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Forced vibration of a functionally graded porous beam resting on viscoelastic foundation

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Abstract. This paper concerns with forced dynamic response of thick functionally graded (FG) beam resting on viscoelastic foundation including porosity impacts. The dynamic point load is proposed to be triangle point loads in time domain. In current analysis the beam is assumed to be thick, therefore, the two-dimensional plane stress constitutive equation is proposed to govern the stress-strain relationship through the thickness. The porosity and void included in constituent is described by three different distribution models through the beam thickness. The governing equations are obtained by using Lagrange's equations and solved by finite element method. In frame of finite element analysis, twelve-node 2D plane element is exploited to discretize the space domain of beam. In the solution of the dynamic problem, Newmark average acceleration method is used. In the numerical results, effects of porosity coefficient, porosity distribution and foundation parameters on the dynamic responses of functionally graded viscoelastic beam are presented and discussed. The current model is efficient in many applications used porous FGM, such as aerospace, nuclear, power plane sheller, and marine structures.

Keywords: dynamic response; functionally graded beam; porosity; viscoelastic foundation; numerical finite element method

1. Introduction

The concept of functionally graded materials (FGMs) was originated in Japan in 1984 during a space plane project, Alshorbagy (2011). FGMs are made from a mixture of metals and ceramics. The main characteristics of these materials are the smooth and continuous variation of their properties in one or more directions. This makes it possible to create structures with very interesting properties, such as a high resistance to temperature shocks, low transverse shear stresses, and high strength-to-weight ratios (Praveen and Reddy 1998). FGMs can be designed for many specific applications, such as thermal barrier coatings for turbine blades, armor protection for military use, fusion energy devices, biomedical materials, automotive and space/aerospace industries, Micro/Nano-electro-mechanical system (MEMS/NEMS) and atomic force microscopes (AFMs) Eltaher *et al.* (2013a, b). With the wide application of FGM structures, understanding the behavior of FGM beams becomes an important task.

Researchers have investigated the behavior of FG beams using the classical beam theory (CBT), called the Euler-Bernoulli theory, the first shear-order or Timoshenko beam theory (FSBT), and higher-order beam theories (HSBTs). In the recent years, many works have been performed on the dynamic and bending, vibration and buckling of FGM beam structures. Aydogdu and Tashkin (2007) studied the free vibration of simply supported FG beams employing the classical and shear-deformation beam theories. Ying *et al.* (2008) obtained the exact solutions for bending and free vibration of FG beams resting on a Winkler-Pasternak elastic foundation based on 2D elasticity theory by assuming that the beam is orthotropic at any point and the material properties vary exponentially along the thickness direction. Şimşek and Kocatürk (2009) investigated free vibration characteristics and dynamic behaviors of a FG simply supported beam under a concentrated moving harmonic load, in which the effects of the material homogeneity, the velocity of the moving harmonic load, and the excitation frequency on the dynamic responses of the beam were discussed. Assie *et al.* (2010a, b) presented a computational finite element model capable of simulation of the impact response of viscoelastic-frictionless bodies. Simşek (2010) studied the dynamic deflections and the stresses of a FG simply-supported beam subjected to a moving mass using different shear deformation theories by considering the centripetal, inertia and Coriolis effects of the moving mass. Assie *et al.* (2011) developed an efficient numerical algorithm for analyzing dynamic response of orthotropic viscoelastic composite laminates in the time domain by using Generalized Wiechert model. By means of the finite-element method and the Euler–Bernoulli beam theory, Alshorbagy *et al.* (2011) studied the free vibration of FG beams using the principle of virtual work to derive the equations of motion.

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Using the finite-element method, Mohanty *et al.* (2011) investigated the dynamic instability of a functionally graded Timoshenko beam on a Winkler foundation. Fallah and Aghdam (2012) presented a piece of research work to investigate the thermo-mechanical buckling and nonlinear free vibration of FGM beams on nonlinear elastic foundation. Their formulations have been based on the classical beam theory (CBT) and Von Karman's assumptions to establish the nonlinear strain-displacement relations. Nguyen *et al.* (2013) proposed an improved first-order shear-deformation beam theory for the static and free vibration analyses of axially loaded FG beams. Based on the concept of position of the neutral surface with a refined trigonometric higher-order beam theory, Fekrar *et al.* (2014), Bourada *et al.* (2015) and Bellifa *et al.* (2016) studied the bending and vibration behavior of FG beam/plate structures. In order to take into account, the transverse shear and normal deformations, using higher-order shear-deformation theories, Tounsi and his co-workers developed several models for investigating the static and dynamic behavior of FGMs beam/plate structures (Draiche *et al.* 2016, Mouffoki *et al.* 2017, Fahsi *et al.* 2017, Younsi *et al.* 2018, Mahmoudi *et al.* 2019 and Rabhi *et al.* 2020.

Utilizing different shear deformation theories, Eltaher and his co-authors presented many pieces of research work where they extensively studied and analyzed buckling, post-buckling, stability, dynamics and vibration characteristics of different types of FG structures (beam, plate, MEMS and NEMS) under various conditions and materials properties, (Hamed *et al.* 2016, Eltaher *et al.* 2018a, 2020a, b, Attia *et al.* 2018, Abdelrahmaan *et al.* 2019, 2020, Almitani *et al.* 2019, Mohamed *et al.* 2020, Asiri *et al.* 2020 and Hamed *et al.* 2020a, b) Akbas (2017a, 2018a) illustrated the dynamic free and forced responses of cracked and uncracked FGM damped micro beam by using modified couple stress theory. Huang *et al.* (2018) illustrated the effect of material graduation through the axial direction on whirling frequencies and critical speeds of a spinning Timoshenko beam. Zouatnia *et al.* (2018) developed an efficient and simple refined shear deformation theory for the free vibration of FG Plates. Khiloun *et al.* (2019) used four-variable quasi 3D HSDT to study static and vibration behaviors of thick FG plates. Ahmed *et al.* (2019) analyzed post-buckling behavior of continuously FG Nano beams with geometrical imperfections by using nonlocal elasticity theory. To study the behavior of FGMs beams/plates resting on elastic foundation, several mathematical models had been developed by (Yaghoobi and Fereidooni 2014, Sobhy 2016, Benahmed *et al.* 2017, Attia *et al.* 2018, Zaoui *et al.* 2019, Eltaher *et al.* 2019, Merzoug *et al.* 2020 and Babaei *et al.* 2020, Akbaş 2014, 2015, 2017b, c, 2018c, 2021).

In fabrication of FG materials, micro voids or pores can occur in them during the sintering process owing to large differences in solidification temperatures between material constituents. On the other hand, porosity or voids may be created inside materials to provide lightweight or enhance energy-absorbing capability. Voids are frequently presented in ceramics phase rather than metallic phase Fahsi *et al.* (2019). Consequently, it is imperative to take into account the effects of these porosities or imperfections on the global behavior of FG beams because materials can lose their strength after a certain porosity ratio.

Recently, lots of researchers focus on this relevant topic. Wattanasakulpong and Ungbhakorn (2014) studied vibration characteristics of FGM porous beams by using differential transformation method (DTM) with different kinds of elastic supports. Hadji *et al.* (2015) exploited a refined exponential shear deformation theory to study free vibration of FG beam considering porosities. Chen *et al.* (2016) illustrated the effect of symmetric and asymmetric porosity distributions on the free and forced vibration response of shear deformable FG beams. Kitipornchai *et al.* (2017) investigated free vibration and buckling of FG porous nanocomposite beams where the internal pores and graphene platelets (GPLs) are layer-wise distributed in the matrix. Eltaher *et al.* (2018b) implemented a modified porosity model that represents porosity and Young's modulus in an implicit form to simulate the vibrational behavior of porous FG nanobeams. Fazzolari (2018) exploited generalized beam theories to study the vibration and stability of porous FG sandwich beams resting on elastic foundations. Akbaş (2018b, 2019) studied harmonic forced vibration sandwich deep FG beams including porous effect in core layer. Hamed *et al.* (2019) presented the effects of porosity models on static behavior of size dependent FG nanobeam by using nonlocal elasticity. Sheng and Wang (2019a, b) studied the nonlinear forced vibration of FG Timoshenko microbeams under thermal and damping effects including von Kármán nonlinear theory, Hamilton's principle and the modified couple stress theory. Jena *et al.* (2020) exploited Navier's technique to study vibration of a functionally graded porous beam embedded in Kerr foundation. Akbaş *et al.* (2020a, b) studied dynamic behavior of viscoelastic functionally graded porous, thick beams under pulse load. Hamed *et al.* (2020b) presented the influence of axial load function on static stability of sandwich functionally graded beams with porous core. Liang *et al.* (2020) investigated wave propagation in porous FGM sandwich plates resting on viscoelastic foundation using a quasi-3D trigonometric shear deformation theory and considering different types of FGM sandwich structures.

In this study, forced vibration responses of a porous functionally graded beam resting on viscoelastic foundation under a triangle point loads are investigated in framework of finite element method. The governing equations are obtained by using Lagrange's equations and solved by finite element method. In frame of finite element analysis, twelve –node 2D plane element is used for modelling of beam. In the open literature, a study about dynamic responses of functionally graded beam resting on viscoelastic foundation with porosity has not been investigated so far. This article tends to fill this gap. In this paper, effects of porosity coefficient, porosity distribution and foundation parameters on the dynamic responses of functionally graded viscoelastic beam are presented and discussed. The rest of article is order as, the problem formulation including the force type, the constitutive equations, and balance equations are presented in section 2. Numerical results and parametric studies to figure out the response of FG porous, thick beam rest on elastic foundation under dynamic load with different

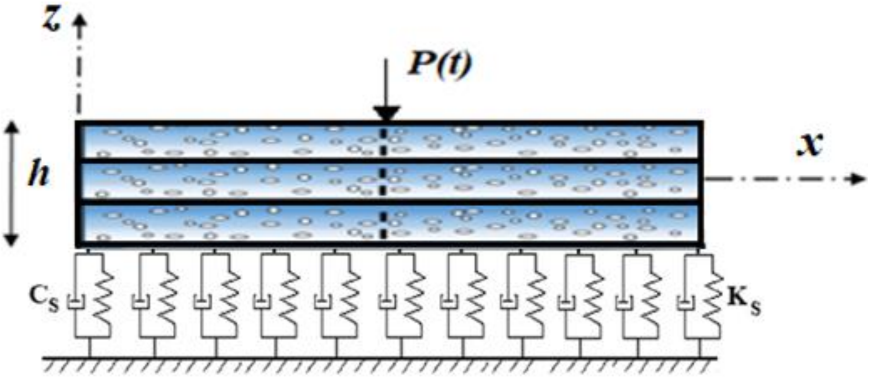

Fig. 1 A functionally graded beam resting on viscoelastic foundation with porosity

porosity coefficient, elastic foundation constant, damping foundation constant, and gradation index have been presented in section 3. Section 4 focuses on the main conclusions and remarks observed through analysis.

2. Problem formulation

A functionally graded beam resting on viscoelastic foundation with porosity under a dynamic point load $P(t)$ at midpoint is shown in Fig. 1. The stiffness and damping coefficients of the two viscoelastic foundation indicate as K_s and C_s, respectively. The dynamic point load $P(t)$ is assumed to be triangle in time domain as following:

$$P(t) = P_0 \qquad 0 \leq t \ll t_0 \tag{1}$$

where, P_0 is the amplitude of the dynamic load.

The beam is rectangular with a length of L and a thickness of h. The material graduation through transverse direction can be described by a power-law distribution as Alshorbagy *et al.* (2011).

$$E(z) = (E_T - E_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + E_B \tag{2a}$$

$$\rho(z) = (\rho_T - \rho_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \rho_B \tag{2b}$$

$$\nu(z) = (\nu_T - \nu_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \nu_B \tag{2c}$$

in which E , ρ and ν are modulus of elasticity, the density and Poisson's ratio, respectively. The positive gradation parameter n and subscripts T and B indicate the top and bottom properties of the beam. To include the impact of porosity on the material properties, three different distribution models are used through analysis. Therefore, equivalent mechanical and physical properties of Eqs. (2) can be adjusted to (Akbaş 2018b),

$$E(z) = (E_T - E_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + E_B - (E_T - E_B)a \tag{3a}$$

Model 1 $$\rho(z) = (\rho_T - \rho_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \rho_B - (\rho_T - \rho_B)a \tag{3b}$$

$$\nu(z) = (\nu_T - \nu_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \nu_B - (\nu_T - \nu_B)a \tag{3c}$$

$$E(z) = (E_T - E_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + E_B - (E_T - E_B)\frac{a}{2}\left(1 - \frac{2|z|}{h}\right) \tag{4a}$$

Model 2 $$\rho(z) = (\rho_T - \rho_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \rho_B - (\rho_T - \rho_B)\frac{a}{2}\left(1 - \frac{2|z|}{h}\right) \tag{4b}$$

$$\nu(z) = (\nu_T - \nu_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \nu_B - (\nu_T - \nu_B)\frac{a}{2}\left(1 - \frac{2|z|}{h}\right) \tag{4c}$$

and

$$E(z) = (E_T - E_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + E_B - (E_T - E_B)\frac{a}{2}\left(\frac{2|z|}{h}\right) \tag{5a}$$

Model 3 $$\rho(z) = (\rho_T - \rho_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \rho_B - (\rho_T - \rho_B)\frac{a}{2}\left(\frac{2|z|}{h}\right) \tag{5b}$$

$$\nu(z) = (\nu_T - \nu_B)\left[\frac{z}{h} + \frac{1}{2}\right]^n + \nu_B - (\nu_T - \nu_B)\frac{a}{2}\left(\frac{2|z|}{h}\right) \tag{5c}$$

where a (a<<1) is the volume fraction of porosities. When a=0, the layer becomes Fully FGM without any porosity constituent. The strain-displacement relations for 2D plane element are as follows;

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ w \end{Bmatrix} \tag{6a}$$

$$\{\varepsilon\} = [B]\{d\} \tag{6b}$$

where u, w indicate the displacements the in x and z directions, respectively. ε_{xx}, ε_{zz} and γ_{xz} indicate the normal strains and shear strain, respectively. The stress - strain- relations for porous viscoelastic FGM are as follows with the Kelvin–Voigt viscoelastic model;

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11}(z,a) & Q_{12}(z,a) & 0 \\ Q_{12}(z,a) & Q_{22}(z,a) & 0 \\ 0 & 0 & Q_{33}(z,a) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

$$\{\sigma\} = [C]\{\varepsilon\} \tag{8}$$

where

$$Q_{11}(z)=Q_{22}(z)=\frac{E(z)}{1-[v(z)]^2}\ ,\ Q_{33}(z)=\frac{E(z)}{2\ [1+v(z)]};\quad Q_{12}(z)=\frac{v(z)E(z)}{1-[v(z)]^2}, \tag{9}$$

where, c is the damping coefficient. Strain energy (U_i), kinetic energy (T) and potential energy of the external loads (U_e) is presented as follows;

$$U_i=\frac{1}{2}\{d\}^T\left(\int_V [B]^T\,[C][B]\right)\{d\}dV \tag{10}$$

$$T=\frac{1}{2}\{\dot{d}\}^T\left(\int_V \rho(z)\right)\{\dot{d}\}dV \tag{11}$$

$$U_e=-P(t)w(x_p,t) \tag{12}$$

in which $\{\dot{d}\}$ is the velocity vector, x_p is the coordinate of the applied load. The potential energy (U_s) and dissipation function (R_s) of the viscoelastic foundation are given as follows;

Potential energy and dissipation function of the supports at any time,

$$U_s=\frac{1}{2}K_s w^2 \tag{13}$$

$$R_s=\frac{1}{2}C_s\dot{w}^2 \tag{14}$$

The Lagrangian functional of the problem is presented as follows;

$$I=T-(U_i+U_e) \tag{15}$$

The proposed model is solved by using finite element method with Twelve-node 2D-plane element model, as illustrated in Fig. 2.
Where L_x and L_y are element lengths in X and Z directions, respectively. The displacement vector for Twelve-node plane element is expressed as:

$$\{d\}=[\emptyset]\{d_n\} \tag{16a}$$

$$[\emptyset]=[\emptyset_1\ \ \emptyset_2\ \ldots.\emptyset_{12}] \tag{16b}$$

where $\{d_n\}$ indicates the node displacement vector.

$$\{d_n\}=\begin{Bmatrix}u_1\\u_2\\.\\.\\u_{12}\\w_1\\w_2\\.\\.\\w_{12}\end{Bmatrix} \tag{17}$$

where $\{d_n\}$ is the node displacement vector, u_i and w_i indicate the displacement components for i node. The displacement of any generic element can be represented by its nodal values and corresponding functions as,

$$u=(u_1\emptyset_1+u_2\emptyset_2+u_3\emptyset_3+u_4\emptyset_4+u_5\emptyset_5+u_6\emptyset_6+u_7\emptyset_7+u_8\emptyset_8+u_9\emptyset_9+u_{10}\emptyset_{10} \tag{18a}$$

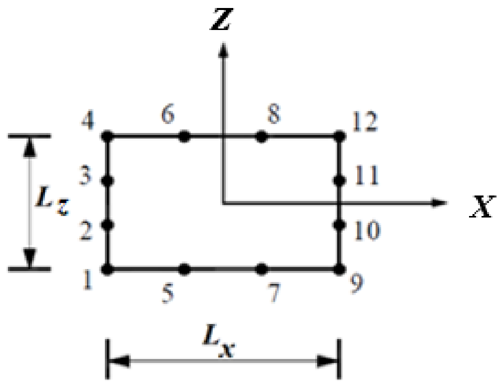

Fig. 2 Twelve-node 2D plane element

$$uw=(w_1\emptyset_1+w_2\emptyset_2+w_3\emptyset_3+w_4\emptyset_4+w_5\emptyset_5+w_6\emptyset_6+w_7\emptyset_7+w_8\emptyset_8+w_9\emptyset_9+w_{10}\emptyset_{10}+w_{11}\emptyset_{11}+w_{12}\emptyset_{12}) \tag{18b}$$

where $\emptyset_i$ is the nonlinear interpolation shape functions, which can be represented as follows;

$$\begin{aligned}
\emptyset_1&=\frac{1}{32}\left(1-\frac{2X}{L_x}\right)\left(1-\frac{2Z}{L_z}\right)\left(-10+9\left(\frac{4X^2}{L_x{}^2}+\frac{4Z^2}{L_z{}^2}\right)\right),\\
\emptyset_2&=\frac{9}{32}\left(1-\frac{2X}{L_x}\right)\left(1-\frac{4Z^2}{L_z{}^2}\right)\left(1-\frac{6Z}{L_z}\right)\\
\emptyset_3&=\frac{9}{32}\left(1-\frac{2X}{L_x}\right)\left(1-\frac{4Z^2}{L_z{}^2}\right)\left(1+\frac{6Z}{L_z}\right),\\
\emptyset_4&=\frac{1}{32}\left(1-\frac{2X}{L_x}\right)\left(1+\frac{2Z}{L_z}\right)\left(-10+9\left(\frac{4X^2}{L_x{}^2}+\frac{4Z^2}{L_z{}^2}\right)\right)\\
\emptyset_5&=\frac{9}{32}\left(1-\frac{2Z}{L_z}\right)\left(1-\frac{4X^2}{L_x{}^2}\right)\left(1-\frac{6X}{L_x}\right),\\
\emptyset_6&=\frac{9}{32}\left(1+\frac{2Z}{L_z}\right)\left(1-\frac{4X^2}{L_x{}^2}\right)\left(1-\frac{6X}{L_x}\right)\\
\emptyset_7&=\frac{9}{32}\left(1-\frac{2Z}{L_z}\right)\left(1-\frac{4X^2}{L_x{}^2}\right)\left(1+\frac{6X}{L_x}\right),\\
\emptyset_8&=\frac{9}{32}\left(1+\frac{2Z}{L_z}\right)\left(1-\frac{4X^2}{L_x{}^2}\right)\left(1+\frac{6X}{L_x}\right)\\
\emptyset_9&=\frac{1}{32}\left(1+\frac{2X}{L_x}\right)\left(1-\frac{2Z}{L_z}\right)\left(-10+9\left(\frac{4X^2}{L_x{}^2}+\frac{4Z^2}{L_z{}^2}\right)\right),\\
\emptyset_{10}&=\frac{9}{32}\left(1+\frac{2X}{L_x}\right)\left(1-\frac{4Z^2}{L_z{}^2}\right)\left(1-\frac{6Z}{L_z}\right)\\
\emptyset_{11}&=\frac{9}{32}\left(1+\frac{2X}{L_x}\right)\left(1-\frac{4Z^2}{L_z{}^2}\right)\left(1+\frac{6Z}{L_z}\right),\\
\emptyset_{12}&=\frac{1}{32}\left(1+\frac{2X}{L_x}\right)\left(1+\frac{2Z}{L_z}\right)\left(-10+9\left(\frac{4X^2}{L_x{}^2}+\frac{4Z^2}{L_z{}^2}\right)\right)
\end{aligned} \tag{19}$$

Substituting Eq. (18), into the energy Eqs. (10)-(14) and Lagrange's equations, and then Lagrange's equations are presented as following:

$$\frac{\partial \mathrm{I}}{\partial \mathrm{q}_\mathrm{k}^{(\mathrm{e})}}-\frac{\partial}{\partial \mathrm{t}}\frac{\partial \mathrm{I}}{\partial \dot{\mathrm{q}}_\mathrm{k}^{(\mathrm{e})}}+\mathrm{Q}_{\mathrm{D}_\mathrm{k}}=0\quad \mathrm{Q}_{\mathrm{D}_\mathrm{k}}=-\frac{\partial \mathrm{R}}{\partial \dot{\mathrm{q}}_\mathrm{k}^{(\mathrm{e})}},\quad \mathrm{k}=1,2,3,\ldots 12 \tag{20}$$

where $\mathrm{Q}_{\mathrm{D}_\mathrm{k}}$ is generalized damping load. After using the Lagrange procedure, the dynamic equilibrium equation is writeen as follows:

$$[K+K_s]\{d_n\}+[C_s]\{\dot{d}_n\}+[M]\{\ddot{d}_n\}=\{F\} \tag{21}$$

where $[K]$, $[M]$, $\{F\}$, $\{d_n\}$, $\{\dot{d}_n\}$, $\{\ddot{d}_n\}$, K_s and C_s indicates the stiffness matrix, mass matrix, load vector, displacement vector, velocity vector, acceleration vector, stiffness and damping matrix of viscoelastic foundation respectively. The expansions of finite element matrices are represented as

$$[K] = b\int_A [H]^T[Q][H]dA \tag{22a}$$

$$[C] = b\int_A \eta[H]^T[Q][H]dA \tag{22b}$$

$$[M] = b\int_A \rho(z)[\emptyset]^T[\emptyset]dA \tag{22c}$$

$$\{F\} = \int_\Gamma \{\delta d_n\}^T[\emptyset]^T P(t)d\Gamma \tag{22d}$$

where

$$[H] = \begin{bmatrix} \frac{\partial}{\partial X} & 0 \\ 0 & \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial Y} & \frac{\partial}{\partial X} \end{bmatrix}[\emptyset] \tag{23}$$

The governing equation of motions Eq. (21) is solved numerically by using implicit Newmark average acceleration method in the time domain. The dimensionless stiffness of foundation is given as follows;

$$\mathrm{k}_{sb} = \frac{KL^3}{E_c I}, \tag{24}$$

where E_b is Young's Modulus of bottom surface, I is inertia of moment, A is the area.

3. Numerical results

In the numerical results, effects of graduation parameter, porosity parameters, porosity models and viscoelastic foundation parameters on the time response of multilayer FGM beam are discussed and presented in detail. The material of FGM is considered as top surface; Aluminium (Al; E=70 GPa, ν=0.3, ρ=2702 kg/m^3) and bottom surface as ; Zirconia (E=151 GPa, ν=0.3, ρ=3000 kg/m^3). Through the results, dynamic load is considered as triangle pulse load and three different porosity distribution model are proposed. The dimensions of the FGM beam are considered as follows: b = 0.1 m, h = 0.1 m, L=1 m in the numerical analysis. The five-point Gauss rule is used for calculation of the integration. In the numerical process, the finite element number is taken as 30 in both X and Z directions. The triangular pulse load with the following properties: t_s=0.2 s and P_0=1000 kN.

3.1 Verification

A comparison study is performed to validate the proposed model with previous published work. In the comparison study, the maximum vertical displacements of a fully Aluminum beam are obtained and compared with SAP2000 structural analysis program for load 1 for L/h=5, P_0=1000 kN, Ω=2 rad/s in Fig. 4. It is seen from Fig. 3, that results of this study are approximately identical with results of SAP2000.

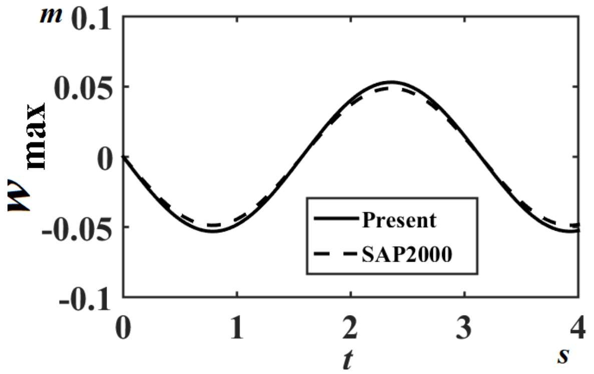

Fig. 3 Comparison study: Time responses of the fully Aluminium beam for load 1for L/h=5, P_0=1000 kN, Ω=2 rad/s

Table 1 Dimensionless of the fundamental frequencies of porous FGM beam for L/h=20

Porosity Models	a	n=0.5		n=2	
		Present	Ebrahimi and Jafari (2016)	Present	Ebrahimi and Jafari (2016)
Model 1	0	4.52911	4.51585	3.5661	3.55529
	0.1	4.5953	4.58213	3.5202	3.50825
	0.2	4.6761	4.66783	3.4606	3.44947
Model 2	0	4.5391	4.51585	3.5643	3.55529
	0.1	4.6182	4.60304	3.5912	3.58170
	0.2	4.7184	4.70024	3.6381	3.61002

In another comparison study, fundamental frequencies of a simply supported FGM porous beam are compared with porosity and FGM parameters in Table 1 with presented results in Ebrahimi and Jafari (2016). The fundamental frequencies are presented as dimensionless values ($\bar{\boldsymbol{\omega}} = \omega\sqrt{\rho_B L^4 / E_B\ h^2}$) based on Reddy beam for L/h=20. With using the geometry and material parameters of Ebrahimi and Jafari (2016), the fundamental frequencies are obtained from the eigenvalue solution of equation 21. As seen from Table 1, the presented results are very close to obtained results published in this work.

3.2 Paramteric studies

Effect of porosity model, porosity factor (a), graduation parameter (n), dimensionless stiffness of foundation (k_{sb}) and viscoelastic coefficient of foundation (Cs) for triangle load on the dynamic response of FGM beam are investigated in detail through these subsections.

3.2.1 Effects of foundation stifness on max deflection

In Figs. 4, 5 and 6, the dynamical displacements of the mid-point of beam versus dimensionless foundation

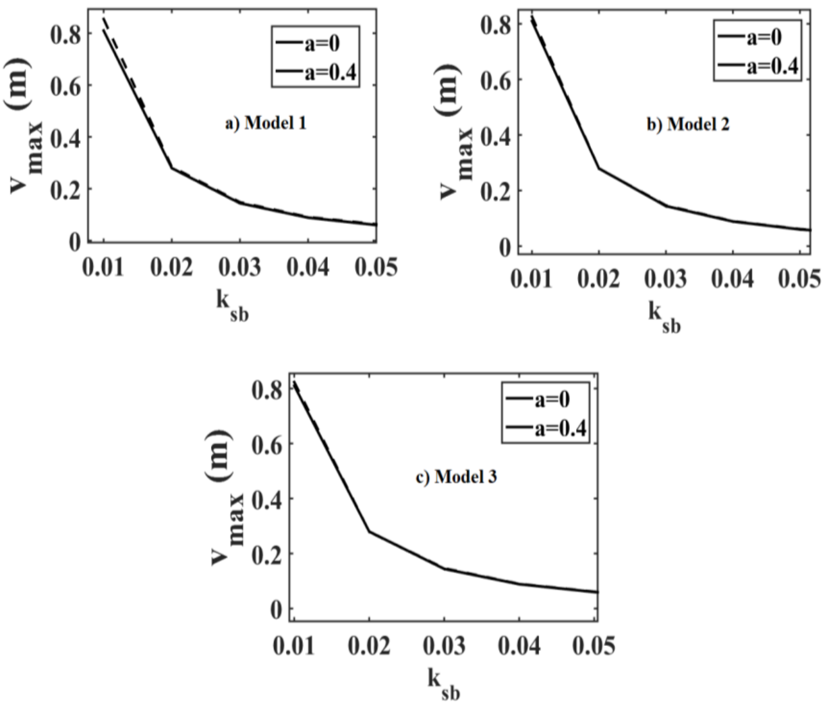

Fig. 4 The dynamical displacements of the mid-point versus dimensionless foundation parameter (k_{sb}) with different porosity parameters for graduation parameter *n*=0 at (a) model 1, (b) model 2 and (c) model 3

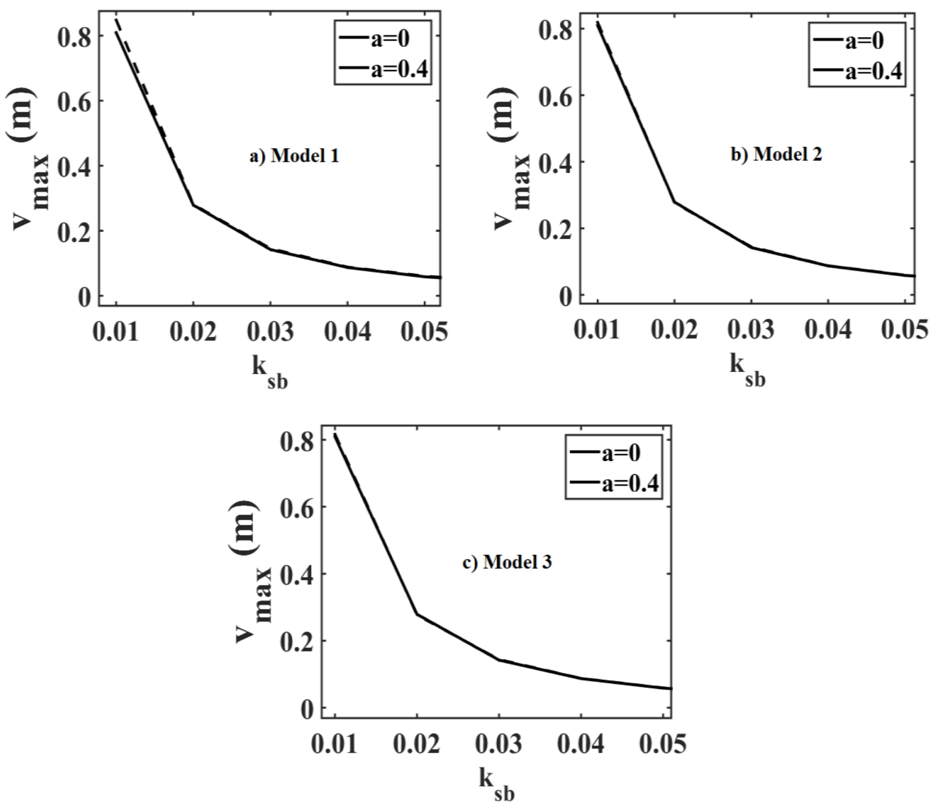

Fig. 5 The dynamical displacements of the mid-point versus dimensionless foundation parameter (k_{sb}) with different porosity parameters for graduation parameter *n*=1 at (a) model 1, (b) model 2 and (c) model 3

parameter (k_{sb}) presented with different porosity parameters and models for viscoelastic coefficient of foundation Cs=0.0001. In these figures, the dynamical displacements are obtained for *t*=0.1 s. As shown in Fig. 4, by increasing the elastic stiffness of foundation the peak amplitude is decreased in a linear form as the stiffness coefficient changed from 0.01 to 0.04. After that, the amplitude changes with small amount as the stiffness of

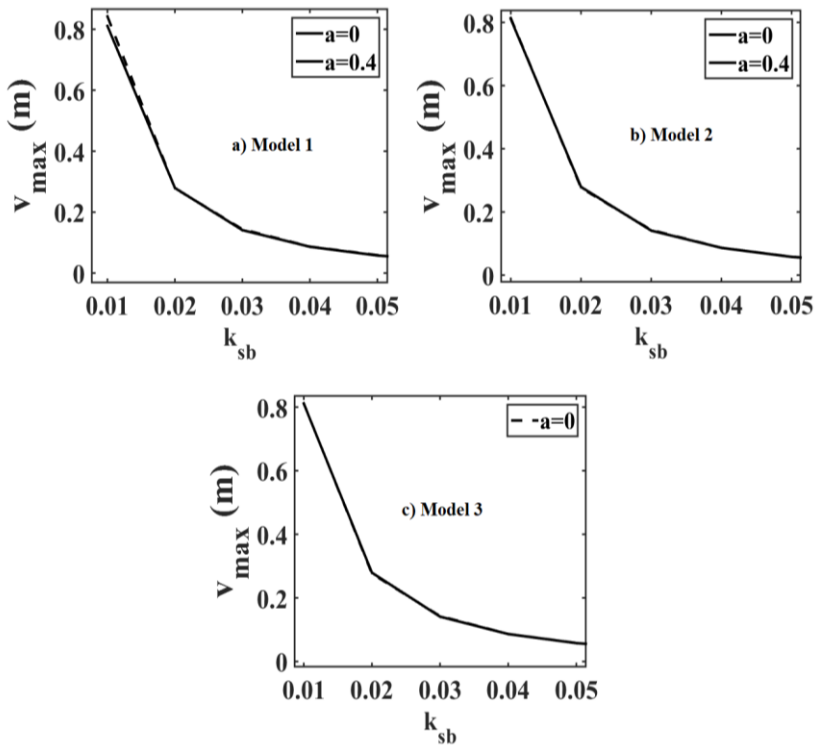

Fig. 6 The dynamical displacements of the mid-point versus dimensionless foundation parameter (k_{sb}) with different porosity parameters for graduation parameter *n*=10 at (a) model 1, (b) model 2 and (c) model 3

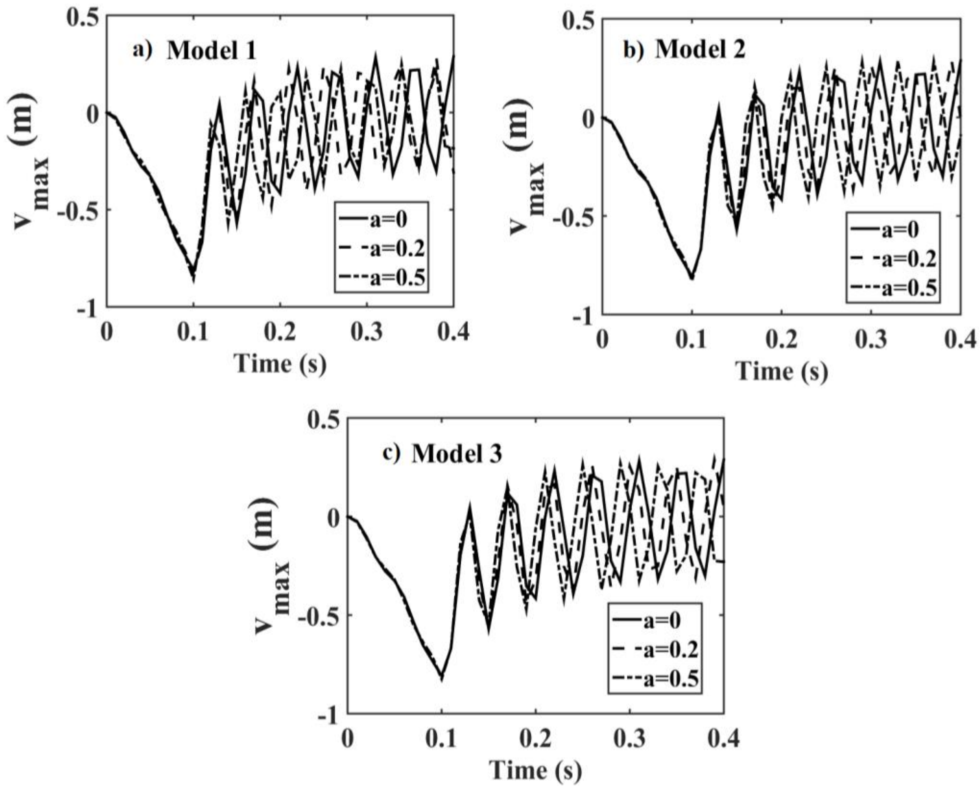

Fig. 7 Time responses of FGM porous beam with different porosity parameters for k_{sb}=0.01 at (a) model 1, (b) model 2 and (c) model 3

foundation increased more than 0.04. It is noted that, within a range of stiffness 0.01 to 0.02, the porosity distribution effect on the maximum amplitude of vibration may be noticed. After 0.02 stiffening value, the influence of porosity disappeared. The same observation concluded at gradation index n=0 from Fig. 4, can be predicted for

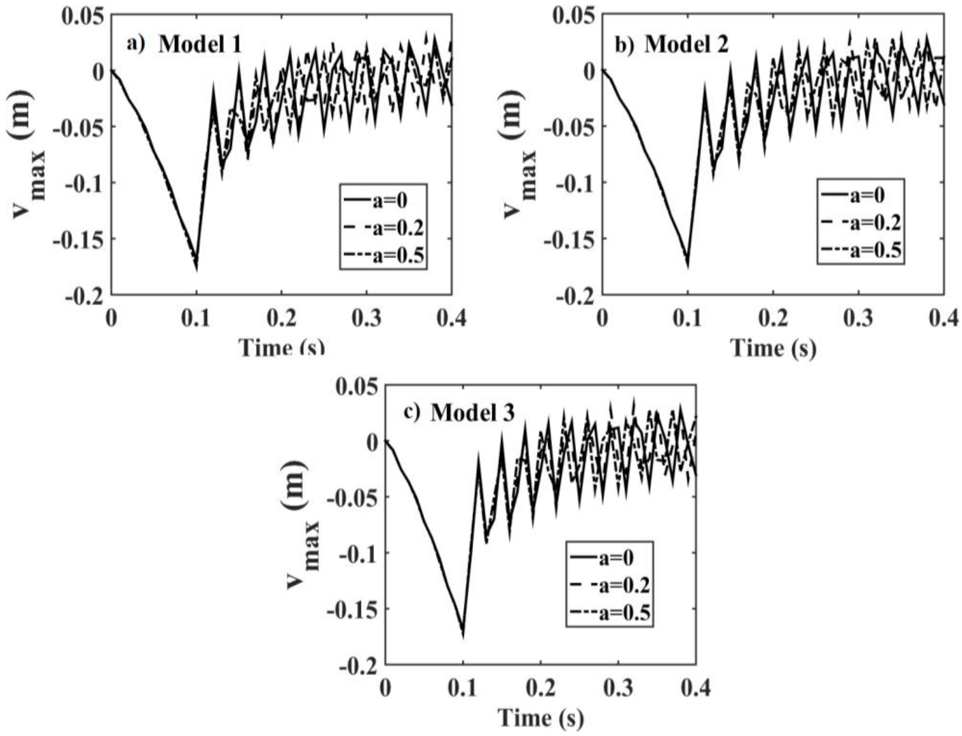

Fig. 8 Time responses of FGM porous beam with different porosity parameters for k_{sb}=0.05 at (a) model 1, (b) model 2 and (c) model 3

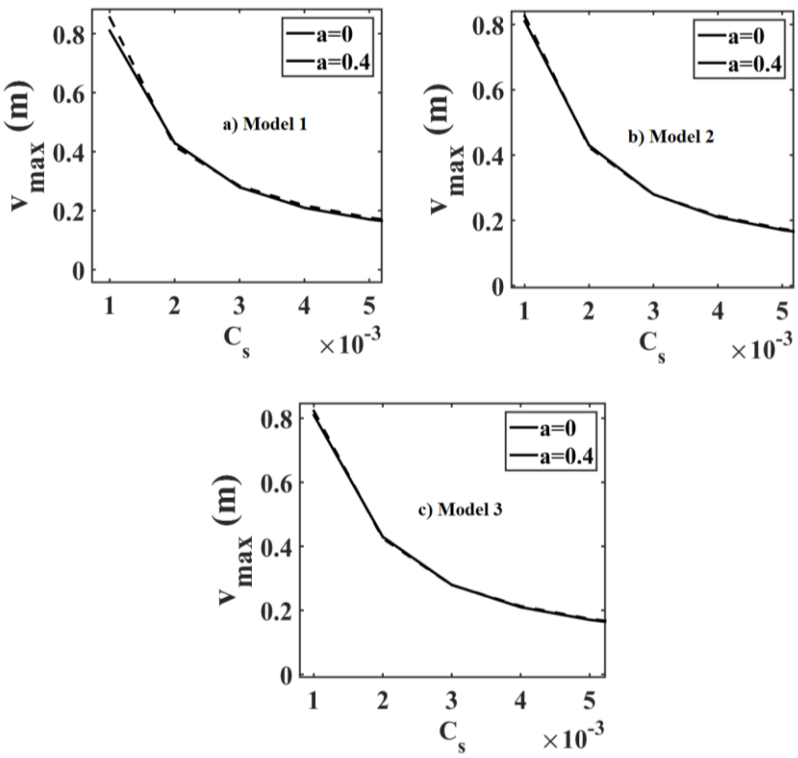

Fig. 9 The dynamical displacements of the mid-point versus viscoelastic coefficient of foundation (Cs) with different porosity parameters for graduation parameter *n*=0 at (a) model 1, (b) model 2 and (c) model 3

different gradation indices n=1 and n=10, as shown in Figs 5 and 6, respectively.

3.2.2 Effects of foundation stifness on the time response

In Figs. 7 and 8, the time history of dynamical displacements at mid-point of beam are plotted for 0.01 and 0.05 values of dimensionless foundation parameter k_{sb}, with different porosity parameters and models for viscoelastic coefficient of foundation Cs=0.0001.

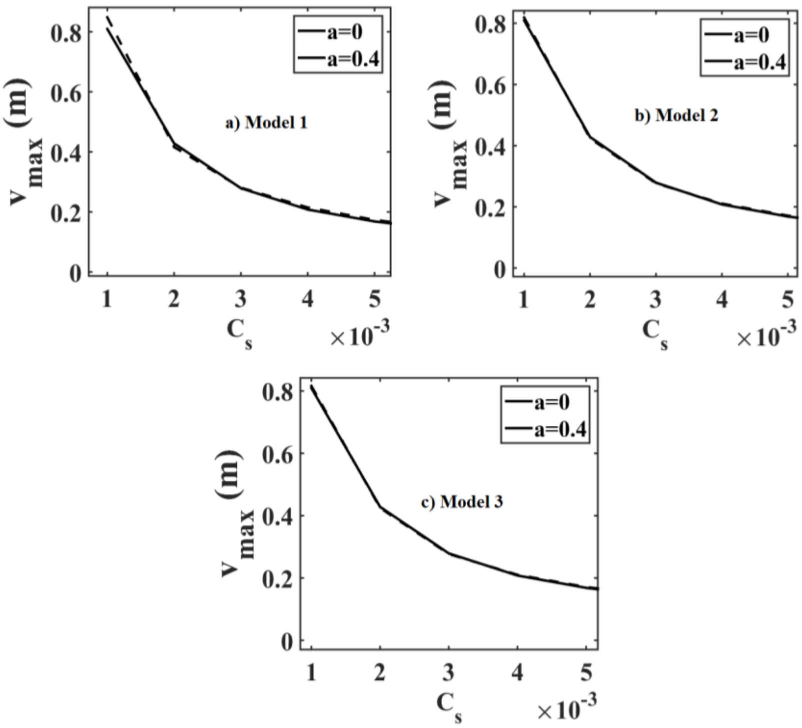

Fig. 10 The dynamical displacements of the mid-point versus viscoelastic coefficient of foundation (Cs) with different porosity parameters for graduation parameter *n*=1 at (a) model 1, (b) model 2 and (c) model 3

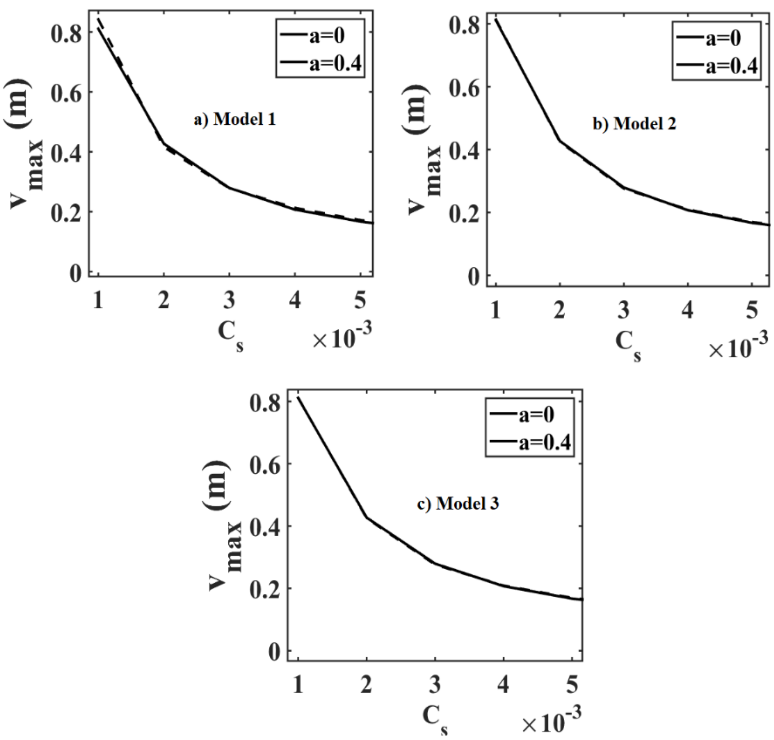

Fig. 11 The dynamical displacements of the mid-point versus viscoelastic coefficient of foundation (Cs) with different porosity parameters for graduation parameter *n*=10 at (a) model 1, (b) model 2 and (c) model 3

In these figures, the dynamical displacements are obtained for t=0.1 s. As presented in these figures, the response composed from two parts, which are the forced response (during the period of applied load) and free response (after removing the load). Through the first part, which is under applying load, the influence of porosity is insignificant, however, the porosity parameter plays and important role on the time period (frequency) of the vibration especially after removing the load. The effect porosity models have the same effect on the dynamic response, even, under the forced application or after removing the force.

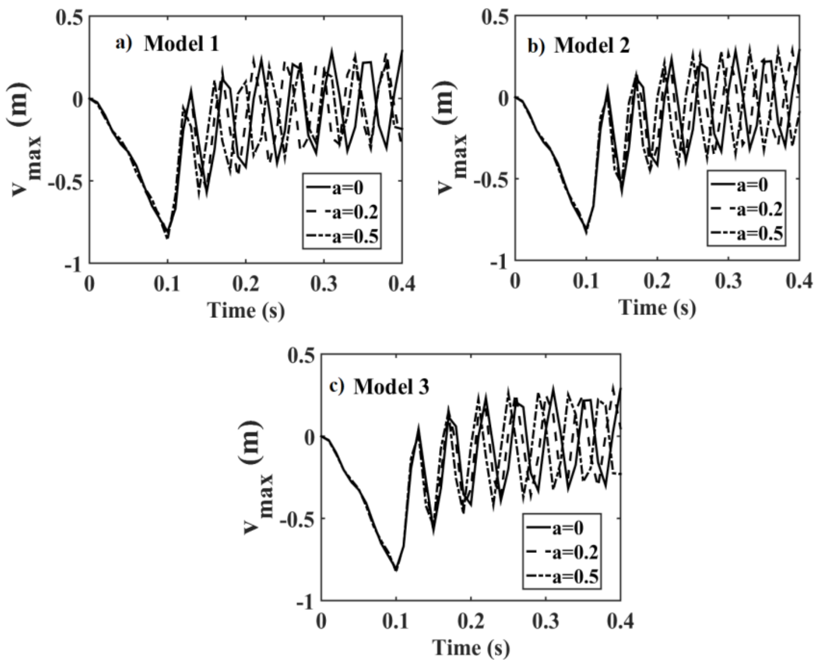

Fig. 12 Time responses of FGM porous beam with different porosity parameters for viscoelastic coefficient of foundation Cs=0,001 at (a) model 1, (b) model 2 and (c) model 3

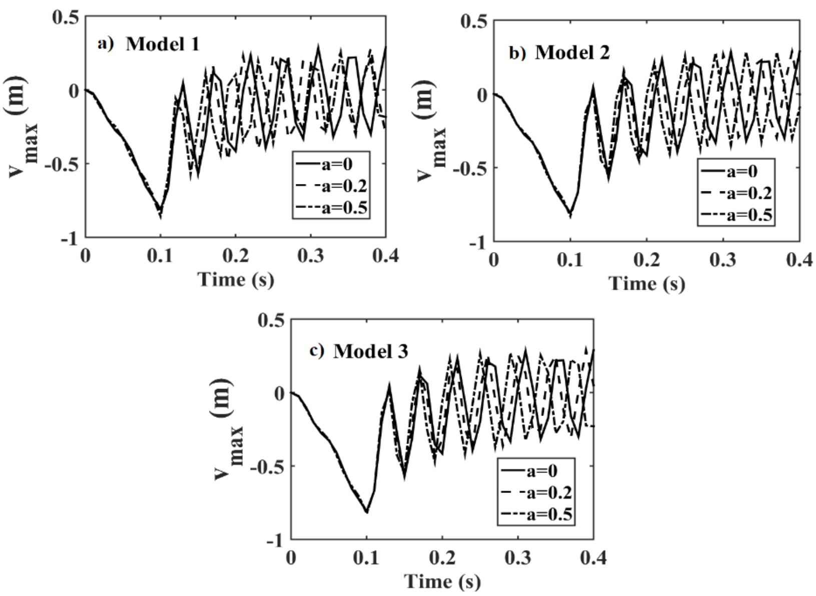

Fig. 13 Time responses of FGM porous beam with different porosity parameters for viscoelastic coefficient of foundation Cs=0,005 at (a) model 1, (b) model 2 and (c) model 3

3.2.3 Effects of foundation vicoelasticity on the on Max deflection

In Figs. 9-11, the dynamical displacements of the mid-point of beam versus viscoelastic coefficient of foundation (Cs) presented with different porosity parameters and models for dimensionless foundation parameter k_{sb}=0.01. In these figures, the dynamical displacements are obtained for t=0.1 s. As presented in Fig. 9, the amplitude of vibration is decreased parabolically by increasing the viscoelastic foundation parameter, which tends to resist the motion and absorb and dissipate the energy induced to the system by force. It is also observed from this figure that, the porosity parameter is significant for the model 1 of distribution, however, for the other cases of model 2 and 3, the porosity parameter is insignificant on the maximum amplitude. The observation noticed in Fig. 9 at n=0, can be predicated by increasing the gradation index to 1 and 10, as shown in Figs. 10 and 11, respectively.

3.2.4 Effects of foundation vicoelasticity on the on the time response

In Figs. 12 and 13, the time history of dynamical displacements at mid-point of beam are plotted for different values of viscoelastic coefficient of foundation (Cs) with different porosity parameters and models for for dimensionless foundation parameter k_{sb} = 0.01. As illustrated through Figs. 12 and 13, the midpoint of beam under the load deflects linearly as the load increased from 0 to 0.1 S. after removing the load, the stiffness of the beam and foundation parameters react adversely to the forced, therefore, the beam will restore its static equilibrium position. However, the beam still has the some of stored energy, therefore, the beam vibrates freely after removing the load (at t=0.1 s) and its decreases due to the present of viscoelastic foundation. It is notice that the response for a specific model has different time period according to different porosity parameter. This means that the natural frequencies of the bam is dependent on the porosity parameter.

4. Conclusions

The dynamical responses of FGM beam resting on viscoelastic foundation under applied dynamic load have been investigated with porosity effects. In this paper, effects of porosity coefficient, porosity distribution and foundation parameters on the dynamic responses of functionally graded viscoelastic beam are presented and discussed. Twelve-node 2D-plane element is used to discretize the space domain and derive the finite element model. Time integration of Newmark is adopted to predict the midpoint vibration with respect to time-domain. The verification example has been done to prove the accuracy of the present model. Many remarks can be inferred from numerical studies as follows:

• The elastic stiffness has a significant influence on the maximum amplitude of vibration. by increasing the elastic stiffness of foundation from 0.01 to 0.04, the maximum amplitude is decreased. More than k_{sb} = 0.04, the maximum amplitude becomes constant.

• Within a range of stiffness 0.01 to 0.02, the porosity distribution effect on the maximum amplitude of vibration may be noticed. After 0.02 stiffening value, the influence of porosity disappeared.

• Under applying load, the influence of porosity is insignificant, however, the porosity parameter plays and important role on the time period (frequency) of the vibration especially after removing the load.

• The natural frequencies of the bam is dependent on the porosity parameter.

• The amplitude of vibration is decreased parabollically by increasing the viscoelastic foundation parameter, which tends to resist the motion and absorb and dissipate the energy induced to the system by force.

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