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Influence of spatial variability on unsaturated hydraulic properties

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Abstract. To investigate the effect of spatial variability on hydraulic properties of unsaturated soils, a numerical model is set up which can simulate seepage process in an unsaturated heterogeneous soil. The unsaturated heterogeneous soil is composed of matrix sand embedded with a small proportion of clay for simulating the heterogeneity. Soil-water characteristic curve and unsaturated hydraulic conductivity curve of the unsaturated soil are expressed by Van Genuchten model. Hydraulic parameters of the matrix sand are considered as random fields. Different autocorrelation lengths (ACLs) of hydraulic parameter of the matrix sand and different proportions of clay are assumed to investigate the influence of spatial variability on the equivalent hydraulic properties of the heterogeneous soil. Four model sizes are used in the numerical experiments to investigate the influence of scale effects and to determine the sizes of representative volume element (RVE) in the numerical simulations. Through a number of Monte Carlo simulations of unsaturated seepage analysis, the means and the coefficients of variations (COVs) of the equivalent hydraulic parameters of the heterogeneous soil are calculated. Simulations show that the ACL and model size has little influence on the means of the equivalent hydraulic parameters, but they have a large influence on the COVs of the equivalent hydraulic parameters. The size of an RVE is mainly affected by the ACL and the proportion of heterogeneity. The influence of spatial variability on the hydraulic parameters of the heterogeneous unsaturated soil can be used as a guidance for geotechnical reliability analysis and design related to unsaturated soils.

Keywords: spatial variability; autocorrelation; random field; hydraulic parameters; van Genuchten model

1. Introduction

Spatial variability of soil properties is inherent, which means that soil parameters vary stochastically in space. Modeling of soil's spatial variability is the basis of reliability analysis in geotechnical engineering, such as evaluating the failure probability of a slope or performing a stochastic analysis of foundation displacements (Lombardi *et al.* 2017, Wang *et al.* 2018).

Many geotechnical structures are built in unsaturated soils. For an unsaturated soil, soil water characteristic curve (SWCC) is a very important curve. Another important curve for an unsaturated soil is unsaturated hydraulic conductivity curve (UHCC). Several properties of unsaturated soils, such as shear strength (Fredlund *et al.* 1995, Vanapalli *et al.* 1996) and deformation characteristics (Pereiral and Fredlund 2000), can be predicted based on these two types of curves. Some nonlinear functions, such as the van Genuchten model (van Genuchten 1980, Vogel *et al*. 2001) and the Fredlund-Xing model (Fredlund and Gitirana 2011, Fredlund and Xing 1994, Fredlund *et al*. 2011), are widely used to express the SWCC and UHCC of unsaturated soils. The fitting parameters of SWCC and UHCC models can be seemed as hydraulic parameters of unsaturated soils.

To study the effect of spatial variability of hydraulic properties on seepage, deformation and stability of geotechnical engineering, some authors carried out numerical simulations by assuming different values of autocorrelation lengths of hydraulic parameters. Le *et al*. (2012) used a stochastic finite element method to model the seepage in a heterogeneous embankment. The soil porosity was chosen as a lognormally distributed random field, and two hydraulic parameters (i.e., saturated hydraulic conductivity (k_s) and a fitting parameter (a) of SWCC) were assumed to be directly correlated to the porosity. Cho (2012) performed a probabilistic analysis to an embankment seepage by considering the spatial variation of saturated hydraulic conductivity in a stratified soil. Zhu *et al*. (2013) explored the influence of spatial variability of saturated hydraulic conductivity on the variability of the pore-water pressures in a slope. In most analyses (Cho 2012, Zhu *et al*. 2013, Srivastava *et al*. 2010, Gui *et al*. 2000, Ahmed 2009), k_s was the only hydraulic parameter which was considered as lognormal random field. Although closed-form formulas for SWCC are widely used, research about the effect of heterogeneity on the fitting parameters of SWCCs is still very limited (Le *et al*. 2012, Tan *et al*. 2017).

Furthermore, because of the differences in depositional

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and weathering environments and mineral compositions, the heterogeneity of a soil is very complicated (Tan *et al.* 2019). A soil may be mainly composed of a matrix material and be embedded with another material. Chen *et al.* (2012a, b) examined the influences of the arrangements and amounts of local heterogeneities on the hydraulic properties of the subsurface porous media by numerical experiments. Their analysis method was deterministic, which means that the embedded local heterogeneities were scattered regularly in the porous media, and all the embedded local heterogeneities had one size and shape in one numerical simulation. In reality, the locations and sizes of the embedded local heterogeneities are scattered randomly in a matrix material. Therefore, a stochastic method should be used to investigate the influences of local heterogeneities on the hydraulic parameters of soils. Griffiths *et al.* (2012) and Paiboon *et al.* (2013) adopted a stochastic finite element method to evaluate the effect of porosity and pore size on the equivalent elastic stiffness of soils. They only carried out Linear elastic analysis for the geomaterials to examine the elastic properties of soils, but their method can be extended to investigate the influence of spatial variability on unsaturated hydraulic properties.

The goal of this study is to investigate the influence of spatial variability on the means and coefficients of variations (COVs) of hydraulic properties of unsaturated heterogeneous soils using a stochastic simulation method. The investigated heterogeneous soil is an unsaturated sand embedded with a proportion of clay. Through seepage analysis and curve fitting of SWCC and UHCC, values of the equivalent hydraulic parameters of the heterogeneous unsaturated soil can be computed, and the statistics of the equivalent hydraulic properties can be obtained based on Monte-Carlo simulation. The influence factors on the statistics of the hydraulic parameters of the heterogeneous soil and the size of representative volume element were analyzed and discussed in detail. The results of this study can be used as a guidance in geotechnical reliability analysis and design related to unsaturated soil.

2. Hydraulic parameters of unsaturated soils

Soil is a porous material consisting of soil skeletons and voids. When the voids of a soil are filled with both water and gas, the soil is unsaturated. Seepage in unsaturated soils is affected by many factors, (e.g., matrix suction, water content, saturation, permeability and particle size distribution). To analyze the unsaturated seepage in unsaturated soils, SWCCs and UHCCs are required. The SWCC and UHCC are two important hydraulic property curves of unsaturated soil (Fredlund and Gitirana 2011, Fredlund and Xing 1994, Heshmati and Motahari 2015, Mahmood *et al.* 2018, Phoon *et al.* 2010, Sillers and Fredlund 2001, Wu *et al.* 2012). The van Genuchten model (VG model) is widely used to depict the SWCC and UHCC of unsaturated soil (van Genuchten 1980, Vogel *et al.* 2001, Kim and Jeong 2017).

SWCC defines the relationship between the saturation (S_e) and matrix suction (hereinafter referred to as suction, ψ) of unsaturated soils. It has been extensively used to estimate unsaturated soil parameters, such as deformation parameters, shear strength parameters, and hydraulic parameters (Fredlund and Houston 2009). The closed-form expression for the VG model (Mualem 1976, van Genuchten 1980) is as follows:

$$S_e = \frac{\theta_w - \theta_r}{\theta_s - \theta_r} = \frac{1}{(1+(\psi/a)^n)^{1-1/n}} \tag{1}$$

where S_e is the degree of saturation; θ_w is the volumetric water content, θ_s is the saturated volumetric water content, and θ_r is the residual volumetric water content; a and n are the fitting parameters of VG model. (a is a suction value of the inflection point of the SWCC and its value is a little larger than the air entry value; n is related to the rate of change of the desaturation zone of the SWCC (Sillers and Fredlund 2001).

UHCC defines the hydraulic conductivity (k_w) of unsaturated soils as a function of the degree of saturation of unsaturated soils. The formula for the UHCC using VG model is (Mualem 1976, van Genuchten 1980):

$$k_w = k_s S_e^{0.5}[1-(1-S_e^{1/m})^m]^2 \tag{2}$$

where k_s is the saturated hydraulic conductivity, $m = 1-1/n$ (n is the fitting parameter of SWCC in Eq. (1)).

It can be seen from Eqs. (1) and (2) that the fitting parameters of SWCC and UHCC are a, n, and k_s. To consider the spatial variability of hydraulic properties of unsaturated soils, the three fitting parameters (a, n, and k_s) were generally considered as random fields with lognormal distribution (Tan *et al.* 2017).

3. Materials and methods

To study the effect of spatial variability on the hydraulic parameters of unsaturated heterogeneous soil, a number of numerical experiments were conducted by FLAC software (Itasca 2008). The influence of scale effect (different model sizes) and proportion of heterogeneity were also investigated.

3.1 Experimental setup

The modeled unsaturated soil in this paper contains predominantly sand embedded with 10%-20% clay to create heterogeneity. To model the heterogeneity realistically, the additive (clay) should be added into the matrix material (sand) randomly. In other word, the locations and sizes of a certain proportion of additives should be randomly distributed in the matrix material. This type of heterogeneous soil can be generated according to the concept of spatial variability and autocorrelation (Griffith *et al.* 2012). Through random discretization, the discrete values of soil properties can be designated to each element of a numerical model. Then, by comparing the discrete values of soil properties with a critical value, which is a function of the proportion of additive (n_p), the location and sizes of the additives can be determined.

After a heterogeneous soil was generated, numerical

Table 1 Statistics of hydraulic parameters of unsaturated soils for the van Genuchten model

Soil type	Statistics	a (kPa)	n	k_s (mm/d)	θ_s	θ_r
Sand	Mean value	0.7	2.68	7128	0.43	0.05
	COV	0.4	0.2	0.6	/	/
	ACL	1 / 10 / 100	1 / 10 / 100	1 / 10 / 100	/	/
Clay	Mean value	22.6	1.27	6	0.53	0.05

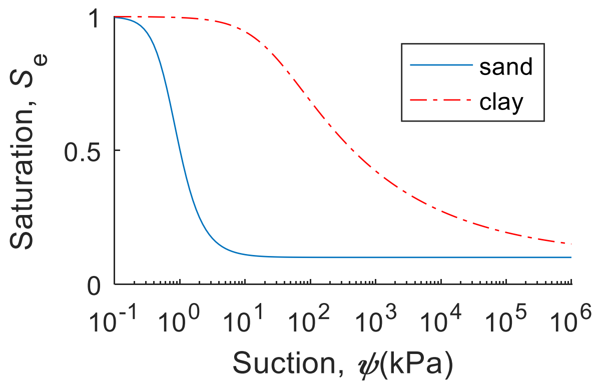


Fig. 1 SWCCs of the unsaturated sand and clay

experiments of unsaturated seepage analysis were conducted to simulate variations of suction (ψ), the degree of saturation (S_e), and the hydraulic conductivity (k_w) in the numerical model under different constant suction boundary conditions. Under each given boundary condition, the mean degree of saturation and the mean hydraulic conductivity of all elements can be calculated. The equivalent hydraulic parameters can be derived by curve fitting of the relationship of S_e–ψ and k_w–ψ for different boundary conditions.

3.1.1 Spatial variability

Vanmarke (1977) suggested that the spatial variability of soil property could be described by a random field. Some important characteristics of a random field are the mean (μ), coefficient of variation (COV or δ), autocorrelation function, and autocorrelation length (ACL or L). An autocorrelation function represents the decrease of autocorrelation with distance, and ACL is the distance within which soil parameters are strongly correlated. A widely used autocorrelation function is the exponential autocorrelation function which is expressed as follows (Ching and Phoon 2013, Cho 2012, Firouzianbandpey *et al.* 2014, Onyejekwe *et al.* 2016):

$$\rho_A(\tau_x, \tau_y) = \exp(-\frac{\tau_x}{L_h} - \frac{\tau_y}{L_v}) \tag{3}$$

where ρ_A is the autocorrelation coefficient of two spatial points; τ_x is the horizontal distance, τ_y is the vertical distance; L_h is the horizontal ACL, and L_v is the vertical ACL.

To investigate the influence of spatial variability on the means and COVs of hydraulic properties of unsaturated soils, the three hydraulic parameters (a, n, and k_s) of the matrix sand were considered as lognormally distributed random fields, and the locations of the addictive (clay) can be controlled by the ACL of the matrix sand. The mean values, COVs and ACLs of the three random fields (a, n, k_s) of sand and the mean values of parameters (a, n, k_s) of clay are shown in Table 1. The mean values of sand are the typical values of sand (Nemes *et al.* 2001), and the mean values of clay was obtained by laboratory tests of Hefei clay (Tan *et al.* 2014). Generally, the COV of a is larger than the COV of n but small than the COV of k_s (Botros *et al.* 2009, Dou *et al.* 2014). According to Tan *et al.* (2017), the COVs of a, n, and k_s of the sand are supposed to be 0.4, 0.2, and 0.6, respectively. The SWCCs for both the sand and clay are plotted in Fig. 1, in which the fitting parameters are the mean values listed in Table 1.

Without loss of generality, we assumed the ACLs of all the three random fields were the same, and the autocorrelation was isotropic (i.e., $L_h = L_v = L$) (Le *et al.* 2012, Srivastava *et al.* 2010, Zhu *et al.* 2013). Three values of ACLs (1, 10, and 100) were assumed, which represent minor, moderate and strong autocorrelation, respectively (Moradi *et al.* 2016). It should be mentioned that although the values of L_h and L_v were assumed to be the same, the method used in this paper can be easily used to the case of anisotropic autocorrelation.

3.1.2 Random discretization

A random field must be discretized into a number of random variables before it can be projected to a numerical model. Three popular types of discretization methods are point discretization method, average-type discretization method, and series expansion method (Ji *et al.* 2012, Sudret and Kiureghian 2002). The point discretization method, which is widely used for its simplicity (Ching and Phoon 2013, Cho 2007, Jiang *et al.* 2014, Papaioannou and Straub 2012), is adopted to discretize the three random fields of a, n, and k_s of the matrix material in this paper. After discretization of a random field using the midpoint method, the discrete value of a random field of each element is equal to the value at the middle of the corresponding element.

For the three lognormally distributed random fields of hydraulic parameters (a, n, and k_s), the main steps of the midpoint discretization method are summarized as follows. Interested readers are referred to Tan *et al.* (2017) for the detail steps and the flowchart of midpoint discretization method.

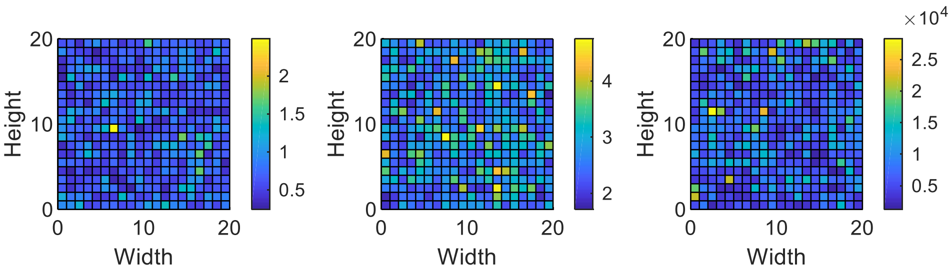
(1) Generate randomly a matrix of $\boldsymbol{U}$ with a size of $N_e \times 3$, where N_e is the number of discrete points in the numerical model. The elements of matrix U are uncorrelated standard normal variables.

(2) Computing the cross-correlation coefficient matrix $\boldsymbol{\rho}_C$ and the autocorrelation coefficient matrix $\boldsymbol{\rho}_{Ak}$ (k=1, 2, 3). The size of matrix $\boldsymbol{\rho}_C$ and $\boldsymbol{\rho}_{Ak}$ are 3×3 and $N_e \times N_e$, respectively.

(3) Perform Cholesky decomposition (Johari and Mehrabani 2016) and compute the lower triangular matrixes $\boldsymbol{L}_1$ and $\boldsymbol{L}_{2k}$ of $\boldsymbol{\rho}_C$ and $\boldsymbol{\rho}_{Ak}$, respectively (e.g., $\boldsymbol{L}_1\boldsymbol{L}_1^T = \boldsymbol{\rho}_C$, $\boldsymbol{L}_{2k}\boldsymbol{L}_{2k}^T = \boldsymbol{\rho}_{Ak}$).

(4) Compute the discretized random field $\boldsymbol{H}_k^0$ using the following equation:

$$\boldsymbol{H}_k^0(x, y) = \boldsymbol{L}_{2k}\boldsymbol{U}\boldsymbol{L}_1^T \tag{4}$$



(a) Random field *a* (b) Random field *n* (c) Random field k_s

Fig. 2 One realization of random fields of matrix material for $L = 1$

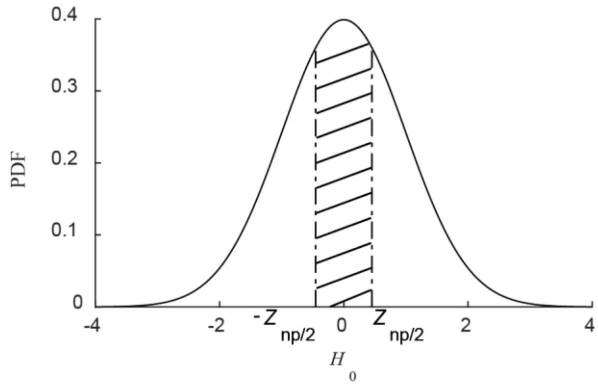
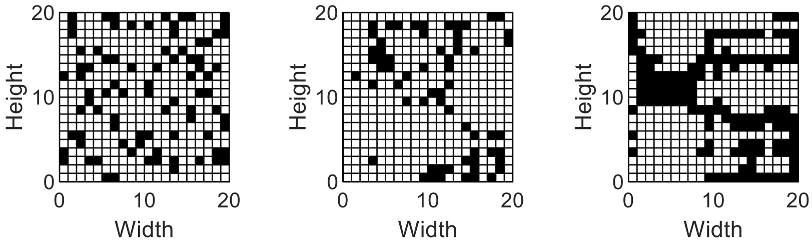


Fig. 3 Target proportion of heterogeneity (n_p) for random field



(a) $L = 1$ (b) $L = 10$ (c) $L = 100$

Fig. 4 Influence of ACL for heterogeneous soil with $n_p = 20\%$

where (x, y) is the spatial position of discrete points.

(5) Compute the discretized random field $\boldsymbol{H}_k$ considering the probabilistic distribution of each random field (k=1, 2, 3). For normally and lognormally distributed random fields, the discretized random field $\boldsymbol{H}_k$ can be computed using Eqs. (5) and (6), respectively.

$$\boldsymbol{H}_k(x,y) = \mu_{X_k} + \sigma_{X_k}\boldsymbol{H}_k^0(x,y) \tag{5}$$

$$\boldsymbol{H}_k(x,y) = \exp(\mu_{\ln X_k} + \sigma_{\ln X_k}\boldsymbol{H}_k^0(x,y)) \tag{6}$$

where $\mu_{\ln Xk}$ is the mean of the logarithm of variable X_k, and $\sigma_{\ln Xk}$ is the standard deviation of the logarithm of variable X_k.

Note that each random field has the same ACL, which corresponds to the same autocorrelation coefficient matrix $\boldsymbol{\rho}_{Ak}$ and the same lower triangular matrixes $\boldsymbol{L}_{2k}$, so each random field has the same discrete results of $\boldsymbol{H}_k^0$ (k=1, 2, 3) according to Eq. (4). However, the final discrete results of each random field ($\boldsymbol{H}_k$) are different because the values of $\boldsymbol{H}_k$ depend on the mean, standard deviation, and probabilistic distribution type of each random field according to Eqs. (5) and (6).

Due to the uncertainty of random variables, the realization of a random field is different in different simulations. Fig. 2 is a realization of random fields *a*, *n*, and k_s, respectively, of matrix sand with ACL = 1 for a square numerical model with a size of 20 × 20. The darker color in Fig. 2 means the smaller value, and the lighter color means the larger value. As shown in Fig. 2, the discrete random field properly explains the local average, which means that the discrete value on each element is the average value of that element.

3.1.3 Embedding of the heterogeneity

The additive (clay) is used to study the influence of heterogeneity on the hydraulic parameters of unsaturated

Fig. 5 Numerical models of different sizes for random field with L = 10 and n_p = 20%

soil. Once the discrete random field values of the matrix material (sand) have been assigned to the mesh, the embedding of a specific proportion of clay can be implemented by comparing the discrete values with a critical value (Z_{np}) (Griffiths *et al.* 2012). For the three random fields of a, n, and k_s, the locations and sizes of the additive should be the same. As analyzed in Section 3.1.2, the final discrete results of each random field ($\boldsymbol{H}_k$) depend on the statistics of each random field, but the middle discrete results $\boldsymbol{H}_k^0$ are the same for different random fields. Therefore, we compared the discrete values of $\boldsymbol{H}_k^0$ with the critical value of Z_{np} for embedding the heterogeneity to each random field.

The value of Z_{np}, which corresponds to a designated proportion of heterogeneity (n_p), can be computed according to the probabilistic distribution function of standard normal distribution. By assuming the area of the shaded part in Fig. 3 as n_p, Z_{np} can be obtained using Eq. (7):

$$Z_{np} = \Phi^{-1}[(1+n_p)/2] \tag{7}$$

where Φ^{-1} is the inverse function of the cumulative distribution function of standard normal variable Z.

After the value of Z_{np} is determined, any element with a discrete value in the range of $-Z_{np} \le \boldsymbol{H}_k^0 \le Z_{np}$ is treated as the heterogeneity (i.e., using clay to replace sand for these elements), and those elements whose random field values are in the range of $\boldsymbol{H}_k^0 > Z_{np}$ or $\boldsymbol{H}_k^0 < -Z_{np}$ remain unchanged. By this way, n_p percentage of clay can be embedded in the matrix sand.

To investigate the influence of the proportion and size of the additivities on the hydraulic parameters of the heterogeneous soil, two proportions of heterogeneity (n_p =10% and 20%) and three autocorrelation distances (ACL =1, 10, and 100) were considered in this study. Fig. 4 shows the influence of ACL for a soil with 20% heterogeneity. Because a small ACL indicates that soil property fluctuates greatly, while a large ACL means that soil property is highly correlated over a large spatial range (Stefanou 2008), the heterogeneities tend to be concentrated with the increase of the ACL. In other word, for the same percentage of heterogeneity, the additives are small and they scatter in the entire numerical model when the ACL is small; but the additives tend to gather together and become large when the ACL is large.

Table 2 Initial suctions for numerical experiments

No.	1	2	3	4	5	6	7	8	9	10	11	12
Suction / kPa	1E-6	0.1	0.5	1	2	3	4	5	10	15	20	30

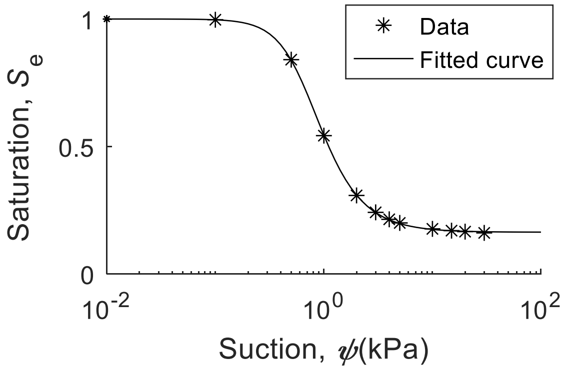
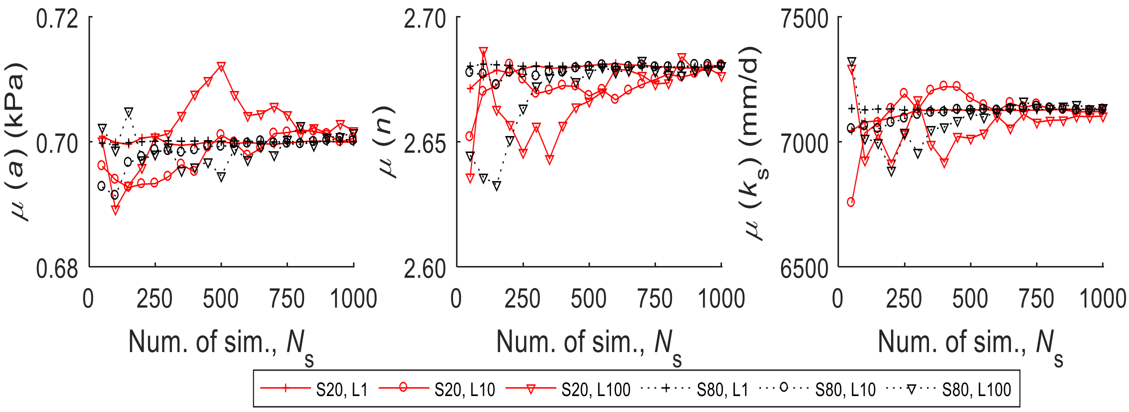


Fig. 6 Soil-water characteristic curve fitted by VG model

3.1.4 Seepage analysis

After a heterogeneous soil was generated, a two-dimensional finite difference model of steady-state flow was set up to simulate variations of suction (ψ), the degree of saturation (S_e), and the hydraulic conductivity (k_w) in the numerical model under different constant suction boundary conditions. Under each given boundary condition, the mean degree of saturation and the mean hydraulic conductivity of all elements can be calculated. The equivalent hydraulic parameters can be derived by curve fitting of the relationship of S_e–ψ and k_w–ψ for different boundary conditions. To study the scale effect of the numerical model, four sizes of square models (20 × 20, 40 × 40, 60 × 60, 80 × 80) were adopted (Fig. 5). The element size was 1 × 1 in each numerical model.

As shown in Fig. 5, the top and the bottom sides of each numerical model are under constant suction (negative water pressure) condition, and the left and right sides of each numerical model were impermeable. Twelve initial suctions were given for the numerical experiments (Table 2) (Chen *et al.* 2012, Wu *et al.* 2012). Through unsaturated seepage analysis, the saturation and the hydraulic conductivity of each element could be obtained, and then the mean value of saturation and the mean value of hydraulic conductivity of



(a) Random field a (b) Random field n (c) Random field k_s

Fig. 7 Variations of mean discrete values with number of simulations for random fields of matrix material

all elements under each specific initial suction condition could be calculated. The equivalent SWCC fitting parameters (a and n) of the heterogeneous soil could be obtained by fitting the relationship between the average saturation and the suction for the SWCC model (Eq. (1)) with nonlinear least square method. A typical SWCC is shown in Fig. 6, in which each data point corresponds to a mean suction and a mean degree of saturation of all elements for the horizontal and vertical coordinates, respectively. Meanwhile, the equivalent saturated hydraulic conductivity (k_s) of the heterogeneous soil could be obtained by fitting the relationship between the average hydraulic conductivity and the suction for the UHCC model (Eq. (2)).

3.2 Monte Carlo simulation

Due to the randomness of the discretization of random fields, each realization of heterogeneous soil is stochastic, which means that the hydraulic parameters obtained from one numerical simulation are also stochastic. To obtain the statistics of the equivalent hydraulic parameters, Monte Carlo simulation (MCS) should be used to repeat the analysis many times until the statistics of the output hydraulic parameters became acceptably reproducible. For each simulation, the discretization of three random fields (a, n, and k_s), the embedding of the heterogeneity, and the seepage analysis were involved as explained previously.

To perform the seepage analysis using the MCS, a MATLAB function was written for discretizing the three random fields of a, n, and k_s, and a FISH function was written using FLAC to run the MCS and to conduct the unsaturated seepage analysis for the numerical model. The discretized data of the three random fields generated by MATLAB were first saved into text files and then read into FLAC code by the FISH function.

4. Results and discussion

Three types of numerical experiments were carried out to investigate the influence of spatial variability, proportion of heterogeneity, and scale effect on the statistics of the equivalent hydraulic properties of the heterogeneous unsaturated soil. As mentioned in Section 3, three values of ACLs (L = 1, 10, and 100), two values of proportions of heterogeneity (n_p = 10% and 20%) and four values of scale effects (S = 20, 40, 60, and 80, where S is the side length of the square numerical model) were considered in the simulations.

4.1 Convergence analysis of Monte Carlo simulation

To obtain stable results of the statistics of the equivalent hydraulic parameters of the simulated heterogeneous unsaturated soil, a large number of MCSs must be conducted. On the other hand, too much simulations will increase the computing efforts. Consequently, the trade-off between accuracy and computational effort should be achieved through convergence analysis.

The relationship between the average discrete data and the number of simulations (N_s) for random fields a, n, and k_s of the matrix material is shown in Fig. 7, for L = 1, 10, and 100, S = 20 and 80 for the matrix material before adding the heterogeneity. Apparently, the curves representing S = 40 and S = 60 lie within those of S = 20 and S = 80, so they are not plotted in Fig. 7 for clarity. As shown in Fig. 7, horizontal axis indicates the number of simulations, and the vertical axis indicates the average value for the first i groups of the discrete data of all discretization points, where i=1, …, 1000.

Fig. 7 shows that the variation of the average discrete data of random fields (a, n, and k_s) decreases with the increase of number of simulations. The three black Lines (representing S = 80) become closer than the three red Lines (representing S = 20) with the increase of the number of simulations, which means that the convergence of the mean hydraulic parameters of larger model size is better than that of smaller model size. The reason is that more elements exist in a large numerical model, then the mean discrete data of more elements will vary weaker than that of fewer elements. One can also find that for the same model size, the variation of the mean discrete data of random fields a, n, or k_s is stronger for larger ACL. For example, the red Lines marked with trigonometric symbols (representing L = 100) vary greater than the red Lines marked with plus signs

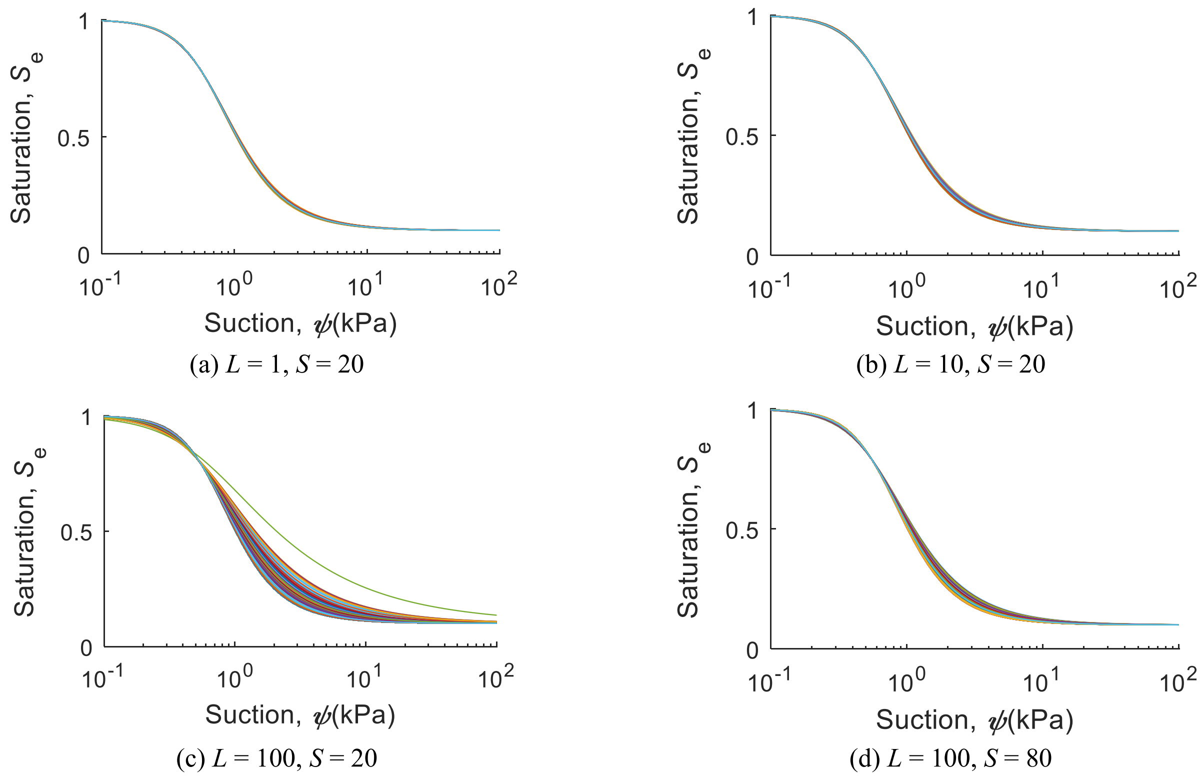


Fig. 8 Fitted SWCCs of the heterogeneous soil for $n_p = 20\%$

(representing $L = 1$). Because a large ACL means that soil property is highly correlated over a large spatial extent, the added heterogeneities tend to concentrate together to form a larger additive, which has been illustrated in Fig. 4. As shown in Fig. 4, the size of the additive increases with the ACL, but the locations of the additives will be more stochastic with the increase of the ACL. However, this will lead to a large variation of the mean discrete data of a random field for different simulations. Therefore, more numbers of simulations are required for larger value of ACL for the same model size.

As shown in Fig. 7, except for the case of $S = 20$ and $L = 100$, average discrete data of random fields (a, n, and k_s) of all the other cases become stable when the number of simulations is 1000. And for case of $S = 20$ and $L = 100$, the three curves in Fig. 7(a), 7(b), and 7(c) are nearly stable. Hence, 1000 MCSs are conducted for each case of numerical experiments in this paper.

4.2 Variation of fitted SWCCs of the heterogeneous soil

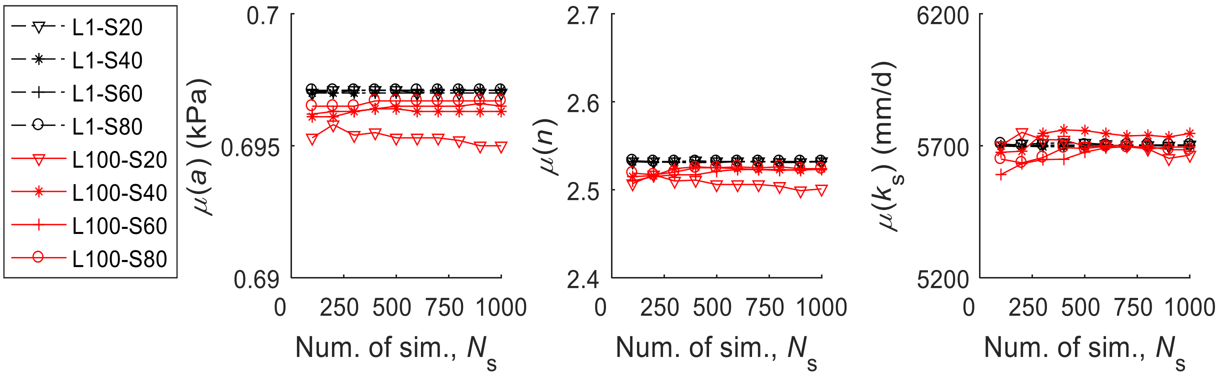
As mentioned in Section 3.1.2, after random fields of the matrix sand were discretized and the heterogeneity was embedded into the matrix, a numerical model could be set up, and twelve seepage analyses were carried out for twelve different suctions listed in Table 1. Then, the equivalent hydraulic parameters of a, n, and k_s of the assumed heterogeneous material could be obtained by curve fitting using Eqs. (1) and (2). One group of a, n, and k_s could be obtained through one MCS. By N_s times' simulations, N_s SWCCs and UHCCs could be fitted and N_s groups of a, n, and k_s could be obtained.

Take the SWCC for example, Fig. 8 shows the fitted SWCCs of unsaturated soil with a proportion of heterogeneity of $n_p = 20\%$. The model sizes of Figs. 8(a) − (c) are 20 × 20. It is evident that 1000 curves almost overlap for $L = 1$ and $S = 20$ (Fig. 8(a)), which means that the variations of the fitted hydraulic parameters (a and n) are very small. By comparison of Fig. 8(a), (b), and (c), one can find the variation range of SWCCs widens with the increase of ACL. This proves that a larger ACL leads to bigger variation of a random field. Fig. 8(d) also shows 1000 SWCCs for $L = 100$, but the model size is 80 × 80. The comparison of Fig. 8(c) and Fig. 8(d) indicates that the variation of the equivalent hydraulic parameters decreases with the increase of model size for the same ACL of soil property, which is same as the conclusion shown in Fig. 7(a). Similar conclusions can be found for other model sizes and proportions of heterogeneity, which are not shown here.

4.3 Influence factors of mean and COV of hydraulic parameters of heterogeneous soil

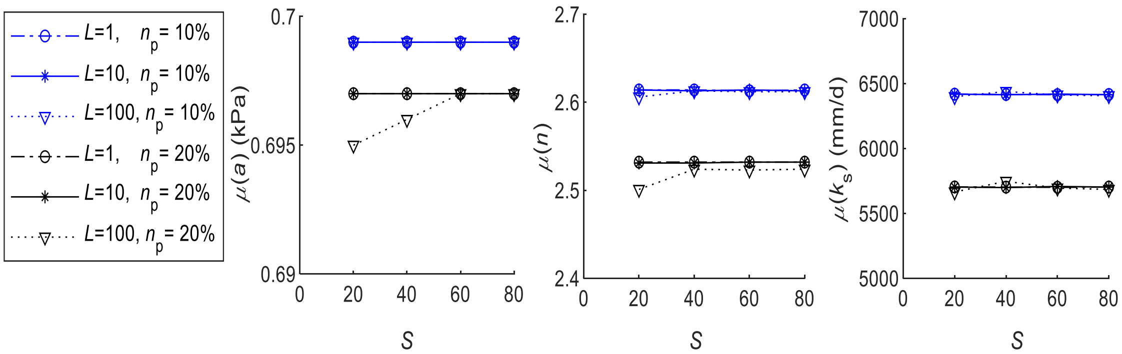
4.3.1 Mean of hydraulic parameters

Based on the results of MCS and curve fitting, the equivalent hydraulic parameters of the heterogeneous soil can be calculated for different cases. Variations of the means of the equivalent hydraulic parameters with the number of simulations are shown in Fig. 9 for a soil with 20% heterogeneity and $L = 1$ and 100. For the clarity of the figure, the curves representing $L = 10$ were not plotted in



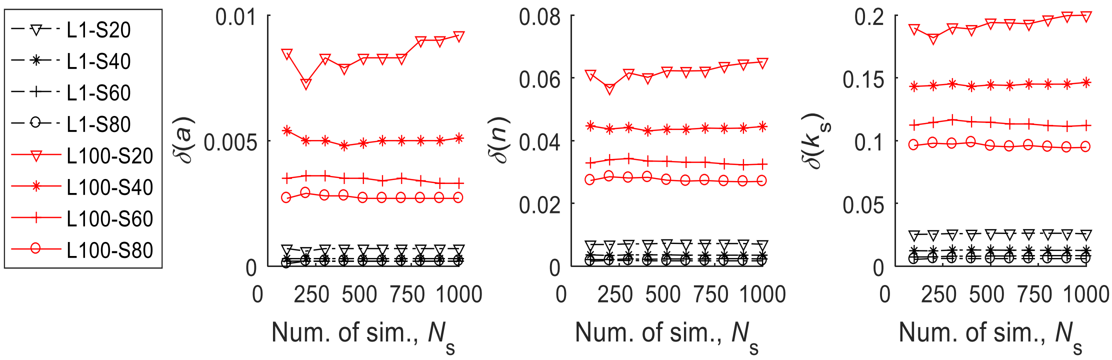
(a) Random field a (b) Random field n (c) Eandom field k_s

Fig. 9 Variations of mean equivalent hydraulic parameters with number of simulations for random fields of heterogeneous soil with $n_p = 20\%$



(a) Random field a (b) Random field n (c) Random field k_s

Fig. 10 Variation of mean equivalent hydraulic parameter with model size, ACL and proportion of heterogeneity for random fields

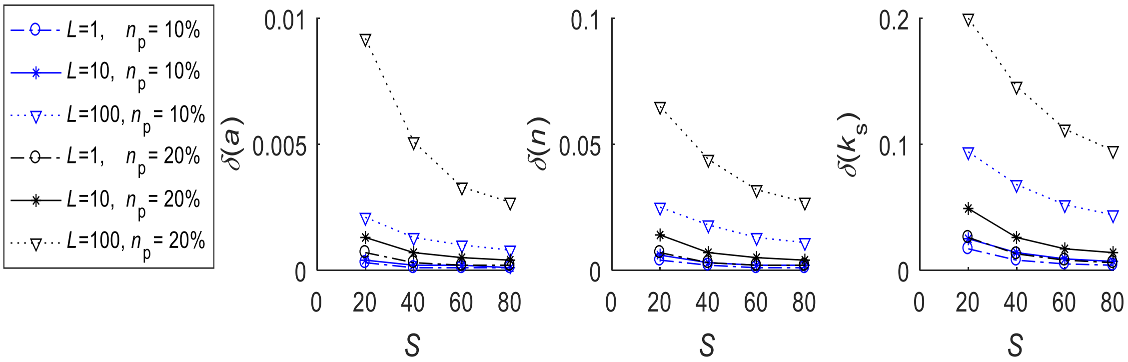


(a) Random field a (b) Random field n (c) Random field k_s

Fig. 11 Variations of COVs of equivalent hydraulic parameters with number of simulations for random fields of heterogeneous soil with np = 20%

this figure because these curves lie between those of $L = 1$ and $L = 100$. In Fig. 9, the horizontal axes indicate the number of simulations, and the vertical axis indicates the average value for the first i groups (i=1, …, 1000) of the means of parameter a, n, and k_s in Fig. 9(a), 9(b), and 9(c), respectively.

It is evident that when the ACL is small, the mean values of the equivalent hydraulic parameters converge quickly with the number of simulations. But when the ACL is large, variations of the mean values of the equivalent hydraulic parameters increase with the decrease of model size. All the curves except for the case of L =100 and S = 20 in Fig. 9 become stable when the number of simulations is larger than 500, and the curve of L =100 and S = 20 is nearly stable when the number of simulations is 1000. This proves again that 1000 simulations are capable of obtaining the statistics of the equivalent hydraulic parameters of the heterogeneous soil.



(a) Random field a (b) Random field n (c) Random field k_s

Fig. 12 Variation of COVs of equivalent hydraulic parameter with model size, ACL and proportion of heterogeneity for random fields

Fig. 10 shows the means of the equivalent hydraulic parameters (a, n, and k_s) of the heterogeneous soil based on 1000 simulations for each case of ACL, proportion of heterogeneity and model size. Most Lines in Fig. 10 are horizontal, and these stable Lines indicate that the equivalent hydraulic parameters of the heterogeneous soil are the same for all the four model sizes with the same proportion of heterogeneity. An apparent exception is the case of L = 100, n_p = 20%, and S= 20. This exception is consistent with those shown in Fig. 7 and Fig. 9. When the ACL and the proportion of heterogeneity are relatively large, a large model size is needed for obtaining a stable value of the mean of the equivalent hydraulic parameters of the heterogeneous soil

As a whole, the means of the equivalent hydraulic parameters are mainly influenced by the proportion of heterogeneity. The influence of the ACL and model size to the means of the equivalent hydraulic parameters of the heterogeneous soil is very small and can be neglected.

4.3.2 COV of hydraulic parameters

Similar to Fig. 9, variations of the COVs of the equivalent hydraulic parameters with the number of simulations are shown in Fig. 11 for a soil with 20% heterogeneity and L = 1 and 100. Fig. 11 shows that nearly all curves are horizontal except for the case of L =100 and S = 20. The horizontal curves indicate that the number of simulations does not influence the COVs of the equivalent hydraulic parameters of the heterogeneous soil. However, Fig. 11 indicates that model size influences the COVs of the equivalent hydraulic parameters. A larger model size will result in a smaller COV of the equivalent hydraulic parameter. If the model size is big enough, the COVs of the equivalent hydraulic parameters of the heterogeneous soil will decrease to a stable value. This phenomenon can be further investigated by Fig. 12.

Fig.12 shows the COVs of the equivalent hydraulic parameters (a, n, and k_s) of the heterogeneous soil based on 1000 simulations for each case of ACL, proportion of heterogeneity and model size. Contrast to Fig. 10, all curves in Fig. 12 decrease with the increase of model size. Fig. 12 also shows the influence of ACL and n_p to the equivalent COV. Generally speaking, larger model sizes are needed for larger ACL and larger n_p to obtain stable values of COVs of the equivalent hydraulic parameters. For example, as shown in Fig. 12(b), the minimum values of S for obtaining stable values of COVs of the equivalent hydraulic parameters for the curves representing L = 1 and n_p = 10%, L = 100 and n_p = 10% , and L = 100 and n_p = 20% are 40, 60, and 80, respectively.

The model size corresponding to a stable value of the COV of equivalent hydraulic parameters can be seen as a representative volumetric element (RVE). An RVE is the smallest element of a material that is large enough to capture the equivalent properties of a heterogeneous material in a reproducible way (Griffith *et al.* 2012). By comparing Figs. 9-12, we can conclude that the sizes of the RVEs for different ACLs and proportions of heterogeneity are not controlled by the means but by the COVs of the equivalent hydraulic parameters. And according to Figs. 11 − 12, the size of an RVE is influenced by the ACL and the proportion of heterogeneity. Larger ACL and larger n_p require a larger RVE to obtain stable values of COVs of the equivalent hydraulic parameters. In a numerical analysis, the model size should be equal or larger than the size of RVE. Otherwise, the COV of the equivalent parameters of the heterogeneous soil will be overestimated.

Generally speaking, spatial variability of soil properties is not considered in most geotechnical reliability analyses (Gong *et al.* 2016, Javankhoshdel *et al.* 2018, Tan *et al.* 2011). Neglecting spatial variability of soil implies that the ACL assumed in a reliability analysis is large (Jimenez and Sitar 2009, Fan and Liang 2013), which will lead to the overestimation of the COVs of the equivalent parameters. And consequently, it will result in an over conservative reliability analysis for geotechnical engineering.

5. Conclusions

The variations of the equivalent hydraulic parameters of heterogeneous unsaturated soil were examined in this study. A stochastic method is used to conduct numerical experiments of unsaturated seepage. By random field discretization and MCS, different degrees of spatial autocorrelation and different proportions of heterogeneity are modelled. The VG model is used to represent unsaturated property curves. The influence of autocorrelation length, proportion of heterogeneity, and model size on the variations of equivalent hydraulic parameters of heterogenous unsaturated soil are examined.

By unsaturated seepage analysis and nonlinear curve fitting using the VG model, a series of SWCCs and UHCCs can be obtained. Simulations show that the variation ranges of SWCCs and UHCCs increase with the increase of ACL for the same model size. On the other hand, the variation ranges of SWCCs and UHCCs decrease slightly with the increase of model size for the same ACL of soil property.

Simulations show that spatial variability and model size have little effect on the mean values of the equivalent hydraulic parameters of the heterogeneous soil, but they have a large effect on the COVs of the equivalent hydraulic parameters. The COVs of the equivalent hydraulic parameters increase with the increase of the ACL and the proportion of heterogeneity. Not considering the spatial variability can lead to an over conservative result. Furthermore, a large ACL of soil property requires a large size of representative volumetric element for the same proportion of heterogeneity.

The results of the variations of the mean and the COV of the equivalent hydraulic parameters, can be used as a guidance for geotechnical reliability analysis and design related with unsaturated soil.

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