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A polynomial mathematical tool for foundation-soil-foundation interaction

Badreddine Sbartai*[1,2]

[1]Department of Civil Engineering, University of Badji Mokhtar-Annaba, BP.25, Annaba 23000, Republic of Algeria
[2]LMGHU, University of 20 Août 1955-Skikda, B.P.26, Skikda 21000, Republic of Algeria

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Abstract. This paper studies the dynamic foundation-soil-foundation interaction for two square rigid foundations embedded in a viscoelastic soil layer. The vibrations come from only one rigid foundation placed in the soil layer and subjected to harmonic loads of translation, rocking, and torsion. The required dynamic response of rigid surface foundations constitutes the solution of the wave equations obtained by taking account of the conditions of interaction. The solution is formulated using the frequency domain Boundary Element Method (BEM) in conjunction with the Kausel-Peek Green's function for a layered stratum, with the aid of the Thin Layer Method (TLM), to study the dynamic interaction between adjacent foundations. This approach allows the establishment of a mathematical model that enables us to determine the dynamic displacements amplitude of adjacent foundations according to their different separations, the depth of the substratum, foundations masss, foundations embedded, and the frequencies of excitation. This paper attempts to introduce an approach based on a polynomial mathematical tool conducted from several results of numerical methods (BEM-TLM) so that practicing civil engineers can evaluation the dynamic foundations displacements more easy.

Keywords: soil-structure interaction; BEM; TLM; dynamic response; nonlinear regression; soil

1. Introduction

The dynamic structure-soil-structure interaction has been studied by a number of researches over the years. The dynamic response of closely spaced structures is affected by the interaction between the soil and each structure. One observes this type of interaction in the adjacent buildings of a big city, in adjacent structures of a nuclear power plant complex, or in a system of close-vicinity machine foundations. The dynamic foundation-soil-foundation interaction phenomenon has long been recognized as an important factor in the seismic and machine vibration response of critical facilities and other closely spaced structures or portions of structures. Rational analysis of the phenomenon requires taking into account the dynamic nature of the interaction between the soil and the foundation. This is essentially a wave-propagation problem with mixed boundary conditions (i.e., rigid body displacement under the foundations and no traction elsewhere). Although a solution of a foundation-soil-foundation interaction problem in most cases involves a straightforward application of any of the well-established soil-structure interaction methods, a relatively small number of 3-D investigations have appeared in the related literature.

This is probably due to the substantial computational effort required by the finite element method and the usual, straightforward boundary element method formulations. Furthermore, there is a noticeable absence of simplified discrete models, which is due, perhaps, to the general lack of rigorous results that could be used for the verification and calibration of such models.

The complicated geometries, loadings, and soil conditions have discouraged, in general, the development of analytical solutions. The case of a rectangular foundation (square) embedded or not in a semi-infinite soil was treated by Dominguez *et al.* (1978). This study also presented analytical forms for stiffness functions corresponding to the six degrees of freedom of the foundation, but only in the case of a foundation on the surface. Wong and Luco (1986) developed a method that permits the evaluation of the impedance or compliance functions for flat, rigid foundations of arbitrary shape placed on the surface of an elastic half-space. The method is based on dividing the contact area between the foundation and the soil into a number of sub-regions and assuming that the contact tractions within each area are uniform but of unknown amplitude. Liou (1989) developed an analytic solution to calculate impedance functions for vibrations of an axial symmetric foundation with an arbitrary shape. Wong and Trifunac (1975) developed an analytic solution based on the exact infinite series solution to study two-dimensional, antiplane, building-soil-building interaction for two or more buildings. Ai *et al.* (2016) developed an analytic solution based on mixed boundary conditions to study the vertical vibration of a rigid strip footing on a transversaly isotropic multilayered half-plane. Aydemir (2013) studied the soil structure interaction effects on structural parameters for stiffness degrading systems built on soft soil sites by using the modified-Clough model.

The finite element method has also been applied by Kausel *et al.* (1975) and Kausel and Roesset (1975) in determining the behavior of rigid foundations resting on or embedded in a stratum over bedrock. However, Gonzalez (1974), Lin and Tassoulas (1987) and Lin *et al.* (1987) used

*Corresponding author, Professor, Ph.D.
E-mail: badreddine.sbartai@univ-annaba.dz

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the finite element method in conjunction with consistent boundaries to analyse the cross-interaction between two foundations subjected to harmonic forces. And Van Nguyen *et al.* (2016) studied the effects of foundation size on the seismic performance of buildings considering the soil-foundation-structure interaction by using the finite element method.

A frequency domain boundary element method formulation has been developed to treat wave-propagation problems, soil-structure problems, and structure-soil-structure problems, which limit the discretization at the soil-foundation interface. In this approach, the field displacement is formulated as an integral equation in terms of Green's functions in Beskos (1987). Using this method in conjunction with a constant element and the half-space Green's function of Apsel and Luco (1987), Wong and Luco (1985) studied the cross-interaction between two rigid, square surface foundations in layered viscoelastic half-space subjected to external harmonic forces. Wang et al (1991) used the boundary element method in conjunction with the constant elements and the half-plane Green's function to study the cross-interaction of two rigid or flexible surface or embedded strip foundations. Qian and Beskos (1996) and Qian *et al.* (1998) studied the harmonic wave response of two 3-D rigid surface foundations using the boundary element method in conjunction with isoparametric elements and the half-space Green's functions. Using the boundary element method in conjunction with a constant element and the full-space and relaxed boundary conditions, Karabalis and Mohammadi (1998) and Mohammadi (1992) studied the 3-D dynamic foundation-soil-foundation interaction on layered soil. Wang and Schmid (1992) analysed the dynamic interaction between 3-D structures by coupling the finite element and boundary element methods. However, Qian *et al.* (1996) studied the dynamic cross-interaction between flexible surface footings by combining the boundary element and finite element methods. And Liang *et al.* (2018) studied the effect of site dynamic characteristics on dynamic soil-structure interaction.

Using a semi-analytical formulation, Gazetas and Roesset (1979) analysed the 2-D problem of strip foundations on a layered half-space. However, Triantafyllidis and Prange (1987, 1989) and Liou (1994) proposed a semi-analytic solution to study an interaction between two circular footings with regular shapes. Waas (1972) and Kausel (1974) developed a semi-discrete analytic method to model the far field with homogenous boundary conditions for two dimensions and axisymmetric problems (thin-layer theory). With the aid of the thin-layer theory, Kausel and Peek (1982) obtained Green's functions for multilayer soil. This semi-discrete analytic model is then combined with the finite element model or the boundary element model of the near field to solve the soil-structure interaction problems in layered media. Using this approach, combined with the boundary element model Boumekik (1985) studied the 3-D dynamic problem of embedded foundation on a layered substratum and Sbartai (2016) studied the 3-D dynamic response between two adjacent embedded foundations on a layered substratum. In addition Sbartai and Boumekik (2008) studied the ground vibration starting from rigid foundation and Messioud et al (2012, 2016, 2019) studied the harmonic seismic response of 3-D embedded foundations. Finally, Sbartai (2018) studied the dynamic response of a square foundation using the same method cited above coupled with an equivalent linear approach.

In this work, the solution is formulated using the frequency domain (BEM) in conjunction with the Kausel-Peek Green's function for a layered stratum and constant elements, with the aid of the (TLM), to study the dynamic interaction between adjacent foundations. For surface foundations, only the foundation-soil-foundation interface is discretized. For the embedded foundations, we discretized the volume of the soil which is occupied by the foundations. Within the discretized medium, the Green's functions are calculated. Using this approach allowed the establishment of a mathematical model that enabled us to determine the dynamic displacement of adjacent foundations. Through the numerical studies, the effects of a set of parameters are considered.

This paper attempts to introduce a new approach based on the use of the polynomial regressions conducted from several results obtained of the used numerical method (BEM-TLM) so that practicing civil engineers would feel more comfortable and easy to use for the evaluation of displacements of adjacent foundations. The mathematical formulation of this approach was fully formed into a computer program developed in Fortran 90 (1990). Using Curve Fitting Toolbox (CFT) of Matlab® (2009), the regressions approach based on least square method were used to obtain mathematical expressions for all studied cases. The novelty of this work is to provide the polynomial relationships which connect the maximum displacement amplitudes of the foundations independently of the frequency of excitation to the four various dimensionless parameters: depth of soil H, distance between foundations D, embedding of foundations E, and mass of foundations M.

2. Formulation of dynamic of displacements by BEM

The model of calculation is represented in Fig. (1). The two foundations considered are supposed to be rigid, of rectangular form (square), placed at the surface or partially embedded in homogeneous soil limited by a substratum. The soil at height H is supposed to be viscoelastic linear characterized by its mass density ρ, its shear modulus G, its damping coefficient β, and Poisson's ratio ν. The foundations are subjected to three harmonic external forces, P_x, P_y, and P_z, and at three harmonic moments M_x, M_y, and M_z. It is assumed that the time dependence of the excitation is of the type $e^{i\omega t}$, in which ω denotes the frequency. For brevity, this time factor will be omitted. The goal is to obtain the compliance functions of the two foundations.

Displacements in an unspecified point "α" of the soil may be obtained from the solution of the wave equation:

$$\left(\left(C_p^2 - C_s^2\right)u_{j,ij} + C_s^2 u_{i,jj} + C_p^2 \omega^2 u_i\right)\rho = 0 \qquad (1)$$

where u_i is the x-component of harmonic displacement

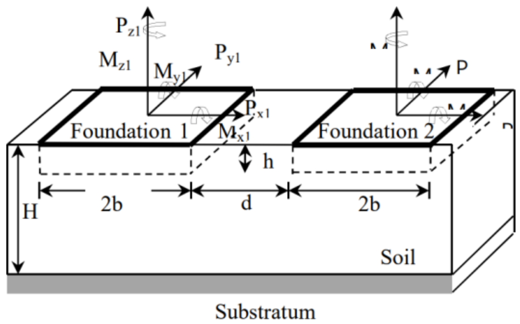


Fig. 1 Model of calculus of two adjacent foundations resting on homogenous soil

amplitude vector; $u_{j,jj}$ is the partial derivative of u_j with respect to x- and y-axes ;

$$u_j(x,\omega) = \int_s G_{ij}(x,\xi,\omega)\cdot t_i(\xi,\omega)\cdot dS(\xi) \tag{2}$$

$u_{i,jj}$ is the second partial derivative of u_i with respect to y-axis ; C_s and C_p are the shear (S)- and compression (P)-wave velocities; and ω is the angular frequency of excitation.

The solution of Eq. (1) can be obtained in form of the following boundary integral equation in frequency domain: Here, G_{ij} represents the Green's function tensor, and t_j the unknown surface traction, ξ.

Eq. (2) remains to solve as long as the domain is a continuum. However, if the domain is discretized in an appropriate form, Eq. (2) can be algebraically evaluated for each element.

In this approach, the discretization principle of the soil mass is represented in Fig. 2. It is based on two types of discretization, one horizontal and other vertical. The horizontal discretization consists of subdividing any horizontal section of the soil mass into square elements. The average displacement of the element is replaced by its centre displacements, for which the distribution of the constraints is supposed to be uniform. The vertical discretization consists of subdividing the solid mass of the soil in the under layers (infinite elements in the horizontal direction), which have rather low thickness compared with the wavelength of Rayleigh (λ/10), in order to linearized the displacement of one under layer to the other. This discretization is characterized by the embedding of the foundation and the depth of the substratum.

In the discretized model, Eq. (2) is expressed in algebraic form as follows:

$$u_j = \sum_{i=1}^{NRT}\int_s G_{ij}\cdot t_i\cdot dS \tag{3}$$

Here, NRT represents the total number of elements, which discretize the free surface and the interface between the soil and the foundations.

3. Determination of Green's functions by TLM

In this work, body A is a layered stratum resting on a substratum base with N horizontal layer interfaces defined by z = $z_1, z_2, \ldots, z_N$ and with layer j defined by $z_n < z < z_{n+1}$, as shown in Fig. 3. The medium within each layer j of thickness h_j is assumed to be homogeneous, isotropic, and linearly elastic. For this body, the frequency domain Green's function is obtained with the aid of the thin-layer theory of Waas (1972) and Kausel and Peek (1982). Actually, the Green's function for body A is obtained by an inversion of the thin-layer stiffness matrix through a spectral decomposition procedure of Kausel and Peek (1982). The advantage of the thin-layer stiffness matrix technique over the classical Haskell-Thomson transfer matrix technique for finite layers Haskell (1953); Thomson (1950) and the finite-layer stiffness matrix technique of Kausel and Roesset (1981) is that the transcendental functions in the layered stiffness matrix are linearize. According to the thin-layer theory of Waas (972), the thickness of each layer is chosen to be small—i.e., less than 1/10 of the Rayleigh wavelength in that layer—so that the displacements within the layer can be assumed to vary linearly with depth and remain continuous in the x direction.

The Green's functions thus obtained are complex and constitute the starting point for the determination of the flexibility matrix of a volume of the arbitrary soil. For more details about the computation of the above Green's functions, one can consult the work of Kausel and Peek (1982).

Viscoelastic soil behaviour can be easily introduced in the present formulation by simply replacing the elastic constants λ and G with their complex values:

$$\lambda^* = \lambda\,(1+2i\beta) \tag{4}$$

$$G^* = G(1+2i\beta) \tag{5}$$

4. Response of the model

The total matrix displacement of the soil is obtained by successive application of the load units distributed on the constituent elements of the discretized soil mass. This matrix includes the terms of flexibility of the soil that will be occupied by the first foundation (medium 1), the soil that will be occupied by the second foundation (medium 2), and the terms of the coupling between the two mediums, which can be written as follows:

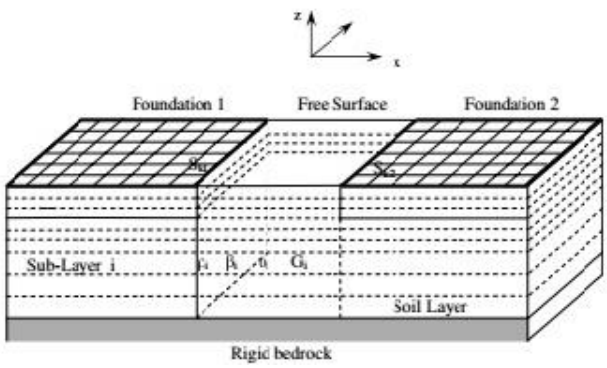


Fig. 2 Horizontal and vertical discretization

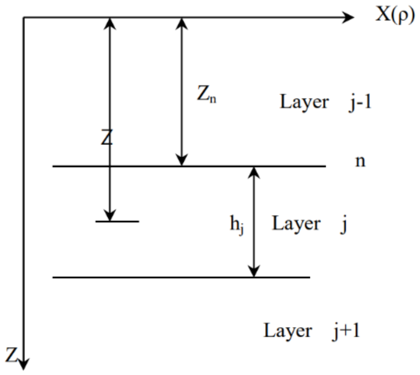


Fig. 3 Geometry of layered stratum

$$[F_t]=\begin{bmatrix} F_1 & F_{12} \\ F_{21} & F_2 \end{bmatrix} \tag{6}$$

where F_1, ($3N_1x3N_1$) is the flexibility matrix of medium 1, and F_2, ($3N_2$ X $3N_2$) is the flexibility matrix of medium 2. F_{12}, ($3N_1$ X $3N_2$) is the flexibility matrix of coupling of medium 1 on medium 2. F_{21}, ($3N_2$ X $3N_1$) is the flexibility matrix of coupling of medium 2 on medium 1. N_1 and N_2 are the number of elements discretizing medium 1 and medium 2, respectively.

The displacements in the two mediums are expressed then by:

$$\{u\} = [F_t]\cdot\{t\} \tag{7}$$

where the vectors {u} and {t} denote the nodal values of displacements and tractions, respectively, at the interfaces. When the two foundations are in place, they impose their displacements on the various sections that will be constrained to move like a rigid body. For all of the elements of the model, we can write the following relations:

$$\{u\} = [R]\cdot\{D\} \tag{8}$$

With the global (6x1) vector $\{D\}=\{U_x,U_y,U_z,\varphi_x,\varphi_y,\varphi_z\}^T$ represents displacements and rotations at the center of each foundation, and $[R]=[R_1,R_2,\ldots,R_k,\ldots,R_N]^T$ is a matrix of transformation of dimension (3N x 6), depending only upon the geometrical characteristics of the discretized volume of the soil of the first foundation and second foundation, where the under matrix is given by:

$$[R]_k=\begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}_k \tag{9}$$

in which x_k, y_k, and z_k are the coordinates of the element k compared with the centre of the foundations.

The resultant forces and moments $\{P\}=\{P_x,P_y,P_z,M_x,M_y,M_z\}^T$ acting at the centre of each foundation, as shown in Fig. 1, can be computed from equilibrium at the interface and expressed as:

$$\{P\}=[R]^T\cdot\{t\} \tag{10}$$

The relation binding the vector directly charges the external $\{P_1\}$ applied to the centre of gravity of the first foundation, and the vector displacements $\{D_1\}$and$\{D_2\}$can be expressed starting from the relations (7), (8), (9) and (10) by:

$$\{P_1\}=[K_1]\cdot\{D_1\}+[K_{12}]\cdot\{D_2\} \tag{11}$$

where

$$[K_1]=[R_1]^T\cdot[A]^{-1}\cdot[R_1] \tag{12}$$

is the dynamic stiffness matrix of the first foundation, with

$$[A]=[F_1]-[F_{12}]\cdot[F_2]^{-1}\cdot[F_{21}] \tag{13}$$

and

$$[K_{12}]=-[R_1]^T\cdot[A]^{-1}\cdot[F_{12}]\cdot[F_2]^{-1}\cdot[R_2] \tag{14}$$

is the coupling matrix of the first foundation on the second foundation. In the same manner, we can obtain the relation binding the external vector charges applied to the centre of gravity of the second foundation to the vector displacements starting from the relations (8), (9), (10) and (11) by:

$$\{P_2\}=[K_2]\cdot\{D_2\}+[K_{21}]\cdot\{D_1\} \tag{15}$$

where

$$[K_2]=[R_2]^T\cdot[F_2]^{-1}\cdot[M]\cdot[R_2] \tag{16}$$

is the dynamic stiffness matrix of the second foundation, with

$$[M]=[I]+[F_{21}]\cdot[A]^{-1}\cdot[F_{12}][F_2]^{-1} \tag{17}$$

and

$$[K_{21}]=-[R_2]^T\cdot[F_2]^{-1}\cdot[F_{21}]\cdot[A]^{-1}\cdot[R_1] \tag{18}$$

is the coupling matrix of the second foundation on the first foundation.

If the second foundation is unloaded ($P_2 = 0$), Eqs. (11)-(15) become:

$$\{P_1\}=[K_1]\cdot\{D_1\}+[K_{12}]\cdot\{D_2\} \tag{19}$$

$$\{0\}=[K_2]\cdot\{D_2\}+[K_{21}]\cdot\{D_1\} \tag{20}$$

From this system, we can write:

$$\{D_1\}=[C_{11}]\cdot\{P_1\} \tag{21}$$

$$\{D_2\}=[C_{12}]\cdot\{P_1\} \tag{22}$$

where

$$[C_{11}]=\left[[K_1]-[K_{12}][K_2]^{-1}\cdot[K_{21}]\right]^{-1} \tag{23}$$

is the compliance matrix of the loaded foundation (foundation 1), and

$$[C_{12}] = -[K_2]^{-1} \cdot [K_{21}][C_{11}] \tag{24}$$

is the compliance matrix of the unloaded foundation (foundation 2).

Using the equations of dynamic equilibrium Eqs. (21) and (22), the dimensionless dynamic amplitudes of displacement of the two foundations are expressed as follows:

$$\left|\frac{\{D_1\}}{\{P_1\}}\right| = \|[C_{11}]\tag{25}$$

$$\left|\frac{\{D_2\}}{\{P_1\}}\right| = |[C_{12}]| \tag{26}$$

where

$$\|[C_{11}] = \left\|[[K_1]\ -[K_{12}]\cdot[K_2]^{-1}\cdot[K_{21}]]^{-1}\right| \tag{27}$$

$$|[C_{12}] = |-[K_2]^{-1}\cdot[K_{21}]\cdot[C_{11}] \tag{28}$$

$|\ .\ |$ corresponds to the modulus operator.

However, to find an analytic formulations which connects the dimensionless displacements amplitude and the various dimensionless parameters (H = h/b, D= d/b, E = e/b, M = m/ρ(2b)^3) independently of the dimensionless frequency a_o, we have first represent the maximum displacements amplitudes according to the variation of each parameter. Second, the regressions approaches, based on least square method, were used to propose polynomial formulations for all studied cases as follows:

$$Max\left|\frac{D_1}{\{P_1\}}\right| = Max|[C_{11}(f)]| \tag{29}$$

$$Max\left|\frac{D_2(f)}{\{P_1\}}\right| = Max|[C_{12}(f)]| \tag{30}$$

where *f = (H, D, E, M)* represented the various parameters.

5. Results and comparisons

The response of two adjacent square, rigid foundations resting on a viscoelastic stratum over rigid bedrock is considered. The geometry and discretization are shown in Fig. 1 and Fig. 2. Due to the space limitations, only the vertical compliances of the foundations are considered according to their different separations, the depth of the substratum, the mass of foundations, the dynamic properties of the soil, and frequency of excitation. On the other hand, the remaining modes will be studied only for the variation in depth of the substratum. Such an application represents a general study that enables the analysis of the influence of different parameters, which we will present in another article later.

5.1 Validation

The results of vertical compliances C_z of this work was

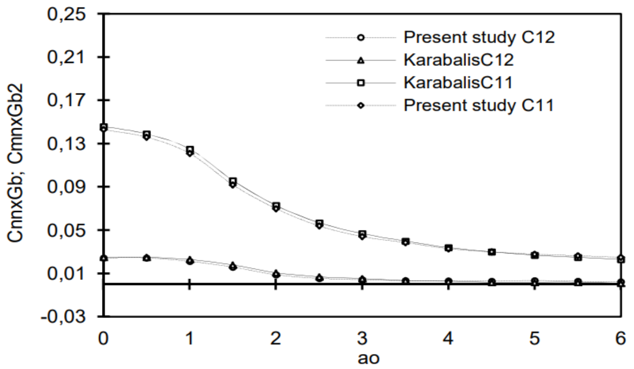


Fig. 4 Validation of vertical compliance of two adjacent square foundations on half-space C_{11} and C_{12} for varying

validated and compared with those obtained by the 3-D frequency domain BEM formulation of (e.g., Karabalis and Mohammadi (1998)). The comparison relates to the case of a square foundation placed at the surface of a viscoelastic and isotropic semi-infinite soil having the following characteristics: ρ = 1; G = 1; Cs = 1; ν = 0.333; and H/b = 16 (to approach the semi-infinite one), with b is the half-width of the foundations. The first foundation is loaded with the unit vertical force P_z = 1, and the second foundation is unloaded.

The soil is discretized horizontally into thirty six quadrilateral constant elements on the soil- foundation interfaces for the lower-frequency case. It is discretized into 64 quadrilateral constant elements on the soil-foundation interfaces for the higher-frequency case. For the vertical discretization, the depth of the substratum will be subdivided into 10 sublayers. The compliances are calculated at a relative distance d/b = 2 between the two foundations versus a different dimensionless frequency a_0 (= ωb/2C_s). It is noted that ω is the circular frequency and C_s is the shear (S)-wave velocities. The results thus presented in Fig. 4 are practically comparable, and the maximum errors are observed only in the static case.

5.2 Parametric analysis

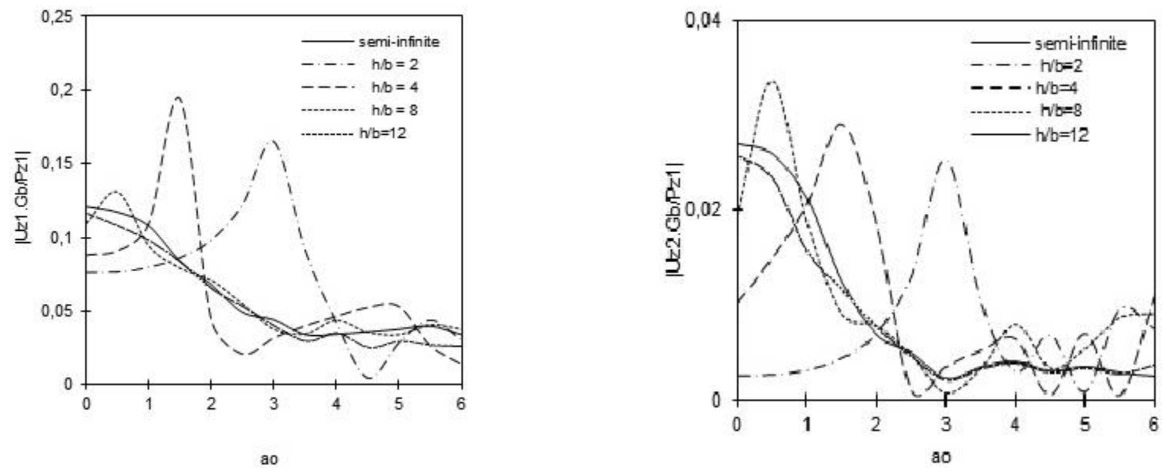
5.2.1 Vertical mode

In the application, the considered foundation is subjected to unit vertical force P_z = 1 for different dimensionless frequencies a_0.

The soil is discretized into thirtysix quadrilateral constant elements on the soil-foundation interfaces for lower frequency case, and it is discretized into 64 quadrilateral constant elements on the soil foundation interfaces for the high-frequency case.

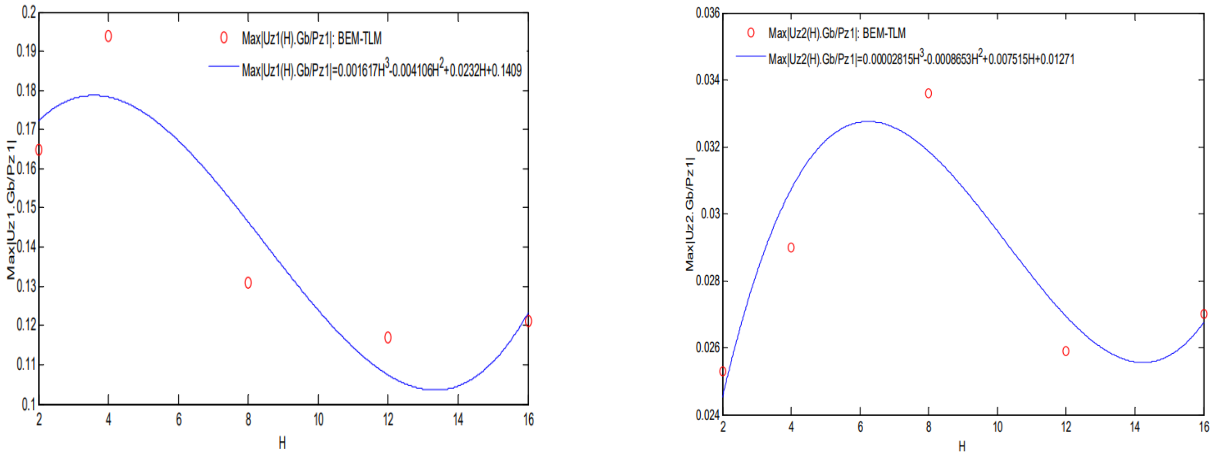
The soil is characterised by ρ = 1, G = 1, ν = 0.333, and β = 0.05. For this, the dimensionless amplitudes of vertical displacements of the loaded and unloaded foundation have been studied for different cases of the ratio of layer (h/b = 2, 4, 8, semi-infinite) versus dimensionless frequency a_o.

In Fig. 5, the effect of the soil layer depth versus frequency on the vertical displacement amplitude is examined while the foundations are massless and the distance ratio between foundations is d/b = 2. While varying



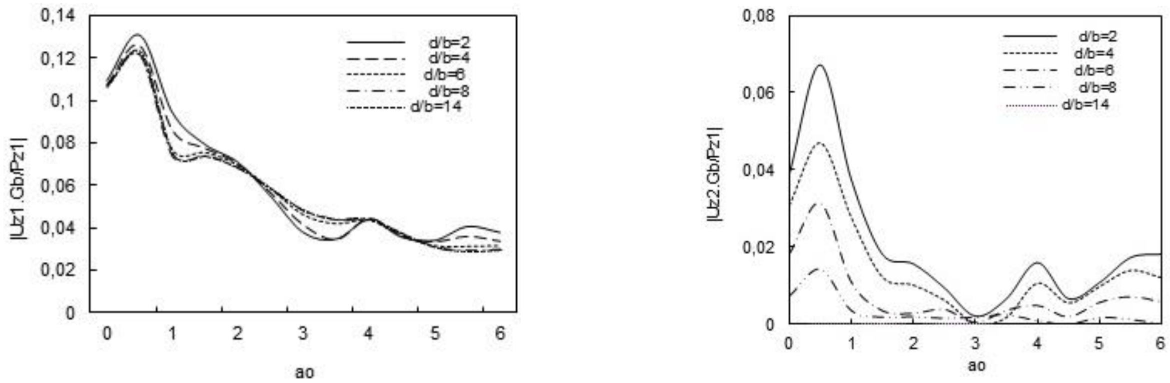
(a) (b)

Fig. 5 (a) Vertical displacement amplitudes versus frequency for varying depths of the stratum: (d/b = 2, β = 0.05)



(a) (b)

Fig. 6 Maxima of the vertical amplitudes versus layerdepths of the stratum (d/b = 2)



(a) (b)

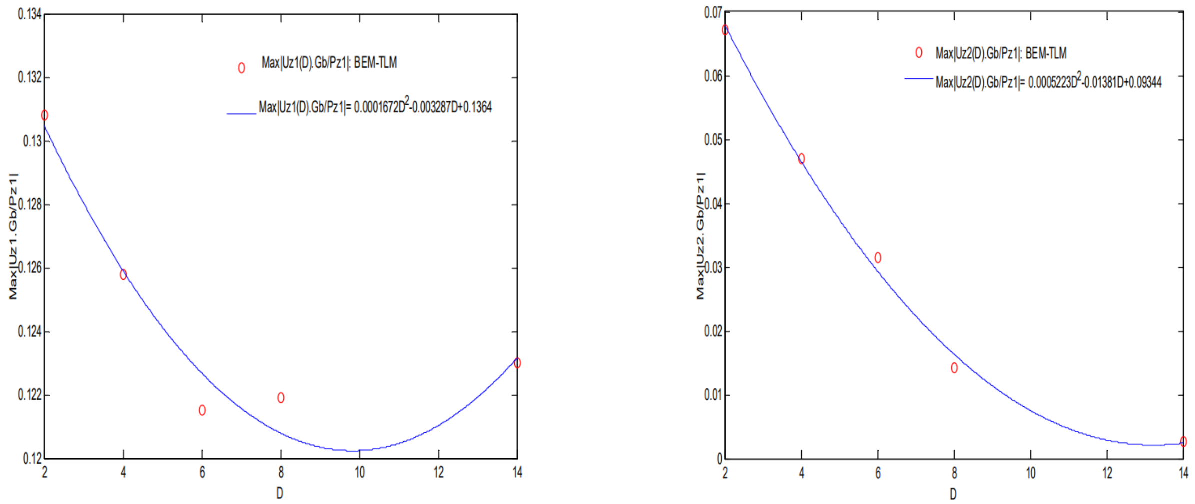
Fig. 7(a) Vertical displacement amplitudes versus frequency for varying distance between foundations (h/b = 8, β = 0.05) and (b) Vertical displacement amplitudes versus frequencyfor varying distance between foundations (h/b = 8, β = 0.05)

the depth of the substratum according to the frequency, it was noted:

• The static response increases when the layer depth increases.

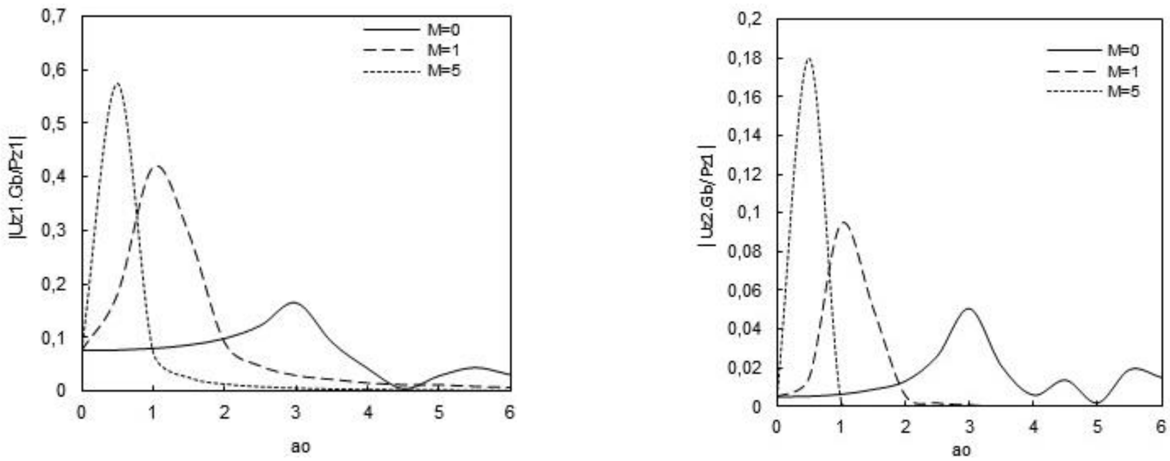
• The response of the foundation on the stratum approached the semi-infinite solution at layer depth increase (h/b ≥ 12).

• There has a remarkable shift in the resonant



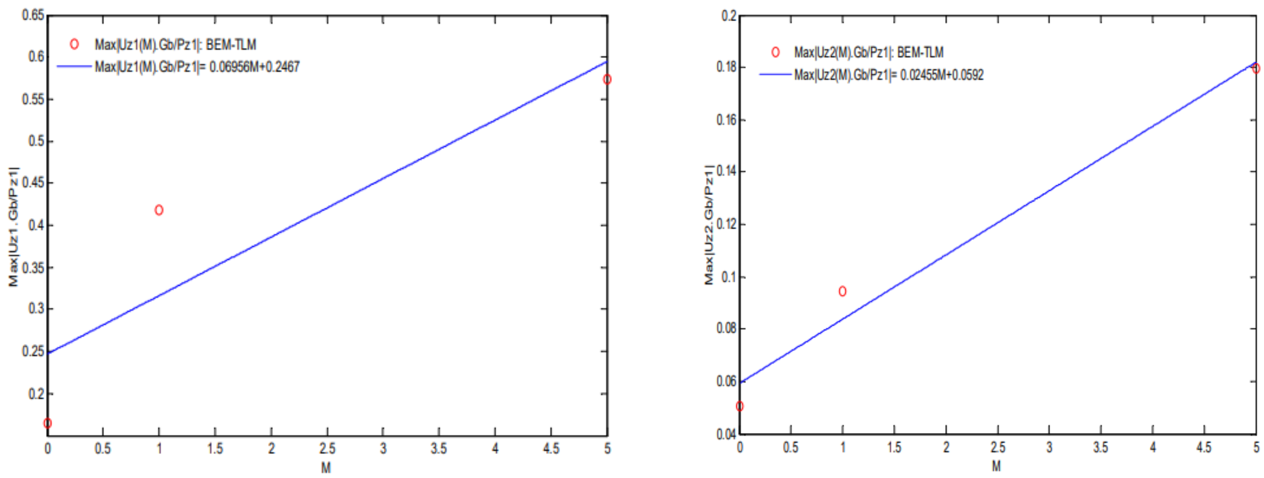
(a) (b)

Fig. 8 Maxima of the vertical amplitudes versus distance between foundations (H= h/b = 8)



(a) (b)

Fig. 9 Vertical displacement amplitudes versus frequency for varying mass of foundations (h/b = 2, β = 0.05)



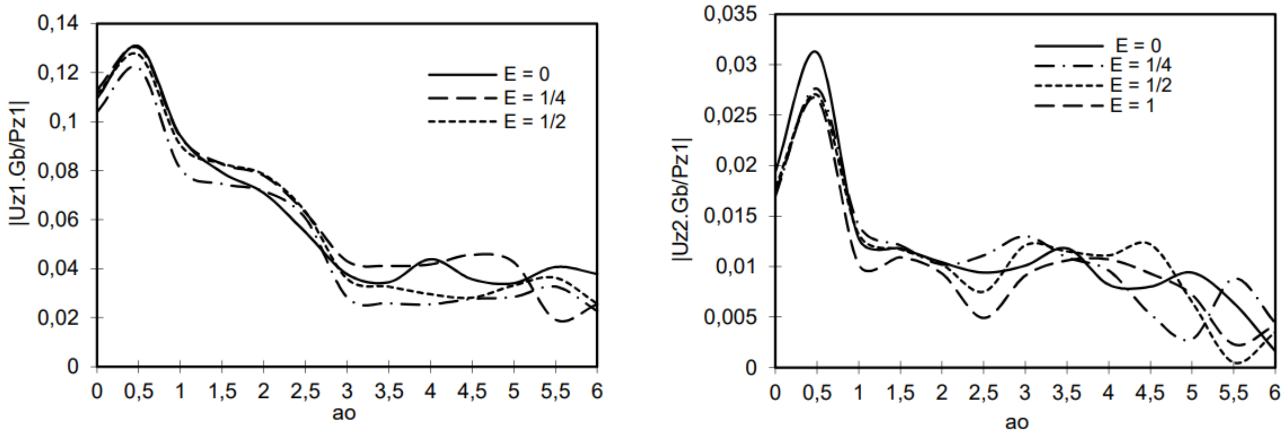
(a) (b)

Fig. 10 Maxima of the vertical amplitudes of displacements versus mass of the foundations (d/b = 2)

frequencies.

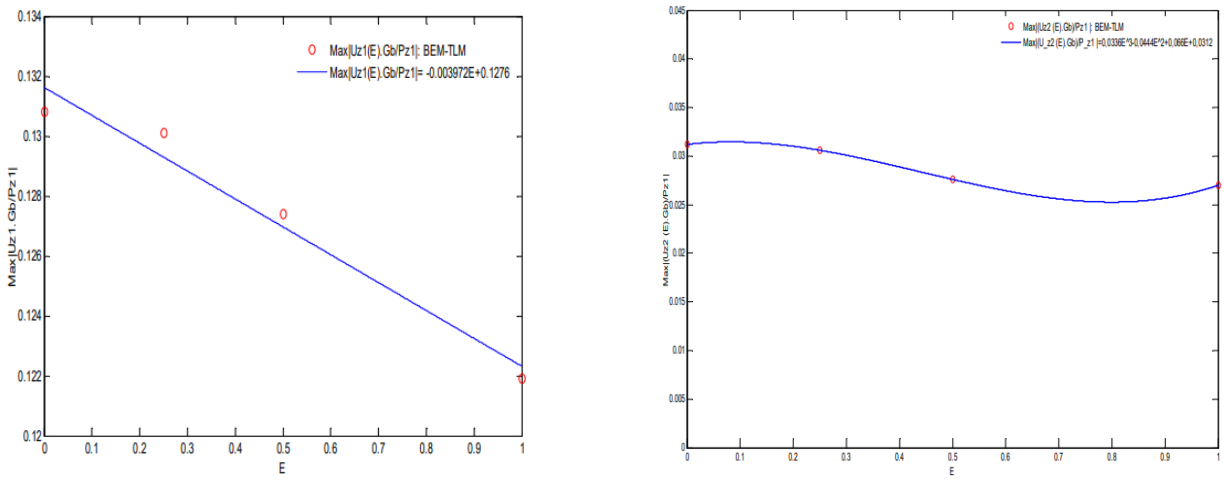
• There has a variation in the peaks of resonance.

• There has an important variation in the magnitude of the level of the resonant frequencies with smaller depths of



(a) (b)

Fig. 11 Vertical displacement amplitude versus frequency for varying embedding of foundations (d/b = 2, h/b = 8 and β = 0.05)



(a) (b)

Fig. 12 Maxima of the vertical amplitudes of displacement versus embedding of the foundations (d/b = 2)

the substratum.

The behaviour of the unloaded foundation is similar to that described above for the loaded foundation, the only difference being that the magnitude of the resonant peak increases with the layer depth increase.

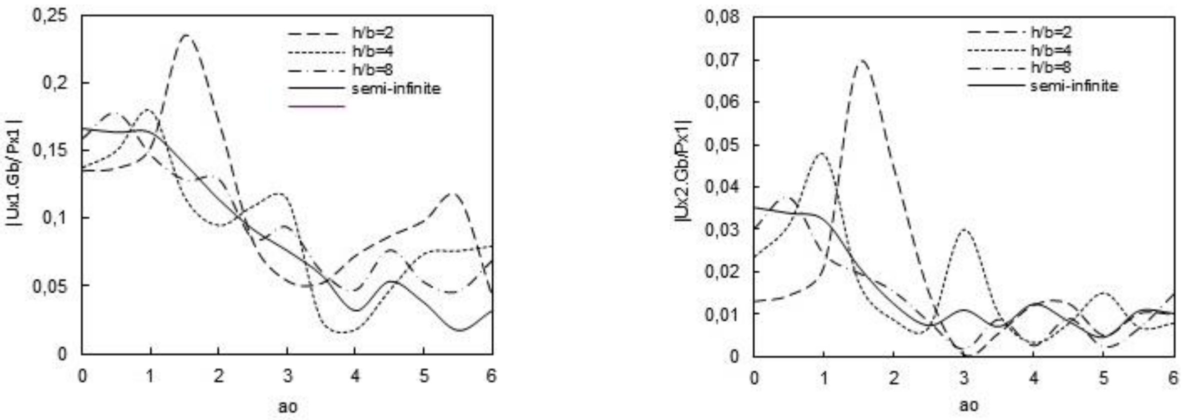
Fig. 6 shows the variation of soil depth ratio (H = h/b) versus maxima of the vertical amplitudes of displacements of the two adjacent foundations under non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two cubic regressions in using the curve fitting tool (CFT) of Matlab (2009). These two approximate formulas have been represented by the Eqs. (31) and (32):

$$Max\left|\frac{U_{z1}(H).Gb}{P_{z1}}\right| = 0.0001617H^3 - 0.004106H^2 + 0.02324\,H + 0.1409 \tag{31}$$

$$Max\left|\frac{U_{z2}(H).Gb}{P_{z1}}\right| = -0.00022815H^3 - 0.0008653H^2 + 0.007515H + 0.01271 \tag{32}$$

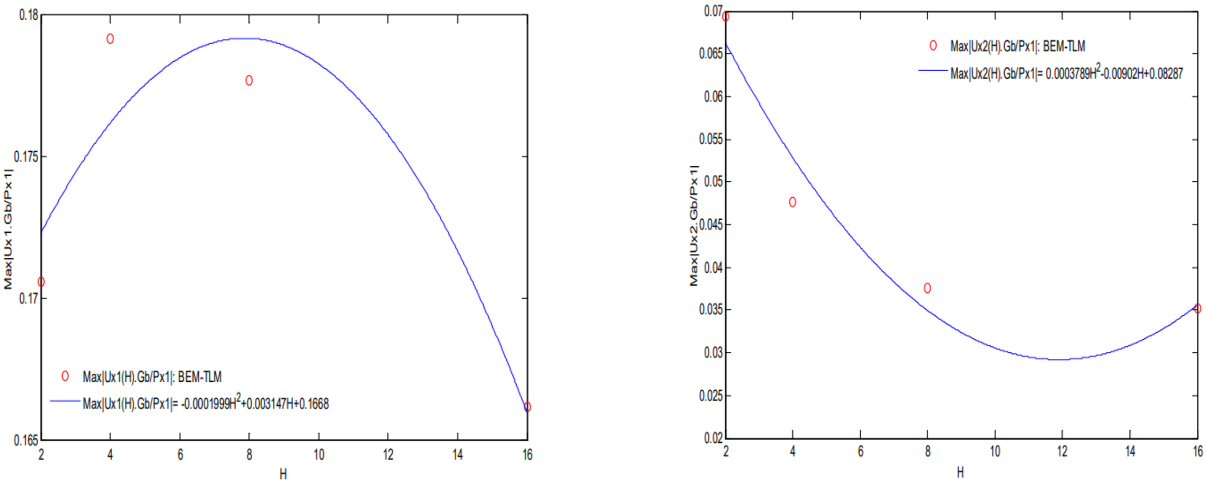
The goodness of fit statistics of both Eqs. (31) and (32) are respectively R^2 = 0.85471, SSE = 6.355e-4, RMSE = 0.02521 and R^2 = 0.8273, SSE = 7.761e-6, RMSE = 0.002786. It's noted that the adjusted R^2 statistic is generally the best indicator of the fit quality when his value is closer to one. Thus, the two proposed formulas (Eqs. 31-32) predict correctly the results obtained by (BEM TLM) method presented in this paper.

In Fig. 7, the influence of the distance between foundations versus frequency is examined when H/b = 8 and the damping level is kept constant at β = 0.05. It can be seen that the variation in the magnitude of the vertical displacement amplitude of the charged foundation is appreciably affected in the vicinity of the maximum values with a reduction in magnitude as the distance between the two foundations increases. However, the displacement amplitude is not significantly affected by the distance. With regard to the displacement amplitude of the non-charged foundation, the same remark applies, but with the magnitudes being much less important. In other words, the displacement amplitude of non-charged foundation is influenced much more by the distance. For the two cases, the displacements amplitude for d/b = 14 overlaps with the



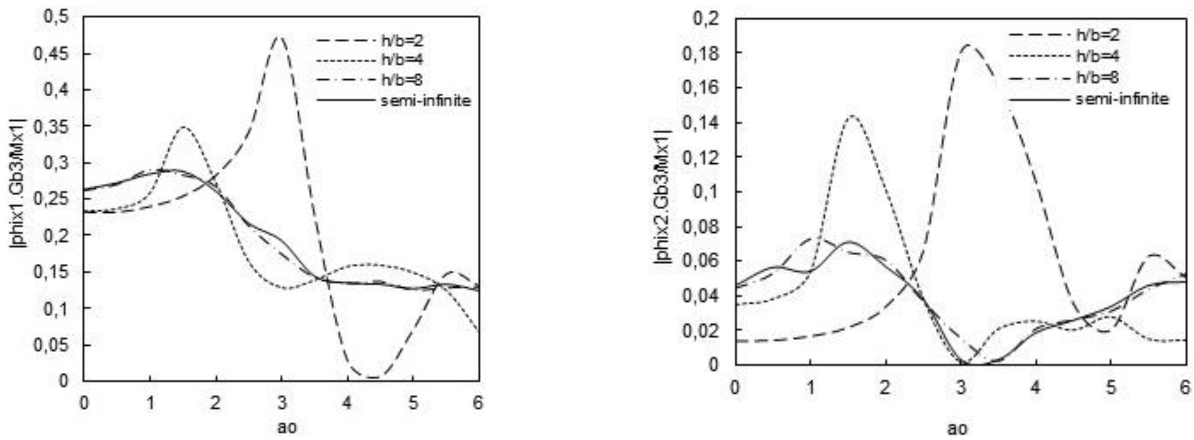
(a) (b)

Fig. 13 (a) Horizontal displacement amplitudes versus frequency for varying depths of the stratum (d/b = 2, β = 0.05) and (b) Horizontal displacement amplitude versus frequency for varying depths of the stratum (D = d/b = 2 and β = 0.05)



(a) (b)

Fig. 14 Maxima of the horizontal amplitudes versus layer depths of the stratum (D = d/b = 2)



(a) (b)

Fig. 15 Rocking rotations amplitudes versus frequency over varying depth of stratum (d/b =2 and β = 0.05)

one for a single-foundation case.

In Fig. 8 shows the variation of distance ratio between foundations (D= d/b) versus maxima of vertical amplitudes of displacements of the two adjacent foundations under non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two quadratic regressions in using the same tool (2009). These approximate formulas have been represented by the

Eqs. (33)-(34).

The goodness of fit statistics of both Eqs. (33)-(34) are respectively R^2 = 0.9534, SSE = 2.764e-06, RMSE = 0.001176 and R^2 = 0.9965, SSE = 9.174e-06, RMSE = 0.002142. Thus, the two proposed formulas (Eqs. 33-34) predict correctly the results obtained by (BEM TLM) method presented in this paper..

$$Max\left|\frac{U_{z1}(D).Gb}{P_{z1}}\right| = -0.00001672D^2 - 0.00387D + 0.1364 \quad (33)$$

$$Max\left|\frac{U_{z2}(D).Gb}{P_{z1}}\right| = 0.0005223D^2 - 0.01381D + 0.09344 \quad (34)$$

In Fig. 9, the influence of the mass of the two foundations versus frequency is examined when H/b = 2, d/b = 2, and β = 0.05. The mass ratio is defined as:

$$M_i = \frac{m_i}{\rho(2b)^3} \qquad i = (1,2) \quad (35)$$

where ρ is the mass density of the soil, and m_i is the mass of the foundations. The dynamic stiffness matrix becomes:

$$[K]_z - \omega^2[M] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}_v - \omega^2\begin{bmatrix} M_{11} & \\ & M_{22} \end{bmatrix} \quad (36)$$

The dynamic compliance matrix can be written as:

$$[C]_z = \left[[K]_z - \omega^2[M]\right]_z^{-1} \quad (37)$$

While varying (increasing) the mass ratio ($M_1 = M_2 = M$ = 0, 1, 5) according to the dimensionless frequency, we noted a remarkable shift in the resonant frequencies toward lower frequencies for displacements amplitude of charged foundation. With regard to the displacements amplitude of the non-charged foundation, the same remark applies, but with the magnitude being much less important.

In Fig.10 shows the influence of the dimensionless mass of the two foundations on the maximum vertical displacements amplitudes of the two adjacent foundations in the non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two linear regressions in using the same tool (2009). These two approximate formulas have been represented by the Eqs. (38) and (39):

$$Max\left|\frac{U_{z1}(M).Gb}{P_{z1}}\right| = 0{,}06956M + 0{,}2467 \quad (38)$$

$$Max\left|\frac{U_{z2}(M).Gb}{P_{z1}}\right| = 0{,}02455M + 0{,}0592 \quad (39)$$

The goodness of fit statistics of both Eqs. (38) and (39) are respectively R^2 = 0.7945, SSE = 0.01752, RMSE = 0.1324 and R^2 = 0.9775, SSE = 1.9415e-04, RMSE = 0.0139. Thus, the two proposed formulas Eqs. (38) and (39) predict correctly the results obtained by (BEM TLM) method presented in this paper. In Fig. 11, the influence of the variation of the embedding of the two foundations (*E/b* = 0, 0.25, 0.5, 1) according to the dimensionless frequency is examined for d/b = 2, H/b = 8 and ν = 0.333. While varying the embedding according to the frequency, it was noted:

• More the embedding increases more the displacement amplitude of the charged foundation decrease especially at the resonance frequencies.

• The static response increases when the embedding of the foundations increases.

• The inverse phenomenon is noted for the displacements amplitude of the second foundation but with less important magnitudes compared to the static response.

• The magnitudes of the displacements amplitude of the non-charged foundations are less important compared to the displacements amplitudes of the charged foundation.

• There has no shift of the resonance frequency.

In Fig. 12 shows the influence of the dimensionless embedding of the two foundations ($E_1 = E_2 = E$) on the maximum vertical displacement amplitude of the two adjacent foundations in the non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two linear regressions in using the same tool (CFT). These two approximate formulas have been represented by the Eqs. (40) and (41):

$$Max\left|\frac{U_{z1}(E).Gb}{P_{z1}}\right| = -0{,}003972E + 0{,}1276 \quad (40)$$

$$Max\left|\frac{U_{z2}(E).Gb}{P_{z1}}\right| = 0{,}0336E^3 - 0{,}0444E^2 + 0{,}066E + 0{,}0312 \quad (41)$$

The goodness of fit statistics of both Eqs. (40) and (41) are respectively R^2 = 0.9657, SSE = 1.682e-06, RMSE = 0.000917 and R^2 = 1, SSE = 2.648e-034, RMSE = 0.

Thus, the two proposed formulas (Eqs. 40-41) predict correctly the results obtained by (BEM TLM) method presented in this paper.

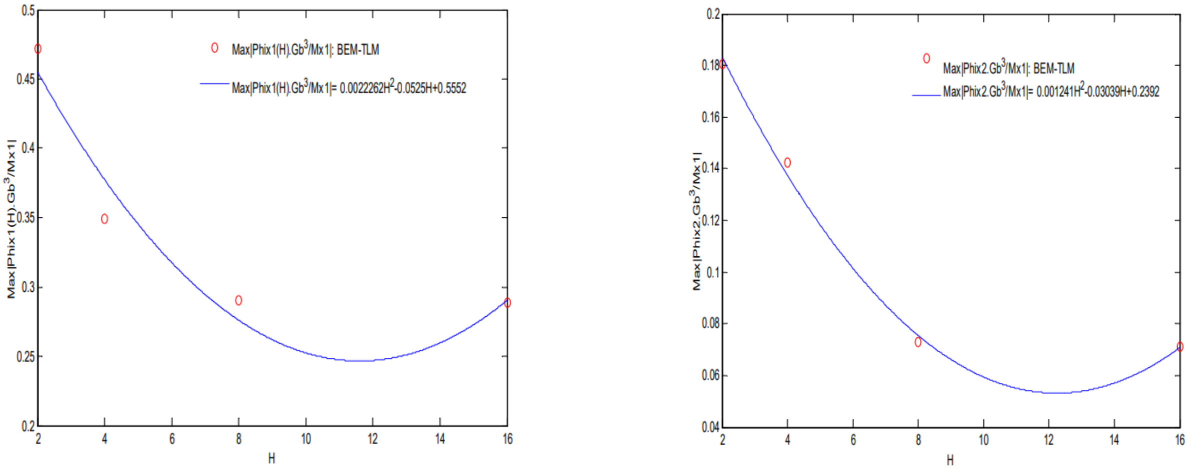
5.2.2 Horizontal mode

For the study of the other modes according to the depth of the substratum, Fig. 13 shows the horizontal displacements amplitudes of the two foundations. The same was established with the vertical mode, with more important values relating to non-charged foundation. That means that the non-charged foundation is more affected by interaction phenomena for the horizontal mode than for the vertical mode.

In Fig. 14 shows the influence of the dimensionless soil layer depth (H) on the maxima of horizontal displacements amplitudes of the two adjacent foundations in the non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two quadratic regressions in using the same tool (2009). These two approximate formulas have been represented by the Eqs. (42) and (43):

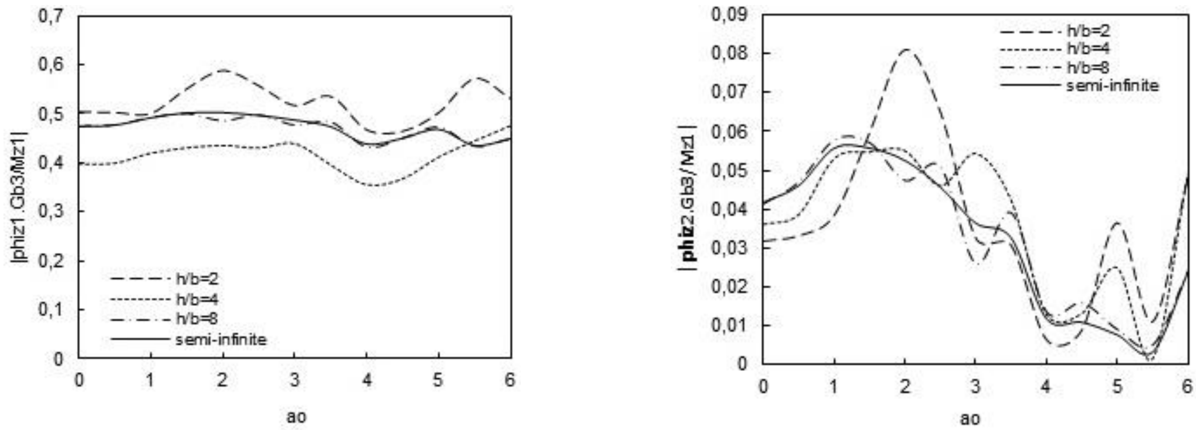
$$Max\left|\frac{U_{x1}(D).Gb}{P_{x1}}\right| = -0.0001999H^2 + 0.003147H + 0.1668 \quad (42)$$

$$Max\left|\frac{U_{x2}(D).Gb}{P_{x1}}\right| = 0.0003789H^2 - 0.00902H + 0.08287 \quad (43)$$



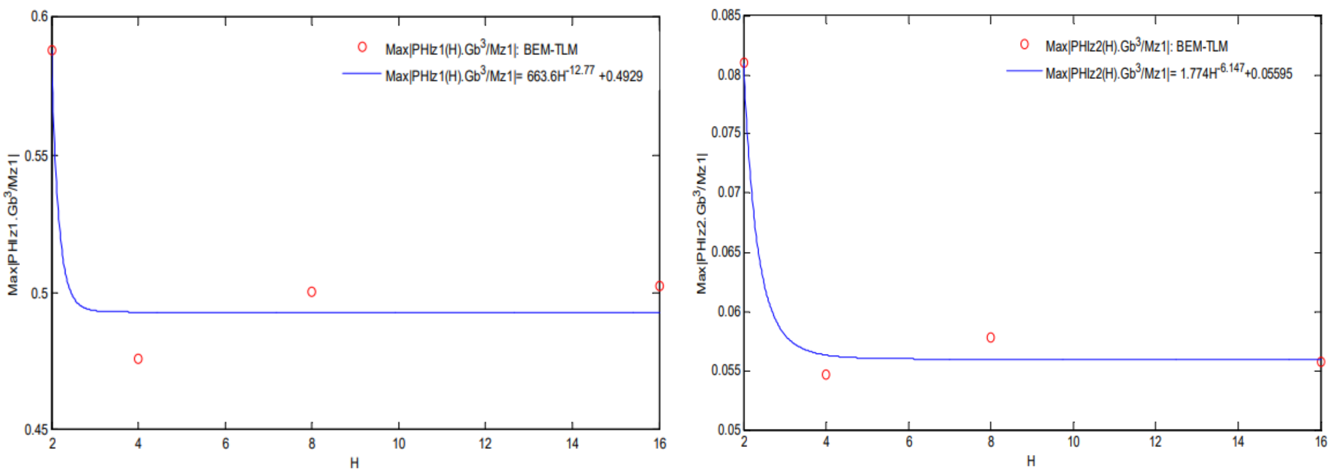
(a) (b)

Fig. 16 (a) Maxima of the rocking amplitudes versus layerd depths of the stratum (d/b = 2) and (b) Maxima of the rocking amplitudes versus layer depths of the stratum (d/b = 2 and β = 0.05)



(a) (b)

Fig. 17 Torsional rotations amplitudes versus frequency for varying depths of stratum (d/b = 2 and β = 0.05)



(a) (b)

Fig. 18 (a) Maxima of the torsion amplitudes versus layer depths of the stratum (D = d/b = 2 and β = 0.05)

The goodness of fit statistics of both Eqs. (42) and (43) are respectively R^2 = 0.8732, SSE = 1.418e-05, RMSE = 0.003766 and R^2 = 0.942, SSE = 4.211e-05, RMSE = 0.006489. Thus, the two proposed formulas (Eqs. (44) and 45) predict correctly the results obtained by (BEM TLM) method presented in this paper.

Table 1 Validation of the proposed formulas

Vertical mode	BEM-TLM Fig. (5)	Proposed Formulas Eqs. (31) and (32)	Error %
$\left\lvert\frac{U_{z1}(H).Gb}{P_{z1}}\right\rvert$	0.165	0.172	4.07
$\left\lvert\frac{U_{z2}(H).Gb}{P_{z1}}\right\rvert$	0.0253	0.024	5.14

5.2.3 Rocking mode

In Fig.15 shows the rocking rotations amplitudes. These are the same as the two modes of translation but with magnitudes being much less important. It can also be seen that starting from the depth of substratum h/b = 8 the of a semi-infinite soil.

In Fig. 16 shows the influence of the dimensionless soil layer depth (H) on the maximum rotation rotations amplitudes join those amplitude of the two adjacent foundations in the non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two quadratic regressions in using the same tool (CFT). These two approximate formulas have been represented by the Eqs. (44) and (45):

$$Max\left\lvert\frac{U_{\varphi 1}(D).Gb^3}{M_{x1}}\right\rvert = 0.002262H^2 - 0.0525H + 0.5512 \quad (44)$$

$$Max\left\lvert\frac{U_{\varphi 2}(D).Gb^3}{M_{x1}}\right\rvert = 0.001241H^2 - 0.03039H + 0.2392 \quad (45)$$

The goodness-of-fit statistics of both Eqs. (44)-(45) are respectively R^2 = 0.9416, SSE = 0.001294, RMSE = 0.0359 and R^2 = 0.9954, SSE = 3.982e-05, RMSE = 0.00631.We note a very good fit with the results obtained by our (BEM-TLM).

5.2.4 Torsion mode

Fig. 17 shows the torsion amplitudes. Again, the same remarks can be made as for the other modes, except that when the depth of the substratum is increased it note a slight variation of rotation amplitude of the charged foundation for both the low and high frequencies, which is not seen in the other modes (i.e., the peaks of resonance disappear). On the other hand, with respect to rotation amplitude of non-charged foundation, rotation amplitude of the peaks of resonances appears in the low frequencies and decrease quickly in the high frequencies, with a shift of the frequency of resonance as well.

In Fig. 18 shows the influence of the dimensionless soil layer depth (H) on the maximum torsion amplitudes of the two adjacent foundations in the non-dimensional form. For finding approximate analytic formulas of this variation, we have selected two power series regressions in using the same tool (CFT). These two approximate formulas have been represented by the Eqs. (46) and (47):

$$Max\left\lvert\frac{U_{z1}(D).Gb^3}{M_{z1}}\right\rvert = 0.663H^{-12.77} + 0.4928 \quad (46)$$

$$Max\left\lvert\frac{U_{z2}(D).Gb^3}{M_{z1}}\right\rvert = 1.774H^{-6.147} + 0.05595 \quad (47)$$

The goodness-of-fit statistics of both Eqs. (46) and (47) are respectively R^2 = 0.9387, SSE = 0.0004428, RMSE = 0.02104 and R^2 = 0.9872, SSE = 6.037e-06, RMSE = 0.002457. Thus, the two proposed formulas (Eqs. (46) and (47)) predict correctly the results obtained by (BEM TLM) method presented in this paper.

5.3 Application example

This example illustrates the use of the new formulations to calculate the response of dynamic foundation-soil-foundation problem. We wish to know the movements of the two three-dimensional foundations loaded by vertical harmonic force. Only the case of one loaded foundation will be studied. In order to have an easier understanding, we will give the details of the numerical calculation for one frequency: f = 217.64Hz.

We have:

Soil:

$G_1 = 54.10^6$ N.m^{-2}, $G_2 = 54.10^8$ N.m^{-2}, ρ = 1850 Kg.m^{-3}, ν = 0.3, h = 1.5

Foundations:

$m_1 = m_2$ = 6243.75Kg, $B_1 = B_2 = 2b$ = 1.5 m, e = 0 m, d = 1.5 m

Force:

$\lvert F\rvert = 4.10^3 N$, $\omega = 2\pi f = 1367 rds^{-1}$

Dimensionless parameters :

$H = h/b = 2$, $D = d/b = 2$, $m/\rho(2b)^3 = 1$, $E = e/b = 0$, $G_1/G_2 = 0.01$, $a_0 = \omega b/2C_s = 3$

From the both results, we note a light difference not exceeded 5%. It shows the efficient and the easy of the proposed analytic formulas.

6. Conclusions

In this work, the dynamic responses (displacements and rotations) of rigid foundations resting (embedded) on viscoelastic soil subjected to harmonic external force excitation has been calculated and fully tested. The solution

has been formulated by employing the frequency-domain boundary-element method (BEM) in conjunction with the Kausel-Peek Green's function for a layered stratum, along with a quadrilateral constant element determined using the thin-layer method (TLM). The study shows well the great importance of the wave-propagation problem in the vicinity of a machine foundations, which proves to be more complicated than the static case. On the basis of the results presented in this paper, the following conclusions can be stated:

• The proposed BEM-TLM formulation provides a very good tool for studying wave propagation and soil-structure interaction problems in multilayered soils.

• Parametric studies have been conducted to assess the effects of the various substratum depths, frequencies, masses ratios, embedded ratios on the two foundations response and to provide some design guidelines and formulations to the engineers.

• The problem of soil-structure interaction has been treated in a highly accurate and efficient way by calculating the displacements (rotations) of a two rigid foundations resting on a semi-infinite soil or soil layer. The case of embedded foundations has also been treated.

It is thus recommended to take into account all of these phenomena in the study of any structure placed in the vicinity of a machine foundation because the transmitted

waves represent a critical factor that influences the behaviour of the adjacent structure and the amount of damage that can be caused to it.

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