Geomechanics and Engineering, *Vol. 23, No. 4 (2020) 393-403*
DOI: https://doi.org/10.12989/gae.2020.23.4.393

Modeling of GN type III with MDD for a thermoelectric solid subjected to a moving heat source

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(Received August 28, 2020, Revised November 5, 2020, Accepted November 11, 2020)

Abstract. We design the Green-Naghdi model type III (GN-III) with widespread thermoelasticity for a thermoelectric half space using a memory-dependent derivative rule (MDD). Laplace transformations and state-space techniques are used in order to find the general solution for any set of limit conditions. A basic question of heat shock charging half space and a traction-free surface was added to the formulation in the present situation of a traveling heat source with consistent heating speed and ramp-type heating. The Laplace reverse transformations are numerically recorded. There are called the impacts of several calculations of the figure of the value, heat source spead, MDD parameters, magnetic number and the parameters of the ramping period.

Keywords: thermoelectric materials; Green-Naghdi of type III; memory-dependent derivative; ramp-type heating; moving heat source; state-space approach; Laplace transforms; numerical result

1. Introduction

Many authors have formulated new theories of thermoelasticity to replace the coupled theory introduced by Biot (1956). The heat equations associated with these theories are hyperbolic and hence automatically eliminate the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity.

Two generalizations introduced to the coupled theory. The first generalization to coupled thermoelasticity is due to Lord and Shulman (1967), who introduced the theory of generalized thermoelasticity with one relaxation time. The second generalization to the coupled theory of thermoelasticity is what is known as the theory of thermoelasticity with two relaxation times which proposed by Green and Lindsay (1972). One can refer to Chandrashekhariah (1998) and Hetnarski and Ignaczak (2000) for a review, presentation of generalized theories. Within the theoretical contributions to the subject are the proofs of uniqueness theorems under different conditions by Sherief (1986). Among the contributions to the subject of generalized thermoelasticity are the works of Marin (1995, 1996, 2009, 2010) and Marin and Stan (2013). A couple of examinations subject to these generalized theories were researched in Refs. Othman *et al.* (2002), Ezzat (2006), Mukhopadhyay and Kumar (2009), Lata *et al.* (2016), Lata and Kaur (2019), and Lata and Singh (2019).

Green and Naghdi (1991, 1992, 1993) assumed another generalized thermoelasticity theory which was viewed as an elective definition of heat transfer and they incorporated the warm heart beat transmission in this hypothesis in all around reliably way. Chandrasekharaiah (1996) has demonstrated unique theories utilizing a vitality strategy. In view of Green-Naghdi hypotheses, the three-stage slack thermoelasticity hypothesis was proposed Choudhuri (2007). Chirita and Ciarletta (2010) set up the corresponding and variational rule in direct thermoelasticity withoutvitality scattering. Ciarletta (2009) built up a hypothesis of micropolar thermoelasticity without vitality dissemination. The enthusiasm for the field of stage slacking heat transport has developed unimaginably as of late in light of the fact that they show great concurrence with the tests over a wide scope of length and time scale (Ghazanfarian *et al.* 2015).

A direct conversion between electricity and heat by using thermoelectric materials has attracted much attention because of their potential applications in Peltier coolers and thermoelectric power generators (See Ref. Rowe (1995)). Thermoelectric gadgets have numerous alluring highlights contrasted and the customary liquid based coolers and power age innovations, for example, long life, no moving part, any commotion, and simple support also, high unwavering quality. Be that as it may, their utilization has been constrained by the moderately low execution of present thermoelectric materials. The productivity of a thermoelectric material is identified with the supposed dimensionless thermoelectric figure-of-merit *ZT* by Tritt (2000). The expansion in *ZT* drives specifically to change in the vitality transformation productivity of thermoelectric generators and in the cooling proficiency of Peltier modules (See Ref. Tritt (2000)). Much exertion has been made to raise the *ZT* of thermoelectric bulk materials for vitality transformation productivity, so there have been a few changes in *ZT*. The thermoelectric figure of legitimacy gives a measure of the nature of such materials for applications and is characterized by Hiroshige *et al.* (2007),

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ISSN: 2005-307X (Print), 2092-6219 (Online)

$ZT = (\sigma_o S^2 / k)T$ with a specific end goal to accomplish a high figure of legitimacy; one requires a high thermopower *S*. Among the commitments in continuum mechanics of thermoelectric materials are crafted by Shercliff (1979) and Ezzat and Youssef (2010). More applications in the magneto-thermoelasticity theory can be found in the references: Biswas *et al.* (2017), Biswas and Mukhopadhyay (2018) and Abd-Elaziz *et al.* (2019), for a review, presentation of thermoelectric materials.

Differential equations of fractional order have been the focal point of numerous examinations because of their continuous appearance in different applications in liquid mechanics, viscoelasticity, science, material science and building. Povstenko (2009) investigated new thermoelasticity models that use fractional derivative. The fractional order theory of thermoelasticity was derived by Sherief *et al.* (2010). As of late, Kothari and Mukhopadhyay (2011), Sherief and Abd El-Latief (2013), Ezzat and El-Bary (2016), Yu *et al.* (2015) displayed a conservative numerical strategy for tackling the two-dimensional non-straight fractional reaction-sub diffusion equations, while Zhang *et al.* (2018) presented a period space ghastly technique for the time-space fragmentary Fokker-Planck condition and its contrary issue. Yu *et al.* (2014, 2020) solved some problems in fractional order generalized thermoelasticity.

The use of memory-dependent derivative (MDD) heat conduction law suggests that the heat transport condition is balanced and therefore the constitutive equations are changed and the new memory-dependent model, Ezzat *et al.* (2015), Lotfy and Sarkar (2017), Tiwari and Mukhopadhyay (2018), Xue *et al.* (2018), Shaw (2019), Biswas (2019a, b) and Sur and Kanoria (2019) for an overview of utilizations of memory-dependent derivative analytics.

2. Derivation of thermoelectric MDD heat equation for GN-III theory

Green and Naghdi (1991) developed a model of thermoelasticity theory with energy dissipation and proposed heat conduction law as given below:

$$\boldsymbol{q}(\boldsymbol{x},t) = -\left[n_1 k\, \nabla T(\boldsymbol{x},t) + n_2 k^* \nabla \upsilon(\boldsymbol{x},t)\right], \tag{1}$$

where n_1, n_2 are key numbers, each equals 0 or 1, $\boldsymbol{q}$ is the heat flux vector and υ is the thermal displacement defined by $\frac{\partial \upsilon}{\partial t} = T$ and $k>0$, $k^*>0$ are the material parameters which are known as the thermal conductivity and conductivity rate, respectively.

The conventional electro-thermoelasticity is based on the principles of GN-III of heat conductivity, in which relates the heat flux vector $\boldsymbol{q}$ and the conduction current density vector $\boldsymbol{J}$ to the temperature gradient (Kaliski and Nowacki 1963):

$$\boldsymbol{q}(\boldsymbol{x},t) = -\left[n_1 k\, \nabla T(\boldsymbol{x},t) + n_2 k^* \nabla \upsilon(\boldsymbol{x},t)\right] + \Pi \boldsymbol{J}(\boldsymbol{x},t), \tag{2}$$

$$\boldsymbol{J} = \sigma_o \left(\boldsymbol{E} + \frac{\partial \boldsymbol{u}}{\partial t} \wedge \boldsymbol{B} - S \nabla T \right). \tag{3}$$

The energy equation in terms of the heat flux vector $\boldsymbol{q}$ is (Biot 1956)

$$\frac{\partial}{\partial t}(\rho C_E T + \gamma T_o \gamma e) = -\nabla \cdot \boldsymbol{q} + Q. \tag{4}$$

By using Taylor-Riemann series expansion of small time-delay ω to expand $\boldsymbol{q}(\boldsymbol{x},t+\omega)$, we get

$$\boldsymbol{q}(\boldsymbol{x},t+\omega) = \boldsymbol{q}(\boldsymbol{x},t) + \omega D_\omega \boldsymbol{q}(\boldsymbol{x},t), \tag{5}$$

where ω is time-delay and $D_\omega f(\boldsymbol{x},t) = \frac{1}{\omega} \int_{t-\omega}^{t} K(t-\xi) f'(\boldsymbol{x},\xi)\, \mathrm{d}\xi$ and $K(t-\omega)$ is the kernel function in which they can be chosen freely.

The constitutive law for the heat flux vector in case of GN-III with memory-dependent derivative theory for elasto-thermoelectric materials, is given by (El-Karamany and Ezzat 2011, 2016)

$$(1+\omega D_\omega)\boldsymbol{q}(\boldsymbol{x},t) = -\left[n_1 k\, \nabla T(\boldsymbol{x},t) + n_2 k^* \nabla \upsilon(\boldsymbol{x},t)\right] + \Pi \boldsymbol{J}(\boldsymbol{x},t), \tag{6}$$

Eq. (6) is more intuitionistic for understanding the physical significance and the comparing memory subordinate differential condition is progressively expressive.

Taking the memory-time derivative of Eq. (4), we get

$$\frac{\partial}{\partial t} D_\omega \left(\rho C_E T + \gamma T_o e \right) = -\nabla \cdot D_\omega \boldsymbol{q} + D_\omega Q. \tag{7}$$

Multiplying Eq. (7) by ω and adding to Eq. (4), we obtain

$$\left(1+\omega D_\omega\right)\left(\rho C_E \frac{\partial}{\partial t} T + \gamma T_o \frac{\partial e}{\partial t} \right) = -\nabla \cdot \left(1+\omega D_\omega\right)\boldsymbol{q} + \left(1+\omega D_\omega\right)Q. \tag{8}$$

Substituting from Eq. (6), we get

$$\left(1+\omega D_\omega\right)\left(\rho C_E \frac{\partial T}{\partial t} + \gamma T_o \frac{\partial e}{\partial t} \right) = n_1 k\, \nabla^2 T + n_2 k^* \nabla^2 \upsilon - \nabla \cdot \Pi \boldsymbol{J} + \left(1+\omega D_\omega\right)Q. \tag{9}$$

Differentiating Eq. (9) with respect to time, we have

$$\left(n_1 k \frac{\partial}{\partial t} + n_2 k^* \right)\nabla^2 T - \nabla \cdot \Pi \frac{\partial \boldsymbol{J}}{\partial t} = \left(1+\omega D_\omega\right)\left(\rho C_E \frac{\partial^2 T}{\partial t^2} + \gamma T_o \frac{\partial^2 e}{\partial t^2} \right) - \frac{\partial}{\partial t}\left(1+\omega D_\omega\right)Q. \tag{10}$$

Eq. (10) is the new generalized energy equation of GN-III with memory-dependent derivative, taking into account the time-delay ω for thermoelectric materials.

Limiting cases

Eq. (10) when $\omega \to 0$ a, so that $\left|D_\omega f(\boldsymbol{x},t)\right| \le \left|\frac{\partial f(\boldsymbol{x},t)}{\partial t}\right| = \left|\lim_{\omega \to 0} \frac{f(\boldsymbol{x},t+\omega) - f(\boldsymbol{x},t)}{\omega}\right|$ leads to the Fourier law for the following theories:

(1) Biot theory (1956), $n_1=1$, $n_2=0$.

(2) Green-Naghdi of type III theory with energy dissipation (1991), $n_1 = n_2 = 1$.

(3) Green-Naghdi of type II without energy dissipation (1993), $n_1=0$, $n_2=1$.

3. Mathematical model

The governing equations for generalized magneto-thermoelasticity when the thermoelectric properties of the material are taken into account consist of:

1- The figure-of-merit ZT_o at some reference temperature T_o (Ezzat and Youssef 2010):

$$ZT_o = \frac{\sigma_o k_o^2}{k} T_o, \tag{11}$$

where k_o the Seebeck coefficient at T_o.

2- The first Thomson relation at T_o

$$\pi_o = k_o\, T_o, \tag{12}$$

where π_o is the Peltier coefficient at T_o .

3-The equation of motion the absence of body forces

$$\sigma_{ji,j} + \mu_o \varepsilon_{ijk} J_k H_j = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{13}$$

where ***B*** magnetic induction vector given by

$$B_i = \mu_o H_i, \tag{14}$$

and modified Ohm's law is defined

$$J_i = \sigma_0 \left(E_i + \mu_o \varepsilon_{ijk} \frac{\partial u_k}{\partial t} H_j - k_o T_{,i} \right). \tag{15}$$

4- The constitutive equations

$$\sigma_{ij} = \lambda e_{kk}\, \delta_{ij} + 2\mu e_{ij} - \gamma\theta\delta_{ij}. \tag{16}$$

5-Heat equation with memory-dependent derivative

$$\begin{aligned} &\left(n_1 k \frac{\partial}{\partial t} + n_2 k^* \right) \theta_{,ii} - \Pi \frac{\partial J_{j,j}}{\partial t} = \rho C_E \frac{\partial^2 \theta}{\partial t^2} + \gamma T_o \frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t} \\ &+ \int_{t-\omega}^{t} K(t-\xi) \left(\rho C_E \frac{\partial^3 \vartheta(x,\xi)}{\partial \xi^3} + \gamma T_o \frac{\partial^3 e(x,\xi)}{\partial \xi^3} - \frac{\partial^2 Q}{\partial \xi^2} \right) d\xi, \end{aligned} \tag{17}$$

where the kernel function $K(t-\omega)$ can be picked unreservedly as:

$$K(t-\xi) = 1 - \frac{2n}{\omega}(t-\xi) + \frac{m^2 (t-\xi)^2}{\omega^2} = \begin{cases} 1 \quad if \;\; m=n=0 \\ 1 - \dfrac{(t-\xi)}{\omega} \quad if \quad , m=0, n=\dfrac{1}{2} \\ 1-(t-\xi) \quad if \;\; m=0 \;\; n=\omega/2 \\ (1-\dfrac{t-\xi}{\omega})^2 \quad if \;\; m=n=1, \end{cases}$$

and $\theta = |T - T_0|$ and $\dfrac{\theta}{T_o} << 1$.

6-Kinematic relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{18}$$

In the above equations, a dot denotes differentiation with respect to time while a comma denotes material derivatives. The summation convention is used.

The preceding equations represent entire device of GN-III theory with memory-dependent derivative of thermoelectric material with energy dissipation in the presence of both a constant magnetic field and a moving heat source.

4. Physical problem

We consider thermoelectric material of limited conductivity σ_o possessing the area $x \geq 0$, where x-hub is taken opposite to the jumping plane of half-space pointing inwards. A consistent attractive field with segments (0, H_o, 0) is saturating the medium without an outside electric field. Presently for the one-dimensional issues, all the saw features will depend just on the space factors x and time t. The displacement vector has aspects

$$u_x = u(x,t),\; u_y = 0,\; u_z = 0. \tag{19}$$

The strain-displacement relation

$$e = \frac{\partial u}{\partial x}. \tag{20}$$

The components of the electromagnetic induction vector are $B_y = B_z = 0,\;\; B_x = \mu_o H_o = B_o(\text{constant})$, while the components of the Lorentz force appearing in Eq. (13) are given by

$$F_x = -\sigma_o B_o^2 \frac{\partial u}{\partial t},\; F_y = F_z = 0. \tag{21}$$

The components of current density vectorare

$$J_x = -\sigma_o k_o \frac{\partial \theta}{\partial x},\;\; J_y = 0,\;\; J_z = \sigma_o B_o \frac{\partial u}{\partial t}. \tag{22}$$

The displacement Eq. (12) reduce to

$$\frac{\partial \sigma}{\partial x} - \sigma_o B_o^2 \frac{\partial u}{\partial t} = \rho \frac{\partial^2 u}{\partial t^2}. \tag{23}$$

The constitutive equation

$$\sigma = (\lambda + 2\mu) e - \gamma\theta. \tag{24}$$

The energy equation in thermoelectric GN-III theory with memory-dependent derivative in the presence of heat sources

$$\begin{aligned} &k\left([n_1 + ZT_o] \frac{\partial}{\partial t} + n_2 \chi \right) \frac{\partial^2 \theta}{\partial x^2} = \rho C_E \frac{\partial^2 \theta}{\partial t^2} + \gamma T_o \frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t} \\ &\qquad + \int_{t-\omega}^{t} K(t-\xi) \left(\rho C_E \frac{\partial^3 \vartheta(x,\xi)}{\partial \xi^3} + \gamma T_o \frac{\partial^3 e(x,\xi)}{\partial \xi^3} - \frac{\partial^2 Q}{\partial \xi^2} \right) d\xi. \end{aligned} \tag{25}$$

Let us introduce the following non-dimensional variables

$$x^* = c_o \eta_o x,\, u^* = c_o \eta_o u,\, t^* = c_o^2 \eta_o t,\; \theta^* = \frac{\gamma\theta}{\rho c_o^2},\; \sigma^* = \frac{\sigma}{\rho c_o^2},\; J^* = \frac{J}{H_o c_o \eta_o},$$

$$q_x^* = \frac{\delta_o}{k T_o c_o \eta_o} q_x,\;\; k_o^* = \frac{\sigma_o \rho c_o^2}{\gamma H_o} k_o,\;\; \chi^* = \frac{\chi}{\eta_o c_o^2},\; Q^* = \frac{\gamma}{k \rho c_o^4 \eta_o^2} Q,\; T_o = \frac{\delta_o \rho c_o^2}{\gamma}.$$

Eqs. (23)-(29) in non-dimensional form become

$$e = \frac{\partial u}{\partial x}, \tag{26}$$

$$J_x = -k_o \frac{\partial \theta}{\partial x},\;\; J_z = \frac{1}{\beta} \frac{\partial u}{\partial t}, \tag{27}$$

$$\frac{\partial^2 \sigma}{\partial x^2} - M \frac{\partial e}{\partial t} = \frac{\partial^2 e}{\partial t^2}, \tag{28}$$

$$\left([n_1 + ZT_o] \frac{\partial}{\partial t} + n_2 \chi \right) \frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2}{\partial t^2}(1 + \omega D_\omega)(\theta + \varepsilon e) - \frac{\partial}{\partial t}(1 + \omega D_\omega) Q, \tag{29}$$

$$\sigma = e - \theta, \tag{30}$$

$$\frac{\partial q_x}{\partial t} = -\left(1-\omega D_\omega\right)\left[\frac{\partial}{\partial t}\left(n_1 + ZT_o\right) + n_2\chi\right]\frac{\partial \theta}{\partial x}. \tag{31}$$

5. Formulation of the model in the Laplace transform domain

Applying the Laplace transform with parameter s characterized by the formula

$$\left.\begin{array}{l} \mathrm{L}\{g(t)\} = \bar{\mathrm{g}}(s) = \int\limits_0^\infty \mathrm{e}^{-st}\mathrm{g}(t)\,\mathrm{d}t \\ \mathrm{L}\{D^{\mathrm{n}}\mathrm{g}(t)\} = s^{\mathrm{n}}\,\mathrm{L}\{\mathrm{g}(t)\} \end{array}\right\}, \quad s > 0$$

to both sides of Eqs. (26)- (31), we get a coupled system of the following equations

$$\bar{e} = D\bar{u}, \tag{32}$$

$$\bar{J}_x = -k_o\frac{\partial\bar{\theta}}{\partial x}, \quad \bar{J}_z = \frac{s}{\beta}\bar{u}, \tag{33}$$

$$D^2\bar{\theta} = s\varpi\bar{\theta} + s\varpi\varepsilon\bar{e} - \varpi\bar{Q}, \tag{34}$$

$$D^2\bar{\sigma} = s(M+s)\bar{e}, \tag{35}$$

$$\bar{\sigma} = \bar{e} - \bar{\theta} \tag{36}$$

$$\bar{q}_x = -\frac{1}{s(1+\Omega)}\left[s^2\left(n_1 + ZT_o\right) + n_2\chi\right]\frac{\partial\bar{\theta}}{\partial x}, \tag{37}$$

where

$$L\{\omega D_\omega f(t)\} = F(s)\begin{cases} [(1-e^{-s\omega})], & m = n = 0 \\ [1 - \frac{1}{\omega s}(1-e^{-s\omega})], & m = 0, n = \frac{1}{2} \\ [(1-e^{-s\omega}) - \frac{1}{s}(1-e^{-s\omega}) + \omega e^{-s\omega}], & m = 0, n = \frac{\omega}{2} \\ [(1-\frac{2}{\omega s}) + \frac{2}{\omega^2 s^2}(1-e^{-s\omega})], & m = n = 1 \end{cases}$$

$$\Omega(s) = (1-e^{-s\omega})\left(1 - \frac{2n}{\omega s} + \frac{2m^2}{\omega^2 s^2}\right) - \left(m^2 - 2n + \frac{2m^2}{\omega s}\right)e^{-s\omega} \tag{38}$$

$$F(s) = L\{\frac{\partial^2\theta}{\partial t^2} + \varepsilon\frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t}\} = s^2(\bar{\theta} + \varepsilon\bar{e}) - s\bar{Q},$$

$$D = \frac{\mathrm{d}}{\mathrm{d}x}, \quad \varpi(s) = \frac{s^2(1+\Omega)}{(n_1 + ZT_o)s^2 + n_2\chi},$$

and all the initial functions are equal to zero.

We consider that the medium is subjected to a moving heat source of consistent quality discharging its vitality ceaselessly while moving along the x-axis in the positive course with a steady speed v. The moving heat source is thought to be of the non-dimensional shape:

$$Q(x,t) = Q_o\,\delta(x - vt), \tag{39}$$

where Q_o is a constant heat. Taking Laplace transform, we obtain

$$\bar{Q}(x,s) = \ell\exp(-hx), \tag{40}$$

where $\ell = Q_o/v$ and $h = s/v$.

Eliminating $\bar{e}$ and $\bar{\theta}$ from Eqs. (42)-(45), we have

$$D^2\bar{\theta} = L_1\bar{\theta} + L_2\bar{\sigma} - L_3\exp(-hx), \tag{41}$$

where $L_1 = s\varpi(1+\varepsilon)$, $L_2 = s\varpi\varepsilon$, $L_3 = \ell\varpi$, and

$$D^2\bar{\sigma} = M_1\left(\bar{\theta} + \bar{\sigma}\right), \tag{42}$$

where $M_1 = s(M+s)$.

6. State space formulation

Picking as state factors that the temperature of heat conduction $\bar{\varphi}$ and the stress component $\bar{\sigma}$ in the x-direction, Eqs. (48) and (49) can be composed in the framework shape as:

$$D^2\bar{G}(x,s) = A(s)\bar{G}(x,s) + F(s)\exp(-hx), \tag{43}$$

where

$$\bar{G}(x,s) = \begin{bmatrix}\bar{\theta}(x,s)\\ \bar{\sigma}(x,s)\end{bmatrix}, \quad A(s) = \begin{bmatrix} L_1 & L_2 \\ M_1 & M_1\end{bmatrix} \text{ and } F(s) = \begin{bmatrix}-\ell\varpi \\ 0\end{bmatrix}.$$

Solutions of Eq. (43) that stay bounded for large x can be written as:

$$\bar{G}(x,s) = \exp[-\sqrt{A(s)}x]\bar{G}_o(s) + D(s)\exp(-hx), \tag{44}$$

where

$$\bar{G}_o(s) = \begin{bmatrix} G_1(s) \\ G_2(s)\end{bmatrix}$$

$$D(s) = \begin{bmatrix} D_1(s) \\ D_2(s)\end{bmatrix} = \left[h^2 I - A(s)\right]^{-1}F(s) \text{ and } I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

We shall use the well-known Cayley-Hamilton theorem to find the form of the matrix $\exp[\sqrt{A(s)}x]$. The characteristic equation of the matrix $A(s)$ can be written as

$$k^2 - (L_1 + M_1)k + M_1(L_1 - L_2) = 0, \tag{45}$$

The roots of this equation, namely, k_1 and k_2, satisfy the following relations:

$$k_1 + k_2 = L_1 + M_1, \tag{46a}$$

$$k_1 k_2 = M_1(L_1 - L_2). \tag{46b}$$

The Taylor series expansion of the matrix exponential in Eq. (51) has the form

$$\exp[-\sqrt{A(s)}x] = \sum_{n=0}^{\infty}\frac{[-\sqrt{A(s)}x]^n}{n!}. \tag{47}$$

Utilizing the Cayley-Hamilton hypothesis, we can express A^2 and higher powers of the matrix A in terms of I and A, where I is the unit matrix of second request. Subsequently, the infinite series in Eq. (44) can be decreased to

$$\exp[-\sqrt{A(s)}x] = a_o(x,s)I + a_1(x,s)A(s)$$

where a_o and a_1 are coefficients relying upon x and s.

By the Cayley–Hamilton hypothesis, the trademark roots k_1 and k_2 of the matrix A must satisfy

$$\exp[-\sqrt{k_1}x] = a_oI + a_1k_1, \quad \exp[-\sqrt{k_2}x] = a_oI + a_1k_2. \tag{48}$$

The solution of the above system of two linear equations is given by

$$a_o = \frac{k_1e^{-\sqrt{k_2}x} - k_2e^{-\sqrt{k_1}x}}{k_1 - k_2}, \text{ and } a_1 = \frac{e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}}{k_1 - k_2}.$$

Hence the entries of the matrix $\exp[-\sqrt{A(s)}x] = L_{ij}(x,s), \quad i,j = 1,2$, are given by

$$\begin{aligned} L_{11} &= \frac{(k_1 - L_1)e^{-\sqrt{k_2}x} - (k_2 - L_1)e^{-\sqrt{k_1}x}}{k_1 - k_2}, \quad L_{12} = \frac{L_2\left(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}\right)}{k_1 - k_2}, \\ L_{22} &= \frac{(k_1 - M_2)e^{-\sqrt{k_2}x} - (k_2 - M_2)e^{-\sqrt{k_1}x}}{k_1 - k_2}, \quad L_{12} = \frac{M_1\left(e^{-\sqrt{k_1}x} - e^{-\sqrt{k_2}x}\right)}{k_1 - k_2}. \end{aligned} \tag{49}$$

Additionally,

$$D_1 = -\frac{\ell^2\varpi(M_1 - h^2)}{(h^2 - k_1)(h^2 - k_2)}, \quad D_2 = \frac{\ell^2\varpi_3 M_1}{(h^2 - k_1)(h^2 - k_2)}.$$

We can compose the solution (44) in the shape

$$\begin{bmatrix} \bar{\theta}(x,s) \\ \bar{\sigma}(x,s) \end{bmatrix} = \begin{bmatrix} L_{11}(x,s) & L_{12}(x,s) \\ L_{21}(x,s) & L_{22}(x,s) \end{bmatrix} \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} + \begin{bmatrix} D_1(s) \\ D_2(s) \end{bmatrix} \exp(-hx) \tag{50}$$

To get $G_1(s)$ and $G_2(s)$ we set x=0on Eq. (57), and we obtain

$$\begin{bmatrix} \bar{\theta}(0,s) \\ \bar{\sigma}(0,s) \end{bmatrix} = \begin{bmatrix} L_{11}(0,s) & L_{12}(0,s) \\ L_{21}(0,s) & L_{22}(0,s) \end{bmatrix} \begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} + \begin{bmatrix} D_1(s) \\ D_2(s) \end{bmatrix}.$$

which implies to

$$\begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} = \begin{bmatrix} \bar{\theta}(0,s) \\ \bar{\sigma}(0,s) \end{bmatrix} - \begin{bmatrix} D_1(s) \\ D_2(s) \end{bmatrix} \tag{51}$$

Henceforth, the exact solution in the Laplace domain for any set of boundary conditions is given by

$$\bar{\theta}(x,s) = \left[\bar{\theta}(0,s) - D_1(s)\right]L_{11}(x,s) + \left[\bar{\sigma}(0,s) - D_2(s)\right]L_{12}(x,s) + D_1(s)\exp(-hx) \tag{52}$$

$$\bar{\sigma}(x,s) = \left[\bar{\theta}(0,s) - D_1(s)\right]L_{21}(x,s) + \left[\bar{\sigma}(0,s) - D_2(s)\right]L_{22}(x,s) + D_2(s)\exp(-hx) \tag{53}$$

It should be noted that the corresponding expressions for Green-Naghdi of type-III with memory-dependent derivative thermoelasticity in the absence of magnetic field can be deduced by setting M=0 in Eqs. (52) and (53).

7. Application

We consider a semi-space homogeneous medium of faultless conductivity having the region x ≥ 0 with quiet starting state and point of confinement conditions in the going with shape:

(i) Thermal boundary condition:

We suppose that the bounding plane x=0, subjected to ramp- type heating

$$\varphi(0,t) = \begin{cases} 0 & 0 \le t \\ \theta_o \dfrac{t}{t_o} & 0 \le t \le t_o \\ \theta_o & t \ge t_o \end{cases} \quad \text{or} \quad \bar{\theta}(0,s) = \frac{\theta_o\left(1 - e^{-st_o}\right)}{t_o s^2}, \tag{54}$$

where t_o is called the ramping parameter and θ_o is a constant.

(ii) Mechanical boundary condition:

The bounding plane x=0 is taken to be traction-free, i.e.,

$$\sigma(0,t) = 0 \text{ or } \bar{\sigma}(0,s) = 0. \tag{55}$$

Hence, we can use the conditions of (61) and (63) in Eqs. (59) and (60) to getthe exact solution for the heat conduction and stress x-component in the Laplace transform domain in the following forms:

$$\bar{\theta}(x,s) = \theta_1(s)\, e^{-\sqrt{k_1}x} - \theta_2(s)\, e^{-\sqrt{k_2}x} + D_1(s)\mathrm{e}^{-hx}, \tag{56}$$

$$\bar{\sigma}(x,s) = \sigma_1(s)\, e^{-\sqrt{k_1}x} - \sigma_2(s)\, e^{-\sqrt{k_2}x} + D_2(s)\mathrm{e}^{-hx}, \tag{57}$$

where

$$\theta_1(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\theta_o\left(1 - e^{-st_o}\right)}{t_o s^2} - D_1\right)(k_1 - M_1) - L_2D_2\right], \tag{58a}$$

$$\theta_2(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\theta_o\left(1 - e^{-st_o}\right)}{t_o s^2} - D_1\right)(k_2 - M_1) - L_2D_2\right], \tag{58b}$$

$$\sigma_1(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\theta_o\left(1 - e^{-st_o}\right)}{t_o s^2} - D_1\right)M_1 - D_2(M_1 - k_2)\right], \tag{59a}$$

$$\sigma_2(s) = \frac{1}{k_1 - k_2}\left[\left(\frac{\theta_o\left(1 - e^{-st_o}\right)}{t_o s^2} - D_1\right)M_1 - D_2(M_1 - k_1)\right], \tag{59b}$$

Clearly, $\bar{\sigma}(0,s) = 0$, in agreement with Eq. (65).

From Eq. (22), the displacement field takes the form:

$$\bar{u}(x,s) = -\left[\frac{1}{\sqrt{k_1}}(\sigma_1 + \theta_1)e^{-\sqrt{k_1}x} - \frac{1}{\sqrt{k_2}}(\sigma_2 + \theta_2)e^{-\sqrt{k_2}x} + \frac{1}{h}(D_2 + D_1)\mathrm{e}^{-hx}\right]. \tag{60}$$

By substituting from Eq. (60) into Eq. (33), we obtained the electric current component $\bar{J}_z$ as:

$$J_z(x,s) = -\frac{s}{\beta}\left[\frac{1}{\sqrt{k_1}}(\sigma_1 + \theta_1)e^{-\sqrt{k_1}x} - \frac{1}{\sqrt{k_2}}(\sigma_2 + \theta_2)e^{-\sqrt{k_2}x} + \frac{1}{h}(D_2 + D_1)\mathrm{e}^{-hx}\right]. \tag{61}$$

Differentiating Eq. (56) with respect to x and substituting by the result into Eqs. (33) and (37) to getthe electric current x-component $\bar{J}_x$ and heat flux x-component $\bar{q}_x$, respectively, as:

$$\bar{J}_x = k_o\left[\sqrt{k_1}\,\theta_1(s)\, e^{-\sqrt{k_1}x} - \sqrt{k_2}\,\theta_2(s)\, e^{-\sqrt{k_2}x} + h\,D_1(s)\mathrm{e}^{-hx}\right] \tag{62}$$

and

$$\bar{q}_x = \frac{1}{s\,(1+\Omega)}\left[s^2\,(a_1 + ZT_o) + a_2\chi\right]\left[\sqrt{k_1}\,\theta_1(s)\,e^{-\sqrt{k_1}x} - \sqrt{k_2}\,\theta_2(s)\,e^{-\sqrt{k_2}x} + h\,D_1(s)\,\mathrm{e}^{-hx}\right]. \qquad (63)$$

Those complete the solution in the Laplace transform domain.

8. Inversion of the Laplace transforms

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{g}(s)$ be the Laplace transform of a function $g(t)$. The inversion formula for Laplace transforms can be written as Honig and Hirdes (1984)

$$g(t) = \frac{e^{dt}}{2\pi}\int_{-\infty}^{\infty} e^{ity}\,\bar{g}(d+iy)\,\mathrm{d}y\,,$$

where d is an arbitrary real number greater than all the real parts of the singularities of $\bar{g}(s)$.

Expanding the function $h(t) = \exp(-dt)g(t)$ in a Fourier series in the interval [0,2 l], we obtain the approximate formula Honig and Hirdes (1984):

$$g(t) \approx g_N(t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k\,, \quad \text{for}\ \ 0 \le t \le 2\ell\,, \qquad (64)$$

where

$$c_k = \frac{e^{dt}}{\ell}\mathrm{Re}\left[e^{ik\pi t/\ell}\,g\left(d + ik\pi/\ell\right)\right]. \qquad (65)$$

Two methods are used to reduce the total error. First, the ‘Korrektur’ method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur-method uses the following formula to evaluate the function $g(t)$

$$g(t) = g_{NK}(t) = g_N(t) - e^{-2d\ell}\,g_{N'}(2\ell + t)\cdot \qquad (66)$$

where N' is an integer such that $N' > N$.

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (64). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$, be the sequence of partial sums of (64). We define the ε-sequence by

$$\varepsilon_{0,m} = 0,\ \varepsilon_{1,m} = s_m, \quad m = 1, 2, 3, \ldots$$

and

$$\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + 1/\left(\varepsilon_{n,m+1} - \varepsilon_{n,m}\right), \quad n, m = 1, 2, 3, \ldots.$$

It can be shown from Honig and Hirdes (1984) that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \ldots, \varepsilon_{N,1}, \ldots$ converges to $g(t) - c_0/2$ faster than the sequence of partial sums.

9. Numerical results and discussion

The technique dependent on a Fourier arrangement extension proposed by Honig and Hirdes (1984) and is created in detail in numerous writings, for example, the numerical code has been readied utilizing Fortran 77 programming language.

So as to translate the numerical calculations, we consider material properties of copper material, whose

Table 1 Values of the constants

$\rho = 8954\ kg/m^3$	$k = 386\ N/K\,s$	$T_o = 293\,\mathrm{K}$
$C_E = 383.1\,m^2/K$	$\lambda = 7.76\,(10)^{10}N/m^2$	$\mu = 3.86\,(10)^{10}N/m^2$
$\gamma = 210\,(10)^4 N/m^2K$	$\eta_0 = 3.36(10)^6\,sec/m^2$	$c_o = 4158\,m/s$
$\mu_o = 1.256\,(10)^{-6}\,Ns^2/C^2$	$k^* = 124W/mKs$	$\alpha_T = 1.78\,(10)^{-5}\,K^{-1}$
$\mu_o H_o = 1T$	$\varepsilon = 0.0168$	

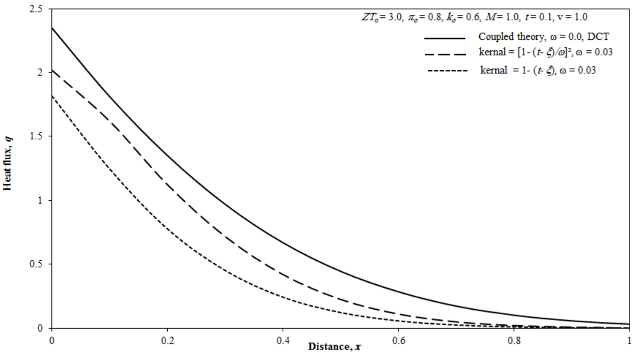

Fig. 1 The variation of heat flux for different forms of kernel function (t, ξ)

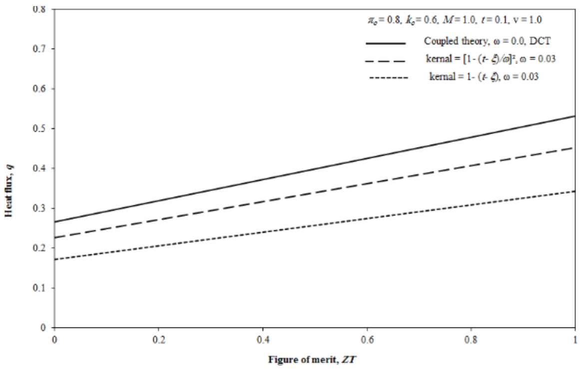

Fig. 2 The variation of heat flux vs. figure of merit for different forms of kernel function $K(t, \xi)$

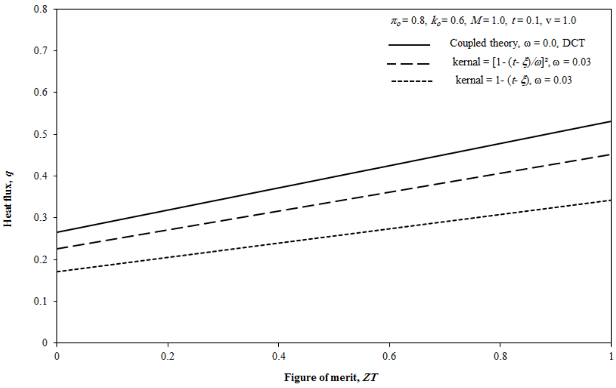

Fig. 3 The variation of temperature for different values of time-delay ω

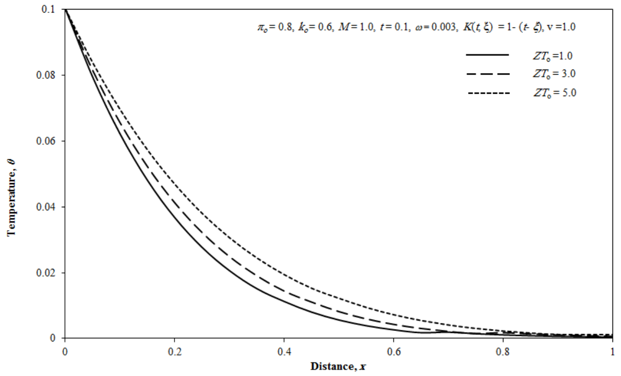

Fig. 4 The variation of temperature for different values of figure-of-merit at room temperature ZT_o

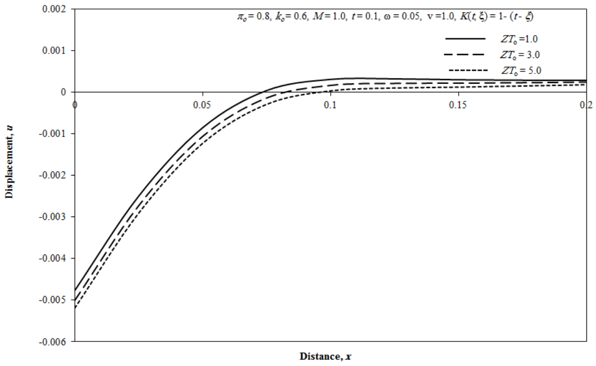

Fig. 5 The variation of displacement for different of figure-of-miret at room temperature ZT_o

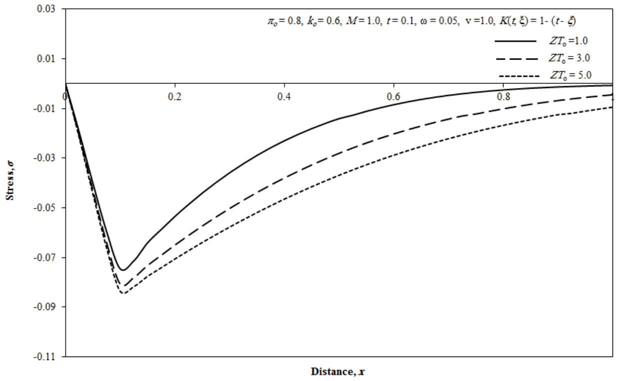

Fig. 6 The variation of stress for different values of figure-of-miret at room temperature ZT_o

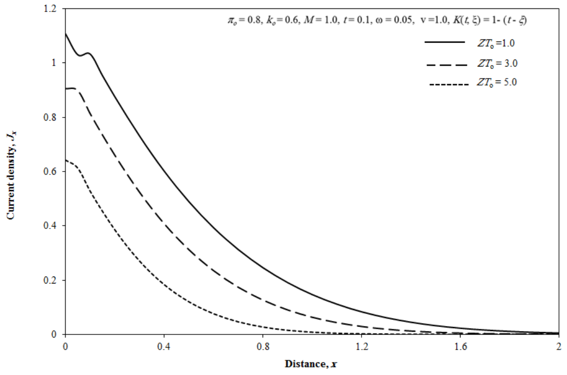

Fig. 7 The variation of current density vs. distance for different values of figure-of-miret ZT_o at room temperature

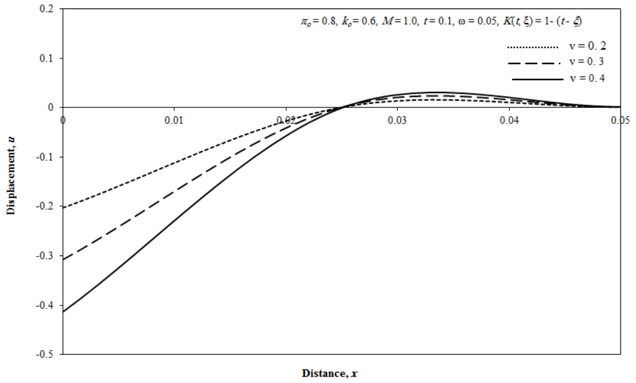

Fig. 8 Displacement distribution for different values of heat source velocity

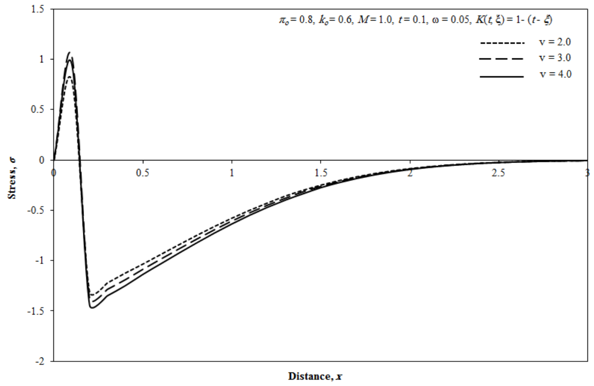

Fig. 9 Stress distribution for different values of the heat source velocity

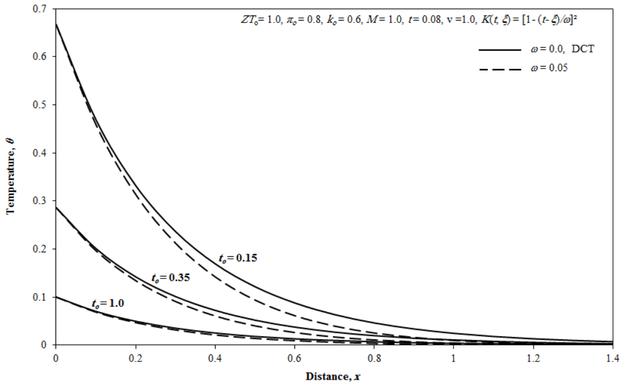

Fig. 10 The variation of temperature for different values of time-delay ω and t_o

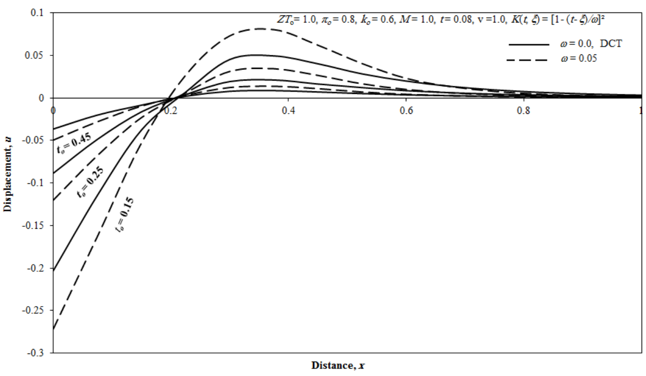

Fig. 11 The variation displacement vs. distance for different values of ramping parameter t_o

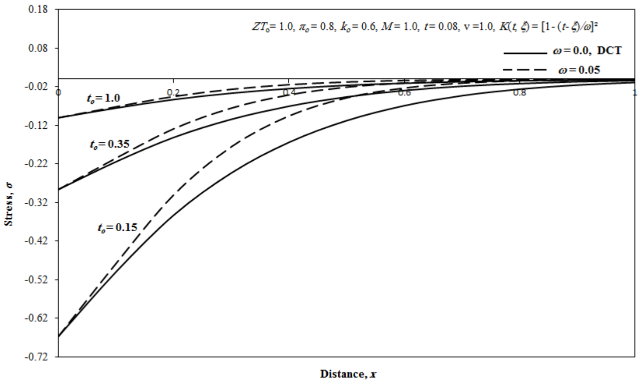

Fig. 12 The variation stress vs. distance for different values of time-delay ω and ramping parameter t_o

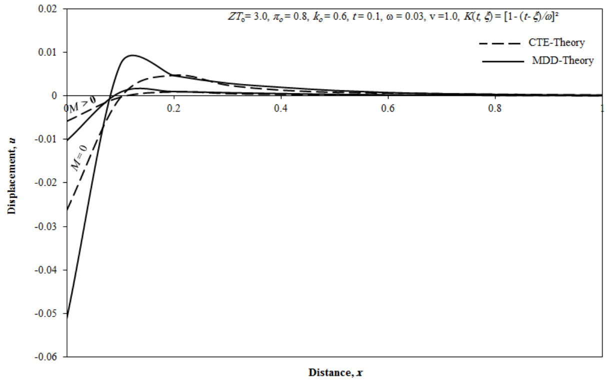

Fig. 13 The variation of displacement vs. distance for different values of magnetic number M

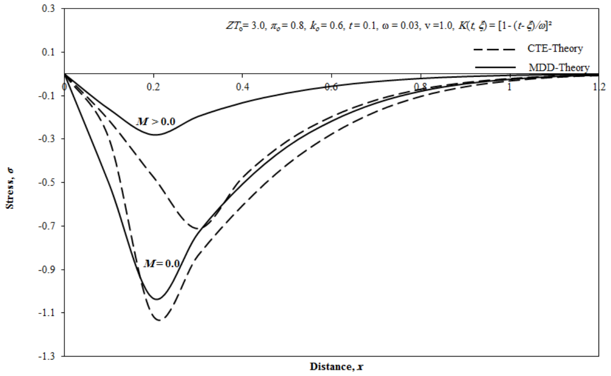

Fig. 14 The variation of stress component vs. distance for different values of magnetic number M

physical information is given in Table 1 (Sherief and Raslan 2016):

Thinking about the above physical information, we have assessed the numerical estimations of the field amounts.

The calculations were performed for an estimation of time, in particular t = 0.1 and for time-delay $\omega = 0.0, 0.003, 0.03, 0.3$. The numerical method laid out above was utilized to get the temperature, displacement and stress appropriations just as the electric flow segments for various estimations of the thought about parameters. The outcomes are shown graphically at various places of x as appeared in Figs. 1-11. While doing analysis of analytical and numerical results,we have found following highlighted results:

Fig. 1 speaks to the dimensionless estimation of temperature for wide scope of outspread separation x ($0 \le x \le 1$) and for different forms of kernel function. In this figure, strong lines speak to the arrangement got in the casing of Biot theory (ω=0) and broken lines speak to the arrangement relating to utilizing generalized electro-thermoelasticity(ω>0) with MDD when the kernel function is taken as the form $[1-(t-\xi)/\omega]^2$, while dotted lines when the kernel function is $1-(t-\xi)$. We learned from this figures that vital wonder saw in these assumes that the arrangement of any of the considered capacity in the new model is confined in a limited locale. Past this area, the varieties of these appropriations try not to occur. This implies to the arrangements concurring the new generalized hypothesis show the conduct of limited rates of wave spread.

Fig. 2 indicates the variation heat flux against the figure-of-merit for Biot theory (ω=0) and for the generalized electro-thermoelasticity theory with MDD (ω=0.03) when the kernel function has two forms, namely, $[1-(t-\xi)/\omega]^2$ and $1-(t-\xi)$. We saw from this assume the proficiency of a thermoelectric material figure-of-merit is relative to the temperature of the strong particles and the choice of the kernel function forms has significant effect on the heat flux field.

Fig. 3 displays the space variety of the temperature dissemination. In this figure, strong line speaks to the arrangement acquired in the casing of dynamic coupled hypothesis (Biot theory, ω=0) and different lines speak to the arrangements got for the situation $\omega = 0.003, 0.03, 0.3$. We observed that the temperature fields have been influenced when delay ω, where the expanding of the estimation of the parameter causes diminishing in temperature fields. The warm waves are consistent capacities, smooth and reach to unfaltering state contingent upon the estimation of time-delay ω, which implies that the particles transport the heat to different particles effectively and this makes the diminishing rate of the temperature more noteworthy than different ones. Additionally, the thermal waves cut x-hub all the more quickly when increments.

Figs. 4-7 show the variety of temperature, displacement, stress and current density circulations in thermoelectric circular depression with spiral separation$r$ for three values of figure-of-merit at room temperature ZT_o, namely, ZT_o = 1, 3 and 5. We noticed that the stress and displacement field has been affected by the figure-of-merit values, where the expanding of the estimation of figure-of-merit causes decreasing in the magnitude of the stress and displacement field while causes increasing in the temperature and current density.

Figs. 8 and 9 depict the space variation of displacement and stress distributions. In these figures the effect of the heat source velocity v on these distributions are studied. We noticed that for different values of the heat source velocity parameter v (2.0, 3.0, 4. 0) have a significant effect on all fields. We also learned from these figures that the increasing of the value of the parameter v causes increasing in the magnitude of stress and displacement distributions.

The expectations of the new hypothesis are examined and compared with dynamic classical coupled theory (Biot theory, ω=0) in Figs. 12-14. The sloping parameter t_o have critical impact on the temperature, displacement and stress distributions, so that he temperature and magnitude of both displacement and stress fields increase when the value of the sloping parameter t_o decreases.

Figs. 13 and 14 display the displacement and stress distributions with distance for two different theories; Biot theory, ω=0 and MDD theory, ω>0 when the magnetic number has two values M (M=0, absent of the magnetic field and in the present of the magnetic field, M>0). We find that the attractive field acts to diminish the displacement and stress fields. This is generally known as attractive damping.

10. Conclusions

The primary objective of this work is to take care of certain issues of thermal excitations in the hypothesis of coupled fields have a place with the thermoelectric elastic materials. The expanding wide use in detecting and activation has pulled in much consideration towards hypotheses about materials displaying couplings between versatile, electric, attractive and warm fields.

The conditions of wave hypothesis of thermoelectric materials exposed to MDD based on the change of the Fourier law was built rough phenomenological conditions of thermo-electromagnetic versatility described by a limited speed of engendering of electromagnetic and flexible excitations.

As per the aftereffects of the work, we can see the nearness of MDD's parameters in Fourier law of heat conduction can assume a crucial job in expanding or diminishing the speed of the wave proliferation of all fields through the thermoelectric medium.

From the considered model we can set up some fundamental hypotheses on the straight coupled and generalized speculations of electro-thermo-viscoelasticity; for example the coupled hypothesis (ω=0) and the generalized case hypothesis (ω>0).

The significance of state-space investigation is perceived in fields where the time conduct of any physical procedure is of premium. The state-space approach is more broad than the traditional Laplace and Fourier transfer systems. Therefore, state space is relevant to all frameworks that can be investigated by necessary changes in time, and is appropriate to numerous frameworks for which change hypothesis separates (Ezzat 2008).

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Nomenclature

λ, μ	Lame's constants
ρ	density
t	time
C_E	specific heat at constant strain
B_i	components of magnetic field strength
E_i	components of electric field vector
J_i	conduction electric density vector
H_i	magnetic field intensity
q_i	components of heat flux vector
H_o	constant component of magnetic field
μ_o	magnetic permeability
σ_o	electric conductivity
ε_{ijk}	permutation symbol
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector $\theta = T - T_o$
T_o	reference temperature chosen so that $\lvert T - T_o \rvert / T_o << 1$
e	$u_{i,i}$ dilatation
k	thermal conductivity
α_T	coefficient of linear thermal expansion
γ	$(3\lambda + 2\mu)\alpha_T$
π_o	Peltier coefficient at T_o
k_o	Seebeck coefficient at T_o
ε	$= \dfrac{\delta_o \gamma}{\rho C_E}$ thermoelastic parameter
M	$= \dfrac{\sigma_o B_o^2}{\eta_o \rho c_o^2}$ magnetic number
η	$= \dfrac{1}{\sigma_o \mu_o}$ magnetic diffusivity
η_o	$= \dfrac{\rho C_E}{k}$
c_o^2	$= \dfrac{\lambda + 2\mu}{\rho}$