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A new innovative 3-unknowns HSDT for buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions

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Abstract. In this study a new innovative three unknowns trigonometric shear deformation theory is proposed for the buckling and vibration responses of exponentially graded sandwich plates resting on elastic mediums under various boundary conditions. The key feature of this theoretical formulation is that, in addition to considering shear deformation effect, it has only three unknowns in the displacement field as in the case of the classical plate theory (CPT), contrary to five as in the first shear deformation theory (FSDT) and higher-order shear deformation theory (HSDT). Material characteristics of the sandwich plate faces are considered to vary within the thickness direction via an exponential law distribution as a function of the volume fractions of the constituents. Equations of motion are obtained by employing Hamilton's principle. Numerical results for buckling and free vibration analysis of exponentially graded sandwich plates under various boundary conditions are obtained and discussed. Verification studies confirmed that the present three -unknown shear deformation theory is comparable with higher-order shear deformation theories which contain a greater number of unknowns.

Keywords: functionally graded materials; sandwich plates; a 3-unknown theory; various boundary conditions; elastic; foundations; free vibration; buckling loads

1. Introduction

In recent years, the use of functionally graded (FG) sandwich structures in engineering fields such as civil engineering, aerospace, shipbuilding and machinery sectors has attracted intensive research interests due to their high strength-to-weight ratio. As a result, the mechanical response of FG sandwich structures is of considerable importance in both academic research and industrial fields (Sofiyev *et al.* 2012, Yaghoobi and Yaghoobi 2013, Swaminathan and Naveenkumar 2014, Kar and Panda 2015, Ahmed *et al.* 2019, Mehar and Panda 2018, Rezaiee-Pajand *et al.* 2018, Dash *et al.* 2018, Karami *et al.* 2019, Mehar *et al.* 2019 and 2020, Avcar 2019, Kirlangiç and Akbaş 2020). Since the shear deformation influences are considerably important in thick plates or plates manufactured of FGMs, shear deformation theories such as first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) should be utilized to investigate the mechanical responses of FG plates.

The FSDT generates reasonable results, but requires the use of a shear correction parameter (Civalek and Acar 2007, Mousavi and Tahani 2012, Malekzadeh and Monajjemzadeh 2013, Avcar 2016, Shokravi 2017, Hamidi *et al.* 2018, Bensattalah *et al.* 2019, Mirjavadi *et al.* 2019a). Whereas, the HSDTs (Touratier 1991, Soldatos 1992, Reddy 2000, Karama *et al.* 2003, Pradyumna and Bandyopadhyay 2008, Talha and Singh 2010, Sobhy 2013, Panda and Katariya 2015, Katariya and Panda 2016, Mehar *et al.* 2016 and 2017, Kar and Panda 2017, Katariya *et al.* 2017, Katariya and Panda 2019ab and 2020, Mehar and Panda 2019, Katariya *et al.* 2018 and 2019, Abdulrazzaq *et al.* 2020) do not need a shear correction parameter, but their governing equations are more complex than those of the FSDT.

Recently, new shear deformation theories involving only four unknowns have been developed (Ghugal and Sayyad 2011, Nguyen 2014, Zhang *et al.* 2015, Sobhy 2016, Adim *et al.* 2016, Abdelmalek *et al.* 2017, Zouatnia *et al.* 2018),

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but these theories lead to the resolution of a number of equations of motion more than of those used in classical plate theory (CPT) (Akgoz and Civalek 2011, Avcar and Mohammed 2018). Thus, needs exist for the development of HSDTs which are simple to use.

In the present work, an efficient and simple formulation with shear deformation effect is developed for the buckling and vibrational analyses of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions. The highlight of this new formulation is that, in addition to incorporating the shear deformation effect, the displacement components are expressed with only three unknowns as the classical plate theory (CPT), which is even less than the FSDT and do not requires shear correction parameter. The sandwich plate is considered to be supported by isotropic or orthotropic two-parameter elastic foundations. The equations of motion of exponentially graded sandwich plates resting on elastic foundation are obtained by utilizing the Hamilton's principle. These equations of motion for the sandwich plate under various boundary conditions are then solved. As a result, critical buckling loads and natural frequencies are determined by solving eigenvalue problem. The performance of the present formulation is verified by comparing it with HSDT's solutions available in literature. It can be concluded that the present method is as accurate as other HSDTs with higher number of unknowns and so deserves attention.

2. Theoretical formulation

Consider a sandwich plate with three layers as depicted in Fig. 1. Two FG face sheets are synthesized of a mixture of a metal and a ceramic, while the core is manufactured from an isotropic homogeneous material. The material properties $\bar{P}$ of FG face sheets such as Young's modulus E and the material density ρ, are supposed to vary continuously within the plate thickness by an exponential law distribution as (Sobhy 2013):

$$\overline{P}^{(n)}(z)=\overline{P}_m\exp\left(\beta\, V^{(n)}\right),\quad \beta=\ln\left(\frac{\overline{P}_c}{\overline{P}_m}\right),\quad (n=1,\,2,\,3) \tag{1}$$

where subscripts m and c refer to metal and ceramic, and the volume fraction $V^{(n)}$ of each layer is given by:

$$V^{(1)}=\left(\frac{2\bar{z}+1}{2\bar{h}_1+1}\right)^k,\qquad -1/2\le\bar{z}\le\bar{h}_1 \tag{2a}$$

$$V^{(2)}=1,\qquad \bar{h}_1\le\bar{z}\le\bar{h}_2 \tag{2b}$$

$$V^{(3)}=\left(\frac{2\bar{z}-1}{2\bar{h}_2-1}\right)^k,\qquad \bar{h}_2\le\bar{z}\le 1/2 \tag{2c}$$

where $\bar{z}=z/h$, $\bar{h}_i=h_i/h$ (i=) and k is the inhomogeneity parameter which takes values greater than or equal to zero. It is noted that the core is independent of the value of k which is fully ceramic.

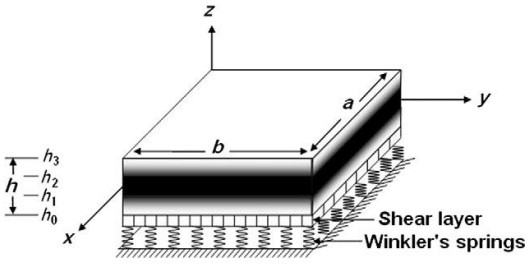

Fig. 1 Geometry of the exponentially graded sandwich plate resting on elastic foundations

2.1 Kinematics and constitutive equations

The displacement field of the novel theory is given as follows:

$$\begin{aligned} u(x,y,z,t)&=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}-\beta\, f(z)\frac{\partial^3 w_0}{\partial x^3}\\ v(x,y,z,t)&=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}-\beta\, f(z)\frac{\partial^3 w_0}{\partial y^3}\\ w(x,y,z,t)&=w_0(x,y,t)\end{aligned} \tag{3}$$

where u_0, v_0, and w_0 are three unknown displacement functions of middle surface of the plate and β is a parameter of the present displacement model.

In this work the displacement field contains a shape functions expressed as:

$$f(z)=\frac{h}{\pi}\sin\frac{\pi z}{h} \tag{4}$$

The nonzero linear strains associated with the displacement field in Eq. (3) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}+\beta\, f(z)\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix},\qquad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\beta\, g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}, \tag{5}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^4 w_0}{\partial x^4}\\ -\dfrac{\partial^4 w_0}{\partial y^4}\\ -\dfrac{\partial^2(\nabla^2 w_0)}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^3 w_0}{\partial y^3}\\ -\dfrac{\partial^3 w_0}{\partial x^3}\end{Bmatrix}, \tag{6}$$

and

$$g(z)=f'(z),\quad \nabla^2 w_0=\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2} \tag{7}$$

For the exponentially graded sandwich plates, the stress–strain relation- ships can be written as:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}^{(n)}=\begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}^{(n)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{8}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The elastic constants C_{ij} are defined as

$$C_{11}^{(n)}=C_{22}^{(n)}=\frac{E^{(n)}(z)}{1-\nu^2},\quad C_{12}^{(n)}=\nu\, C_{11}^{(n)},$$
$$C_{44}^{(n)}=C_{55}^{(n)}=C_{66}^{(n)}=\frac{E^{(n)}(z)}{2(1+\nu)}, \tag{9}$$

2.2 Equations of motion

Hamilton's principle is utilized herein to determine equations of motion. The principle can be stated in an analytical form as follows (Ebrahimi and Barati 2017a, Eltaher *et al.* 2018, Hadji *et al.* 2019, Sahouane *et al.* 2019, Fenjan *et al.* 2019ab, Safa *et al.* 2019, Mirjavadi *et al.* 2019b, Hamed *et al.* 2020, Eltaher and Mohamed 2020):

$$0=\int_0^T\left(\delta U+\delta V-\delta K\right)dt \tag{10}$$

where δU is the variation of strain energy; δV is the variation of work induced by the external forces; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is calculated by

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right]dV\\&=\int_A\Big[N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x\delta k_x+M_y\delta k_y+M_{xy}\delta k_{xy}\\&\quad+\beta(S_x\delta\eta_x+S_y\delta\eta_y+S_{xy}\delta\eta_{xy}+Q_{yz}\delta\gamma_{yz}^0+Q_{xz}\delta\gamma_{xz}^0)\Big]dA=0\end{aligned} \tag{11}$$

where A is the top surface and the stress resultants N, M, S and Q are defined by:

$$(N_i,M_i,S_i)=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}(1,z,\beta f)(\sigma_i)dz,\ (i=x,y,xy)\ \text{ and}$$
$$Q_i=\sum_{n=1}^{3}(\tau_i)\,\beta\, g(z)dz,\ (i=xz,yz) \tag{12}$$

The variation of work done by the applied loads can be expressed as

$$\delta V=-\int_A(P-f_e)\delta\left(w_b+w_s\right)dA \tag{13}$$

where f_e is the density of reaction force of foundation. For the Pasternak foundation model:

$$f_e=K_W w-K_{S1}\frac{\partial^2 w}{\partial x^2}-K_{S2}\frac{\partial^2 w}{\partial y^2} \tag{14}$$

where K_W is the modulus of subgrade reaction (elastic coefficient of the foundation) and K_{S1} and K_{S2} are the shear moduli of the subgrade (shear layer foundation stiffness). If foundation is homogeneous and isotropic, we will get K_{S1}= K_{S2}=K_S. If the shear layer foundation stiffness is neglected, Pasternak foundation becomes a Winkler foundation with

$$P=\left[P_x^0\frac{\partial^2 w}{\partial x^2}+P_y^0\frac{\partial^2 w}{\partial y^2}+2P_{xy}^0\frac{\partial^2 w}{\partial x\partial y}\right] \tag{15}$$

The variation of kinetic energy is written as

$$\begin{aligned}\delta K&=\int_V\left[\dot u\delta\dot u+\dot v\delta\dot v+\dot w\delta\dot w\right]\rho(z)dV\\&=\int_A\Big\{I_0\left[\dot u_0\delta\dot u_0+\dot v_0\delta\dot v_0+\dot w_0\delta\dot w_0\right]\\&\quad-I_1\left(\dot u_0\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial x}\delta\dot u_0+\dot v_0\frac{\partial\delta\dot w_0}{\partial y}+\frac{\partial\dot w_0}{\partial y}\delta\dot v_0\right)\\&\quad-J_1\beta\left(\dot u_0\frac{\partial^3\delta\dot w_0}{\partial x^3}+\frac{\partial^3\dot w_0}{\partial x^3}\delta\dot u_0+\dot v_0\frac{\partial^3\delta\dot w_0}{\partial y^3}+\frac{\partial^3\dot w_0}{\partial y^3}\delta\dot v_0\right)\\&\quad+I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right)\\&\quad+K_2\beta^2\left(\frac{\partial^3\dot w_0}{\partial x^3}\frac{\partial^3\delta\dot w_0}{\partial x^3}+\frac{\partial^3\dot w_0}{\partial y^3}\frac{\partial^3\delta\dot w_0}{\partial y^3}\right)\\&\quad+J_2\beta\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial^3\delta\dot w_0}{\partial x^3}+\frac{\partial^3\dot w_0}{\partial x^3}\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial y}\frac{\partial^3\delta\dot w_0}{\partial y^3}+\frac{\partial^3\dot w_0}{\partial y^3}\frac{\partial\delta\dot w_0}{\partial y}\right)\Big\}dA\end{aligned} \tag{16}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\left(1,z,f,z^2,zf,f^2\right)\rho(z)dz \tag{17}$$

Using the expressions for δU, δV, and δK from Eqs. (11), (14), and (16) into Eq. (10) and integrating by parts, and collecting the coefficients of δu_0, δv_0 and δw_0, the following equations of motion of the plate are obtained

$$\begin{aligned}&\delta u_0:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot u_0-I_1\frac{\partial\ddot w_0}{\partial x}-\beta J_1\frac{\partial^3\ddot w_0}{\partial x^3}\\&\delta v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot v_0-I_1\frac{\partial\ddot w_0}{\partial y}-\beta J_1\frac{\partial^3\ddot w_0}{\partial y^3}\\&\delta w_0:\ \frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}\\&\quad+\beta\left(\frac{\partial^4 S_x}{\partial x^4}+\frac{\partial^4 S_{xy}}{\partial x^3\partial y}+\frac{\partial^4 S_{xy}}{\partial y^3\partial x}+\frac{\partial^4 S_y}{\partial y^4}-\frac{\partial^3 Q_{xz}}{\partial x^3}-\frac{\partial^3 Q_{yz}}{\partial y^3}\right)\\&\quad-P-f_e=I_0\ddot w_0+I_1\left(\frac{\partial\ddot u_0}{\partial x}+\frac{\partial\ddot v_0}{\partial y}\right)-I_2\left(\frac{\partial^2\ddot w_0}{\partial x^2}+\frac{\partial^2\ddot w_0}{\partial y^2}\right)\\&\quad+\beta J_1\left(\frac{\partial^3\ddot u_0}{\partial x^3}+\frac{\partial^3\ddot v_0}{\partial y^3}\right)-2\beta J_2\left(\frac{\partial^4\ddot w_0}{\partial x^4}+\frac{\partial^4\ddot w_0}{\partial y^4}\right)-\beta^2K_2\left(\frac{\partial^6\ddot w_0}{\partial x^6}+\frac{\partial^6\ddot w_0}{\partial y^6}\right)\end{aligned} \tag{18}$$

Substituting Eq. (5) into Eq. (8) and the subsequent results into Eqs. (12), the following constitutive equations are found:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ S_x \\ S_y \\ S_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & \beta B_{11}^s & \beta B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & \beta B_{12}^s & \beta B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & \beta B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & \beta D_{11}^s & \beta D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & \beta D_{12}^s & \beta D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & \beta D_{66}^s \\ \beta B_{11}^s & \beta B_{12}^s & 0 & \beta D_{11}^s & \beta D_{12}^s & 0 & \beta^2 H_{11}^s & \beta^2 H_{12}^s & 0 \\ \beta B_{12}^s & \beta B_{22}^s & 0 & \beta D_{12}^s & \beta D_{22}^s & 0 & \beta^2 H_{12}^s & \beta^2 H_{22}^s & 0 \\ 0 & 0 & \beta B_{66}^s & 0 & 0 & \beta D_{66}^s & 0 & 0 & \beta^2 H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \\ \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} \tag{19a}$$

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \beta^2 \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{19b}$$

where

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} C_{11}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{20a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) \tag{20b}$$

$$A_{44}^s = A_{55}^s = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} C_{44}\left[g(z)\right]^2 dz, \tag{20c}$$

The equations of motion of the present three-unknown trigonometric shear deformation theory can be written in terms of displacements (u_0, v_0 and w_0) by substituting Eq.(6) into Eqs. (19) and the subsequent results into Eq. (18)

$$\begin{aligned} &A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y} - B_{11}\frac{\partial^3 w}{\partial x^3} \\ &-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w}{\partial x \partial y^2} - \beta\left(B_{66}^s\frac{\partial^5 w}{\partial x^3 \partial y^2} + \left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 w}{\partial x \partial y^4} + B_{11}^s\frac{\partial^5 w}{\partial x^5}\right) = \\ &I_0\ddot{u}_0 - I_1\frac{\partial \ddot{w}_0}{\partial x} - \beta J_1\frac{\partial^3 \ddot{w}_0}{\partial x^3} \end{aligned} \tag{21a}$$

$$\begin{aligned} &A_{22}\frac{\partial^2 v_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + \left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y} - B_{22}\frac{\partial^3 w}{\partial y^3} \\ &-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w}{\partial x^2 \partial y} - \beta\left(B_{66}^s\frac{\partial^5 w}{\partial x^2 \partial y^3} + \left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 w}{\partial x^4 \partial y} + B_{22}^s\frac{\partial^5 w}{\partial y^5}\right) = \\ &I_0\ddot{v}_0 - I_1\frac{\partial \ddot{w}_0}{\partial y} - \beta J_1\frac{\partial^3 \ddot{w}_0}{\partial y^3} \end{aligned} \tag{21b}$$

$$\begin{aligned} &B_{11}\frac{\partial^3 u_0}{\partial x^3} + \left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x \partial y^2} + \left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2 \partial y} \\ &+B_{22}\frac{\partial^3 v_0}{\partial y^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} - 2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} \\ &-D_{22}\frac{\partial^4 w_0}{\partial y^4} + \beta\left[B_{11}^s\frac{\partial^5 u_0}{\partial x^5} + \left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 u_0}{\partial x \partial y^4}\right. \\ &\left.+\left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 v_0}{\partial x^4 \partial y} + B_{22}^s\frac{\partial^5 v_0}{\partial y^5} + B_{66}^s\frac{\partial^5 u_0}{\partial x^3 \partial y^2} + B_{66}^s\frac{\partial^5 v_0}{\partial x^2 \partial y^3}\right. \\ &\left.-2D_{11}^s\frac{\partial^6 w_0}{\partial x^6} - 2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^6 w_0}{\partial x^2 \partial y^4}\right. \\ &\left.-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^6 w_0}{\partial x^4 \partial y^2} - 2D_{22}^s\frac{\partial^6 w_0}{\partial y^6}\right] - \beta^2\left[H_{11}^s\frac{\partial^8 w_0}{\partial x^8}\right. \\ &\left.+2\left(H_{12}^s+H_{66}^s\right)\frac{\partial^8 w_0}{\partial x^4 \partial y^4} + H_{66}^s\frac{\partial^8 w_0}{\partial x^6 \partial y^2} + H_{66}^s\frac{\partial^8 w_0}{\partial x^2 \partial y^6} + H_{22}^s\frac{\partial^8 w_0}{\partial y^8}\right. \\ &\left.-A_{44}^s\frac{\partial^6 w_0}{\partial x^6} - A_{55}^s\frac{\partial^6 w_0}{\partial y^6}\right] - P - f_e = I_0\ddot{w}_0 + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) \\ &-I_2\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2} + \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) + \beta J_1\left(\frac{\partial^3 \ddot{u}_0}{\partial x^3} + \frac{\partial^3 \ddot{v}_0}{\partial y^3}\right) - 2\beta J_2\left(\frac{\partial^4 \ddot{w}_0}{\partial x^4} + \frac{\partial^4 \ddot{w}_0}{\partial y^4}\right) \\ &-\beta^2 K_2\left(\frac{\partial^6 \ddot{w}_0}{\partial x^6} + \frac{\partial^6 \ddot{w}_0}{\partial y^6}\right) \end{aligned} \tag{21c}$$

3. Analytical solutions

The analytical solution of Eq. (21) can be determined for sandwich plates under various boundary conditions by employing the following expansions of generalized displacements:

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \begin{Bmatrix} U_{mn}\dfrac{\partial X_m(x)}{\partial x}Y_n(y)\,e^{i\,\omega\,t} \\ V_{mn}X_m(x)\dfrac{\partial Y_n(y)}{\partial y}\,e^{i\,\omega\,t} \\ W_{mn}X_m(x)Y_n(y)\,e^{i\,\omega\,t} \end{Bmatrix} \tag{22}$$

where $i=\sqrt{-1}$, (U_{mn}, V_{mn}, W_{mn}) are coefficients, and $\omega=\omega_{mn}$ denotes the eigenfrequency associated with (m, n)$^{\text{th}}$ eigenmode. The functions $X_m(x)$ and $Y_n(y)$ are suggested by Sobhy (2013) to satisfy various boundary conditions, and they are listed in Table 1 noting that $\lambda=m\pi/a$ and $\mu=n\pi/b$.

Substituting expressions (22) into Eqs. (21) and multiplying each equation by the corresponding eigenfunction then integrating over the domain of solution, we can determine, after some mathematical manipulations, the following expressions:

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} - \omega^2\begin{bmatrix} m_{11} & 0 & m_{13} \\ 0 & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

in which:

$$\begin{aligned} S_{11} &= A_{11}\alpha_{12} + A_{66}\alpha_8 \\ S_{12} &= \left(A_{12}+A_{66}\right)\alpha_8 \\ S_{13} &= -B_{11}\alpha_{12} - \left(B_{12}+2B_{66}\right)\alpha_8 \\ &\quad -\beta\left(B_{11}^s\alpha_{15} + \left(B_{12}^s+B_{66}^s\right)\alpha_{14} + B_{66}^s\alpha_{28}\right) \\ S_{21} &= \left(A_{12}+A_{66}\right)\alpha_{10} \\ S_{22} &= A_{22}\alpha_4 + A_{66}\alpha_{10} \\ S_{23} &= -B_{11}\alpha_4 - \left(B_{12}+2B_{66}\right)\alpha_{10} \\ &\quad -\beta\left(B_{22}^s\alpha_{17} + \left(B_{12}^s+B_{66}^s\right)\alpha_{33} + B_{66}^s\alpha_{16}\right) \\ S_{31} &= B_{11}\alpha_{13} + \left(B_{12}+2B_{66}\right)\alpha_{11} \\ &\quad -\beta\left(B_{11}^s\alpha_{18} + \left(B_{12}^s+B_{66}^s\right)\alpha_{19} + B_{66}^s\alpha_{26}\right) \\ S_{32} &= B_{22}\alpha_5 + \left(B_{12}+2B_{66}\right)\alpha_{11} \\ &\quad -\beta\left(B_{22}^s\alpha_{27} + \left(B_{12}^s+B_{66}^s\right)\alpha_{26} + B_{66}^s\alpha_{19}\right) \end{aligned} \tag{24}$$

Table 1 The admissible functions $X_m(x)$ and $Y_n(y)$

	Boundary conditions		The functions X_m and Y_n	
	At x=0,a	At y=0, b	$X_m(x)$	$Y_n(y)$
SSSS	$X_m(0)=X_m''(0)=0$	$Y_n(0)=Y_n''(0)=0$	$\sin(\lambda x)$	$\sin(\mu y)$
	$X_m(a)=X_m''(a)=0$	$Y_n(b)=Y_n''(b)=0$		
CSSS	$X_m(0)=X_m'(0)=0$	$Y_n(0)=Y_n''(0)=0$	$\sin(\lambda x)\left[\cos(\lambda x)-1\right]$	$\sin(\mu y)$
	$X_m(a)=X_m''(a)=0$	$Y_n(b)=Y_n''(b)=0$		
CSCS	$X_m(0)=X_m'(0)=0$	$Y_n(0)=Y_n'(0)=0$	$\sin(\lambda x)\left[\cos(\lambda x)-1\right]$	$\sin(\mu x)\left[\cos(\mu x)-1\right]$
	$X_m(a)=X_m''(a)=0$	$Y_n(b)=Y_n''(b)=0$		
CCSS	$X_m(0)=X_m'(0)=0$	$Y_n(b)=Y_n''(b)=0$	$\sin^2(\lambda x)$	$\sin(\mu y)$
	$X_m(a)=X_m'(a)=0$	$Y_n(b)=Y_n''(b)=0$		
CCCC	$X_m(0)=X_m'(0)=0$	$Y_n(0)=Y_n''(0)=0$	$\sin^2(\lambda x)$	$\sin^2(\mu y)$
	$X_m(a)=X_m'(a)=0$	$Y_n(b)=Y_n'(b)=0$		
FFCC	$X_m''(0)=X_m'''(0)=0$	$Y_n(0)=Y_n'(0)=0$	$\cos^2(\lambda x)\left[\sin^2(\lambda x)+1\right]$	$\sin^2(\mu y)$
	$X_m''(a)=X_m'''(a)=0$	$Y_n(b)=Y_n'(b)=0$		

()' Denotes the derivative with respect to the corresponding coordinates

$$
\begin{aligned}
&S_{33}=-D_{11}\alpha_{13}-2\left(D_{12}+2D_{66}\right)\alpha_{11}-D_{22}\alpha_5-\beta D_{11}^s\alpha_{18}\\
&-\beta D_{22}^s\alpha_{27}-2\beta\left(\left(D_{12}^s+2D_{66}^s\right)\alpha_{19}+\left(D_{12}^s+2D_{66}^s\right)\alpha_{26}\right)-\\
&\beta^2\begin{pmatrix}H_{11}^s\alpha_7+2\left(H_{12}^s+H_{66}^s\right)\alpha_{21}+H_{66}^s\left(\alpha_{20}+\alpha_{22}\right)\\+H_{22}^s\alpha_{23}+A_{44}^s\alpha_{18}+A_{55}^s\alpha_{27}\end{pmatrix}\\
&+K_{S1}\alpha_9+K_{S2}\alpha_3-K_W\alpha_1-P_x^0\alpha_9-P_y^0\alpha_3
\end{aligned}
\tag{24}
$$

and

$$
\begin{aligned}
m_{11}&=-I_0\alpha_6\\
m_{13}&=I_1\alpha_6+J_1\alpha_{12}\\
m_{22}&=-I_0\alpha_2\\
m_{23}&=I_1\alpha_2+J_1\alpha_4\\
m_{31}&=-I_1\alpha_{31}+\beta J_1\alpha_{13}\\
m_{32}&=-I_1\alpha_{31}+\beta J_1\alpha_5\\
m_{33}&=-I_0\alpha_1+I_2\left(\alpha_3+\alpha_9\right)+2\beta J_2\left(\alpha_{11}+\alpha_5\right)\\
&+\beta^2 K_2\left(\alpha_{27}+\alpha_{18}\right)
\end{aligned}
\tag{25}
$$

with

$$
\begin{aligned}
&\left(\alpha_1,\alpha_3,\alpha_5\right)=\int_0^b\int_0^a\left(X_mY_m,X_mY'',X_mY_m^{(IV)}\right)X_mY_n\,dxdy\\
&\left(\alpha_2,\alpha_4,\alpha_{10},\alpha_{33},\alpha_{16},\alpha_{17}\right)=\\
&\int_0^b\int_0^a\left(X_mY_n',X_mY_n''',X_m''Y_n',X_m^{(IV)}Y_n',X_m''Y_n''',X_mY_n^{(V)}\right)X_mY_n'dxdy\\
&\left(\alpha_6,\alpha_8,\alpha_{12}\right)=\int_0^b\int_0^a\left(X_m'Y_n,X_m'Y_n'',X_m'''Y_n\right)X_m'Y_n\,dxdy\\
&\left(\alpha_{28},\alpha_{14},\alpha_{15}\right)=\int_0^b\int_0^a\left(X_m'''Y_n'',X_m'Y_n^{(IV)},X_m^{(V)}Y_n\right)X_m'Y_n\,dxdy
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\left(\alpha_7,\alpha_9,\alpha_{11},\alpha_{13},\alpha_{18},\alpha_{19},\alpha_{20}\right)=\\
&\int_0^b\int_0^a\left(X_m^{(VIII)}Y_n,X_m''Y_n,X_m''Y_n'',X_m^{(IV)}Y_n,X_m^{(VI)}Y_n,X_m''Y_n^{(IV)},X_m^{(VI)}Y_n''\right)X_mY_n\,dxdy\\
&\left(\alpha_{27},\alpha_{21},\alpha_{22},\alpha_{23},\alpha_{26},\alpha_{31}\right)=\\
&\int_0^b\int_0^a\left(X_mY_m^{(VI)},X_m^{(IV)}Y_m^{(IV)},X_m''Y_m^{(VI)},X_mY_m^{(VIII)},X_m^{(IV)}Y'',X_m'Y_n\right)X_mY_n\,dxdy
\end{aligned}
\tag{26}
$$

The non-trivial solution is determined when the determinant of Eq. (23) equals zero. For the free vibration problem, we have $P_x^0=P_y^0=P_{xy}^0=0$. While for the buckling analysis, we put $\omega=P_{xy}^0=0$; $P_x^0=\overline{P}$ and $P_y^0=\xi\,\overline{P}$, i.e., $\xi=P_y^0/P_x^0$.

It is noted that the present solution depends on the choice of parameter β of the proposed theory. In this study, the adequate value is taken as a solution of the eigenvalue problem $\left|S_{ij}\right|=0$.

4. Numerical results and discussions

For checking the accuracy of the developed model, several comparisons and parametric studies are presented in the following. All presented results are in dimensionless form as

$$
\omega^*=\frac{\omega a^2}{h};\frac{\overline{P}a^2}{100\,h^3};K_0=\frac{a^4K_W}{D};J_0=\frac{a^2K_{S1}}{D}=\frac{b^2K_{S2}}{D};D=\frac{h^3E_C}{12(1-\nu^2)}
\tag{27}
$$

The properties of the materials used in the FG-sandwich plate are

Ceramic (Alumina, Al_2O_3): E_c=380 GPa, ν=0.3, ρ_c=3800 kg/m^3

Metal (Aluminium, Al): Em= 70 GPa, ν=0.3, ρ_c=2707 kg/m^3

The configurations (1-0-1, 1-1-1, 1-2-1 and 1-3-1) are

Table 2 Comparison of dimensionless frequency "ω^*" of simply supported EGM sandwich square plates "k=1.5" resting on Pasternak's elastic foundations

Scheme	Theory	$K_0 = J_0 = 0$			$K_0 = 100, J_0 = 0$			$K_0 = 100, J_0 = 100$		
		$a/h=5$	$a/h=10$	$a/h=20$	$a/h=5$	$a/h=10$	$a/h=20$	$a/h=5$	$a/h=10$	$a/h=20$
1-0-1	Model 1 [(a)]	0.9655	1.0200	1.0356	1.4125	1.4633	1.4781	4.7808	4.8854	4.9135
	Model 2 [(a)]	0.9647	1.0198	1.0356	1.4121	1.4632	1.4781	4.7808	4.8854	4.9136
	Present	0.9656	1.0201	1.0357	1.4129	1.4633	1.4782	4.7956	4.8857	4.9136
1-1-1	Model 1 [(a)]	1.0811	1.1396	1.1563	1.4697	1.5248	1.5407	4.6537	4.7517	4.7789
	Model 2 [(a)]	1.0807	1.1395	1.1563	1.4695	1.5247	1.5407	4.6537	4.7517	4.7789
	Present	1.0812	1.1396	1.1563	1.4700	1.5248	1.5407	4.6629	4.7519	4.7789
1-2-1	Model 1 [(a)]	1.1872	1.2578	1.2781	1.5392	1.6045	1.6236	4.6099	4.7076	4.7357
	Model 2 [(a)]	1.1872	1.2578	1.2781	1.5392	1.6045	1.6236	4.6100	4.7076	4.7357
	Present	1.1873	1.2578	1.2781	1.5395	1.6045	1.6236	4.6180	4.7078	4.7357
1-3-1	Model 1 [(a)]	1.2663	1.3479	1.3716	1.5954	1.6703	1.6924	4.5910	4.6901	4.7192
	Model 2 [(a)]	1.2666	1.3480	1.3716	1.5956	1.6704	1.6924	4.5911	4.6901	4.7192
	Present	1.2665	1.3479	1.3716	1.5957	1.6703	1.6924	4.5987	4.6902	4.7192

[(a)]Given from Ait Amar Meziane *et al.* (2014)

Table 3 Comparison of dimensionless frequency "ω^*" of (1-1-1) EGM sandwich plates with various boundary conditions (b/a=2, K_0=J_0=10)

B.C	Theory	k=0			k=0.5			k=3.5		
		a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20
FFCC	Model 1 [(a)]	2.4747	2.8519	2.9944	2.1360	2.3371	2.4055	1.8894	1.9923	2.0248
	Model 2 [(a)]	2.4736	2.8516	2.9943	2.1352	2.3368	2.4054	1.8880	1.9918	2.0247
	Present	2.4229	2.7793	2.9660	2.0792	2.2983	2.3926	1.8507	1.9742	2.0194
CCCC	Model 1 [(a)]	2.3473	2.6053	2.6953	2.0305	2.1658	2.2093	1.8189	1.8906	1.9124
	Model 2 [(a)]	2.3467	2.6052	2.6952	2.0301	2.1657	2.2093	1.8182	1.8904	1.9123
	Present	2.3448	2.6035	2.6947	2.0285	2.1649	2.2091	1.8178	1.8902	1.9123
CSCS	Model 1 [(a)]	2.3349	2.5270	2.5906	2.0360	2.1399	2.1722	1.8539	1.9148	1.9328
	Model 2 [(a)]	2.3345	2.5269	2.5906	2.0358	2.1398	2.1721	1.8535	1.9146	1.9328
	Present	2.3123	2.5164	2.5875	2.0227	2.1351	2.1708	1.8474	1.9128	1.9323
CCSS	Model 1 [(a)]	2.2746	2.5175	2.6015	1.9691	2.0959	2.1364	1.7665	1.8334	1.8536
	Model 2 [(a)]	2.2740	2.5173	2.6014	1.9687	2.0957	2.1363	1.7658	1.8331	1.8535
	Present	2.2644	2.5133	2.6003	1.9636	2.0940	2.1358	1.7640	1.8326	1.8534
CSSS	Model 1 [(a)]	2.2207	2.3931	2.4496	1.9408	2.0334	2.0619	1.7729	1.8270	1.8429
	Model 2 [(a)]	2.2204	2.3930	2.4495	1.9406	2.0334	2.0619	1.7725	1.8269	1.8429
	Present	2.1906	2.3803	2.4458	1.9245	2.0278	2.0604	1.7653	1.8247	1.8423
SSSS	Model 1 [(a)]	1.5388	1.5947	1.6113	1.3990	1.4310	1.4401	1.3376	1.3594	1.3655
	Model 2 [(a)]	1.5387	1.5948	1.6113	1.3990	1.4310	1.4401	1.3375	1.3594	1.3655
	Present	1.5438	1.5963	1.6117	1.4010	1.4316	1.4403	1.3384	1.3597	1.3655

[(a)]Given from Ait Amar Meziane *et al.* (2014)

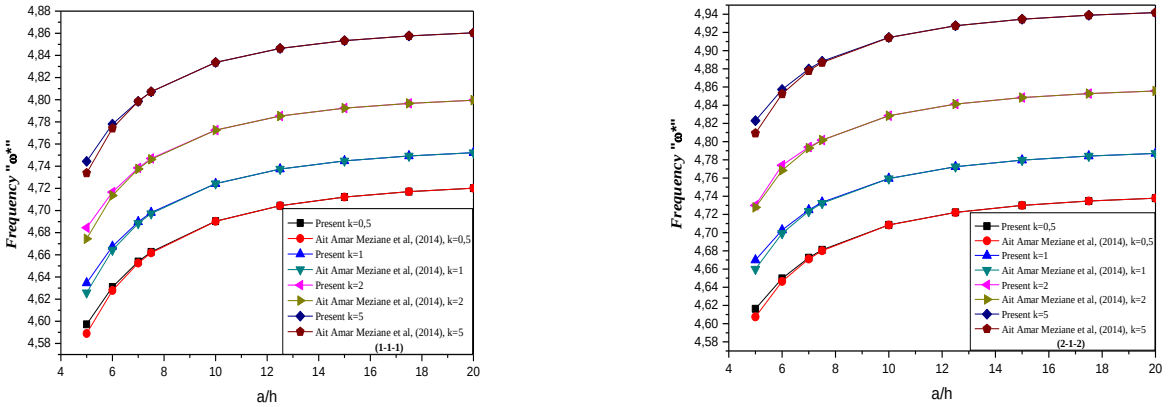

Fig. 2 Dimensionless frequency "ω*" versus the ratio "a/h" for various values of the inhomogeneity parameter "k" and various types of simply- supported EGM sandwich square plates resting on elastic foundations "K_0=J_0=100"

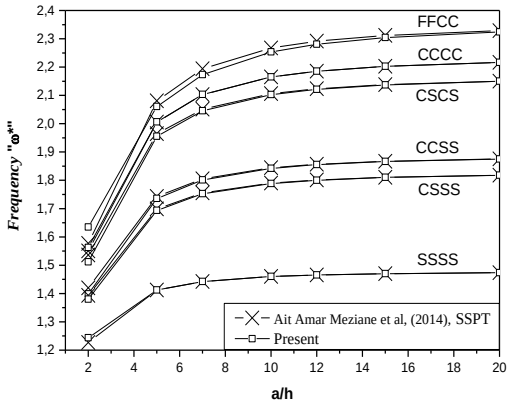

Fig. 3 Dimensionless frequency "ω*" versus the ratio "a/h" of the (2-1-2) E-FG sandwich square plates resting on Winkler's elastic foundation with various boundary conditions(K0=100, k=2)

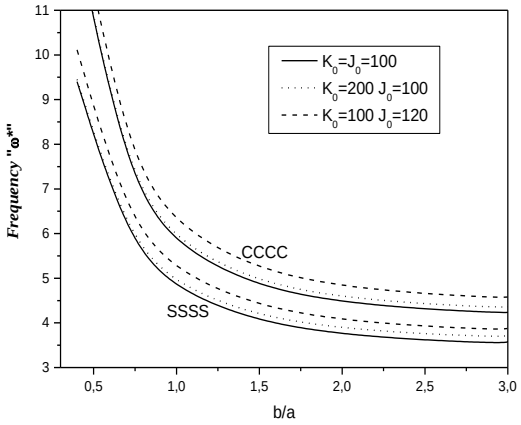

Fig. 4 Dimensionless frequency "ω*" versus the aspect ratio "b/a" of simply-supported and clamped E-FG sandwich plate (1-2-1) for different values of foundation stiffness's "K_0" and "J_0" (a/h=10, k=2)

the layers thicknesses ratio defined as

• 1-0-1 the plate is made of two layers of equal thickness without a core.

• 1-1-1 the plate has the same thicknesses of the layers (core and faces sheets)

• 1-2-1 the plate has the thickness of the core is twice the thickness of face sheets.

• 1-3-1 the plate has the thickness of the core is thrice the thickness of face sheets.

4.1 Free vibration of the FG sandwich plate

In this first part, several numerical results of the free vibrational analysis of the FG-sandwich plate with and without elastic foundation computed by the current three unknowns shear deformation theory are presented in the form of explicit tables and graphs.

To verify the accuracy of the current model, the obtained results of the free vibrational analysis of the simply supported E-FG sandwich plate resting on elastic foundations are compared with the models existing in the literature (Ait Amar Meziane *et al.* 2014). From the results of the fundamental frequency of the E-FG sandwich plate are given in the Table 2, it can be seen that the present model is in good agreement with the two models developed by Ait Amar Meziane *et al.* (2014). We can note from the results that the thick E-FG sandwich plate give the smaller value of the fundamental frequencies. Also, it is clear that the configuration 1-3-1 give the biggest values of the fundamental frequency.

The Table 3 shows the comparison of the fundamental frequency "ω*" of the (1-1-1) rectangular E-FG sandwich plate on the Winkler-Pasternak elastic foundation "J_1=J_2=10" versus the material index "k" and geometry ratio "a/h" with various boundary conditions (FFCC, CCCC, CSCS, CCSS, CSSS and SSSS). From the comparison made in the Table 3, it is clear that the current results obtained using the present three unknowns shear deformation theory are in good agreement those computed by Ait Amar Meziane *et al.* (2014) with four variable refined shear deformation model.

From the table, we can see that the increasing in the values of the material index "k" lead to decrease the fundamental frequencies "ω*" for different boundary conditions.

It can be also concluded that the FG sandwich plate with two free edges and two opposite clamped edges (FFCC) give the biggest values of the frequency "ω*"

The variation of the non-dimensional fundamental

Table 4 Comparison of critical buckling load "$\bar{N}$" of simply supported EGM sandwich square plates "k=0.5" resting on Pasternak's elastic foundations "ξ=1"

Scheme	Theory	K_0=J_0=0			K_0=100, J_0=0			K_0=100, J_0=100		
		a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20
1-0-1	Model 1 [a]	2.5618	2.8127	2.8834	4.3247	4.5756	4.6463	39.1232	39.3741	39.4449
	Model 2 [a]	2.5592	2.8120	2.8833	4.3221	4.5749	4.6462	39.1206	39.3734	39.4447
	Present	2.5618	2.8127	2.8835	4.3247	4.5757	4.6464	39.1233	39.3742	39.4449
1-1-1	Model 1 [a]	3.1030	3.4156	3.5040	4.8659	5.1785	5.2669	39.6644	39.9770	40.0655
	Model 2 [a]	3.1015	3.4152	3.5039	4.8644	5.1781	5.2668	39.6629	39.9766	40.0654
	Present	3.1030	3.4156	3.5040	4.8659	5.1785	5.2670	39.6645	39.9771	40.0655
1-2-1	Model 1 [a]	3.5165	3.9026	4.0129	5.2795	5.6655	5.7758	40.0780	40.4640	40.5744
	Model 2 [a]	3.5166	3.9027	4.0130	5.2795	5.6656	5.7759	40.0780	40.4641	40.5744
	Present	3.5166	3.9026	4.0129	5.2795	5.6655	5.7759	40.0780	40.4641	40.5744
1-3-1	Model 1 [a]	3.8243	4.2743	4.4030	5.5872	6.0364	6.1659	40.3857	40.8349	40.9644
	Model 2 [a]	3.8253	4.2738	4.4031	5.5882	6.0368	6.1660	40.3868	40.8353	40.9646
	Present	3.8243	4.2735	4.4030	5.5872	6.0364	6.1659	40.3858	40.8349	40.9645

[a] Given from Ait Amar Meziane *et al.* (2014)

Table 5 Comparison of critical buckling load "$\bar{N}$" of (1-1-1) EGM sandwich plates with various boundary conditions (b/a=2, K_0=J_0=10)

B.C	Theory	k=0			k=0.5			k=3.5		
		a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20
FFCC	Model 1 [a]	15.7316	20.6940	22.6679	10.9709	12.9380	13.6121	7.8557	8.5665	8.7850
	Model 2 [a]	15.7165	20.6896	22.6667	10.9609	12.9346	13.6112	7.8423	8.5621	8.7838
	Present	15.2671	19.6685	22.2406	10.4701	12.5168	13.4665	7.5660	8.4128	8.7379
CCCC	Model 1 [a]	13.0716	15.8553	16.8365	9.1520	10.1974	10.5257	6.7161	7.0785	7.1835
	Model 2 [a]	13.0640	15.8532	16.8360	9.1467	10.1958	10.5252	6.7091	7.0764	7.1829
	Present	13.0641	15.8351	16.8293	9.1407	10.1897	10.5233	6.7099	7.0759	7.1827
CSCS	Model 1 [a]	10.9052	12.4673	12.9712	7.7552	8.3183	8.4846	5.8764	6.0659	6.1185
	Model 2 [a]	10.9012	12.4662	12.9709	7.7523	8.3175	8.4843	5.8726	6.0648	6.1182
	Present	10.7534	12.3673	12.9397	7.6747	8.2825	8.4742	5.8427	6.0539	6.1152
CCSS	Model 1 [a]	12.9080	15.5750	16.5080	9.0485	10.0467	10.3584	6.6581	7.0033	7.1028
	Model 2 [a]	12.9010	15.5729	16.5074	9.0435	10.0451	10.3580	6.6514	7.0013	7.1023
	Present	12.8160	15.5259	16.4928	9.0059	10.0293	10.3535	6.6412	6.9975	7.1013
CSSS	Model 1 [a]	10.6425	12.0761	12.5336	7.5981	8.1123	8.2630	5.7919	5.9643	6.0120
	Model 2 [a]	10.6389	12.0752	12.5334	7.5955	8.1116	8.2628	5.7885	5.9633	6.0117
	Present	10.4029	11.9501	12.4954	7.4871	8.0681	8.2505	5.7483	5.9497	6.0080
SSSS	Model 1 [a]	7.5261	7.9091	8.0175	5.7942	5.9239	5.9590	4.8267	4.8685	4.8795
	Model 2 [a]	7.5253	7.9089	8.0175	5.7936	5.9238	5.9590	4.8259	4.8684	4.8795
	Present	7.5804	7.9249	8.0217	5.8129	5.9291	5.9604	4.8328	4.8702	4.8799

[a]Given from Ait Amar Meziane *et al.* (2014)

frequencies "ω*" of simply supported E-FG sandwich plate on elastic foundations versus geometry ratio "a/h" and index "k" are plotted in the Fig. 2. The aspect ratio is taken "a/b=1". From the graphs it can be seen that the non-dimensional fundamental frequencies "ω*" is in direct correlation relation with the both the index "k" and "a/h"

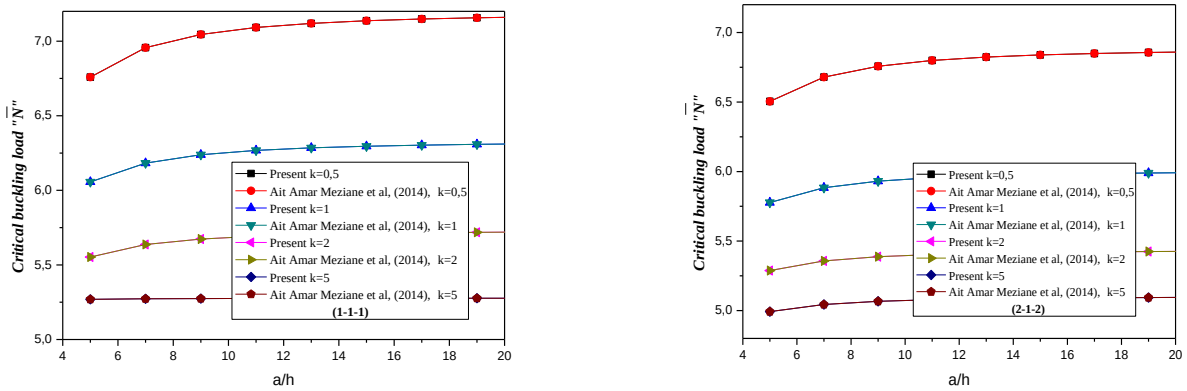

Fig. 5 Critical buckling load "$\bar{N}$" versus the ratio "a/h" for various values of the inhomogeneity parameter "k" and various types of simply- supported EGM sandwich square plates resting on elastic foundations(K_0=J_0=10, ξ=1, a=b)

ratio and this is the same for the 1-1-1 and 2-1-2 FG plate. It can be confirmed again that the results are in good agreement with those given in the literature (Ait Amar Meziane *et al.* 2014).

Fig. 3 plot the non-dimensional frequency "ω*" of 2-1-2 FG-sandwich plate on elastic foundations with "K_0=100, k=2" versus the type of boundary conditions and geometry ratio "a/h". The obtained results are compared with those given by Ait Amar Meziane *et al.* (2014). A good agreement is confirmed between current results and those computed by Ait Amar Meziane *et al.* (2014). From the plotted curves it can be noted that the boundary condition has an important role on the non-dimensional frequency "ω*" it is clear from the graphs that the biggest values of the non-dimensional frequency "ω*" are obtained for FFCC FG-sandwich plate. It is confirmed again that the non-dimensional frequency "ω*" increase with increasing in the values of the "a/h".

Fig. 4 gives the variation of the frequencies "ω*" versus the aspect ratio b/a, boundary conditions and foundations parameters "K_0, J_0" of the 1-2-1 E-FG sandwich plate with "a/h=10, k=2". From the plotted graphs, we can see clearly that the FG-plate with four clamped edges gives the bigger values of the frequencies "ω*" than simply supported FG-plate. It can be observed also that the smaller values of the frequency "ω*" are obtained of the plate with foundations parameters (K_0, J_0=100). Also an inverse relation is observed between the aspect ratio and frequency "ω*".

4.2 Stability of the FG sandwich plate

The second part is reserved to numerical results of the buckling analysis of the E-FG sandwich plate with and without elastic foundation with various boundary conditions.

Table 4 present a comparison of the critical buckling load "$\bar{N}$" of simply supported E-FG sandwich square plates "k=0.5" reposed on Winkler-Pasternak elastic foundations. From the computed results, it can be seen that the current model is in good agreement with those obtained by Ait Amar Meziane *et al.* (2014) for various type of simply supported FG-sandwich plate (1-0-1, 1-1-1, 1-2-1 and 1-3-1). It can be observed from the results that the presence of the Winkler-Pasternak elastic foundations lead to an increase of the critical buckling load of the FG-sandwich

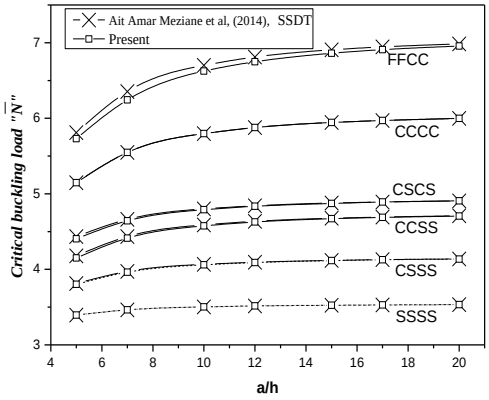

Fig. 6 Critical buckling load "$\bar{N}$" versus the ratio "a/h"of the (2-1-2) EGM sandwich square plates resting on Winkler's elastic foundation with various boundary conditions (K_0=100, k=2, ξ=1)

plate. From the comparison of the various types of the FG-sandwich plate, it is clear that the 1-3-1 FG-plate gives the higher values of the critical buckling load.

Table 5 shows the comparison of the critical buckling load "$\bar{N}$" of the (1-1-1) rectangular E-FG sandwich plate on the elastic foundation "K_0=J_0=10" as function the parameters "k" and "a/h" with different boundary conditions (FFCC, CCCC, CSCS, CCSS, CSSS and SSSS). From the table, it is can be confirmed again that the present model with only three unknown give almost the same results of the critical buckling load "$\bar{N}$" of FG-sandwich plate as those computed by Ait Amar Meziane *et al.* (2014) with RPT model. From the results, we can observe that the critical buckling load "$\bar{N}$" is in inverse relation with the material index "k" for various boundary conditions. It can be also concluded that the smaller values of the critical buckling load "$\bar{N}$" are given by a simply-supported FG-sandwich plate.

The critical buckling load "$\bar{N}$" versus the ratio "a/h" and inhomogeneity parameter "k" of simply- supported E-FG sandwich plates seated on elastic foundations with (K_0=J_0=10, ξ=1, a=b) is illustrated in Fig. 5. From the plotted curves, it can be noted that the current results are almost the same with those given by Ait Amar Meziane *et al.* (2014). It can also be observed that the increasing in the ratio "a/h" leads to an increase in the critical buckling

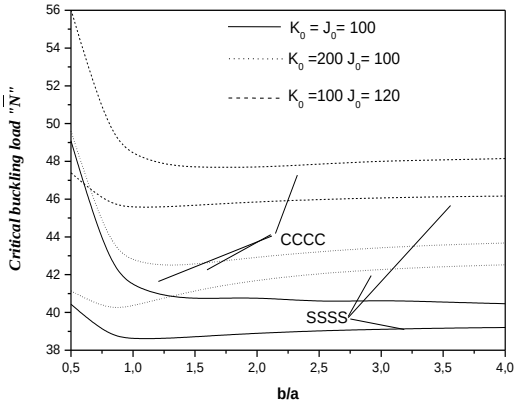

Fig. 7 Critical buckling load "$\bar{N}$" versus the aspect ratio "b/a" of simply-supported and clamped E-FG sandwich plate (1-1-1) for different values of foundation stiffness's "K_0" and "J_0" (a/h=10, k=2, ξ=1)

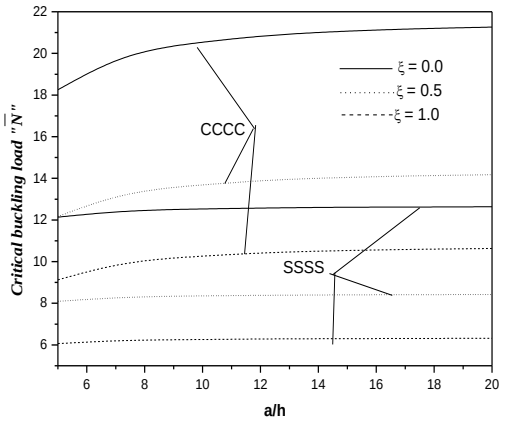

Fig. 8 Critical buckling load "$\bar{N}$" versus the ratio "a/h" of simply-supported and clamped E-FG sandwich square plate (1-2-1) for different values of "ξ" (K_0=J_0=10, k=2)

load "$\bar{N}$". But this last decreases when the inhomogeneity parameter "k" increases.

Fig. 6 illustrates the variation of critical buckling load "$\bar{N}$" of the 2-1-2 FG-sandwich plate seated elastic foundation with (K_0=100, k=2) for various boundary conditions (FFCC, CCCC, CSCS, CCSS, CSSS and SSSS). The FG sandwich plate is supposed under biaxial compressive loads "ξ=1".

The present results of the critical buckling load "$\bar{N}$" are in good agreement with those given by Ait Amar Meziane *et al.* (2014). From the graphs it can be observed that smaller values of the critical buckling load "$\bar{N}$" are obtained for SSSS FG-sandwich plate.

Fig. 7 presents the variation of the critical buckling load "$\bar{N}$" of the clamped and simply supported 1-1-1 E-FG sandwich plate with (a/h=10, k=2) as foundations parameters "K_0, J_0". From the obtained graphs, it can be noted that the biggest values of the critical buckling load "$\bar{N}$" are obtained for parameters "K_0=100, J_0=120" and this is confirmed for both SSSS and CCCC boundary conditions.

The variations of the critical buckling load "$\bar{N}$" of the 1-2-1 FG-sandwich plate on elastic foundation versus loads ratio "ξ"are illustrated in the Fig. 8. From the plotted curves, it can be noted that the smaller values of the critical buckling load "$\bar{N}$" are obtained for simply supported E-FG with biaxial compressive load "ξ=1". But the larger values are obtained for clamped FG-sandwich plate under uniaxial compressive load along x-axis.

5. Conclusions

A simplified innovative trigonometric higher orders shear deformation with only three unknowns variable was developed for free vibrational and mechanical stability analysis of E-FG sandwich plate with various boundary conditions. The equations of motion are obtained from the Hamilton's principle. The accuracy and efficiency of the developed model has been checked for the stability and dynamic analysis of FG-sandwich plate. Several parametric studies has been examined and discussed to show the various parameters influencing the fundamental frequency and critical buckling load of the plate.

Finally it can be concluded that, the present theory can improve the numerical computational cost due to their reduced degrees of freedom and it can be used to study other structures made of different types of materials (Sedighi and Shirazi 2013, Avcar 2014 Sedighi *et al.* 2015, Panjehpour *et al.* 2018, Othman and Fekry 2018, Dihaj *et al.* 2018, Belmahi, *et al.* 2018, YaylacI *et al.* 2019, Al-Maliki *et al.* 2020, López-Chavarría *et al.* 2019, Nikkhoo *et al.* 2019, Kossakowski and Uzarska 2019, Zouatnia and Hadji 2019, Bakhshi and Taheri-Behrooz 2019, Belmahi *et al.* 2019, Fládr *et al.* 2019, Hamad *et al.* 2019, Mohamed *et al.* 2019, Jothi Saravanan *et al.* 2019, Khater *et al.* 2020, Singh and Kumari 2020, Rezaiee-Pajand and Karimipour 2020, Ghadimi 2020, Shokrieh and Kondori 2020, Ghannadpour and Mehrparvar 2020, Lee *et al.* 2020, Yüksel and Akbaş 2019, Karami and Janghorban 2019, Selmi 2019, Al-Basyouni *et al.* 2020, Safarpour *et al.* 2020, Kunche *et al.* 2019, Eltaher *et al.* 2020, Motezaker *et al.* 2020, Ghabussi *et al.* 2020, Timesli 2020).

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