation of TLBO (Eq. (27)) is used for new populations. The flow chart of the proposed algorithm is illustrated in Fig. 2.

5. Design examples

The scope of this study, 11 the eleven different algorithms are used to compare the performance of Hybrid Teaching-Learning Based Optimization algorithm in the optimum design of the reinforced concrete cantilever retaining walls. In order to compare the pros and cons of the several optimization algorithms in detail simultaneously, Rao-1 and Rao-2 algorithms are added to the nine optimization algorithms used for design of the retaining walls, previously. These algorithms are BB-BC (Camp and Akin 2012), BBO (Aydogdu 2017), FPA (Mergos and Mantoglou 2019), GWO (Kalemci *et al.* 2020), HS (Kaveh and Abadi 2011), PSO (Gandomi *et al.* 2015), TLBO (Temur and Bekdaş 2016), JA (Öztürk *et al.* 2020), and RAO-3 (Kalemci and Ikizler 2020).

For all considered algorithms, the total number of analyses is set to 10000. The population size is used as suggested in the earlier studies in the literature. For determining the appropriate common control parameter of the proposed algorithm, the population size was used as 10, 20, 30, 40, and 50. The maximum iteration number is determined by the total number of analyzes.

Three design examples are considered for evaluating the efficiency of the all considered algorithms. The lower and upper boundaries for the cross-sectional dimensions of the retaining walls are presented in Table 4. The dimensions were determined by both continuous and discrete variables. An incremental step of 0.05 *m* is considered for the cross-sectional dimensions in terms of the applicability in discrete variables. All the compared algorithms are performed 100 independent runs for statistical analysis.

Table 4 Lower and upper boundaries of retaining wall dimensions (Sarıbaş and Erbatur 1996)

Design variable	Lower bound	Upper bound
X_1	$0.4 \cdot H \cdot (12 \cdot 11)$	$(0.7 \cdot H)/0.9$
X_2	$[0.4 \cdot H \cdot (12 \cdot 11)]/3$	$[(0.7 \cdot H)/0.9]/3$
X_3	0.2	$(H/0.9)/10$
X_4	0.2	0.5
X_5	$[H \cdot (12 \cdot 11)]/12$	$(H/0.9)/10$
X_6	0.5	$0.8 \cdot H$
X_7	0.0	0.4
X_8	0.0	0.9

Table 5 Input parameters for numerical examples

Input parameters	Symbol	Unit	Values		
			Example-1	Example-2	Example-3
Height of stem	H	m	3.0	4.5	4.5
Backfill slope angle	α	°	10	0	15
Surcharge load	q	kPa	20	30	30
Depth of the soil in front of wall	D_f	m	0.50	0.3	0.75
Retained soil					
Internal friction angle of retained soil	ϕ_R	°	36	28	36
Cohesion of retained soil	c_R	kPa	0	0	0
Unit weight of retained soil	γ_R	kN/m³	17.5	18.5	17.5
Sub-base soil					
Internal friction angle of base soil	ϕ_B	°	0	34	34
Cohesion of base soil	c_B	kPa	125	0	100
Unit weight of base soil	γ_B	kN/m³	18.5	17	18.5
Reinforcing steel					
Yield strength of steel	f_y	MPa	400	400	400
Unit weight of steel	γ_s	kg/m³	7849	7849	7849
Unit cost of steel	C_s	$/kg	0.40	0.40	0.40
Concrete					
Compressive strength of concrete	f'_c	MPa	21	21	21
Unit weight of concrete	γ_c	kN/m³	23.5	23.5	23.5
Unit cost of concrete	C_c	$/m³	40	40	40
Reinforced concrete					
Concrete cover	c_c	cm	7	7	7
Design load factor	LF	-	1.7	1.7	1.7
Unit cost of formwork	C_f	$/m²	4.29	4.29	4.29
Shrinkage and temperature reinforcement percentage	ρ_{st}	-	0.0020	0.0020	0.0020
Factors of safety					
Safety factor for overturning stability	$SF_{O,design}$	-	1.5	1.5	1.5
Safety factor for sliding stability	$SF_{S,design}$	-	1.5	1.5	1.5
Safety factor for bearing capacity	$SF_{B,design}$	-	3.0	3.0	3.0

Table 6 Statistical comparison for Example-1

Algorithm	PGA	Stem Height	Total Cost ($) (*Continuous*)				Total Cost ($) (*Discrete*)			
			Min	Max	Mean	St. Dev.	Min	Max	Mean	St. Dev.
BB-BC	0.0	3.0	**116.17**	124.05	117.64	1.94	**121.74**	128.13	122.53	1.70
BBO	0.0	3.0	**116.17**	116.53	116.24	0.08	**121.74**	125.21	122.64	1.40
FPA	0.0	3.0	**116.17**	117.05	116.22	0.19	**121.74**	122.32	121.83	0.21
HS	0.0	3.0	**116.17**	116.27	116.19	0.02	**121.74**	**121.74**	**121.74**	**0.00**
PSO	0.0	3.0	**116.17**	127.85	116.77	1.33	**121.74**	132.83	122.20	1.65
GWO	0.0	3.0	116.18	118.79	116.28	0.33	**121.74**	**121.74**	**121.74**	**0.00**
JA	0.0	3.0	**116.17**	121.12	116.61	0.85	**121.74**	122.32	121.75	0.08
RAO-1	0.0	3.0	**116.17**	126.77	117.16	2.17	**121.74**	**121.74**	**121.74**	**0.00**
RAO-2	0.0	3.0	**116.17**	116.20	**116.17**	**0.00**	**121.74**	122.32	121.75	0.08
RAO-3	0.0	3.0	**116.17**	116.20	**116.17**	**0.00**	**121.74**	122.32	121.75	0.06
TLBO	0.0	3.0	**116.17**	117.46	116.19	0.16	**121.74**	122.32	121.79	0.16

Table 6 Continued

Algorithm	PGA	Stem Height	Total Cost ($) (*Continuous*)				Total Cost ($) (*Discrete*)			
			Min	Max	Mean	St. Dev.	Min	Max	Mean	St. Dev.
HTLBO-p10	0.0	3.0	**116.17**	117.64	116.27	0.26	**121.74**	122.32	121.81	0.19
HTLBO-p20	0.0	3.0	**116.17**	116.21	**116.17**	0.01	**121.74**	122.32	121.75	0.08
HTLBO-p30	0.0	3.0	**116.17**	116.20	**116.17**	0.01	**121.74**	**121.74**	**121.74**	**0.00**
HTLBO-p40	0.0	3.0	**116.17**	116.20	**116.17**	**0.00**	**121.74**	**121.74**	**121.74**	**0.00**
HTLBO-p50	0.0	3.0	**116.17**	**116.19**	**116.17**	**0.00**	**121.74**	**121.74**	**121.74**	**0.00**

Minimum results for each category are marked as bold

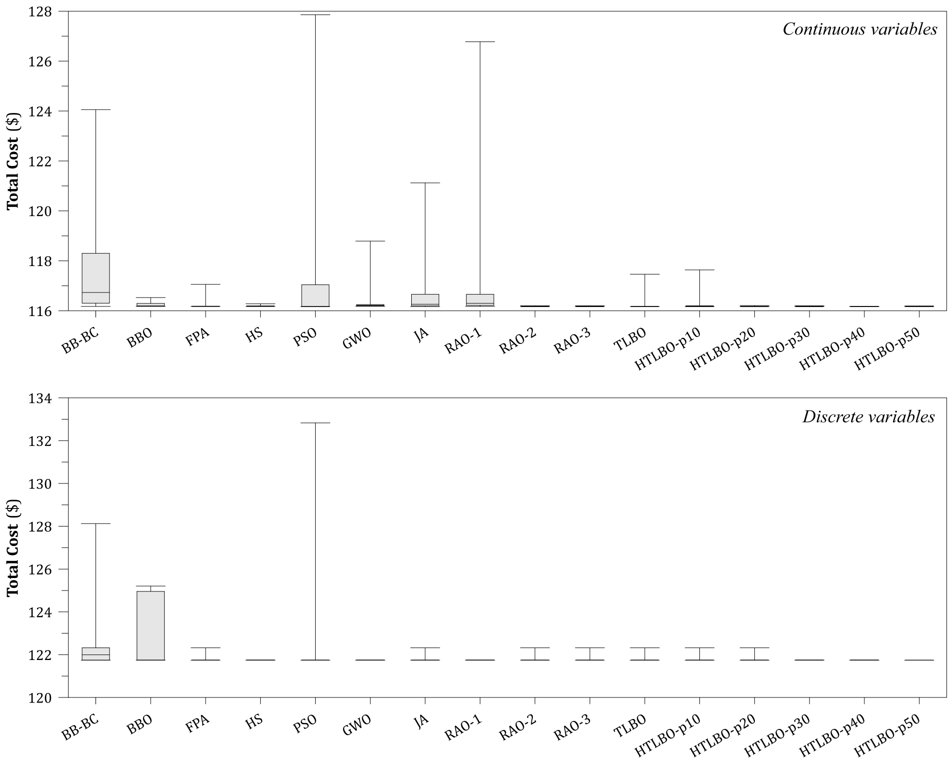


Fig. 3 Box plots of the results of all the algorithms considered for Example-1

Table 7 The optimum design parameters for Example-1

Variables Type	Dimensions (*m*)					Reinforcements (*size/spacing*)			Total
	X_1	X_2	X_3	X_4	X_5	R_1	R_5	R_9	Cost ($)
Continuous	1.80	0.65	0.24	0.20	0.27	ϕ10/120	ϕ10/110	ϕ10/110	116.17
Discrete	1.85	0.75	0.25	0.20	0.30	ϕ14/240	ϕ12/140	ϕ12/140	121.74

5.1 Example-1: The retaining wall without shear key under static loads

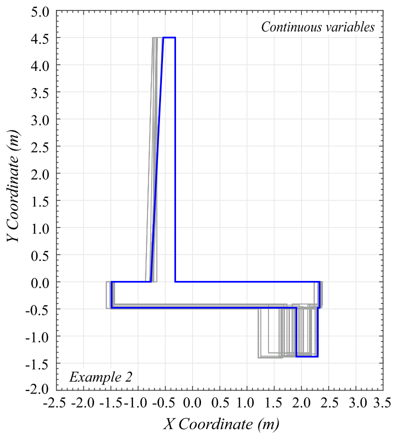
The optimum design of RC cantilever retaining wall without shear key under static loads is analyzed in Example-1 (Sarıbaş and Erbatur 1996). The input parameters of the example are shown in the Table 5. The statistics belong to the costs obtained from 100 independent runs are presented in the Table 6. The population size used for HTLBO are specified next to the algorithm name in the comparison tables (e.g., *HTLBO-p10 for 10 populations*). According to these results, the minimum cost values for continuous and discrete variables were obtained as $116.17 and $121.74, respectively. All the compared algorithms except GWO in the continuous variables achieved these results. The margin of the error of the minimum cost obtained with GWO is negligible. If the minimum, maximum, average cost, and standard deviation values are considered, the best results are obtained with the HTLBO algorithm with 50 populations.

The results of Example-1 from all the compared algorithms were visualized in Fig. 3 by using box plots. In this figure, it is obviously seemed that in the cases where both continuous and discrete variables are used, the most

Table 8 Statistical comparison for Example-2

Algorithm	PGA	Stem Height	Total Cost ($) (*Continuous*)				Total Cost ($) (*Discrete*)			
			Min	Max	Mean	St. Dev.	Min	Max	Mean	St. Dev.
BB-BC	0.0	4.5	317.22	364.00	328.34	8.05	**325.09**	367.19	337.11	8.32
BBO	0.0	4.5	**316.76**	342.76	325.41	7.28	**325.09**	356.00	336.90	5.22
FPA	0.0	4.5	**316.76**	358.19	322.49	7.40	**325.09**	353.41	328.40	5.57
HS	0.0	4.5	**316.76**	331.81	317.42	2.01	**325.09**	331.46	325.20	0.70
PSO	0.0	4.5	**316.76**	576.37	326.50	26.50	**325.09**	508.91	333.26	21.41
GWO	0.0	4.5	**316.76**	**319.18**	317.43	**0.43**	**325.09**	332.75	325.28	0.76
JA	0.0	4.5	**316.76**	515.48	338.54	29.75	**325.09**	486.47	338.55	22.59
RAO-1	0.0	4.5	**317.36**	481.98	332.08	22.30	**325.09**	447.34	335.76	16.56
RAO-2	0.0	4.5	**316.76**	328.79	317.65	2.42	**325.09**	332.75	325.93	1.86
RAO-3	0.0	4.5	**316.76**	346.50	318.66	4.37	**325.09**	349.33	326.46	3.03
TLBO	0.0	4.5	**316.76**	330.60	320.26	4.70	**325.09**	332.75	326.72	2.57
HTLBO-p10	0.0	4.5	**316.76**	466.25	323.85	15.49	**325.09**	349.33	328.99	5.23
HTLBO-p20	0.0	4.5	**316.76**	347.22	319.00	4.66	**325.09**	332.75	326.74	2.42
HTLBO-p30	0.0	4.5	**316.76**	328.79	317.95	3.24	**325.09**	332.75	325.64	1.42
HTLBO-p40	0.0	4.5	**316.76**	328.79	317.27	2.32	**325.09**	332.75	325.28	0.93
HTLBO-p50	0.0	4.5	**316.76**	328.79	**317.04**	1.70	**325.09**	**328.03**	**325.19**	**0.50**

Minimum results for each category are marked as bold



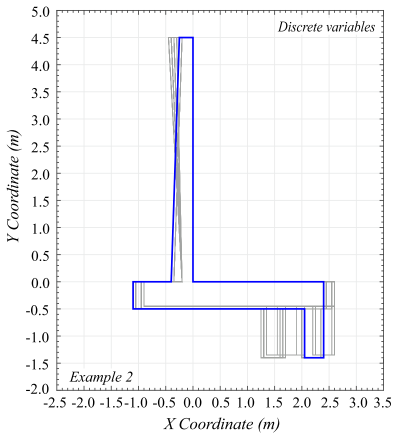


Fig. 4 The best solutions and the final optimum shape in the optimization process for Example-2

Table 9 Optimum design parameters for Example-2

Variables Type	Dimensions (*m*)								Reinforcements (*size/spacing*)				Total
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	R_1	R_5	R_9	R_{13}	Cost ($)
Continuous	3.50	0.66	0.41	0.20	0.48	3.11	0.36	0.90	ϕ16/135	ϕ16/140	ϕ14/70	ϕ10/75	316.76
Discrete	3.50	0.70	0.40	0.25	0.50	3.10	0.35	0.90	ϕ14/100	ϕ12/75	ϕ14/75	ϕ10/70	325.09

robust results are obtained with the size of 30, 40, and 50 populations of the HTLBO algorithm.

The optimum cross-section dimensions and the reinforcement values of the minimum cost design obtained as a result of the optimization processes are presented in Table 7.

5.2 Example-2: The retaining wall with shear key under static loads

In this example, a retaining wall of 4.5 m height with shear key is considered. The design parameters of the example are expressed in Table 5 (Kalemci *et al.* 2020). The cost statistics obtained from the all considered algorithms using 100 independent runs are presented in Table 8. The minimum cost values for continuous and discrete variables were calculated as $316.76 and $325.09, respectively. The minimum cost value for discrete variables can be achieved with all considered algorithms as in the first example. In the case of continuous variables, a similar result was obtained except for BB-BC. The margin of error of the minimum

Table 10 The statistical comparison for Example-3

Algorithm	PGA	Stem Height	Total Cost ($) (*Continuous*)				Total Cost ($) (*Discrete*)			
			Min	Max	Mean	St. Dev.	Min	Max	Mean	St. Dev.
BB-BC	0.4	4.5	346.32	406.80	364.23	12.24	**347.64**	424.33	368.30	12.36
BBO	0.4	4.5	**345.63**	369.98	346.64	2.58	348.51	379.21	356.32	6.36
FPA	0.4	4.5	**345.63**	348.09	345.71	0.39	**347.64**	348.51	347.65	0.09
HS	0.4	4.5	**345.63**	347.84	346.07	0.66	**347.64**	349.01	348.05	0.53
PSO	0.4	4.5	**345.63**	423.87	351.07	12.60	**347.64**	394.62	351.61	8.30
GWO	0.4	4.5	**345.63**	350.63	347.74	1.15	**347.64**	352.94	349.15	1.35
JA	0.4	4.5	**345.63**	367.37	346.12	3.06	**347.64**	371.35	348.43	4.07
RAO-1	0.4	4.5	**345.63**	348.32	345.77	0.54	**347.64**	**347.64**	**347.64**	**0.00**
RAO-2	0.4	4.5	**345.63**	356.80	345.77	1.14	**347.64**	371.34	348.15	2.80
RAO-3	0.4	4.5	**345.63**	347.51	345.65	0.19	**347.64**	356.54	347.82	1.25
TLBO	0.4	4.5	**345.63**	346.80	345.74	1.12	**347.64**	**356.54**	**347.82**	**1.25**
HTLBO-p10	0.4	4.5	**345.63**	368.11	346.67	4.39	**347.64**	356.54	348.39	1.96
HTLBO-p20	0.4	4.5	**345.63**	345.79	345.64	0.04	**347.64**	371.34	348.12	3.24
HTLBO-p30	0.4	4.5	**345.63**	**345.72**	**345.63**	0.02	**347.64**	369.99	347.87	2.23
HTLBO-p40	0.4	4.5	**345.63**	**345.72**	**345.63**	0.02	**347.64**	**347.64**	**347.64**	**0.00**
HTLBO-p50	0.4	4.5	**345.63**	**345.72**	**345.63**	**0.01**	**347.64**	**347.64**	**347.64**	**0.00**

Minimum results for each category are marked as bold

Table 11 Optimum design parameters for Example-3

Variables Type	Dimensions (*m*)								Reinforcements (*size/spacing*)				Total
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	R_1	R_5	R_9	R_{13}	Cost ($)
Continuous	4.02	0.97	0.41	0.20	0.60	2.25	0.00	0.00	ϕ12/95	ϕ16/105	ϕ18/90	-	345.63
Discrete	4.05	0.85	0.40	0.20	0.55	2.85	0.00	0.00	ϕ14/125	ϕ18/150	ϕ18/75	-	347.64

cost obtained with BB-BC is at a negligible rate of 0.2%. If the minimum, maximum, average cost, and standard deviation values are considered, the best results are obtained from the HTLBO algorithm with 50 populations, and GWO.

The design variables of the minimum cost design obtained from the optimization processes are presented in Table 9. The best solutions and the final optimum cross-section of the wall in the optimization process with HTLBO (*for population size = 50*) are illustrated in Fig. 4. Based on this figure, although the lower limit of the shear key is defined as zero (Table 4), the formation of the shear key is inevitable due to the dimensional boundaries and the forces acting on the wall.

5.3 Example-3: The retaining wall under seismic loads

An example of the 4.5 m high cantilever retaining wall, whose input parameters are presented in Table 5, was used to investigate the optimization of the retaining walls under seismic loads. Peak ground acceleration (PGA) value is taken as 0.4 g, to compare the performance of the considered algorithms. The shear key was considered for cross-section design in the optimization process. Due to the increase of the applied loads in this example, the dimensional boundaries cause both the overturning stability condition is not satisfied and the shear capacity for the base of the wall is insufficient. Therefore, the upper boundaries of the X_1 and X_5 variables were increased by 20% for this example.

At the end of the 100 independent optimization process for the all considered algorithms, the minimum total cost was calculated as $345.63 and $347.64 for continuous and discrete variables, respectively. According to the optimization results presented in Table 10, all considered algorithms can obtain the minimum cost values except BB-BC for continuous variables and BBO for discrete variables. Considering other statistical parameters such as maximum cost, mean cost, and standard deviation, only HTLBO (*with 50 populations*) can achieve the minimum values for both continuous and discrete variables. In addition, RAO-1, TLBO, and HTLBO (*with 40 populations*) can also achieve minimum cost value in all independent optimization processes for only discrete variables.

The total cost values obtained as a result of 100 independent runs were given in Fig. 5 by using the box charts to compare the robustness of all considered algorithms. In this plot, it is clearly understood that in the cases where both continuous and discrete variables are

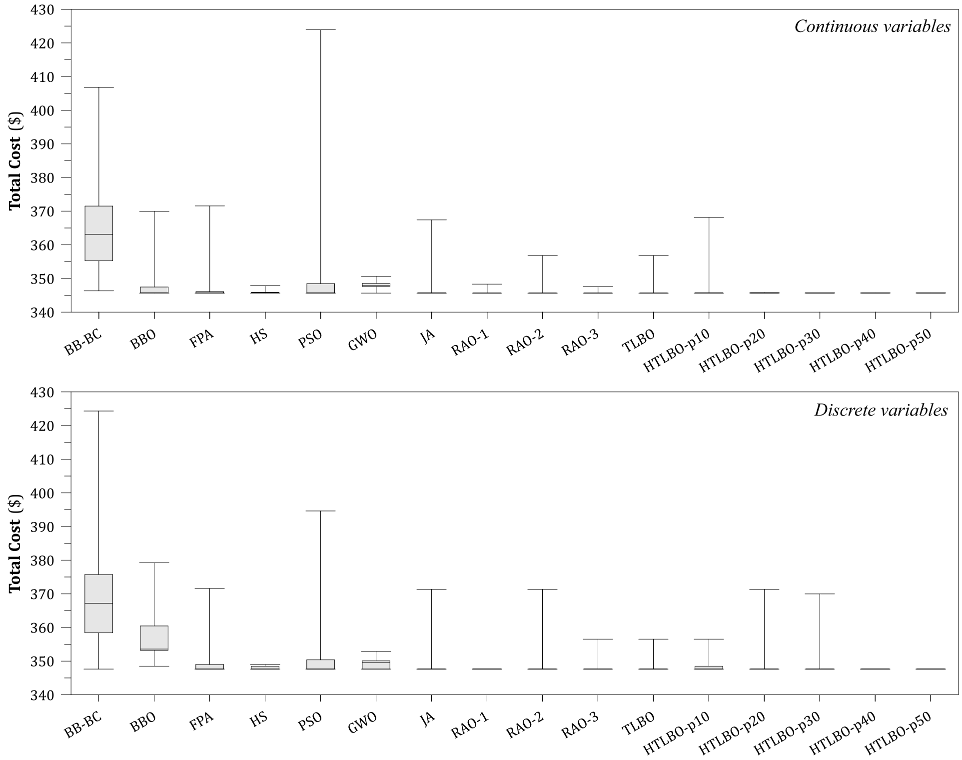


Fig. 5 Box plots of the results of all the algorithms considered for Example-3

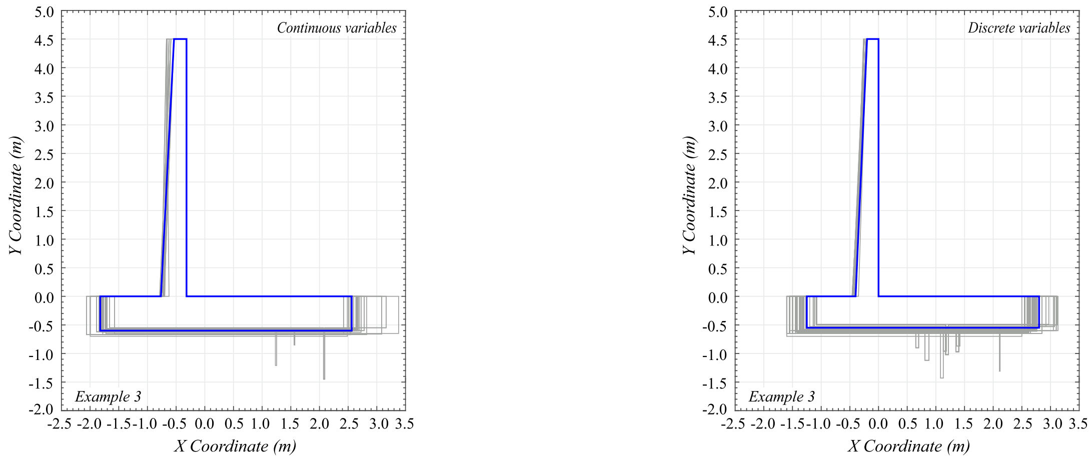


Fig. 6 The candidate solutions and the final optimum cross-section for Example-3

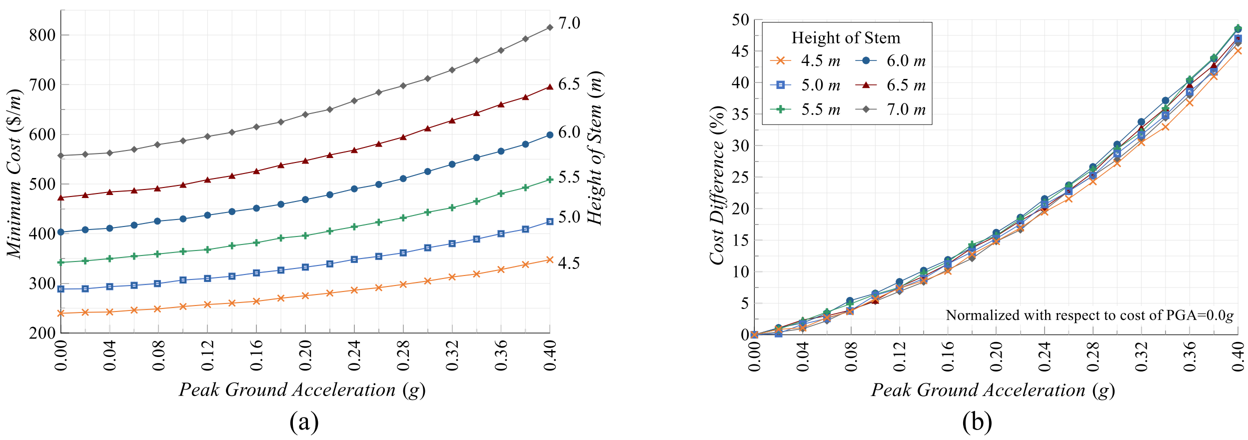
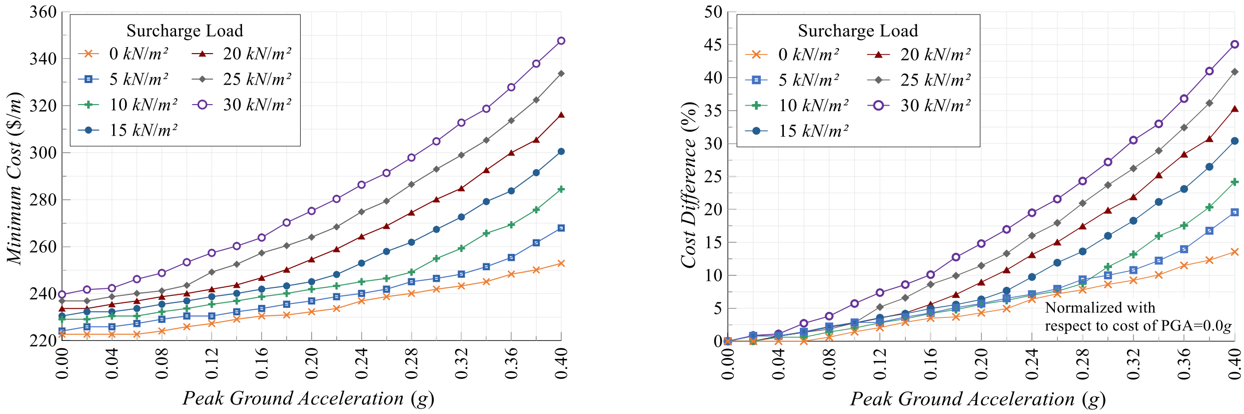
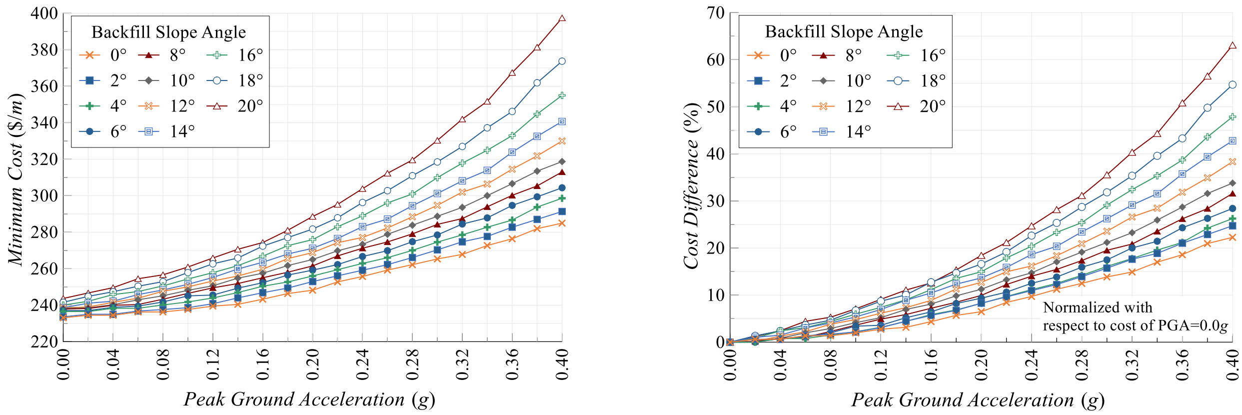


Fig. 7 Effects of PGA on minimum cost considering the height of stem



(a) (b)

Fig. 8 Effects of PGA on minimum cost considering surcharge loads



(a) (b)

Fig. 9 Effects of PGA on minimum cost considering backfill slope angle

used, the most robust results are achieved with the 40 and 50 populations of the HTLBO algorithm.

The candidate solutions obtained during the optimization processes and the final optimum cross-section are illustrated in Fig. 6. As can be seen in the figure, however candidate cross-sections with the shear key are generated during the optimization process, the minimum cost can be achieved with a cross-section without the shear key. The cross-section dimensions and the reinforcement details for minimum cost designs are presented in Table 9.

The effect of the PGA value on total cost was examined by using various stem heights, backfill slope angles, and surcharge loads. Accordingly, 6 different stem heights from 4.5 to 7 meters in intervals of 0.5 m, 7 different surcharge loads from 0 to 30 kN/m² in intervals of 5 kN/m², and 21 different backfill slope angles from 0 to 20° in intervals of 1° were considered. In all these cases, PGA value increases from 0 to 0.4 g at 0.02 g intervals. Due to its robustness in the previous examples, HTLBO algorithm with 50 populations was preferred in optimization processes. According to the minimum cost values obtained, it can be said that the effect of the PGA value on the total cost is similar for all considered stem heights (Fig. 7(a)). The difference in cost compared to PGA=0.0*g* depending on PGA values is shown in Fig. 7(b). These values indicate that the average cost increased by 5.4% for 0.1 g, 14.8% for 0.2 g, 27.4% for 0.3 g, and 44.8% for 0.4 g. Therefore, it can be said that the difference in the surcharge loads affect the minimum total cost change more than different stem height.

Figs. 8 and 9 show the effect of the PGA increase on the minimum cost on different surcharge loads and backfill slope angles, respectively. As can be seen in the Fig. 8(b), the effect of PGA on total cost increases as the surcharge loads increase, unlike the effect of the stem height. Compared with the total cost from PGA=0.0 g, the minimum total costs achieved with various surcharge loads increase between 1.5% to 5.7% for PGA=0.1g, between 4.3% to 14.8% for PGA=0.2 g, between 8.6% to 27.2% for PGA=0.3 g, and between 13.6% to 45.1% for PGA=0.4 g. The effect of backfill slope angle on minimum total cost change is similar to the effect of surcharge loads (Fig. 9).

6. Conclusions

This article studies the cost optimization of the reinforced cantilever retaining wall under the excitation of the seismic loads, the influence of peak ground acceleration on the minimum cost, and the performance of the Hybrid Teaching-Learning Based Optimization Algorithm on the optimum design of the RC cantilever retaining walls. Three different design examples were optimized 100 times for each case using 12 different meta-heuristic algorithms and their results were compared statistically.

According to the optimization results, minimum cost design has been achieved with all considered algorithms except GWO, BB-BC, and BBO. The minimum costs achieved with these excluded algorithms have an acceptable margin of error. Therefore, it is possible to achieve low-cost design with all considered algorithms.

In order to determine the performance of the HTLBO algorithm; the statistical results obtained with HTLBO were compared with the results obtained with other algorithms. In all cases, the minimum average cost was obtained with the 50 populations HTLBO algorithm. The minimum values were obtained with 50 populations HTLBO, except for the case where Example-2 continuous variables, were used by considering the maximum cost and standard deviation values. Therefore, the most robust results among all the algorithms considered were obtained with 50 populations of HTLBO. When the effect of the population size on optimum cost is examined in HTLBO algorithm, it is determined that the population size and standard deviation value for this problem are inversely correlated.

Although Rao algorithms have achieved minimum costs as a result of 100 independent runs in all design examples, they are not sufficiently robust considering the maximum values. Therefore, the modifications are required to increase the performance of the Rao algorithms in the optimization of designing cantilever retaining wall.

Depending on the increase in the peak ground acceleration value, it has been observed that in cases with different stem heights, approximately the same rate of the cost increase. In the case of using various backfill slope angle and surcharge loads, when the effect of PGA value is examined. As a result, it is determined that as the values of these parameters increase, the effect of PGA on the cost increases.

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