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A rock physical approach to understand geo-mechanics of cracked porous media having three fluid phases

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Abstract. The role of precise prediction of subsurface fluids and discrimination among them cannot be ignored in reservoir characterization and petroleum prospecting. A suitable rock physics model should be build for the extraction of valuable information form seismic data. The main intent of current work is to present a rock physics model to analyze the characteristics of seismic wave propagating through a cracked porous rock saturated by a three phase fluid. Furthermore, the influence on wave characteristics due to variation in saturation of water, oil and gas were also analyzed for oil and water as wet cases. With this approach the objective to explore wave attenuation and dispersion due to wave induce fluid flow (WIFF) at seismic and sub-seismic frequencies can be precisely achieved. We accomplished our proposed approach by using BISQ equations and by applying appropriate boundary conditions to incorporate heterogeneity due to saturation of three immiscible fluids forming a layered system. To authenticate the proposed methodology, we compared our results with White's mesoscopic theory and with the results obtained by using Biot's poroelastic relations. The outcomes reveals that, at low frequencies seismic wave characteristics are in good agreement with White's mesoscopic theory, however a slight increase in attenuation at seismic frequencies is because of the squirt flow. Moreover, our work crop up as a practical tool for the development of rock physical theories with the intention to identify and estimate properties of different fluids from seismic data.

Keywords: exploration geophysics; mathematical geophysics; seismic methods; seismology; waves and wave analysis; geomechanics measurements and monitoring

1. Introduction

The importance of hydraulic and geophysical characterization of subsurface (saturated fractured/cracked) rocks cannot be ignored in the many scientific fields, like the sustainable use of aquifers, the secure storage of nuclear waste and CO_2, recovery of geothermal energy and for optimized production of oil and gas (Zhu *et al.* 2014, Elyasi *et al.* 2016, Sun *et al.* 2019). From White's (1975) pioneering work, it is evident that along with rock properties the characteristics of seismic waves are greatly influenced by the variation in fluid properties. Previous studies also reveals that, variation in characteristics of both fluid and solid rock create pressure gradient which ultimately results into wave induced fluid flow (WIFF) and considered as a dominant cause of wave attenuation and dispersion (Müller *et al.* 2010). Analysing the characteristics of seismic waves is among the focused points during interpretation of seismic field data. Also, establishing a relationship between fluid saturation and seismic attribute is the key task of rock physics modelling (Hefner and Jackson 2010).

In his well-publicized poroelastic theory, Biot (1956a, b, 1962) accounted the Visco-inertial mechanism between the fluid and solid which he considered as a major cause of wave attenuation and dispersion at wavelength scale (macroscopic scale). In his scheme, Biot assumed pore bearing isotropic rock, whereas pores are filled with a single phase fluid. Adhesiveness between the fluid and solid was the proposed cause of energy loss and also supposed that the macroscopic flow is in a line with the direction of wave propagation. Accompanying these presumptions, Biot predicted two longitudinal waves (P1, P2) and one shear wave. The consequence of variation in viscosity and permeability, predicted by Biot, contradict with the empirical findings of Jones (1986). Also, the Energy loss and change in direction of velocity, predicted by Biot seems to be at a higher frequency, i.e., at ultrasonic frequencies, but potently misjudge the attenuation between seismic

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frequency band, i.e., 1-100 Hz (Rubino and Holliger 2012, Li and Tao 2015, Wang *et al.* 2015, Zhao *et al.* 2015).

In order to avoid the short coming within the description provided about attenuation and velocity dispersion at macroscopic scale, heterogeneities, present within the subsoil, at the local scale (microscopic) were incorporated for the prediction of wave attenuation and velocity dispersion, in a more precise way. Such kind of, inhomogeneities are supposed to occur due to pores of different shapes and size, micro cracks, connection between the cracks and pores and damaged grain contacts which results fluid flow at micro scale (Mavko and Nur 1975, Budiansky and O'Connell 1976, Mavko and Nut 1979, Mavko *et al.* 1998). The proposed mechanism of squirt flow in the above-mentioned situation is supposed to occur due to the creation of pressure gradient within the rock having disparate features at local scale. Both, the Biot and squirt flow can be occur simultaneously; this thought provokes the idea of proposing Biot-Squirt (BISQ) model (Dvorkin 1994, Dvorkin *et al.* 1995) and for a period of time, these phenomenon were considered suitable at high frequencies (because of having insufficient time for pressure gradient to be stable. With the passage of time, we came to know through well-reputed theories that it can also cause high attenuation at seismic frequencies (Low) i.e., 1-100 Hz (Pride 2004, Müller *et al.* 2010, Guo *et al.* 2015).

In consequences of having high predicting ability within seismic frequency band mesoscopic scale theory emerged as an alternative approach (apart from macroscopic and microscopic scale theory) to compute energy loss due to heterogeneities at intermediate scale i.e., scale larger than local scale but smaller than global scale (Pride and Berryman 2003, Pride *et al.* 2003). Also, variation in characteristics of both rocks and fluids at that intermediate scale is presumed to be the major cause of attenuation and dispersion within a seismic frequency band (Müller *et al.* 2010). White explored the phenomenon of wave attenuation and velocity dispersion, (White 1975a, White *et al.* 1975) due to presence of spherical inhomogeneities within the fluids at mesoscopic scale. White overlooked the fluid/solid couple dynamics (Biot effect) which was then incorporated by upcoming researchers. Keeping the same limitations of geometry as in White's work, Dutta and Ode (Dutta and Odé 1979a, b) incorporated the Biot's effect in the White framework. Also, for arbitrary geometry White's work was reformulated by Johnson (2001). The concept of double porosity and dual permeability (mesoscopic scale) was taken into account by upcoming author's (Pride *et al.* 2004, Ba *et al.* 2011, Ba *et al.* 2015). Mesoscopic flow is considered as the dominant cause of wave attenuation and dispersion at seismic frequencies (Pride 2004, Chapman 2009, Müller *et al.* 2010, Sun *et al.* 2015). After the White's innovative work, many rock physics models were developed to compute wave attenuation and dispersion, considering the effects of fractures and random distribution of fluid patches. Squirt flow theory (Dvorkin 1993) raised the importance of influence of squirt flow from individual and network of connected fracture's (Rubino and Holliger 2013, Rubino *et al.* 2014, Subramaniyan *et al.* 2014, Sun *et al.* 2019). The combined effect of both the theories with some useful modifications opened new horizons for the hydrological and geophysical characterization of subsurface rocks.

All above mentioned schemes regarding wave propagation through porous media have some sort of fluctuation with one another, referring to the kind of disparities in rock frame, nature of pore fluids and with reference to their action at different frequencies. In spite of that differences they are well controlled, by a single phenomenon named as wave-induced fluid flow (Müller *et al.* 2010). Some practical steps were initiated to study the combined effects on wave propagation by interlayer-fluid flow at different scales because of the different kind of heterogeneities within the respective medium (Dutta and Seriff 1979, Dvorkin 1994, Chapman 2003, Tang 2011, Rubino and Holliger 2013, Kumar *et al.* 2017, Haghnejad *et al.* 2018, Manna *et al.* 2018, Bouanati *et al.* 2019). Recently, a new research has been done to analyze geotechnical data through soft computing (machine learning). In this study coal and gas hydrate bearing sediments were analyzed through a novel neural network approach (Jiang *et al.* 2020). In literature many solutions were presented to compute wave attenuation and dispersion due to interlayer-fluid flow due to saturation of two phase fluid (White 1975a, Carcione and Picotti 2006, Subramaniyan *et al.* 2014). However, very few solutions were presented to compute the effect of three phase fluid saturation on wave attenuation and dispersion (Frehner and Quintal 2012, Ahmad *et al.* 2017, Ahmad *et al.* 2019).

For Geoscientist and engineers basic foundation of rock physics play a fundamental role to precisely describe the physical processes that administrate the response of subsurface rocks to the applied stress essential for reservoir characterization, monitoring and various enhance oil recovery techniques like water and gas flooding. Keeping this in mind, the soul focus of this work is to present a solution for the computation of wave attenuation and dispersion due to interlayer-fluid flow in cracked porous rock saturated with a three phase fluid. We have reiterated the combined effect of mesoscopic and squirt flow on wave characteristics at sub-seismic and seismic frequencies. Subsequently, we analyze the effects of change in saturation of fluids on characteristics of seismic waves during their propagation through a cracked porous rock saturated with three phase fluid.

2. Modeling part

In previous rock physics models, wave attenuation and velocity dispersion were well computed by considering Biot's theory (macroscopic scale), Squirt flow theory (microscopic scale), Biot-Squirt flow theory (macro + micro scopic scale) and White's theory (mesoscopic scale). In all above mentioned theories the effect on wave propagation due to rock saturated with single or double fluid were computed and analyzed. We are seeking to expand the methods for the computation of wave attenuation and velocity dispersion, by considering the effects on wave propagation characteristics due to saturation of three phase fluid in a cracked porous rock. In current work, we are keen to compute the influence of squirt flow due to the presence of crack and the mesoscopic

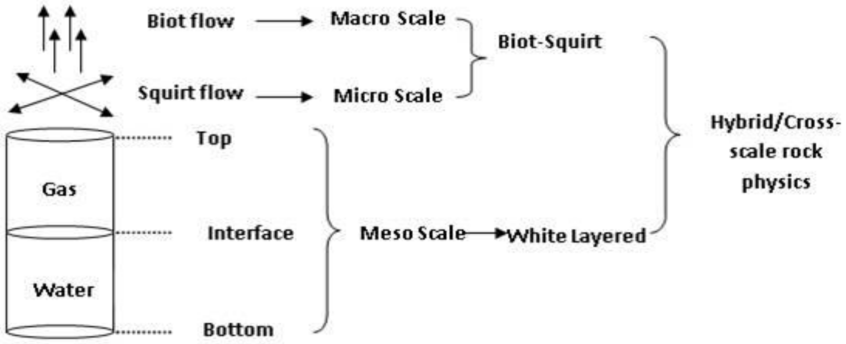

Fig. 1 Basic concept behind the proposed model

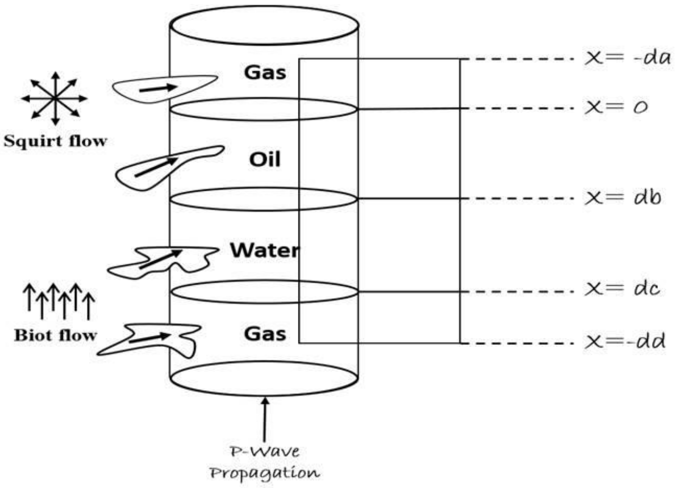

Fig. 2 Conceptual scheme of the proposed model

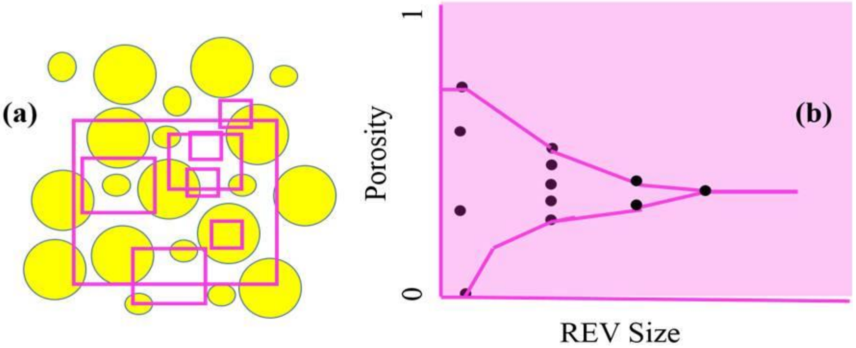

Fig. 3 Representation of the procedure to average the effects of complex pore geometries by selecting the size of RVE in such a way that the properties are averaged out over the RVE. In (a) yellow color circles represents the solid grains while white part is the representation of pore volume. The Rectangles representing the RVE in pink color is the total volume at which we are averaging over. In (b) the dark blue circles are representing the value of porosity corresponding to the RVE. With taking bigger size RVE means we get an averaging region where the volume of the pore space to the total volume become constant, representing as a straight line

flow due to compressibility difference among fluids of different nature. Squirt flow may occur due to heterogeneities because of cracks and fractures of at local scale (micro scale) and the mesoscopic flow may occur due to the presence of different nature of fluids (Layered form) at mesoscopic scale (i.e., larger than pore scale but smaller than global scale) (Müller *et al.* 2010). In our proposed model, a cracked porous medium is considered to be saturated by three immiscible fluids (Gas, Oil, and Water) forming a layered system. We have demonstrated our proposed model through word-by-word description of renowned phenomenon's regarding wave propagation in a porous media, as shown in Fig. 1.

For precise computation of effects on wave propagation in a porous medium due to combined effect of wave induced fluid flow at macro, micro, and mesoscopic scale. Aforementioned method is established for a cracked rock, which is here in Fig. 2 is supposed to be in the form of a cylinder saturated by three immiscible fluids. A rectangular case enclosing the heterogeneities due to saturation of three immiscible fluids within the cracked porous medium containing length ‘L’ in total is considered as a representative element. That representative element is placed within the porous medium in such a manner, that its upper end is situating within the middle of layer a, and its bottom end rests in the center of layer d. The radius of the cylinder ‘R’, being the characteristic squirt-flow length, is independent of frequency and nature of the fluid.

The proposed model symbolizes the hydrocarbon reservoirs having oil or water as a wetting fluids which usually exist in a region of low porosity. In order to upscale for the continuum of the representative volume element, we have averaged out the properties of saturated medium. Fig. 3 shows the graphical representation to show the volume average of properties over RVE. The main aim to do so is to get an averaging region that is ten times bigger than the pore size, which is indicated with a straight line along the horizontal axis indicating the size of selected RVE.

Dutta and Ode’s (1979a, b) have done a noteworthy work by developing an equation for poly-phase media, causing wave attenuation and velocity dispersion. This later on established by Dvorkin (1993) as an effective equation for the measurement of attenuation and dispersion caused by the combined effect of wave-induced fluid flow at macroscopic and microscopic scale. Relationship among different parameters accomplished by Dvorkin (1993) can be written in form as given below:

$$(1-\phi)\,\rho_s u_{tt} + \phi\,\rho_f v_{tt} = M u_{xx} + \alpha\,D\left(v_{xx} + \frac{\gamma}{\phi}u_{xx}\right) \tag{1}$$

$$\phi\rho_f v_{tt} - \rho_a\left(u_{tt} - v_{tt}\right) - \frac{\mu\phi^2}{\kappa}\left(u_t - v_t\right) = \phi D\left(v_{xx} + \frac{\gamma}{\phi}u_{xx}\right) \tag{2}$$

Within these relationships “u_{tt}” and “v_{tt}” indicate two-time derivative of solid and fluid displacement, “μ” is the viscosity, “κ” is the permeability, “ϕ” is the porosity, “ρ_s, ρ_f” are the solid and fluid density, “ρ_a” is the additional coupling density, “u_{xx}” and “v_{xx}” are the two-time partial derivatives (with respect to ‘x’) of solid and fluid motion. Where “M” is the plane wave modulus, “$\gamma=\alpha-\phi$”, the relationships among moduli and other reservoir parameters as given below.

$$D = \left(\frac{1}{K_f} + \frac{1}{\phi q}\right)^{-1}\left(1 - 2J_1\frac{\lambda R}{\lambda R J_0(\lambda R)}\right) \quad \alpha = 1 - \frac{K_d}{K_s}$$

$$q = \frac{K_s}{\left(1 - \phi - \frac{K_d}{K_s}\right)}$$

where K_f, K_s and K_d are the fluid, solid and dry rock bulk modulus and “R” is the characteristic squirt flow length, independent from the frequency and fluid characteristics as in Divorkin’s work.

By putting the above values in Eqs. (1) and (2), these can be rewritten in the form:

$$\rho_b u_{tt} + \rho_f W_{tt} = H u_{xx} + \frac{\alpha D}{\phi}W_{xx} \tag{3}$$

$$\rho_f u_{tt} + m W_{tt} = \frac{\alpha D}{\phi}u_{xx} + \frac{D}{\phi}W_{xx} - \frac{\mu}{\kappa}W_t \tag{4}$$

where “W_{tt}” is the two times derivatives of fluid/solid coupled displacement, “$\rho_b = \left[(1-\phi) + \rho_s + \phi\rho_f\right]$” is the equivalent media density, “W_{xx}” is the two time partial derivatives (with respect to ‘x’) of fluid/solid coupled motion i.e., W=ϕ(v-u), “$m = \frac{\rho_f}{\phi} + \frac{\rho_a}{\phi^2}$” and “$H = M + \alpha^2\frac{D}{\phi}$”.

The relationship between, solid displacement u, stress σ and fluid/solid coupled displacement W are mentioned below.

$$u = u(x)\,e^{iwt} \qquad W = W(x)\,e^{iwt}$$
$$u = u_c + u_d \qquad W = W_c + W_d$$
$$u_c = \sigma_c W_c \qquad u_d = \sigma_d W_d$$

Within the aforementioned values, coupling displacement and solid displacement can be written in the form.

$$\begin{aligned} W_c &= K1\cos(k_c x) + K2\sin(k_c x) \\ W_d &= K3\cos(k_d x) + K4\sin(k_d x) \\ W &= W_c + W_d \\ W &= K1\cos(k_c x) + K2\sin(k_c x) \\ &\quad + K3\cos(k_d x) + K4\sin(k_d x) \\ u_c &= \sigma_c W_c\,, \quad u_d = \sigma_d W_d \end{aligned} \tag{5}$$

$$\begin{aligned} u &= u_c + u_d \\ u &= \sigma_c K1\cos(k_c x) + \sigma_c K2\sin(k_c x) \\ &\quad + \sigma_d K3\cos(k_d x) + \sigma_d K4\sin(k_d x) \end{aligned} \tag{6}$$

A representative volume element (RVE) in the form of a rectangular medium in considered to be enclosing both the microscopic heterogeneity due to cracked rock and mesoscopic heterogeneity due to saturation of fluid of different nature. The considered RVE is normally subjected by a time harmonic compressional stress at the top of the representative volume element by the wave passing transversely to the representative volume element, which will result into time harmonic vertical strain (displacement) within the RVE. In order to accurately compute the change in complex plane wave modulus, a situation of stress, strain continuity at top and bottom of RVE is assumed i.e., no fluid can be enter or leave the RVE. The ratio of applied compression and resulting deformation will give complex plane wave modulus, which will be useful in computing attenuation and velocity dispersion. Displacement created by passing wave can be calculated by using following equations:

$$u_a = K_1\sigma_c\cos(k_c l_a) + K_2\sigma_c\sin(k_c l_a) + K_3\sigma_d\cos(k_d l_a) + K_4\sigma_d\sin(k_d l_a) \tag{7}$$

$$u_b = K_5\sigma_c\cos(k_c l_b) + K_6\sigma_c\sin(k_c l_b) + K_7\sigma_d\cos(k_d l_b) + K_8\sigma_d\sin(k_d l_b) \tag{8}$$

$$u_c = K_9\sigma_c\cos(k_c l_c) + K_{10}\sigma_c\sin(k_c l_c) + K_{11}\sigma_d\cos(k_d l_c) + K_{12}\sigma_d\sin(k_d l_c) \tag{9}$$

$$u_d = K_{13}\sigma_c\cos(k_c l_d) + K_{14}\sigma_c\sin(k_c l_d) + K_{15}\sigma_d\cos(k_d l_d) + K_{16}\sigma_d\sin(k_d l_d) \tag{10}$$

In above equations, K1- K16 are the unknown parameters and kc and kd represent the fast and slow p-wave number for a layer a, b, c, and d, which can be computed by applying, below mentioned boundary conditions on them.

Continuity of stress and pore pressure at the boundary between layer a and b, b and c, and between layer c and d shows that

At x=0

$$\tau_{\perp a} = \tau_{\perp b} \tag{11}$$

$$\hat{p}_a = \hat{p}_b \tag{12}$$

At x=l_b

$$\tau_{\perp b} = \tau_{\perp c} \tag{13}$$

$$\hat{p}_b = \hat{p}_c \tag{14}$$

At x=l_c

$$\tau_{\perp c} = \tau_{\perp d} \tag{15}$$

$$\hat{p}_c = \hat{p}_d \tag{16}$$

Condition for continuity of solid velocity and fluid velocity at the interface between layers demands

At x=0

$$\phi\,(v_a - u_a) = \phi\,(v_b - u_b) \tag{17}$$

$$u_a = u_a \tag{18}$$

At x=l_b

$$\phi(v_b - u_b) = \phi\,(v_c - u_c) \tag{19}$$

$$u_b = u_c \tag{20}$$

At x=l_c

$$\phi\,(v_c - u_c) = \phi\,(v_d - u_d) \tag{21}$$

$$u_c = u_d \tag{22}$$

Condition of stress continuity is applied both at the top and bottom of the respective element

At x=-l_a

$$\hat{p}_a + \tau_{\perp a} = -p \tag{23}$$

At x=l_d

$$\hat{p}_d + \tau_{\perp d} = -p \tag{24}$$

As, normal stress is applied on the RVE and the solid displacement is zero at top and bottom of the RVE, also solid displacement is zero at the lateral boundaries of the RVE and the fluid is not allowed to get in or to flow out of the RVE.

At x=-l_a

$$\phi\,(v_a - u_a) = 0 \tag{25}$$

At x=l_d

$$\phi\,(v_d - u_d) = 0 \tag{26}$$

where the above mentioned fluid pressure and stress given by Dvorkin (Dvorkin and Nur 1993) can be related with rock and fluids parameters in the form given below:

$$p_f = -D\left(w_x + \frac{\gamma u_x}{\phi q}\right) \tag{27}$$

$$\tau_{\perp} = M u_x - \alpha p \tag{28}$$

In above equations "γ=α− ϕ" and "M" is plane wave modulus. By utilizing Eqs. (27) and (28) and the boundary conditions, we will get sixteen equations having above-discussed parameters. For the computation of the unknown parameters we have arranged the displacement equations into matrix form, mentioned as under

$$\sum_{i,j=1}^{16} A_{ij} B_i = C_i \tag{29}$$

where, "A_{ij}" is a matrix of coefficients, given in Eqs. (7)-(10) and "B_j" is a column vector of the unknown parameters and C_i is a vector describing stress values at top and bottom of the representative volume element, as describe below.

$$B = \left(K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K11, K12, K13, K14, K15, K16\right)^T \tag{30}$$

$$C = p_e(0,0,0,0,0,0,0,0,0,0,0,0,-1,0,-1,0)^T \tag{31}$$

The normally subjected time harmonic wave will cause vertical displacement in the solid part $\Delta u\ e^{i\omega t}$ which ultimately results into time harmonic vertical strain having amplitude of the form $\varepsilon = \frac{\Delta u}{L}$. Keeping the dynamic-equivalent premises, the equivalent complex plane-wave modulus P can be defined by providing the relation between applied stress $\tau_{\perp}$ and the resulted strain ε as given below

$$P = \frac{-\tau_{\perp}}{\varepsilon} \tag{32}$$

The ratio of real and imaginary part of plane-wave modulus will gives the inverse quality factor (attenuation) while the velocity dispersion can be computed by taking the square root of the ratio between plane-wave modulus and the density of the fluid saturated rock $\rho = (\sum_{i=1,2,3} S_i \phi_i \rho_i) + (1-\phi_i)\rho_s$. Where "$S_i$" represents the saturation of each fluid, "ϕ_i" is the porosity, "ρ_i" is the density of each layer contacting the fluid and "ρ_s" is the density of the solid rock. The above mention process can be summarized in following way: Solving Eq. (29), we get sixteen unknown parameters K1 to K16, then the solid displacements "u_a" and "u_a" is computed to obtain total strain "ε" within the respective medium. By putting the value of "ε" in equation (32) the required complex plane-wave modulus "P" can be obtained. Finally, P-wave attenuation and velocity dispersion will be measured by using equation.

$$Q_p^{-1} = \frac{imag\_P}{real\_P} \tag{33}$$

Table 1 Physical properties of solid and pore fluid (Dvorkin and Nur 1993, Vogelaar and Smeulders 2007, Frehner and Quintal 2012)

Skeleton	**Gas**	**Water**	**Oil**
$K_s = 33.4*10^9{}_{pa}$	$K_f = 9.6*10^6{}_{pa}$	$K_f = 2.2*10^9{}_{pa}$	$K_f = 1*10^9{}_{pa}$
$\rho_s = 2700_{kg/m}{}^3$	$\rho_f = 70_{kg/m}{}^3$	$\rho_f = 1000_{kg/m}{}^3$	$\rho_f = 700_{kg/m}{}^3$
$\phi = 0.30$	$\eta_f = 15*10^{-6}{}_{kg/m/s}$	$\eta_f = 6*10^{-4}{}_{kg/m/s}$	$\eta_{f3} = 400*10^{-4}{}_{kg/m/s}$
$\mu = 1.40*10^9{}_{pa}$			
$R = 0.0017$			
$\kappa = 1*10$^(-12)m^2			

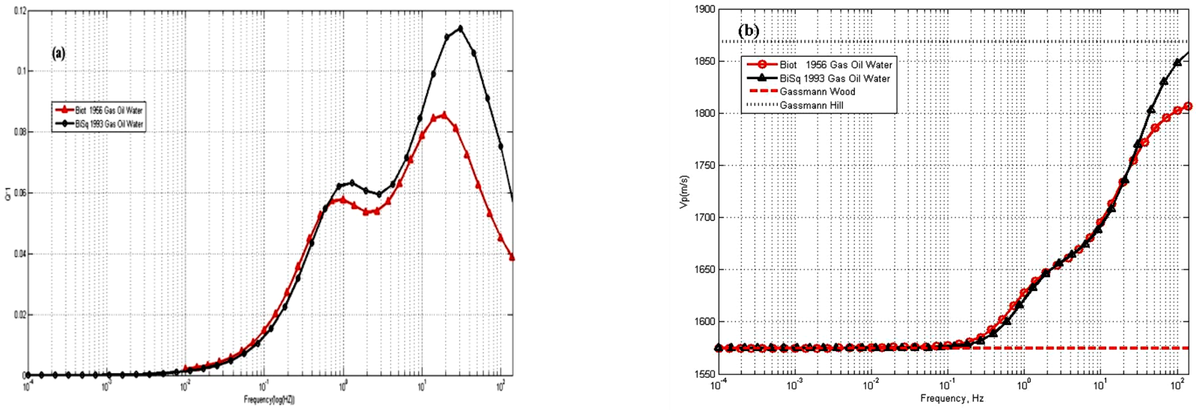

Fig. 4 The attenuation and velocity dipersion by using Biot (1956) and BiSq (Dvorkin and Nur 1993) equations, as a function of frequency are given in the figure. (a) Attenuation as a function of frequency (Hz) and (b) Velocity dispersion as a function of frequency (Hz)

3. Results and discussions

In literature, several authors focused on analysing the characteristics of propagating wave through fluid saturated media, in their conclusions they revealed that the characteristics of propagating waves are strongly influenced by the variation in solid and fluid properties (Johnson 2001, Müller *et al.* 2010). Although, the theory of wave attenuation and velocity dispersion get matured enough, it still lack in providing a numerical solution to compute wave attenuation and velocity dispersion in a porous rock saturated with co-existing water, oil and gas Frehner and Quintal (2012). We proposed a numerical method to study the behaviour of a poroelastic rock having mesoscopic heterogeneity due to saturation of three different fluids and microscopic heterogeneity due to presence of crack. In this regard, poorly consolidated fractured sandstone having spatially variable distribution of Gas, Oil and water, is considered. Wave attenuation and velocity dispersion as function of frequency were computed and analysed at both seismic and sub-seismic frequencies. Table 1 gives the physical properties of the solid matrix and the saturating fluids. Where, "K_s, ρ_s, K_f, ρ_f" represents bulk modulus and density of solid skeleton and fluid, R is the squirt flow length and "η_f" represent viscosity of the fluid, ϕ, μ and 'κ' are porosity, shear modulus and permeability of solid skeleton respectively.

Influence on wave characteristics due to cracked porous rock saturated with three immiscible fluids

A numerical approach is proposed for the computation of wave attenuation and the velocity dispersion due to both mesoscopic and microscopic heterogeneity in a cracked porous rock having fluid saturation due to coexistence of water, oil and gas, as shown in Fig. 1. From the computed results demonstrated in (Fig. 4a), it can be concluded that, the first attenuation peak arises due to the occurrence of compressibility contrast between gas and oil while the second attenuation peak is the result of combined effect of fluid flow due to compressibility contrast between water and gas and also due to the fluid flow owing to squirt flow from the cracks within the solid part. The variation in velocity dispersion curves (Fig. 4(b)) are at the same frequencies where attenuation is at its peak reinforcing the provided reason of wave attenuation. Furthermore, the proposed methodology is validated by comparing the results with the numerical results obtain by using Biot poroelastic equations (Biot 1956a, b, 1962) for rock having spatial variation at mesoscopic scale, due to presence of three immiscible fluids (Qazi *et al.* 2017). It can be inferred from Fig. 4(a) that, at low frequency the attenuation computed by using BISQ equation is in good agreement with the attenuation values measured by Biot equation. In can be seen that, there is a slight increase in the amplitude at gas-oil interface, this because of incorporation of the squirt effect. Also, at the water-gas interface there is an increase in attenuation amplitude in BISQ curve which is again due to the influence of squirt effect combined with the mesoscopic flow due to the occurrence of pressure gradient due to the compressibility contrast between water and gas. It can also be deduced from the figure that the attenuation curve

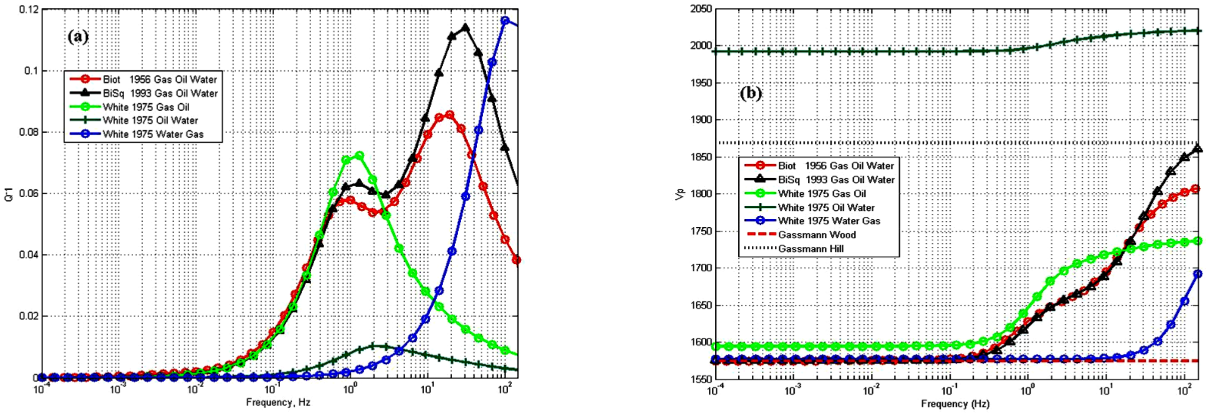

Fig. 5 wave attenuation and velocity dispersionn as a function of frequency by using Biot (1956), White 1975 and BiSq (Dvorkin and Nur 1993) equations are given in the figure. (a) Attenuation as a function of frequency (Hz) and (b) Velocity dispersion as a function of frequency (Hz)

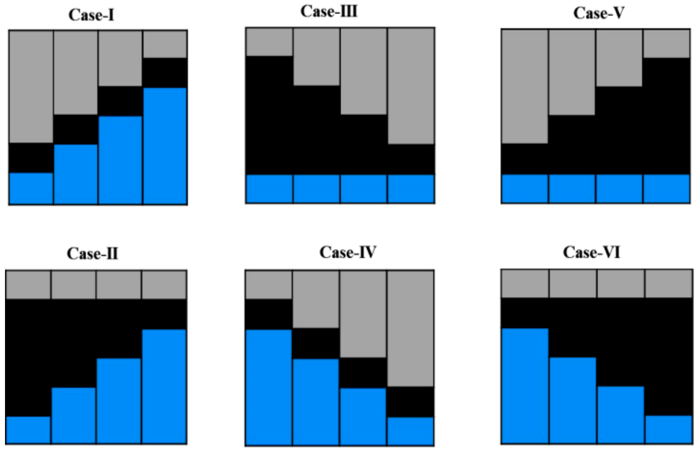

Fig. 6 Six different scenario we may face in different stages of enhanced oil recovery process are demonstrated.

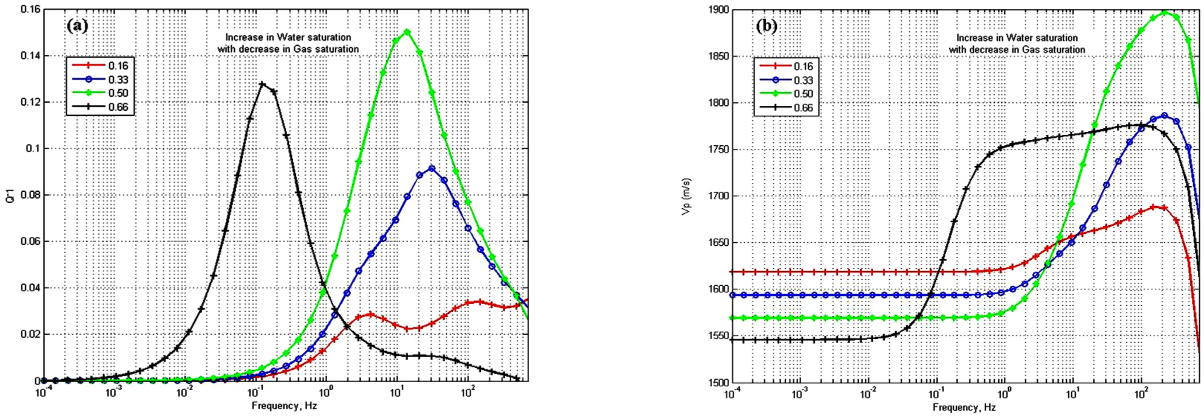

Fig. 7 (a) Wave attenuation as a function of frequency (Hz) and (b) Velocity dispersion as a function of frequency (Hz), for the case when water saturation get increassed (for example during water flooding) with the decrease in gas saturation at constant oil saturation.

shifted towards the lower frequency. Fig. 4(b) reveals that the velocity dispersion phenomenon as function of frequency and it also get increased by incorporating the effect of squirt flow along with the mesoscopic flow in Biot’s equations (Biot 1956a, b).

The proposed methodlogy, for the computation of wave attenuation and dispersion in a cracked porous rock saturated with three immiscible fluids is further validated with the White’s mesoscopic theory for rock saturated with two different fluids (Fig. 5(a)). From the outcomes of comparison it can be concluded that, at low frequecy the proposed mehtodly is in good agreement with the White’s theory the slight increase in White’s results for gas-oil case is because gas saturation is 50 percent in that case, while for

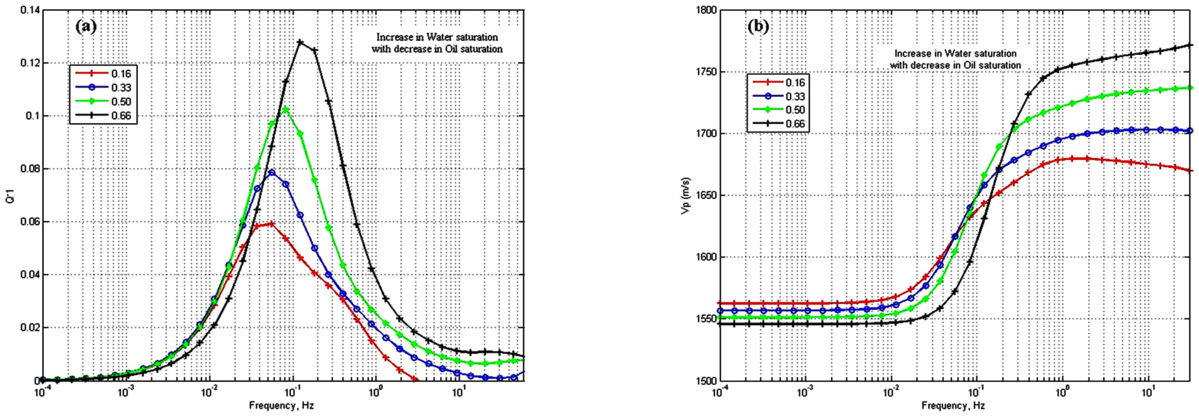

Fig. 8 Mesoscopic and microscopic heterogeneity. (a) Attenuation as a function of frequency (Hz) and (b) Velocity dispersion as a function of frequency (Hz)

other cases its 33 percent. White's theory results for water-gas case attenuation peak appear at higher frequecies while the results computed by using Biot (1956a, b) and BISQ, J Dvorkin and Nur (1993) equations arises at lower frequencies with having high modulus dispersion at higher frequencies, Vogelaar and Smeulders (2007). These differences from White's theory results is because of neglecting the fluid solid coupled motion in White's theory. Similar trends in velocity dispersion curves confirms the athentication of the proposed methodology. Moreover, the attenuation curve for oil-water interface seems to disappear in results of both Biot and BISQ equations, this is because of negligible copressibility contrast in the moduli of the oil and water which make it disapper in the wave attenuation curve in case of three phase fluid. The comparison among velocity dispersion curve's (Fig. 5(b)) also show similar trend at the point of compressibility constrast. The initial velocity for gas-oil case in white's theory is quit higher then the velocities computed by using Biot (1956a, b) and BiSq equaitons, Dvorkin and Nur (1993). This is because of the difference in degree of saturation in both the cases.

As one of the key task of rock physics modelling is to find the relationships between hydrocarbon saturation and seismic attributes. In this paper, we proposed a numerical solution for characterization of seismic velocities and attenuation for the fractured rock, saturated with three fluid phases. Furthermore, we explained how our model is applicable for the determination of variation in hydrocarbon saturation. For that, six different cases of saturation are considered (Fig. 6) by which we discussed how our proposed rock physics model can predict seismic velocities and attenuation for different saturation degrees of water, oil and gas.

Influence on wave characteristics due to variation in fluid saturation

In order to enhance the production from a hydrocarbon reservoir, improved oil recovery strategies such as thermal flooding, gas cap expansion, gas injection, and water flooding are applied which results in variation in saturation of hydrocarbon and non-hydrocarbon fluids (Fig. 6) which ultimately results in complex seismic characteristics (Zhang *et al.* 2019). In case-I the seismic responses through a cracked rock having variation in gas and water saturation were computed i.e., increase in water saturation with decrease in gas saturation while oil saturation remain constant. The computed results (Fig. 7(a) and 7(b)) reveals that, at low water saturation, with lower attenuation amplitude there will be decrease in wave energy at both gas/oil and water/gas interface and the velocity will be dispersed within very small limits (red curve) (Fig. 7(b)). In a situation when the water saturation is about 33% and oil saturation is fixed at 16% while gas saturation is 51%, a single peak attenuation curve will appear due to the interface between water and gas, also the peak moves towards much lower frequencies. The shift towards the lower frequencies will be keep going with further increase in the water saturation in case of constant oil saturation and decrease in gas saturation. For a situation when gas saturation is around 5% the attenuation and velocity dispersion will be maximum, Vogelaar (2009) and will be at sub-seismic and seismic frequencies (black and green curve) which is also concluded in previous theoretical works (White 1975b, Vogelaar and Smeulders 2007).

In second case we demonstrated situations for variation in oil and water saturation while the gas has a constant saturation (Fig. 7) i.e., increase in water saturation with the decrease in oil saturation. The computed result of wave attenuation and velocity dispersion in above mentioned cases reveals that, at constant gas saturation, the increase in water saturation with the decrease in oil saturation results a shift in attenuation curve towards higher frequencies (Fig. 8a) and is at its maximum when the gas saturation is 16% in our case. Variations in higher and lower values of velocity also get increased and maximum dispersion occur (Fig. 8) when the water saturation is at its maximum (66% in current case). The initial velocity is higher for low water saturation which gets decrease with the increase in water saturation which is because of water higher density. At constant gas saturation, the increase in water saturation with the decrease in oil saturation results in shift of attenuation peak towards higher frequencies however, the attenuation amplitude get increase.

Three phase flow occur both naturally and in case of an enhanced oil recovery process i.e., gas cap expansion, surfactant flooding, and gas injection. Monitoring the dynamics of seismic waves in a cracked porous media

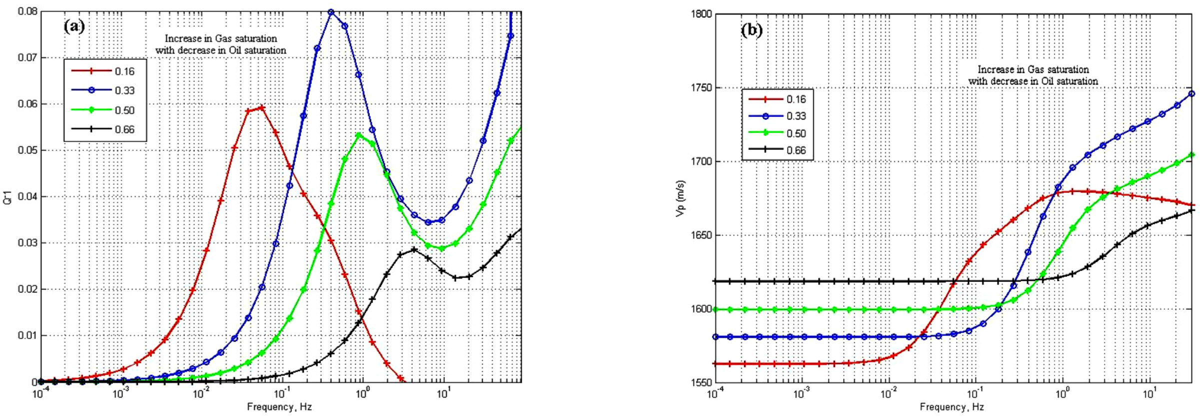

Fig. 9 The attenuation and Velocity dispersion as a function of frequency for the case when the gas saturation get increase (for example gas injection) with decrease in oil saturation at constant water saturation. (a) Attenuation as a function of frequency (Hz) and (b) Velocity dispersion as a function of frequency (Hz)

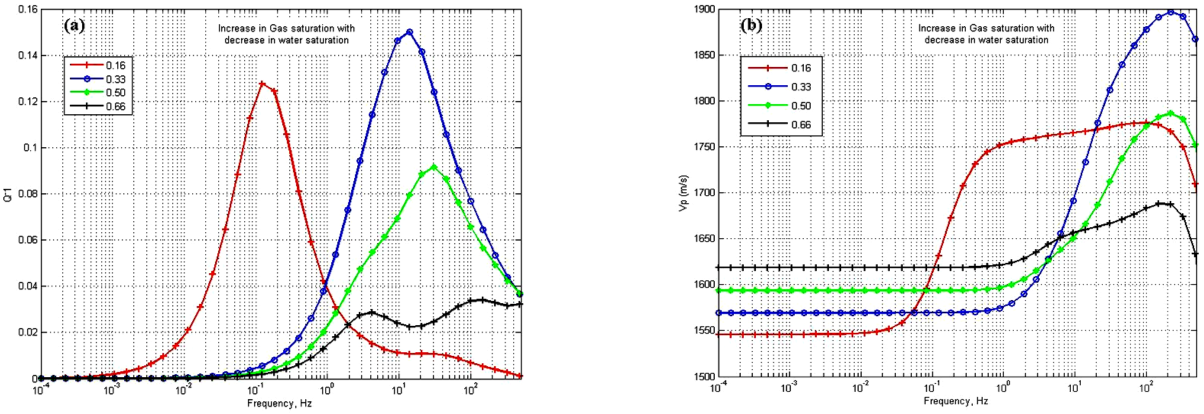

Fig. 10 Wave attenuation and Velocity dispersion as a function of frequency for case-IV where gas saturation increases with the decrease in water saturation while the oil saturation remain constant. (a) Attenuation as a function of frequency (Hz) and (b) Phase velocity as a function of frequency (Hz)

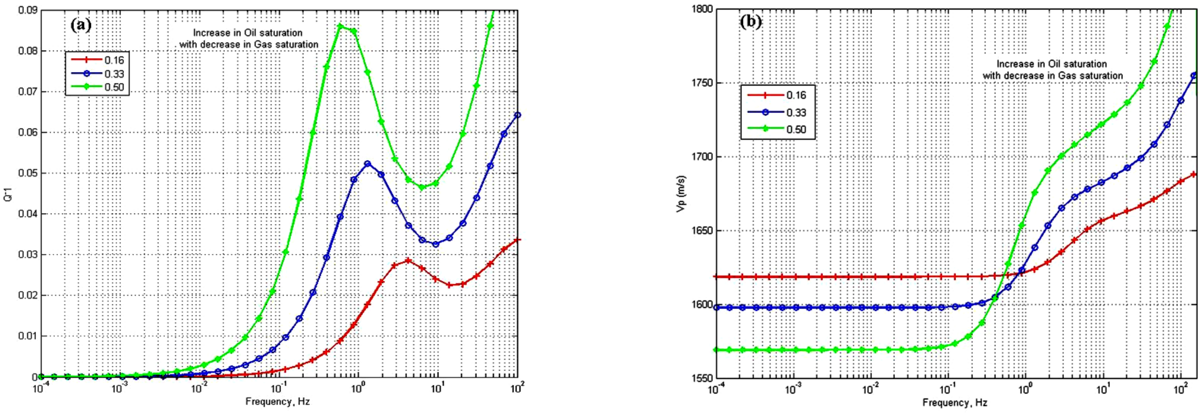

Fig. 11 (a) Attenuation as a function of frequency (Hz) and (b) Phase velocity as a function of frequency (Hz) For the case when the oil saturation increases with the decrease in gas saturation while the water saturation remain constant

saturated with three phase fluids (gas, oil and water) is an essential part in various optimized enhanced oil recovery. In case-III, we demonstrated a situation where gas saturation gradually increased with decrease in oil saturation while the water saturation remains constant (Fig. 6). The computed seismic responses in that scenario reveal that with decrease in gas saturation the attenuation peaks moves towards lower frequencies (Fig. 9) which agree with previous theoretical work. In most naturally accruing three phase flow water act as a wetting fluid, Hui (2000) and the water saturation remain constant, the characteristics of seismic waves through such media reveals that with increase in gas saturation the attenuation peak due to the fluid flow at the interface between water and gas get demolish and turned

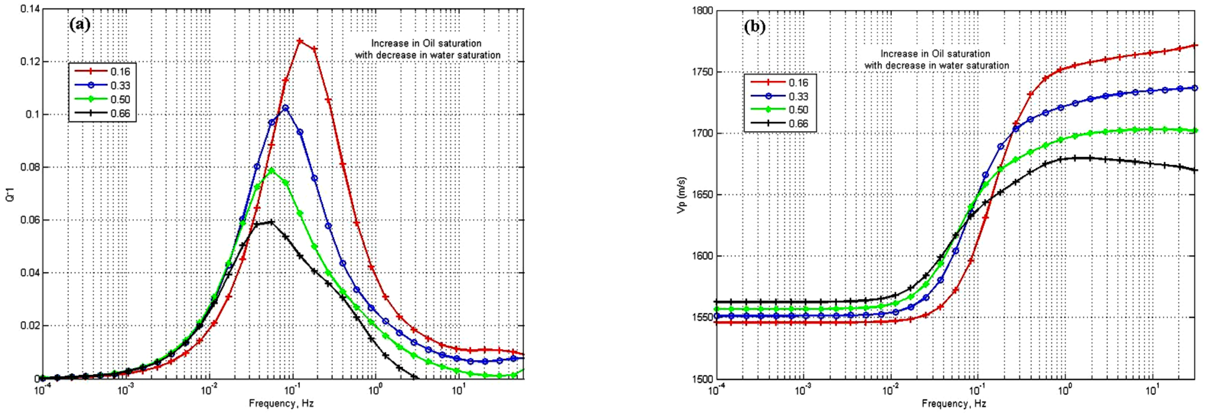

Fig. 12 For the case when at constant gas saturation oil saturation increases with the decrease in water saturation. (a) Attenuation as a function of frequency (Hz) and (b) Phase velocity as a function of frequency (Hz)

into a single attenuation peak.

In enhanced oil recovery, gas displaces oil from the largest pores and water towards the smallest pores, in case-IV a similar situation where at constant oil saturation, the increase in gas saturation with decrease in water saturation is demonstrated (Fig. 6). It can be inferred from Fig. 10 that acoustic properties are strongly influenced by change in gas variation even very small amount of gas saturated in a porous rock have significant influence on wave characteristics within seismic frequency band (1-500 Hz), Vogelaar and Smeulders (2007). As can be seen from the Fig. 10a, attenuation peak moves towards high frequencies with increase in gas saturation along with decrease in water saturation while the oil saturation remain constant and at 33% gas saturation maximum attenuation peak is obtain at about 10 Hz. It is also interesting to notice that the at high water saturation attenuation is maximum below seismic frequencies with only single peak due to fluid flow at the interface between gas and water, the possible reason for that seems to be due to situation where the oil act as a wetting fluid when gas displace both oil and water (Dicarlo *et al.* 2000). Furthermore, with the decrease in water saturation the attenuation peak moves towards seismic frequencies and when oil and water saturation are almost identical we have two low amplitude attenuation peaks indicating the fluid flow at both gas/oil and water gas interface. Although velocity has minimum values when saturation of low compressible fluid (water) is at its maximum which gradually increased with the decrease in water saturation but velocity dispersion is higher when the fluids have high compressibility contrast (33% gas saturation). From velocity dispersion curves (Fig. 10(b)), it can be infer that at frequency where attenuation is maximum, velocity also get more dispersed.

In an oil wet system, at low oil saturation, oil remain in small pores while water in the large pores in case of two-flow phase. So when the gas get injected in such a reservoir for recovery, gas displace the water in the largest pores and become non wetting to both oil and water (Dicarlo *et al.* 2000), for that case oil saturation become maximum. With the gas injection, water get displace first then the oil get displaced with the decrease in gas saturation, while the water saturation remain constant. We demonstrated this scenario in case-V (Fig. 6) and computed the influence of this variation in fluid saturation on acoustic properties. The obtained results thus obtained that with increase in oil saturation the attenuation peaks (Fig. 11(a)) moves towards lower frequencies and the energy loss at the interface between water and gas get disappear which means, at seismic frequencies being highly saturated oil is more responsible in loss of energy then water. Velocity dispersion is also high at maximum oil saturation (Fig. 11(b)).

Case-VI describe the situation where for two phase flow, oil act as a wetting fluid and exist in the small pores while water resides in the large pores. During gas injection water get displace and gas become intermediate wet (Dicarlo *et al.* 2000). The effects of increase in oil saturation on seismic characteristics is computed at seismic and below seismic frequencies. The obtained results reveals that initially at high water saturation attenuation is at its maximum (Fig. 12(a)) and get decrease with the decrease in water saturation. Also, velocity dispersion (Fig. 12(b)) is at its maximum at high water saturation and get decrease with the increase in oil saturation. We can resemble these with the results for case-II where with the increase in water saturation attenuation get increase.

4. Conclusions

On the basis of poroelasticity analysis, it is now conceptualized that wave attenuation and dispersion within the porous media is ruled by varying the nature of the fluid and solid as well as by the fundamental mechanism of wave-induced fluid flow at three different scales. Theory about 'WIFF' reveals that it is derived due to the interaction between fluid and solid and between fluids of a disparate nature at macro, meso, and micro scale. For the computation of P-wave attenuation and dispersion within the seismic frequency band, we have accomplished a rock physics model. In current work we have presented a novel solution to analyse the effect on P-wave attenuation and modulus dispersion by incorporating the effect of multi-scale heterogeneities on wave attenuation and velocity dispersion. By this, we have analysed the effect of WIFF on P-wave attenuation and velocity dispersion within seismic and low seismic frequencies.

From the outcomes of numerical experiments on a periodic model meso-scopically saturated with three phase fluid, some useful conclusions can be drawn: Increase in water saturation is associated with decrease in saturation of both oil and gas and attenuation peak moves to lower frequency and is maximum when gas saturation is at 16%. Increase in gas saturation is associated with decrease in saturation of both oil and water and attenuation peak move towards higher frequencies, Increase in oil saturation is associated with decrease in saturation of both gas and water and attenuation peak moves towards low frequencies. Moreover, it is also concluded that the analysis of P-wave attenuation and velocity dispersion in a complex media (i.e., saturation of three fluid phases) is a potential indicator of hydrocarbon saturation.

As, in current study, we have analysed a more realistic and more complex media, by incorporating the influence of mesoscopic and microscopic fluid flow, and the variation of fluid saturation, on P-wave characteristics. This numerical approach can be a potential tool in various time lapse seismic cases like in hydrocarbon production, improved oil recovery by water flooding or by gas injection. It can also be utilized in investigating the low-frequency anomalies in passive seismic data, which can be further correlated with location of hydrocarbon reservoir and also in testing the mechanical behaviour of medical instruments at low seismic frequencies.

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