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Investigation of wave propagation in anisotropic plates via quasi 3D HSDT

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Abstract. A free vibration analysis and wave propagation of triclinic and orthotropic plate has been presented in this work using an efficient quasi 3D shear deformation theory. The novelty of this paper is to introducing this theory to minimize the number of unknowns which is three; instead four in other researches, to studying bulk waves in anisotropic plates, other than it can model plates with great thickness ratio, also. Another advantage of this theory is to permits us to show the effect of both bending and shear components and this is carried out by dividing the transverse displacement into the bending and shear parts. Hamilton's equations are a very potent formulation of the equations of analytic mechanics; it is used for the development of wave propagation equations in the anisotropic plates. The analytical dispersion relationship of this type of plate is obtained by solving an eigenvalue problem. The accuracy of the present model is verified by confronting our results with those available in open literature for anisotropic plates. Moreover Numerical examples are given to show the effects of wave number and thickness on free vibration and wave propagation in anisotropic plates.

Keywords: wave propagation; functionally graded plate; quasi 3D HSDT

1. Introduction

Since 19th century (Achenbach 1976), several researchers have been attracted by investigating wave propagation in solids. Non-destructive testing is one of the main applications of wave propagation in plates. It should be noted that several works have been carried out on the propagation of waves in the structures. Nayfeh and Chimenti (1989) presented the study on the free wave propagation in anisotropic plates. They considered a formal wave investigation in a triclinic symmetry plate. Zerwer *et al.* (2000) analyzed the motion of Rayleigh waves in thin Plexiglas sheets. Rovetta *et al.* (2006) proposed a thorough study of the propagation of elastic waves in thin plates. They have shown that, in the considered frequency range and for modest plate thicknesses, the only waves that can be excited and propagated in the structure are guided waves. Edalati *et al.* (2006) investigated the ability of ultrasonic testing of lamb waves testing for defect detection and sizing in a thin aluminum plate. Quintanilla *et al.* (2015) used a spectral collocation technique to deal with the more complex and realistic waveguide problems needed for non-destructive evaluation; many pitfalls and limitations encountered in root search routines based on the partial wave method. The effect of transverse shear is crucial and important in modeling the propagation of ultrasonic waves in composites (Maio *et al.* 2015). A finite element method based on the theory of first-order shear deformation was employed by Maio *et al.* (2015) to assess the propagation speed of the first anti-symmetric Lamb wave mode in graphite/epoxy composite plates by numerical approach. Rauter and Lammering (2015) analyzed nonlinear cumulative wave propagation by taking into account micro-structural cracks in thin linear elastic isotropic plates. Recently, new shear deformation theories have been developed by several scientific for bending, buckling and vibration problems of structures (Abualnour *et al.* 2018, Ahmed*et al.* 2019, Boutaleb *et al.* 2019, Tlidji *et al.* 2019, Shahsavari *et al.* 2018, Belabed *et al.* 2018, Karami *et al.* 2018a, Bouadi *et al.* 2018, Bakhadda *et al.* 2018abc, Khiloun *et al.* 2018, Zouatnia *et al.* 2018, Katariya and Panda 2018, Bouhadra *et al.* 2018, Shokravi 2017, Sekkal *et al.* 2017ab, Ait Atmane et al. 2017, Bellifa *et al.* 2017ab, Kolahchi *et al.* 2017abc, Houari *et al.* 2016,Daouadji and Hadji 2015, Zemri *et al.* 2015, Belkorissat *et al.* 2015, Ahmed 2014, Belabed *et al.* 2014, Hebali *et al.* 2014). Thus, it is also interesting to apply these theories to wave propagation in structures. Indeed, Yahia *et al.* (2015) used various higher-order shear deformation plate theories for wave propagation in FG plates with porosities. Fourn *et al.* (2018) proposed a novel four variable refined plate theory for wave propagation in FG plates. Benadouda *et al.* (2017)

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presented an efficient shear deformation theory for wave propagation in FG beams with porosities. Aminipour and Janghorban (2017) studied the wave propagation in anisotropic plates using trigonometric shear deformation theory. Other beam/plate theories can be consulted in many works (Bousahla *et al.* 2014, Kar *et al.* 2015, Kar and Panda 2015a, b, 2016, Madani *et al.* 2016, Kolahchi *et al.*2016a, b, Arani and Kolahchi 2016, Bilouei *et al.* 2016, Kar *et al.* 2016, Kolahchi and Moniri Bidgoli 2016, Kar *et al.* 2017, Klouche *et al.* 2017, Kolahchi and Cheraghbak 2017, Mahapatra *et al.* 2017, Hajmohammad *et al.* 2017 and 2018a, b, c, Kolahchi 2017, Zidi *et al.* 2017, Zamanian *et al.* 2017, Kar and Panda 2017a, b, Kolahchi *et al.* 2017, Fakhar and Kolahchi 2018, Zine *et al.* 2018, Amnieh *et al.* 2018, Hosseini and Kolahchi 2018, Mouli *et al.* 2018, Younsi *et al.* 2018, Golabchi et al. 2018, Kadari *et al.* 2018, Kaci *et al.* 2018, Chaabane *et al.* 2019, Karami *et al.* 2019a, Zaoui *et al.* 2019, Karakoti and Kar 2019, Meksi *et al.* 2019, Adda Bedia *et al.* 2019).

In this work, the bulk waves in anisotropic plates are studied investigated based on the quasi 3D HSDT. This type of quasi 3D HSDT has only three unknowns but it can model a thick plate. In addition, this theory requires no shear correction coefficient similar to first-order plate theory. The influences of some parameters such as wave number on frequencies, phase velocities and group velocities are also investigated. Note that in the current work, several comparisons between triclinic and orthotropic results are also performed to show if it is interesting to consider the triclinic elastic constants instead of orthotropic elastic constants.

2. Governing equations

2.1 Kinematics equations

The displacements for the quasi 3D HSDT are expressed in Shimpi (2002), Hadji *et al.* (2011), Meziane *et al.* (2014) and Boukhari (2016)

$$u(x,y,z,t) = -z\frac{\partial w_b}{\partial x} - z\left[-\frac{1}{4}+\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \tag{1a}$$

$$v(x,y,z,t) = -z\frac{\partial w_b}{\partial y} - z\left[-\frac{1}{4}+\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \tag{1b}$$

$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) + \left[\frac{5}{4}-5\left(\frac{z}{h}\right)^2\right]\varphi(x,y,t) \tag{1c}$$

From above relations, one can find that the displacements in the thickness direction are not independent of z direction in contrast to several other plate theories. According to the above displacement field, the strain-displacement relations can be obtained as

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w_b}{\partial x^2} - z\left[-\frac{1}{4}+\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_s}{\partial x^2} \tag{2a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w_b}{\partial y^2} - z\left[-\frac{1}{4}+\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_s}{\partial y^2} \tag{2b}$$

$$\varepsilon_y = \frac{\partial w}{\partial z} = -z\frac{10}{h^2}\varphi \tag{2c}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} = -2z\frac{\partial^2 w_b}{\partial x\partial y} - 2z\left[-\frac{1}{4}+\frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_s}{\partial x\partial y} \tag{2d}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y} = \left[\frac{5}{4}-5\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} + \left[\frac{5}{4}-5\left(\frac{z}{h}\right)^2\right]\frac{\partial \varphi}{\partial y} \tag{2e}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} = \left[\frac{5}{4}-5\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} + \left[\frac{5}{4}-5\left(\frac{z}{h}\right)^2\right]\frac{\partial \varphi}{\partial x} \tag{2f}$$

2.2 Equations of motion

Hamilton's principle is here in utilized to determine the equations of motion (Beldjelili *et al.* 2016, Besseghier *et al.* 2017, Abdelaziz *et al.* 2017, Avcar and Mohammed 2018)

$$0 = \int_0^t (\delta U - \delta K)\,dt \tag{3}$$

where δU is the variation of strain energy and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by (Al-Basyouni *et al.* 2015, LarbiChaht *et al.* 2015, Ahouel *et al.* 2016, Draiche *et al.* 2016, Benahmed *et al.* 2017, Benchohra *et al.* 2018, Yazid *et al.* 2018, Mokhtar *et al.* 2018, Bourada *et al.* 2018)

$$\begin{aligned}\delta U &= \int_V \left[\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right]dV \\ &= \int_A \Big[N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b \\ &\quad + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta\gamma_{xz}^0\Big]dA = 0\end{aligned} \tag{4}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$M_i^b = \int_{-h/2}^{h/2}\sigma_i z\,dz, \quad (i = x, y, xy), \tag{5a}$$

$$M_i^s = \int_{-h/2}^{h/2}\sigma_i\left[-\frac{1}{4}+\frac{5}{3}\left(\frac{z}{h}\right)^2\right]z\,dz \quad (i = x, y, xy) \tag{5b}$$

$$V_i^s = \int_{-h/2}^{h/2}\sigma_i\left[\frac{5}{4}-5\left(\frac{z}{h}\right)^2\right]dz \quad (i = xz, yz) \tag{5c}$$

$$V_z^s = \int_{-h/2}^{h/2}\sigma_z\left(\frac{10}{h^2}\right)z\,dz \tag{5d}$$

The variation of kinetic energy of the plate is expressed as (Bourada *et al*. 2015, Mahi *et al*. 2015, Bennoun *et al*. 2016, Bounouara *et al*. 2016, Bouafia *et al*. 2017, Hachemi *et al*. 2017, Boukhlif *et al*. 2019, Bourada *et al*. 2019, Draoui *et al*. 2019)

$$\begin{aligned}\delta K &= \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C}\int_A \left[\dot{u}\delta\dot{u}+\dot{v}\delta\dot{v}+\dot{w}\delta\dot{w}\right]\rho(z_{ns})\,dA\,dz_{ns} \\ &= \int_A \Big\{ I_0\left[\dot{u}_0\delta\dot{u}_0+\dot{v}_0\delta\dot{v}_0+(\dot{w}_b+\dot{w}_s)(\delta\dot{w}_b+\delta\dot{w}_s)\right] \\ &+ I_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x}+\frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\dot{w}_b}{\partial y}\right)+\frac{I_2}{84}\left(\frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x}+\frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\dot{w}_s}{\partial y}\right)\Big\}dA\end{aligned} \quad (6)$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; and (I_0, I_2) are mass inertias defined as

$$(I_0, I_2) = \int_{-h/2}^{h/2}(1, z^2)\rho(z)dz \quad (7)$$

Substituting the expressions for δU and δK from Eqs. (4) and (6) into Eq. (3) and integrating by parts, and collecting the coefficients of δw_b, δw_s and $\delta\varphi$, the following governing equations for studying wave propagation of the plate are obtained

$$\begin{aligned}&\delta w_b:\ -\frac{\partial^2 M_x^b}{\partial x^2}-2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}-\frac{\partial^2 M_y^b}{\partial y^2}= \\ &I_0\left(\ddot{w}_b+\ddot{w}_s+\frac{5}{6}\ddot{\varphi}\right)-I_2\left(\frac{\partial^2\ddot{w}_b}{\partial x\partial y}+\frac{\partial^2\ddot{w}_b}{\partial x\partial y}\right) \\ &\delta w_s:\ -\frac{\partial^2 M_x^s}{\partial x^2}-2\frac{\partial^2 M_{xy}^s}{\partial x\partial y}-\frac{\partial^2 M_y^s}{\partial y^2}-\frac{\partial V_{xz}^s}{\partial x}-\frac{\partial V_{yz}^s}{\partial y}= \\ &I_0\left(\ddot{w}_b+\ddot{w}_s+\frac{5}{6}\ddot{\varphi}\right)-\frac{I_2}{84}\left(\frac{\partial^2\ddot{w}_s}{\partial x\partial y}+\frac{\partial^2\ddot{w}_s}{\partial x\partial y}\right) \\ &\delta\varphi:\ -V_z^s-\frac{\partial V_{xz}^s}{\partial x}-\frac{\partial V_{yz}^s}{\partial y}=\frac{5}{6}I_0(\ddot{w}_b+\ddot{w}_s+\ddot{\varphi})\end{aligned} \quad (8)$$

The object of this article is to study the wave propagation in anisotropic plates. In general; thus, the stress-strain relationships can be expressed as follows,

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{31}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&C_{14}&C_{15}&C_{16}\\ C_{21}&C_{22}&C_{23}&C_{24}&C_{25}&C_{26}\\ C_{31}&C_{32}&C_{33}&C_{34}&C_{35}&C_{36}\\ C_{41}&C_{42}&C_{43}&C_{44}&C_{45}&C_{46}\\ C_{51}&C_{52}&C_{53}&C_{54}&C_{55}&C_{56}\\ C_{61}&C_{62}&C_{63}&C_{64}&C_{65}&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{Bmatrix} \quad (9)$$

The elastic constants used in present work for triclinic and orthotropic materials are chosen from Batra *et al* (2004)

$$Triclinic=\begin{bmatrix}98.84&53.92&50.78&0&0&0.03\\ 53.92&99.19&50.78&0&0&0.03\\ 50.78&50.78&87.23&0&0&0.02\\ 0&0&0&21.14&0.07&0\\ 0&0&0&0.07&21.10&0\\ 0.03&0.03&0.02&0&0&22.55\end{bmatrix}GPa \quad (10)$$

$$Orthotropic=\begin{bmatrix}98.84&53.92&50.78&0&0&0\\ 53.92&99.19&50.78&0&0&0\\ 50.78&50.78&87.23&0&0&0\\ 0&0&0&21.14&0.07&0\\ 0&0&0&0.07&21.10&0\\ 0&0&0&0&0&22.55\end{bmatrix}GPa \quad (11)$$

Now, according to relations (2) and (9), the stresses in terms of displacements are obtained as below,

$$\begin{aligned}\sigma_x &= z\left(-C_{11}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{13}\varphi}{h^2}-2C_{16}\frac{\partial^2 w_b}{\partial x\partial y}-C_{12}\frac{\partial^2 w_b}{\partial y^2}\right) \\ &+f(z)\left(-C_{12}\frac{\partial^2 w_s}{\partial y^2}-C_{11}\frac{\partial^2 w_s}{\partial x^2}-2C_{16}\frac{\partial^2 w_s}{\partial x\partial y}\right) \\ &+g(z)\left(C_{15}\frac{\partial w_s}{\partial x}+C_{15}\frac{\partial\varphi}{\partial x}+C_{14}\frac{\partial w_s}{\partial y}+C_{14}\frac{\partial\varphi}{\partial y}\right)\end{aligned} \quad (12a)$$

$$\begin{aligned}\sigma_y &= z\left(-C_{12}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{23}\varphi}{h^2}-2C_{26}\frac{\partial^2 w_b}{\partial x\partial y}-C_{22}\frac{\partial^2 w_b}{\partial y^2}\right) \\ &+f(z)\left(-C_{22}\frac{\partial^2 w_s}{\partial y^2}-C_{12}\frac{\partial^2 w_s}{\partial x^2}-2C_{26}\frac{\partial^2 w_s}{\partial x\partial y}\right) \\ &+g(z)\left(C_{25}\frac{\partial w_s}{\partial x}+C_{25}\frac{\partial\varphi}{\partial x}+C_{24}\frac{\partial w_s}{\partial y}+C_{24}\frac{\partial\varphi}{\partial y}\right)\end{aligned} \quad (12b)$$

$$\begin{aligned}\sigma_z &= z\left(-C_{13}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{33}\varphi}{h^2}-2C_{36}\frac{\partial^2 w_b}{\partial x\partial y}-C_{23}\frac{\partial^2 w_b}{\partial y^2}\right) \\ &+f(z)\left(-C_{23}\frac{\partial^2 w_s}{\partial y^2}-C_{13}\frac{\partial^2 w_s}{\partial x^2}-2C_{36}\frac{\partial^2 w_s}{\partial x\partial y}\right) \\ &+g(z)\left(C_{35}\frac{\partial w_s}{\partial x}+C_{35}\frac{\partial\varphi}{\partial x}+C_{34}\frac{\partial w_s}{\partial y}+C_{34}\frac{\partial\varphi}{\partial y}\right)\end{aligned} \quad (12c)$$

$$\begin{aligned}\sigma_{yz} &= z\left(-C_{14}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{34}\varphi}{h^2}-2C_{46}\frac{\partial^2 w_b}{\partial x\partial y}-C_{24}\frac{\partial^2 w_b}{\partial y^2}\right) \\ &+f(z)\left(-C_{24}\frac{\partial^2 w_s}{\partial y^2}-C_{14}\frac{\partial^2 w_s}{\partial x^2}-2C_{46}\frac{\partial^2 w_s}{\partial x\partial y}\right) \\ &+g(z)\left(C_{45}\frac{\partial w_s}{\partial x}+C_{45}\frac{\partial\varphi}{\partial x}+C_{44}\frac{\partial w_s}{\partial y}+C_{44}\frac{\partial\varphi}{\partial y}\right)\end{aligned} \quad (12d)$$

$$\begin{aligned}\sigma_{xz} &= z\left(-C_{15}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{35}\varphi}{h^2}-2C_{56}\frac{\partial^2 w_b}{\partial x\partial y}-C_{25}\frac{\partial^2 w_b}{\partial y^2}\right) \\ &+f(z)\left(-C_{25}\frac{\partial^2 w_s}{\partial y^2}-C_{15}\frac{\partial^2 w_s}{\partial x^2}-2C_{56}\frac{\partial^2 w_s}{\partial x\partial y}\right) \\ &+g(z)\left(C_{55}\frac{\partial w_s}{\partial x}+C_{55}\frac{\partial\varphi}{\partial x}+C_{45}\frac{\partial w_s}{\partial y}+C_{45}\frac{\partial\varphi}{\partial y}\right)\end{aligned} \quad (12e)$$

$$\begin{aligned}\sigma_{xy} &= z\left(-C_{16}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{36}\varphi}{h^2}-2C_{66}\frac{\partial^2 w_b}{\partial x\partial y}-C_{26}\frac{\partial^2 w_b}{\partial y^2}\right) \\ &+f(z)\left(-C_{26}\frac{\partial^2 w_s}{\partial y^2}-C_{16}\frac{\partial^2 w_s}{\partial x^2}-2C_{66}\frac{\partial^2 w_s}{\partial x\partial y}\right) \\ &+g(z)\left(C_{56}\frac{\partial w_s}{\partial x}+C_{56}\frac{\partial\varphi}{\partial x}+C_{46}\frac{\partial w_s}{\partial y}+C_{46}\frac{\partial\varphi}{\partial y}\right)\end{aligned} \quad (12f)$$

Now by integrating from Eqs. (7)-(12) according to relations (4), we have

$$M_x^b = A_1\left(-C_{11}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{13}\varphi}{h^2} - 2C_{16}\frac{\partial^2 w_b}{\partial x\partial y} - C_{12}\frac{\partial^2 w_b}{\partial y^2}\right) + A_2\left(-C_{12}\frac{\partial^2 w_s}{\partial y^2} - C_{11}\frac{\partial^2 w_s}{\partial x^2} - 2C_{16}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_3\left(C_{15}\frac{\partial w_s}{\partial x} + C_{15}\frac{\partial \varphi}{\partial x} + C_{14}\frac{\partial w_s}{\partial y} + C_{14}\frac{\partial \varphi}{\partial y}\right) \tag{13a}$$

$$M_y^b = A_1\left(-C_{12}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{23}\varphi}{h^2} - 2C_{26}\frac{\partial^2 w_b}{\partial x\partial y} - C_{22}\frac{\partial^2 w_b}{\partial y^2}\right) + A_2\left(-C_{22}\frac{\partial^2 w_s}{\partial y^2} - C_{12}\frac{\partial^2 w_s}{\partial x^2} - 2C_{26}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_3\left(C_{25}\frac{\partial w_s}{\partial x} + C_{25}\frac{\partial \varphi}{\partial x} + C_{24}\frac{\partial w_s}{\partial y} + C_{24}\frac{\partial \varphi}{\partial y}\right) \tag{13b}$$

$$M_{xy}^b = A_1\left(-C_{16}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{36}\varphi}{h^2} - 2C_{66}\frac{\partial^2 w_b}{\partial x\partial y} - C_{26}\frac{\partial^2 w_b}{\partial y^2}\right) + A_2\left(-C_{26}\frac{\partial^2 w_s}{\partial y^2} - C_{16}\frac{\partial^2 w_s}{\partial x^2} - 2C_{66}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_3\left(C_{56}\frac{\partial w_s}{\partial x} + C_{56}\frac{\partial \varphi}{\partial x} + C_{46}\frac{\partial w_s}{\partial y} + C_{46}\frac{\partial \varphi}{\partial y}\right) \tag{13c}$$

$$M_x^s = A_2\left(-C_{11}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{13}\varphi}{h^2} - 2C_{16}\frac{\partial^2 w_b}{\partial x\partial y} - C_{12}\frac{\partial^2 w_b}{\partial y^2}\right) + A_4\left(-C_{12}\frac{\partial^2 w_s}{\partial y^2} - C_{11}\frac{\partial^2 w_s}{\partial x^2} - 2C_{16}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_5\left(C_{15}\frac{\partial w_s}{\partial x} + C_{15}\frac{\partial \varphi}{\partial x} + C_{14}\frac{\partial w_s}{\partial y} + C_{14}\frac{\partial \varphi}{\partial y}\right) \tag{13d}$$

$$M_y^s = A_2\left(-C_{12}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{23}\varphi}{h^2} - 2C_{26}\frac{\partial^2 w_b}{\partial x\partial y} - C_{22}\frac{\partial^2 w_b}{\partial y^2}\right) + A_4\left(-C_{22}\frac{\partial^2 w_s}{\partial y^2} - C_{12}\frac{\partial^2 w_s}{\partial x^2} - 2C_{26}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_5\left(C_{25}\frac{\partial w_s}{\partial x} + C_{25}\frac{\partial \varphi}{\partial x} + C_{24}\frac{\partial w_s}{\partial y} + C_{24}\frac{\partial \varphi}{\partial y}\right) \tag{13e}$$

$$M_{xy}^s = A_2\left(-C_{16}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{36}\varphi}{h^2} - 2C_{66}\frac{\partial^2 w_b}{\partial x\partial y} - C_{26}\frac{\partial^2 w_b}{\partial y^2}\right) + A_4\left(-C_{26}\frac{\partial^2 w_s}{\partial y^2} - C_{16}\frac{\partial^2 w_s}{\partial x^2} - 2C_{66}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_5\left(C_{56}\frac{\partial w_s}{\partial x} + C_{56}\frac{\partial \varphi}{\partial x} + C_{46}\frac{\partial w_s}{\partial y} + C_{46}\frac{\partial \varphi}{\partial y}\right) \tag{13f}$$

$$V_{yz}^s = A_3\left(-C_{14}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{34}\varphi}{h^2} - 2C_{46}\frac{\partial^2 w_b}{\partial x\partial y} - C_{24}\frac{\partial^2 w_b}{\partial y^2}\right) + A_5\left(-C_{24}\frac{\partial^2 w_s}{\partial y^2} - C_{14}\frac{\partial^2 w_s}{\partial x^2} - 2C_{46}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_6\left(C_{45}\frac{\partial w_s}{\partial x} + C_{45}\frac{\partial \varphi}{\partial x} + C_{44}\frac{\partial w_s}{\partial y} + C_{44}\frac{\partial \varphi}{\partial y}\right) \tag{13g}$$

$$V_{xz}^s = A_3\left(-C_{15}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{35}\varphi}{h^2} - 2C_{56}\frac{\partial^2 w_b}{\partial x\partial y} - C_{25}\frac{\partial^2 w_b}{\partial y^2}\right) + A_5\left(-C_{25}\frac{\partial^2 w_s}{\partial y^2} - C_{15}\frac{\partial^2 w_s}{\partial x^2} - 2C_{56}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_6\left(C_{55}\frac{\partial w_s}{\partial x} + C_{55}\frac{\partial \varphi}{\partial x} + C_{45}\frac{\partial w_s}{\partial y} + C_{45}\frac{\partial \varphi}{\partial y}\right) \tag{13h}$$

$$V_z^s = A_1\frac{10}{h^2}\left(-C_{13}\frac{\partial^2 w_b}{\partial x^2} - \frac{10C_{33}\varphi}{h^2} - 2C_{36}\frac{\partial^2 w_b}{\partial x\partial y} - C_{23}\frac{\partial^2 w_b}{\partial y^2}\right) + A_2\frac{10}{h^2}\left(-C_{23}\frac{\partial^2 w_s}{\partial y^2} - C_{13}\frac{\partial^2 w_s}{\partial x^2} - 2C_{36}\frac{\partial^2 w_s}{\partial x\partial y}\right) + A_3\frac{10}{h^2}\left(C_{35}\frac{\partial w_s}{\partial x} + C_{35}\frac{\partial \varphi}{\partial x} + C_{34}\frac{\partial w_s}{\partial y} + C_{34}\frac{\partial \varphi}{\partial y}\right) \tag{13i}$$

where

$$A_1 = \int_{-h/2}^{h/2} z^2 dz, \quad A_2 = \int_{-h/2}^{h/2} z\, f(z) dz, \quad A_3 = \int_{-h/2}^{h/2} z\, g(z) dz, \quad A_4 = \int_{-h/2}^{h/2} [f(z)]^2 dz, \quad A_5 = \int_{-h/2}^{h/2} f(z)\, g(z) dz, \quad A_6 = \int_{-h/2}^{h/2} [g(z)]^2 dz \tag{14}$$

and

$$f(z) = z\left[-\frac{1}{4} + \frac{5}{3}\left(\frac{z}{h}\right)^2\right], \quad g(z) = \left[\frac{5}{4} - 5\left(\frac{z}{h}\right)^2\right] \tag{15}$$

With considering the equilibrium equations, governing equations based on Eq. (13) for studying wave propagation in plates are derived as follows

$$\begin{aligned}
&-A_1\left(-C_{11}\frac{\partial^4 w_b}{\partial x^4} - \frac{10C_{13}}{h^2}\frac{\partial^2 \varphi}{\partial x^2} - 2C_{16}\frac{\partial^4 w_b}{\partial x^3\partial y} - C_{12}\frac{\partial^4 w_b}{\partial x^2\partial y^2}\right) \\
&- A_2\left(-C_{12}\frac{\partial^4 w_s}{\partial x^2\partial y^2} - C_{11}\frac{\partial^4 w_s}{\partial x^4} - 2C_{16}\frac{\partial^4 w_s}{\partial x^3\partial y}\right) \\
&- A_3\left(C_{15}\frac{\partial^3 w_s}{\partial x^3} + C_{15}\frac{\partial^3 \varphi}{\partial x^3} + C_{14}\frac{\partial^3 w_s}{\partial x^2\partial y} + C_{14}\frac{\partial^3 \varphi}{\partial x^2\partial y}\right) \\
&- A_1\left(-C_{12}\frac{\partial^4 w_b}{\partial x^2\partial y^2} - \frac{10C_{23}}{h^2}\frac{\partial^2 \varphi}{\partial y^2} - 2C_{26}\frac{\partial^3 w_b}{\partial x\partial y^2} - C_{22}\frac{\partial^4 w_b}{\partial y^4}\right) \\
&- A_2\left(-C_{22}\frac{\partial^4 w_s}{\partial y^4} - C_{12}\frac{\partial^4 w_s}{\partial x^2\partial y^2} - 2C_{26}\frac{\partial^4 w_s}{\partial x\partial y^3}\right) \\
&- A_3\left(C_{25}\frac{\partial^3 w_s}{\partial x\partial y^2} + C_{25}\frac{\partial^3 \varphi}{\partial x\partial y^2} + C_{24}\frac{\partial^3 w_s}{\partial y^3} + C_{24}\frac{\partial^3 \varphi}{\partial y^3}\right) - \\
&2A_1\left(-C_{16}\frac{\partial^4 w_b}{\partial x^3\partial y} - \frac{10C_{36}}{h^2}\frac{\partial^2 \varphi}{\partial x\partial y} - 2C_{66}\frac{\partial^4 w_b}{\partial x^2\partial y^2} - C_{26}\frac{\partial^4 w_b}{\partial x\partial y^3}\right) \\
&- 2A_2\left(-C_{26}\frac{\partial^4 w_s}{\partial x\partial y^3} - C_{16}\frac{\partial^4 w_s}{\partial x^3\partial y} - 2C_{66}\frac{\partial^4 w_s}{\partial x^2\partial y^2}\right) \\
&- 2A_3\left(C_{56}\frac{\partial^3 w_s}{\partial x^2\partial y} + C_{56}\frac{\partial^3 \varphi}{\partial x^2\partial y} + C_{46}\frac{\partial^3 w_s}{\partial x\partial y^2} + C_{46}\frac{\partial^3 \varphi}{\partial x\partial y^2}\right) \\
&= -I_2\left(\frac{\partial^4 w_b}{\partial x^2\partial t^2} + \frac{\partial^4 w_b}{\partial y^2\partial t^2}\right) + I_0\left(\left(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}\right) + \frac{5}{6}\frac{\partial^2 \varphi}{\partial t^2}\right)
\end{aligned} \tag{16a}$$

$$\begin{aligned}
&-A_2\left(-C_{11}\frac{\partial^4 w_b}{\partial x^4} - \frac{10C_{13}}{h^2}\frac{\partial^2 \varphi}{\partial x^2} - 2C_{16}\frac{\partial^4 w_b}{\partial x^3\partial y} - C_{12}\frac{\partial^4 w_b}{\partial x^2\partial y^2}\right) \\
&- A_4\left(-C_{12}\frac{\partial^4 w_s}{\partial x^2\partial y^2} - C_{11}\frac{\partial^4 w_s}{\partial x^4} - 2C_{16}\frac{\partial^4 w_s}{\partial x^3\partial y}\right) \\
&- A_5\left(C_{15}\frac{\partial^3 w_s}{\partial x^3} + C_{15}\frac{\partial^3 \varphi}{\partial x^3} + C_{14}\frac{\partial^3 w_s}{\partial x^2\partial y} + C_{14}\frac{\partial^3 \varphi}{\partial x^2\partial y}\right) \\
&- A_2\left(-C_{12}\frac{\partial^4 w_b}{\partial x^2\partial y^2} - \frac{10C_{23}}{h^2}\frac{\partial^2 \varphi}{\partial y^2} - 2C_{26}\frac{\partial^4 w_b}{\partial x\partial y^3} - C_{22}\frac{\partial^4 w_b}{\partial y^4}\right)
\end{aligned} \tag{16b}$$

$$-A_4\left(-C_{22}\frac{\partial^4 w_s}{\partial y^4}-C_{12}\frac{\partial^4 w_s}{\partial x^2\partial y^2}-2C_{26}\frac{\partial^4 w_s}{\partial x\partial y^3}\right)$$
$$-A_5\left(C_{25}\frac{\partial^3 w_s}{\partial x\partial y^2}+C_{25}\frac{\partial^3 \varphi}{\partial x\partial y^2}+C_{24}\frac{\partial^3 w_s}{\partial y^3}+C_{24}\frac{\partial^3 \varphi}{\partial y^3}\right)$$
$$-A_3\left(-C_{14}\frac{\partial^3 w_b}{\partial x^2\partial y}-\frac{10C_{34}}{h^2}\frac{\partial \varphi}{\partial y}-2C_{46}\frac{\partial^3 w_b}{\partial x\partial y^2}-C_{24}\frac{\partial^3 w_b}{\partial y^3}\right)$$
$$-A_5\left(-C_{24}\frac{\partial^3 w_s}{\partial y^3}-C_{14}\frac{\partial^3 w_s}{\partial x^2\partial y}-2C_{46}\frac{\partial^3 w_s}{\partial x\partial y^2}\right)$$
$$-A_6\left(C_{45}\frac{\partial^2 w_s}{\partial x\partial y}+C_{45}\frac{\partial^2 \varphi}{\partial x\partial y}+C_{44}\frac{\partial^2 w_s}{\partial y^2}+C_{44}\frac{\partial^2 \varphi}{\partial y^2}\right)$$
$$-A_3\left(-C_{15}\frac{\partial^3 w_b}{\partial x^3}-\frac{10C_{35}}{h^2}\frac{\partial \varphi}{\partial x}-2C_{56}\frac{\partial^3 w_b}{\partial x^2\partial y}-C_{25}\frac{\partial^3 w_b}{\partial y^2\partial x}\right)$$
$$-A_5\left(-C_{25}\frac{\partial^3 w_s}{\partial y^2\partial x}-C_{15}\frac{\partial^3 w_s}{\partial x^3}-2C_{56}\frac{\partial^3 w_s}{\partial x^2\partial y}\right) \quad (16b)$$
$$-A_6\left(C_{55}\frac{\partial^2 w_s}{\partial x^2}+C_{55}\frac{\partial^2 \varphi}{\partial x^2}+C_{45}\frac{\partial^2 w_s}{\partial x\partial y}+C_{45}\frac{\partial^2 \varphi}{\partial x\partial y}\right)$$
$$-2A_2\left(-C_{16}\frac{\partial^4 w_b}{\partial x^3\partial y}-\frac{10C_{36}}{h^2}\frac{\partial^2 \varphi}{\partial x\partial y}-2C_{66}\frac{\partial^4 w_b}{\partial x^2\partial y^2}-C_{26}\frac{\partial^4 w_b}{\partial x\partial y^3}\right)$$
$$-2A_4\left(-C_{26}\frac{\partial^4 w_s}{\partial y^3\partial x}-C_{16}\frac{\partial^4 w_s}{\partial x^3\partial y}-2C_{66}\frac{\partial^4 w_s}{\partial x^2\partial y^2}\right)$$
$$-2A_5\left(C_{56}\frac{\partial^3 w_s}{\partial x^2\partial y}+C_{56}\frac{\partial^3 \varphi}{\partial x^2\partial y}+C_{46}\frac{\partial^3 w_s}{\partial x\partial y^2}+C_{46}\frac{\partial^3 \varphi}{\partial x\partial y^2}\right)$$
$$=-\frac{I_2}{84}\left(\frac{\partial^4 w_s}{\partial x^2\partial t^2}+\frac{\partial^4 w_s}{\partial y^2\partial t^2}\right)+I_0\left(\left(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2}\right)+\frac{5}{6}\frac{\partial^2 \varphi}{\partial t^2}\right)$$

$$-A_1\frac{10}{h^2}\left(-C_{13}\frac{\partial^2 w_b}{\partial x^2}-\frac{10C_{33}\varphi}{h^2}-2C_{36}\frac{\partial^2 w_b}{\partial x\partial y}-C_{23}\frac{\partial^2 w_b}{\partial y^2}\right)$$
$$-A_2\frac{10}{h^2}\left(-C_{23}\frac{\partial^2 w_s}{\partial y^2}-C_{13}\frac{\partial^2 w_s}{\partial x^2}-2C_{36}\frac{\partial^2 w_s}{\partial x\partial y}\right)$$
$$-A_3\frac{10}{h^2}\left(C_{35}\frac{\partial w_s}{\partial x}+C_{35}\frac{\partial \varphi}{\partial x}+C_{34}\frac{\partial w_s}{\partial y}+C_{34}\frac{\partial \varphi}{\partial y}\right)$$
$$-A_3\left(-C_{14}\frac{\partial^3 w_b}{\partial x^2\partial y}-\frac{10C_{34}}{h^2}\frac{\partial \varphi}{\partial y}-2C_{46}\frac{\partial^3 w_b}{\partial x\partial y^2}-C_{24}\frac{\partial^3 w_b}{\partial y^3}\right)$$
$$-A_5\left(-C_{24}\frac{\partial^3 w_s}{\partial y^3}-C_{14}\frac{\partial^3 w_s}{\partial x^2\partial y}-2C_{46}\frac{\partial^3 w_s}{\partial x\partial y^2}\right)$$
$$-A_6\left(C_{45}\frac{\partial^2 w_s}{\partial x\partial y}+C_{45}\frac{\partial^2 \varphi}{\partial x\partial y}+C_{44}\frac{\partial^2 w_s}{\partial y^2}+C_{44}\frac{\partial^2 \varphi}{\partial y^2}\right) \quad (16c)$$
$$-A_3\left(-C_{15}\frac{\partial^3 w_b}{\partial x^3}-\frac{10C_{35}}{h^2}\frac{\partial \varphi}{\partial x}-2C_{56}\frac{\partial^3 w_b}{\partial x^2\partial y}-C_{25}\frac{\partial^3 w_b}{\partial y^2\partial x}\right)$$
$$-A_5\left(-C_{25}\frac{\partial^3 w_s}{\partial y^2\partial x}-C_{15}\frac{\partial^3 w_s}{\partial x^3}-2C_{56}\frac{\partial^3 w_s}{\partial x^2\partial y}\right)$$
$$-A_6\left(C_{55}\frac{\partial^2 w_s}{\partial x^2}+C_{55}\frac{\partial^2 \varphi}{\partial x^2}+C_{45}\frac{\partial^2 w_s}{\partial x\partial y}+C_{45}\frac{\partial^2 \varphi}{\partial x\partial y}\right)$$
$$=\frac{5}{6}I_0\left(\frac{\partial^2 w_b}{\partial t^2}+\frac{\partial^2 w_s}{\partial t^2}+\frac{\partial^2 \varphi}{\partial t^2}\right)$$

Now to investigate the bulk waves in anisotropic plates, following assumptions are made according to Nami and Maziar (2014), Yahia *et al*. (2015), Karami *et al.* (2017), Karami *et al.* (2018d) and Karami *et al.* (2019b, c)

$$\begin{Bmatrix} w_b(x,y,t) \\ w_s(x,y,t) \\ \varphi(x,y,t) \end{Bmatrix}=\begin{Bmatrix} W_b\exp[i(k_1+k_2-\omega t)] \\ W_s\exp[i(k_1+k_2-\omega t)] \\ \Phi\ \exp[i(k_1+k_2-\omega t)] \end{Bmatrix} \quad (17)$$

where k_1 and k_2 are the wave numbers in x and y directions, respectively. The substitution of Eq. (17) in Eq. (16) leads to the following problem

$$\left([K]-\omega^2[M]\right)\{\Delta\}=\{0\} \quad (18)$$

with

$$\{\Delta\}=\{W_b,W_s,\Phi\}^T, \quad (19a)$$

$$[K]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},\quad [M]=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad (19b)$$

in which

$$a_{11}=A_1\begin{pmatrix} 2C_{12}k_1^2\,k_2^2+4C_{26}k_1\,k_2^3+C_{22}k_2^4 \\ +4C_{16}k_1^3k_2+4C_{66}k_1^2\,k_2^2+C_{11}k_1^4 \end{pmatrix}$$
$$a_{12}=A_3\begin{pmatrix} IC_{12}k_1^3+IC_{14}k_2\,k_1^2+2IC_{56}k_1^2\,k_2^4 \\ +2IC_{46}k_1k_2^2+IC_{25}k_1\,k_2^2+IC_{24}k_2^3 \end{pmatrix}$$
$$+A_2(2C_{12}k_1^2k_2^2+4C_{26}k_1\,k_2^3+C_{22}\,k_2^4+4C_{16}k_1^3k_2+C_{11}\,k_1^4)$$
$$a_{13}=A_3\begin{pmatrix} IC_{15}k_1^3+IC_{14}k_2\,k_1^2+2IC_{56}k_1^2\,k_2 \\ +2IC_{46}k_1k_2^2+IC_{25}k_1\,k_2^2+IC_{24}k_2^3 \end{pmatrix}$$
$$+A_1\left(-\frac{10C_{13}k_1^2}{h^2}-\frac{10C_{23}k_2^2}{h^2}-\frac{20C_{36}k_1k_2}{h^2}\right)$$

$$a_{21}=A_3\begin{pmatrix} -IC_{24}k_2^3-IC_{14}k_2\,k_1^2-2IC_{46}k_1\,k_2^2 \\ -IC_{25}k_1k_2^2-IC_{15}\,k_1^3-IC_{15}k_1^3-2IC_{56}k_1^2k_2 \end{pmatrix}$$
$$+A_2\begin{pmatrix} 2C_{12}k_1^2k_2^2+4C_{26}k_1\,k_2^3+C_{22}\,k_2^4 \\ +4C_{16}k_1^3k_2+4C_{66}\,k_1^2k_2^2+C_{11}k_1^4 \end{pmatrix}$$
$$a_{22}=A_4\begin{pmatrix} 2C_{12}k_1^2\,k_2^2+4C_{26}k_1\,k_2^3+C_{22}k_2^4 \\ +4C_{16}k_1^3k_2+4C_{66}k_1^2\,k_2^2+C_{11}k_1^4 \end{pmatrix} \quad (20)$$
$$+A_6\left(2C_{45}k_1k_2+C_{44}k_2^2+C_{55}k_1^2\right)$$
$$a_{23}=A_2\left(-\frac{10C_{13}k_1^2}{h^2}-\frac{10C_{23}k_2^2}{h^2}-\frac{20C_{36}k_1k_2}{h^2}\right)$$
$$+A_3\left(\frac{10IC_{34}k_2}{h^2}+\frac{10IC_{35}k_1}{h^2}\right)$$
$$+A_5\begin{pmatrix} IC_{15}k_1^3+IC_{14}k_2\,k_1^2+2IC_{56}k_1^2\,k_2 \\ +2IC_{46}k_1k_2^2+IC_{25}k_1\,k_2^2+IC_{24}k_2^3 \end{pmatrix}$$
$$+A_6\left(2C_{45}k_1k_2+C_{44}\,k_2^2+C_{55}k_1^2\right)$$

$$a_{31}=A_3\begin{pmatrix} -IC_{24}k_2^3-IC_{14}k_2\,k_1^2-2IC_{46}k_2^2\,k_1 \\ -IC_{25}k_1k_2^2-IC_{15}\,k_1^3-2IC_{56}k_1^2k_2 \end{pmatrix}$$
$$-10A_1\left(\frac{C_{13}k_1^2}{h^2}+\frac{2C_{36}k_1k_2}{h^2}+\frac{C_{23}k_2^2}{h^2}\right)$$

$$a_{32} = -10A_2\left(\frac{C_{13}k_1^2}{h^2} + \frac{2C_{36}k_1k_2}{h^2} + \frac{C_{23}k_2^2}{h^2}\right)$$
$$-10A_3\left(\frac{IC_{35}k_1}{h^2} + \frac{IC_{34}k_2}{h^2}\right)$$
$$+A_5\begin{pmatrix} -IC_{24}k_2^3 - IC_{14}k_2k_1^2 - 2IC_{46}k_2^2k_1 \\ -IC_{25}k_1k_2^2 - IC_{15}k_1^3 - 2IC_{56}k_1^2k_2 \end{pmatrix}$$
$$+A_6\left(2C_{45}k_1k_2 + C_{44}k_2^2 + C_{55}k_1^2\right)$$
$$a_{33} = A_1\frac{100C_{33}}{h^4} + A_3\begin{pmatrix} \dfrac{10IC_{35}k_1}{h^2} + \dfrac{10IC_{34}k_2}{h^2} \\ -\dfrac{10(IC_{35}k_1 + IC_{34}k_2)}{h^2} \end{pmatrix} \tag{20}$$
$$+A_6\left(2C_{45}k_1k_2 + C_{44}k_2^2 + C_{55}k_1^2\right)$$
$$m_{11} = -\left(I_0 + I_2\left(k_1^2 + k_2^2\right)\right)$$
$$m_{12} = -I_0 = m_{21}$$
$$m_{22} = -\left(I_0 + \frac{1}{84}I_2\left(k_1^2 + k_2^2\right)\right)$$
$$m_{33} = -\frac{5}{6}I_0$$

3. Numerical results and discussions

In this part, in order to obtain results of frequencies and velocities one must go through the resolution of a system of three equations on eigenvalues problem; the anisotropic plate made from triclinic and orthotropic material with ρ=7750 kg/m^3 and ν=0.3, the elastic constants are given in Eqs. (10) and (11), according to the reference Batra et al (2004). The thickness of the anisotropic plate is taken h=0.01 m. various numerical examples are presented and discussed to check the accuracy of present theory in investigating the wave propagation of this kind of plates.

In table 1, the circular frequencies based on our methodology are compared with the results extracted from a closed-form solution published by Hooman and Maziar (2017) based on the trigonometric shear plate theory for anisotropic plates.

A good agreement has been observed for the bending vibration frequencies as well as for the shear frequencies for different propagation wave numbers. However it is worth noting that in the present model the number of unknowns in the displacement field is 3 leading thus to a 3×3 system of equations to be solved, whereas this number of unknowns is 4 in Hooman and Maziar (2017).

It is worth also to be noted that this theory enables one to consider thick plates, because warping has been taken into account in the refined shear deformation function.

In Table 2 it is also shown a comparison between phase velocities for orthotropic and triclinic plates for $k_1 \neq 0$ only (k_2=0). When only a wave number is considered, results are also in good agreement, for both bending and shear phase velocities for different x-direction propagation wave numbers. Omitting the number of waves according to y-coordinate; it can be seen that the differences between the

Results for triclinic and orthotropic materials are negligible.

Fig. 1 represents the effects of the wave number and the thickness on the frequencies of a rectangular triclinic plate. According to this figure, it is clear that with the increase of the number of mode, the frequencies increase. However, it

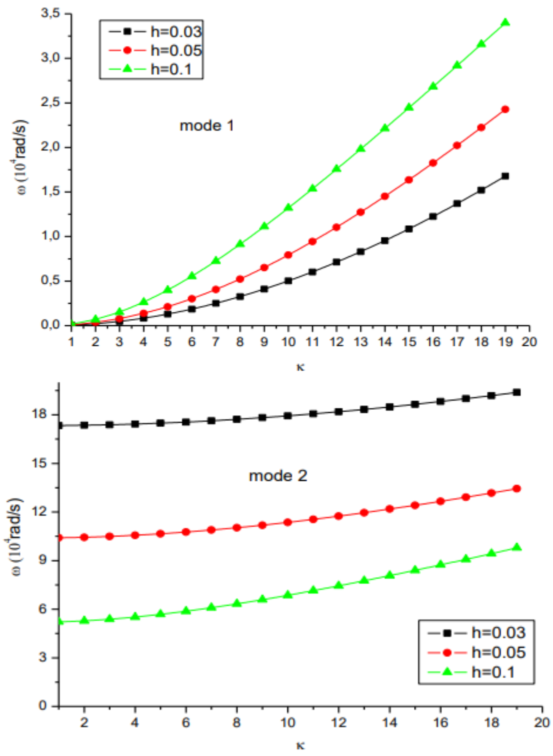

Fig. 1 The effects of wave numbers and thickness on the frequencies of triclinic rectangular plate (k_1=k_2)

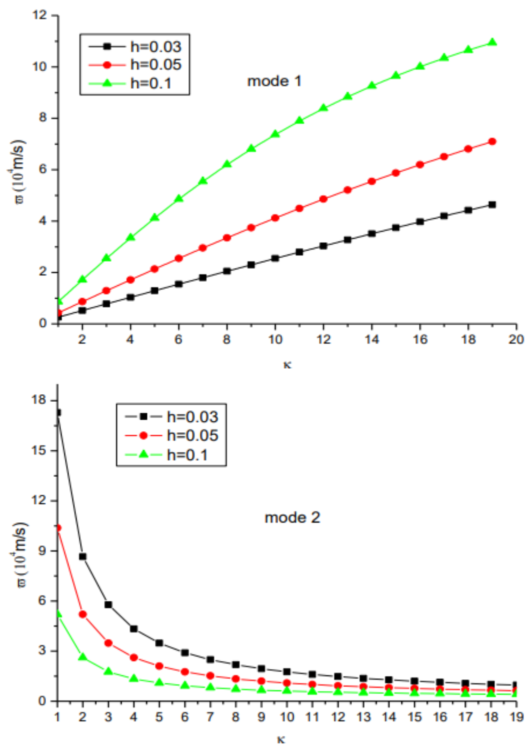

Fig. 2 The effects of wave numbers and thickness on the phase velocities of triclinic rectangular plate ($k = \sqrt{k_x^2 + k_y^2}, k_y = 0$)

Table 1 Comparison of the circular frequencies for orthotropic and triclinic materials (h=0.01 m, k_1=k_2)

		Aminipour and Janghorban (2017)				Present		
		ω_W	ω_Q	ω_ψ	ω_ξ	ω_{Wb}	ω_s	$\omega\varphi$
Triclinic	k_1=9	**1399.54**	518191.50	521359.40	1180042.90	**1395.433185**	521437.4910	2598788.023
	k_1=18	**5544.14**	519564.76	526888.43	1179167.44	**5529.975058**	526266.3894	2599023.272
Orthotropic	k_1=9	**1399.16**	519028.40	520523.85	1180043.14	**1395.054179**	520579.5699	2598787.284
	k_1=18	**5542.51**	520419.32	526034.88	1179168.44	**5528.354161**	525412.0634	2599022.309

Table 2 Comparison of the phase velocities for orthotropic and triclinic materials $(h=0.01m,\varpi=\frac{\omega}{k}\,{}^{rad}/_{s.m})$, $k=\sqrt{k_1^2+k_2^2},k_2=0$

		Aminipour and Janghorban (2017)				Present		
		ϖ_W	ϖ_Q	ϖ_ψ	ϖ_ξ	ϖ_{Wb}	ϖ_{Ws}	ϖ_φ
Triclinic	k_1=7	**60.5422748**	74015.22	74260.88	168606.713874	**60.35877991**	74171.15697	371247.6023
	k_1=22	**188.615437**	23637.59	23822.70	53612.025283	**188.1098465**	23797.22404	118133.7222
Orthotropic	k_1=7	**60.5422753**	74133.60	74142.71	168606.713870	**60.35877991**	74171.15697	371247.6023
	k_1=22	**188.615451**	23646.22	23814.12	53612.025269	**188.1098465**	23797.22404	118133.7222

can be seen that the evolution of the frequencies according to the mode number, for the bending mode (ω_b), is quite significant, whereas it is lower and negligible for the shear mode (ω_s), at least for the range of wave numbers considered here. From this figure, it can be concluded, too, that the thickness of the plate is a significant parameter that cannot be ignored in the study of the wave propagation behavior.

In Fig. 2, the influences of the thickness and the wave number on the phase velocity of a triclinic plate are presented. It has been shown that the relative phase velocity of the bending mode (ω_b) increases with the increase of the wave number; on the other hand, for the shear mode (ω_s),

The increase of the wave number leads to a decrease of the phase velocity.

4. Conclusions

In this work, the natural frequencies and the wave propagation in anisotropic plates are studied with a refined shear deformation theory that reduces the number of equations of motion to three, as well as the mass inertia to two, by means of an analytical approach.

More particularly, triclinic and orthotropic materials are investigated. The equations of motion are derived using the Hamilton principle. The analytical dispersion relation of the anisotropic plates is obtained as a solution to an eigenvalues problem.

A comparative study has been made between a triclinic material and another orthotropic one. It may be particularly noticed that it is not necessary to consider the triclinic concept instead of the orthotropic one for the calculation of frequencies and phase velocities, especially, when waves are considered in a single direction.

An improvement of present formulation will be considered in the future work to consider the thermal effect (Tounsi *et al.* 2013, Bouderba *et al.* 2013 and 2016, Zidi *et al.* 2014, Hamidi *et al.* 2015, Attia *et al.* 2015 and 2018, Bousahla *et al.* 2016, Chikh *et al.* 2017, El-Haina *et al.* 2017, Mouffoki *et al.* 2017, Menasria *et al.* 2017, Khetir *et al.* 2017, Fahsi *et al.* 2017, Cherif *et al.* 2018, Semmah *et al.* 2019).

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