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Improvement in uplift capacity of horizontal circular anchor plate in undrained clay by granular column

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Abstract. A numerical study has been conducted to examine the improvement achieved in the ultimate pullout capacity of horizontal circular anchor plates embedded in undrained clay, by constructing granular columns of varying diameter over the anchor plates. The analysis has been carried out by using lower bound theorem of limit analysis and finite elements in combination with linear programming. The improvement in uplifting capacity of anchor plate is expressed in terms of an efficiency factor (ξ). The efficiency factor (ξ) has been defined as the ratio of ultimate vertical pullout capacity of anchor plate having diameter D embedded in soft clay reinforced by granular column to the vertical pullout capacity of the anchor plate with same diameter D embedded in soft clay only. The variation of efficiency factor (ξ) for different embedment ratios and different diameter of granular column has been studied considering a wide range of softness of clay and different value of soil internal friction angle (ϕ) of the granular material. It is observed that ξ increases with an increase in diameter of the granular column (D_t) and increase in friction angle of granular material. Also, the effectiveness of the usage of granular column increases with decrease in cohesion of the clay.

Keywords: circular anchor plate; granular column; limit analysis; efficiency; anchor in clay; uplift resistance

1. Introduction

Anchor plate is generally used as a foundation component to resist tensile uplifting load in structures like transmission tower, buried pipe under water etc. A significant number of studies have been reported in literature for anchors embedded in cohesive soils (Kupferman 1965, Yu 2000, Merifield *et al*. 2003, Khatri and Kumar 2009a, Demir and Ok 2015). The uplift resistance offered by horizontal anchor plates embedded in soft clays is found to be significantly low compared to the uplift resistance provided by horizontal anchor plates of same diameter embedded at the same depth in granular soil (Merifield *et al*. 2006, Bhattacharya and Kumar 2015, Niroumand and Kassim 2014, Keskin 2015). For practical design purpose it may sometimes be required to increase resistance against uplifting force by anchor plate (i.e., the ultimate uplift capacity of anchor plate) embedded in soft undrained clay.

The most widely employed solution to increase the capacity is to preload the clay using a

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definite surcharge over the soil mass. This induces extra stresses on the cohesive soil. The extra stresses sets up pore water pressure gradient leading to subsequent drainage, reduction of void ratio and strength gain of the clay. However, increase of strength using preloading technique involves a lot of time and money.

The usage of granular column to improve the bearing capacity of footing placed on soft/weak cohesive soils (e.g., Bouassida *et al*. 1995, Bouassida and Porbaha 2004, Bouassida *et al*. 2015) is common in engineering practice. Recently researchers have used the granular column and granular pile anchors in order to improve the vertical pullout capacity of anchor plate (Kumar and Rao 2000, Rao *et al*. 2007, Sivakumar *et al*. 2013, O'Kelly *et al*. 2014). Kumar and Rao (2000) conducted experimental investigation by using base-geosynthetics just above the base anchor plate of the granular pile to increase its pullout resistance. Rao *et al*. (2007) carried out field test to investigate the pullout behavior of granular pile anchor (GPA) in a group on expansive clay. Sivakumar *et al*. (2013) conducted a number of field tests and theoretical analysis to study the performance of granular anchor consisted of either a single plate or double-plates along with the granular materials. Sivalumar *et al*. (2013) performed the theoretical investigation considering only the shaft resistance and localized end bulging. O'Kelly *et al*. (2014) conducted full scale field trials by installing granular anchors with different length and of different diameters ranging from 0.15 m to 0.219 m installed in Brown Dublin Boulder Clay layer. A numerical finite element analysis was also carried out to investigate the uplift capacity of granular anchors of different length to diameter ratio by PLAXIS 2D 2010.

It is understood from the literature that the usage of granular anchor is an emerging area of research in the field of ground improvement where granular anchor can be used as foundation element to increase the resistance against uplifting load and for ground improvement. In the present article, the concept of granular anchor is used to study the improvement of uplift capacity of the anchor plate with diameter D embedded in undrained soft clay by using a granular column with diameter D_t ($0.4 \leq D_t/D \leq 3$) above the anchor plate. For this, a column can be excavated and it can subsequently be backfilled with granular materials. The ultimate uplifting load provided by an anchor plate has been determined by employing lower bound theorem of limit analysis with finite elements. The lower bound solution provides the safe estimates of design load for a material following an associated flow rule. Researchers have used the numerical axisymmetric lower bound limit analysis successfully to determine: (i) bearing capacity of footings (Khatri and Kumar 2009b, Kumar and Khatri 2011); (ii) ultimate pullout capacity of circular anchor plate (Khatri and Kumar 2009a). In the present work, the improvement of uplifting resistance offered with the usage of the granular column has been expressed in terms of an efficiency factor (ξ) of the system.The study has been carried out for six different embedment ratios, ranging from 2 to 7, and using six different values of normalized cohesion ($c_0/\gamma_{clay}D$) of *in-situ* clay lying between 0.5 and 10 where c_0 is the cohesion of clay deposit at the ground surface and γ_{clay} is the unit weight of undrained clay. The friction angle of the granular column has been considered for the analysis is equal to 40° and 45°. In the present numerical work, a linear increase of soil cohesion with depth has also been considered for a few cases as the cohesion of normally consolidated clay and slightly over consolidated clay increases linearly with depth (Bishop 1966). The failure patterns have also been studied for anchor plate embedded at different depths without and with granular column.

2. Problem definition

A circular anchor plate, of diameter D, is embedded horizontally in undrained clay stratum at a depth H from the ground surface of the clay stratum which is reinforced with a granular column having diameter D_t and height H from the ground surface. The thickness of the anchor plate is assumed to be negligible compared to its diameter (D). A schematic diagram of the chosen problem for $D_t = D$ is shown in Fig. 1(a). For a given H/D and D_t/D, it is required to determine the vertical uplift resistance of the circular anchor with and without the reinforcing granular column. In the present analysis, two cases are considered: (i) the unit weights of both the clay (γ_{clay}) and the granular material (γ_{sand}) are same; i.e., $\gamma_{clay} = \gamma_{sand}$ and (ii) the unit weights of both the clay (γ_{clay}) and the granular material (γ_{sand}) are different, i.e., $\gamma_{clay} \neq \gamma_{sand}$. The normalized undrained cohesion ($c_0/\gamma_{clay}D$) of the clay has been assumed to vary between 0.5 and 10 where c_0 is the cohesion of the undrained clay at ground level. "Immediate Breakaway" condition is assumed in the analysis, i.e., no normal stress (σ_z) and shear stress (τ_{rz}) are allowed to exist along the interface between the bottom surface of anchor plate and the underlying undrained clay bed. The resistance along the interface between the upper surface of the plate and overlying soil mass (either clay or granular soil) is assumed to be governed by the respective soil shear strength parameters. It is assumed that the soil mass obeys the Mohr-Coulomb failure criterion and an associated flow rule.

2.1 Domain, finite element mesh and stress boundary conditions

A circular anchor plate with diameter D is kept at an embedment depth of H in a rectangular domain EFLK in r-z plane, as shown in Fig. 1(a). The entire domain is symmetric about the vertical axis IJ passing through the center of the anchor plate. Therefore, only one half of the total domain (EFLK) in r-z plane, that is, the zone IJKL has been considered for the present analysis. The other half of the domain, i.e., EFJI has not been considered for the computation. The vertical boundary (KL) located on the right hand side of the domain IJKL and the bottom boundary (JL) of the domain must be kept at sufficient distances away from the anchor plate. The horizontal distance (L_r) between the right edge of the anchor plate and the vertical boundary KL is varied from 15D to 30D. On the other hand, the vertical extent of the domain (L_d), that is, the distance between the anchor plate and the horizontal boundary JL is varied between 2D and 10D depending on the values of H/D and strength of clay.

The stress boundary conditions that are applicable along the different boundaries of the domain are presented in Fig. 1(a). The values of normal and shear stresses are kept equal to zero along the ground surface (IK). No stress boundary conditions need to be imposed along the boundary KL and JL. Since IJ represents the axis of symmetry, therefore, the shear stress along IJ needs to be equal to zero. Immediate breakaway condition along the interface between bottom surface of the anchor plate and the underlying clay is considered. It means the anchor plate in cohesive soil bed is no longer in contact with the underneath soil lying below it. Therefore, the shear (τ_{rz}) and normal (σ_z) stresses, acting at nodes which lying along the interface between bottom surface of the anchor plate and underneath clay are kept equal to zero. A shear slip is permitted along the top surface of the anchor plate and surrounding soil mass. Along the anchor-soil interface, for top surface, the following stress boundary condition is imposed

$$|\tau_{rz}| \leq (c \cot\phi - \sigma_z)\tan\delta \tag{1}$$

where c is the cohesion of the clay soil at the level of anchor plate.

The negative sign associated with σ_z arises due to the fact that tensile normal stresses are considered as positive in the present analysis. The value of δ has been chosen equal to the internal friction angle of soil. In undrained clay, the internal soil friction angle, $\phi_u = 0$. On the other hand,

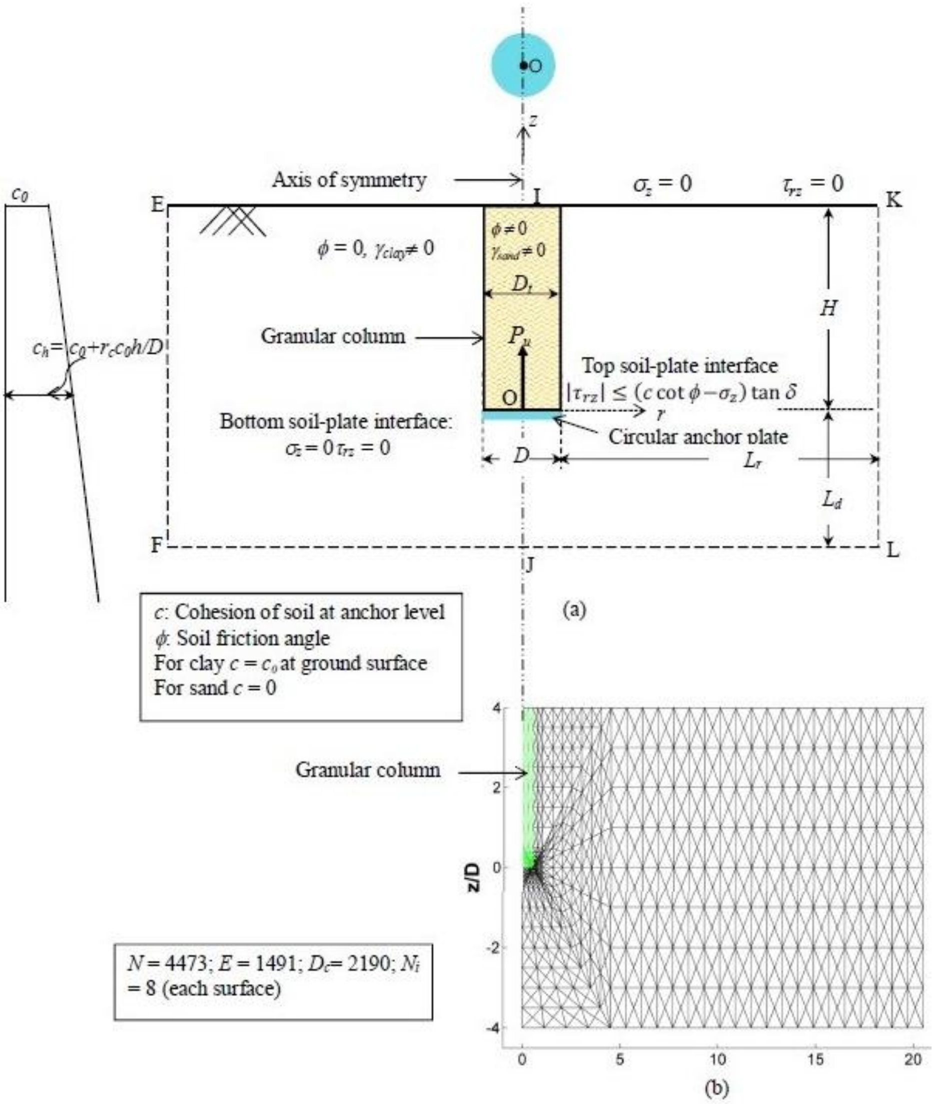


Fig. 1 (a) Problem statement showing the boundary conditions; and (b) Typical finite element mesh for $H/D = 4$ and $D_t/D = 1$

for materials used in granular column, the soil friction angle (ϕ) chosen for present analysis is 40° and 45°.

The domain is discretized by a number of three noded triangular elements such that the sizes of the elements get reduced continuously towards both right and left edges of the anchor plate. The finite element meshes, with embedment ratio (H/D) equals to 4 and D_t/D equal to 1, is shown in Fig. 1(b); in this figures, N, E, D_c and N_i are total number of nodes, elements, stress discontinuities and nodes along the soil-anchor interface (each side), respectively. The values of L_r and L_d are chosen such that (i) none of the yielded elements touch any of the chosen domain boundaries (JL and KL); and (ii) a further increment in the size of the domain does not cause any change in the magnitude of the collapse load.

3. Analysis

3.1 Numerical formulation for lower bound limit analysis with finite elements

Lower bound theorem of limit analysis in combination with finite elements and linear programming has been used in the study. The guideline for such an analysis was proposed by Sloan (1988) to solve plane strain problems and Kumar and Khatri (2011) to solve axi-symmetric problems. Axisymmetric formulation has been employed for the analysis of circular anchor in this case. The nodal stresses σ_r, σ_z, τ_{rz} and σ_θ are considered as the basic unknown stress variables in the axisymmetric case. The element equilibrium conditions are satisfied everywhere in the domain. The equilibrium equations for an axisymmetric formulation are shown below

$$\frac{\partial\sigma_r}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2a}$$

$$\frac{\partial\tau_{rz}}{\partial r} + \frac{\partial\sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = \gamma \tag{2b}$$

where γ is the unit weight of soil mass. γ for sand and clay are represented by γ_{sand} and γ_{clay}, respectively.

Statically admissible stress discontinuities are allowed along all the common edges shared by any two adjacent elements. In this process continuity of shear and normal stresses are maintained for two nodes having same coordinates and being part of a common edge, whereas there is a discontinuity of tangential stresses along the same edge. The Mohr-Coulomb yield condition also needs to be satisfied everywhere in the statically admissible stress field. The Mohr-Coulomb failure envelope takes the shape of a circle when it is plotted using $X = \sigma_r - \sigma_z$ and $Y = 2\tau_{rz}$ in the *X*-*Y* plane. In order to use linear optimization the original Mohr-Coulomb yield function is linearized (Bottero *et al.* 1980) by a regular polygon of sides *p* inscribed to the parent yield circle in *X*-*Y* plane. Thus, each nonlinearity constraint is replaced by *p* number of equality constraints. The value of *p* has been taken equal to 21.

In the axisymmetric case, the value of Hoop stress, σ_θ, other than σ_r, σ_z and τ_{rz}, should be closer to the minor principal stress (σ_3) at failure in *r*-*z* plane by following Harr-von Karman hypothesis (1909). This condition is taken care of by employing following three inequality constraints at each node in the domain

$$\sigma_{\theta j} \geq \sigma_{rj} \tag{3a}$$

$$\sigma_{\theta j} \geq \sigma_{zj} \tag{3b}$$

$$\sigma_{\theta j} \leq \sigma_{3fj} \tag{3c}$$

where *j* implies the node number and σ_{3f} is the minor principal stress at failure. The expression for σ_{3f} is given below

$$\sigma_{3f} = \frac{\sigma_z + \sigma_r}{2} + \left\{-\frac{\sigma_z + \sigma_r}{2} + c_h \cot\phi\right\}\sin\phi \tag{3d}$$

where c_h is the cohesion of the soil at any depth *h* below the ground surface.

The magnitude of the collapse load (P_u) in any statically admissible stress field can be determined by integrating the normal stresses acting along the top interface of the circular anchor plate by using the following expression

$$P_u = \int_{Top\ interface} (-\sigma_z dA) \tag{4}$$

The magnitude of the collapse load (P_u) is maximized subjected to a set of equality and inequality linear constraints in lower bound limit analysis. Therefore, the linear optimization problem can finally be defined by the following form

$$\text{Maximize the objective function:} \quad -\{g\}^T\{\sigma\} \tag{5a}$$

$$\text{Subjected to (i) equality constraints:} \quad \{A_{eq}\}\{\sigma\} = \{b_{eq}\} \tag{5b}$$

$$\text{(ii) inequality constraints:} \quad \{A_{ineq}\}\{\sigma\} \leq \{b_{ineq}\} \tag{5c}$$

The LINPROG function available in MATLAB 2012 was used to perform the necessary linear optimization.

3.2 Definition of efficiency factor (ξ) for improvement of uplift capacity

The efficiency factor (ξ) for improvement of uplift capacity can be defined as a ratio of the ultimate resistance ($P_{u,clay\ with\ granular\ column}$) offered by an anchor plate of diameter *D* against uplifting at an embedment depth *H* in soft clay reinforced with granular column to the ultimate resistance ($P_{u,clay}$) offered by the same anchor plate (with diameter *D*) embedded at same depth *H* in soft clay only.

$$\xi = \frac{P_{u,clay\ with\ granular\ column}}{P_{u,clay}} \tag{6}$$

4. Results and comparison

4.1 Comparison of normalized uplift pressure along anchor plate embedded in clay

A comparison is made among present lower bound solutions of normalized value of uplift pressure (p_u/c_0) with the solutions reported by: (i) Kupferman (1965) based on experimental investigation; (ii) Yu (2000) based on cavity expansion theory; (iii) Merifield *et al*. (2003) by using three dimensional lower bound limit analysis; and (iv) Khatri and Kumar (2009a) on the basis of axisymmetric lower bound limit analysis. The comparison is shown in Fig. 2. For comparison purpose, uplift pressure has been determined for circular anchor plate embedded in undrained clay and the clay is assumed to be weightless (i.e., $\gamma_{clay} = 0$) for the numerical and analytical works presented in Fig. 2. It is observed that in all cases the normalized ultimate uplift pressure resisted by the circular anchor plate increases with an increase in embedment ratio (*H*/*D*) up to a critical embedment ratio beyond which the normalized pressure becomes constant. The value of critical embedment ratio is found to be different in different solutions. Present work and the solution

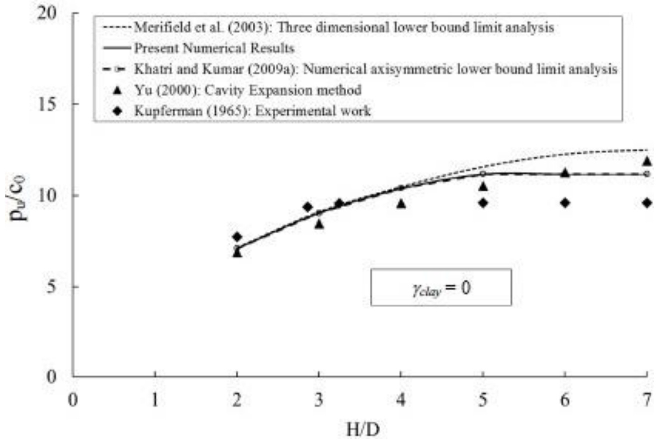


Fig. 2 A comparison of present numerical results with available results in literature

provided by Khatri and Kumar (2009a) match well and it has been noticed in both results that the normalized uplift pressure becomes constant at embedment ratio lying between 4.5 and 5.

The difference in normalized uplift pressure observed between present work and the results reported by Merifield *et al*. (2003) lies within 10%. Note that Merifield *et al*. (2003) used three dimensional lower bound limit analysis formulation by Tresca criterion along with the usage of nonlinear programming. The present results are also compared well with the results of Yu (2000) based on cavity expansion theory where difference observed between the two results is up to 6%. Although the trends of variation of normalized uplift pressure observed in present work and Kupferman (1965) based on laboratory model test match well, -3% to +16% differences between present work and experimental work has been found to exist in Fig. 2. The difference may be attributed due to the uncertainty in measurement of the shear strength parameters in the experiment, and variation of the suction along the anchor-soil interface.

4.2 Variation of efficiency factor (ξ)

Variation of efficiency factor (ξ) of circular anchor plate with the normalized diameter of granular column has been studied for different embedment depth.

4.2.1 Variation of efficiency factor (ξ) with normalized diameter (D_t/D) of granular column for $\gamma_{clay} = \gamma_{sand}$

Variation of efficiency factor (ξ) due to change of diameter of granular column in terms of D_t/D has been plotted in Figs. 3 and 4 for a wide range of clay having $c_0/\gamma_{clay}D$ equal to 0.5 to 10 with two different internal friction angles (say, $\phi = 40°$ and $45°$) of granular material. Uniform cohesion for clay throughout the depth has been considered in this case. Following observations are made based on the present analysis:

- For anchor plates embedded in soft clay, at a particular embedment ratio of $H/D > 3$, it is observed that the efficiency factor increases with increasing value of normalized diameter of the reinforcing granular column keeping ϕ and $c_0/\gamma_{clay}D$ values constant.
- For $H/D \leq 3$, the efficiency factor has found to decrease with an increase in D_t/D for stiff

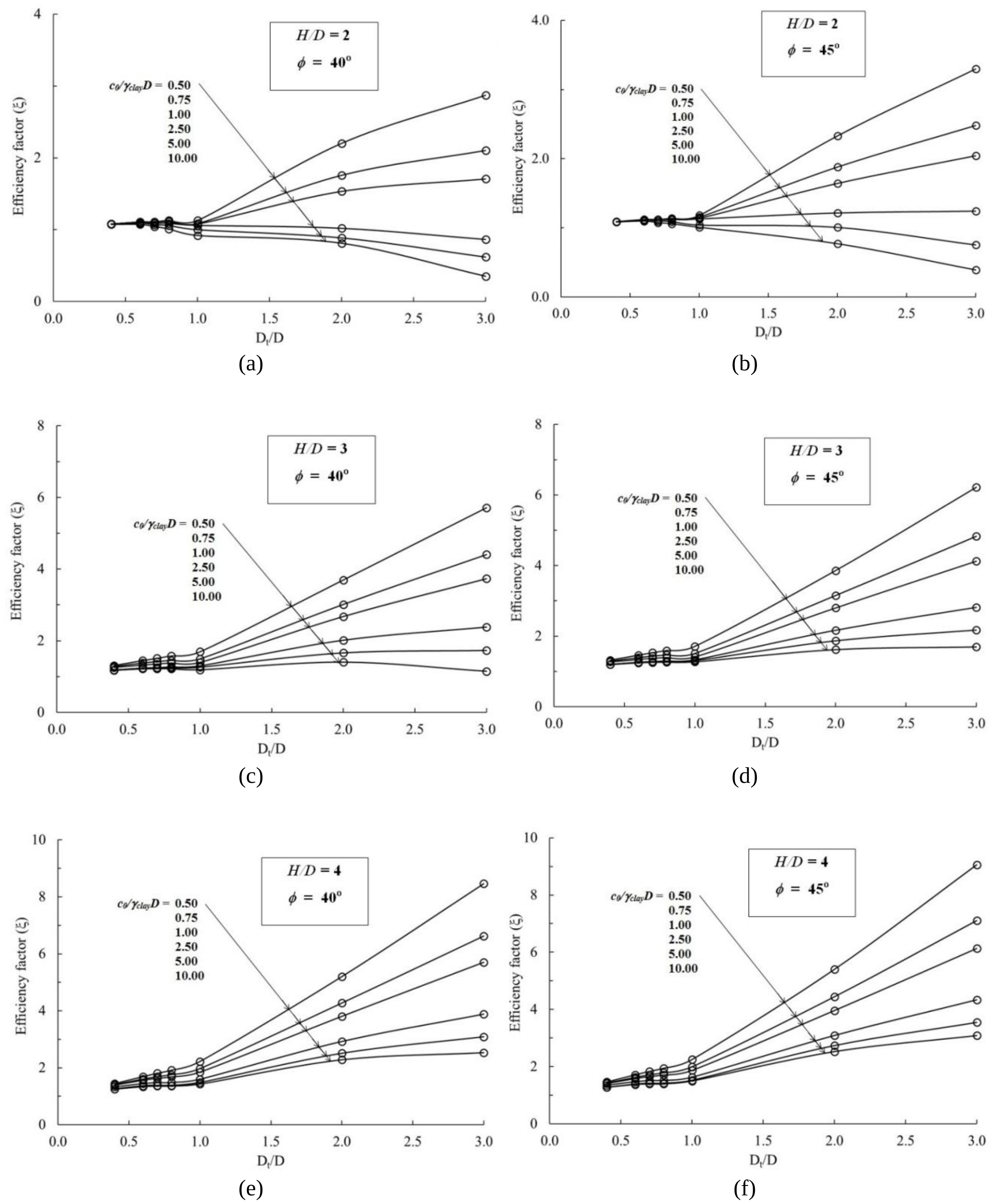


Fig. 3 Variation of efficiency factor (ξ) of circular anchor with D_t/D for $\gamma_{sand}/\gamma_{clay}$ = 1 and (a) H/D = 2 and ϕ = 40°; (b) H/D = 2 and ϕ = 45°; (c) H/D = 3 and ϕ = 40°; (d) H/D = 3 and ϕ = 45°; (e) H/D = 4 and ϕ = 40°; and (f) H/D = 4 and ϕ = 45°

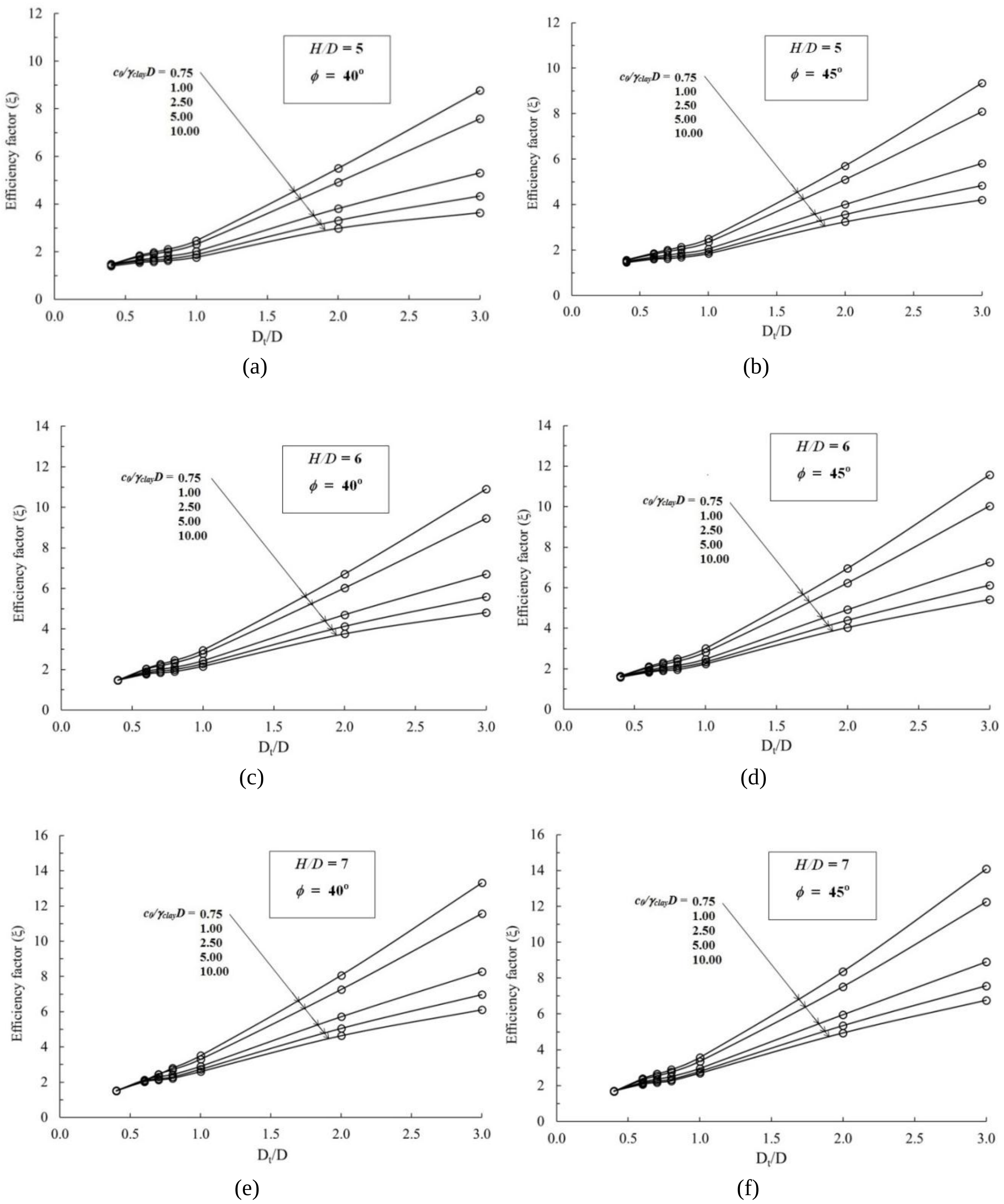


Fig. 4 Variation of efficiency factor (ξ) of circular anchor with D_t/D for $\gamma_{sand}/\gamma_{clay}$ = 1; and (a) H/D = 5 and ϕ = 40°; (b) H/D = 5 and ϕ = 45°; (c) H/D = 6 and ϕ = 40°; (d) H/D = 6 and ϕ = 45°; (e) H/D = 7 and ϕ = 40°; and (f) H/D = 7 and ϕ = 45°

clays with both ϕ values. This is because the strength due to cohesion of the original clay is more than the strength of the material used as the granular column. Therefore replacement of the stiffer clay by the comparatively low strength granular material may cause the reduction of the efficiency factor with the increase in D_t/D. For such cases, the efficiency factor may become even less than 1 also. For example, at $H/D = 2$, ξ values determined for clay $c_0/\gamma_{clay}D = 10$ with granular column of $D_t/D = 1$ and 2 and $\phi = 40°$ are found to be equal to 0.92 and 0.81, respectively.

- For all cases it has been noticed that the rate of increment of efficiency factor is higher for $D_t/D \geq 1$ compared to the rate of increment of efficiency factor at $D_t/D < 1$.
- It has also been observed that at a particular value of granular column diameter and embedment depth, a marginally higher efficiency factor can be achieved by using granular material with higher ϕ value instead of using granular material of lower ϕ value. For example, at $D_t/D = 1$ and $H/D = 6$, the efficiency factors estimated for ϕ equal to 40° and 45° are 2.47 and 2.51, respectively.
- The usage of granular column is more effective in terms of the resistance capacity against pullout load for very soft to soft clays and at higher embedment depth. For example, by using granular column with $\phi = 40°$, $D_t/D = 1$, the efficiency factors determined for circular anchor plate embedded in clays with $c_0/\gamma_{clay}D$ equals to 0.75 and 10 at $H/D = 7$ are 3.51 and 2.60, respectively. On the other hand, by using the same granular column with $\phi = 40°$ and $D_t/D = 1$, the efficiency factors for circular anchor plate embedded at $H/D = 2$ in clays with $c_0/\gamma_{clay}D$ equals to 10 are found to be equal to 0.92.

4.2.2 Variation of efficiency factor (ξ) with normalized diameter (D_t/D) of granular column for $\gamma_{clay} \neq \gamma_{sand}$

Variation of efficiency factor (ξ) with normalized diameter of granular column has been investigated considering three different values of $\gamma_{sand}/\gamma_{clay}$ (say, 1, 1.13 and 1.27) with $c_0/\gamma_{clay}D$ equal to 0.75 and 2.5 for H/D equals to 4 and 6. The results are presented in Fig. 5. Following observations have been made from Fig. 5.

The efficiency factor (ξ) increases with increasing value of $\gamma_{sand}/\gamma_{clay}$ from 1 to 1.27 keeping other parameters say, H/D, D_t/D, and internal friction angle (ϕ) of granular column constant. The rate of increase of efficiency factor due to an increase in $\gamma_{sand}/\gamma_{clay}$ seems to be higher at higher values of D_t/D keeping H/D and internal friction angle (ϕ) of granular column material constant.

- At a particular embedment depth or H/D, the degree of improvement of efficiency factor and hence the uplift resistance due to increase in $\gamma_{sand}/\gamma_{clay}$ is more significant for the weaker/soft clay layer ($c_0/\gamma_{clay}D = 0.75$) than the same in stiffer clay layer ($c_0/\gamma_{clay}D = 2.5$). For example, for $H/D = 4$, $D_t/D = 1$, ξ in soft clay with $c_0/D\gamma_{clay} = 0.75$ increases by 6.76% due to change in $\gamma_{sand}/\gamma_{clay}$ from 1 to 1.27 whereas ξ in stiffer clay with $c_0/\gamma_{clay}D = 2.5$ increases by 3.21% only.
- At a particular clay layer, the degree of improvement of efficiency factor and hence the uplift capacity of plate anchor due to increase in $\gamma_{sand}/\gamma_{clay}$ is slightly higher at low embedment depth than the same at high embedment depth. For example, efficiency factor (ξ) increases 6.76% due to an increase in $\gamma_{sand}/\gamma_{clay}$ from 1 to 1.27 for granular column with $D_t/D = 1$ and $\phi = 40°$ placed over the anchor plate at $H/D = 4$ whereas ξ increases 6.45% due to

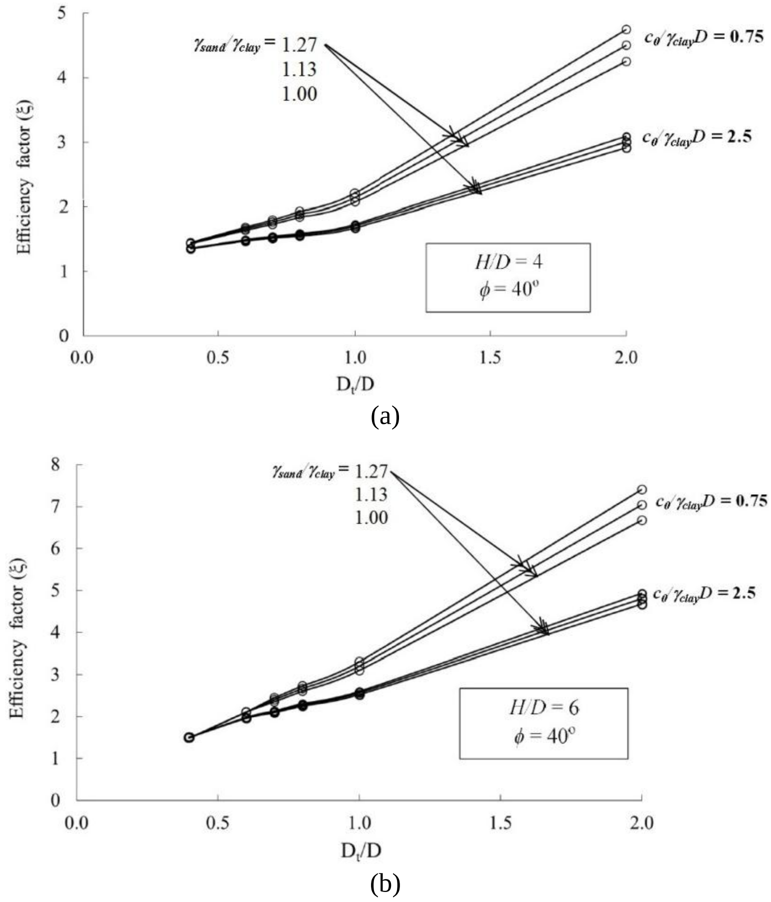


Fig. 5 Variation of efficiency factor (ξ) of circular anchor with D_t/D for different values of $\gamma_{sand}/\gamma_{clay}$ with (a) H/D = 4 and ϕ = 40°; and (b) H/D = 6 and ϕ = 40°

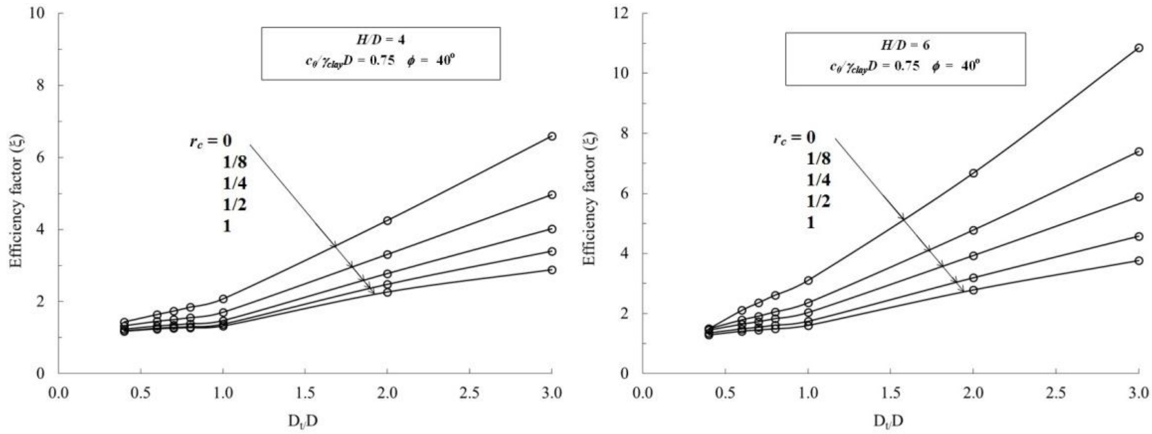
same amount of increase in $\gamma_{sand}/\gamma_{clay}$ (i.e., from 1 to 1.27) for granular column with D_t/D = 1 and ϕ = 40° placed over anchor plate at H/D = 6.

4.2.3 Effect of variation of cohesion with depth on efficiency factor (ξ)

The constant cohesion throughout the depth for the undrained clay has been considered to solve the present problem in previous sections. However the cohesion of normally consolidated clay and slightly over consolidated clay increases linearly with the depth below ground surface (Bishop 1966). Therefore the effect of variation of cohesion with depth on efficiency factor has been studied in this section and illustrated in Fig. 6. The cohesion of soil mass is assumed to increase linearly with depth (h) following Eq. (7) as shown below

$$c_h = c_0 + r_c c_0\, h/D \tag{7}$$

where c_h and c_0 are the cohesion of undrained clay at depth h below the ground surface and cohesion at the ground surface, respectively, r_c is a non-dimensional number which defines the rate



(a) (b)

Fig. 6 Variation of efficiency factor (ξ) of circular anchor with D_t/D for different values of rate of increase of cohesion (r_c) with depth and $\gamma_{sand} = \gamma_{clay}$ for (a) $H/D = 4$ and $\phi = 40°$; and (b) $H/D = 6$ and $\phi = 40°$

at which cohesion increases linearly with depth below ground surface.

Following observations are made from Fig. 6 by considering the linearly increase of cohesion with depth:

- The efficiency factor decreases with an increase in the rate of cohesion (r_c) keeping other parameters say H/D, D_t/D, ϕ and $\gamma_{sand}/\gamma_{clay}$ constant. For example, efficiency factors (ξ) determined in the present analysis for anchor plate embedded at $H/D = 6$ and reinforced with granular column of $D_t/D = 1$ and $\phi = 40°$ are approximately equal to 3.1, 2.03, 1.74 and 1.6 for r_c = 0, 1/4, 1/2 and 1, respectively. As the rate of change of cohesion with depth increases the pullout capacity of the circular anchor plate in clay also increases. Thus, improvement of pullout capacity of anchor plate in undrained clay with the granular column may become less in comparison with the case where uniform cohesion has been considered throughout the depth. It is worthy to mention here that $\gamma_{sand}/\gamma_{clay}$ is chosen equal to 1 in this section.
- The effect of considering increase in cohesion with depth on the efficiency factor is more prominent at lower rate of increase of cohesion (r_c) keeping other parameters constant. However, as magnitude of r_c increases, the impact of r_c on efficiency factor is found to get reduced. For example, in previous example the efficiency factor has been reduced approximately 43% and 8% due to increase of r_c from 0 to 0.5 and 0.5 to 1.0, respectively.
- The reduction in efficiency factor due to consideration of linear increase in cohesion with depth instead of uniform cohesion has been found to be higher at higher embedment depth keeping other parameters D_t/D, ϕ and $\gamma_{sand}/\gamma_{clay}$ constant. For example, the decrease of efficiency factor (ξ) due to considering increase in r_c from 0 to 1/4 for anchor plate embedded in soft clay with $c_0/\gamma_{clay}D = 0.75$ and reinforcing by a granular column with $D_t/D = 1$ and $\phi = 40°$ are 29% and 34.5% for embedment depth (H) equal to $4D$ and $6D$, respectively.

4.3 Failure pattern

After finding out the statically admissible stress field for optimum value of uplift load, the proximity of the stress state to shear failure has been found out in terms of a ratio, a/d, where $a = (\sigma_r - \sigma_z)^2 + (2\tau_{rz})^2$ and $d = (2c_h \cos\phi - (\sigma_r + \sigma_z)\sin\phi)^2$. The a/d value becomes equal to unity at point subjected to shear failure. The proximity of stress state of soil domain to failure has been shown in Figs. 7 and 8 for circular plate anchor embedded in saturated clay with $c_0/\gamma_{clay}D = 1$ and $c_0/\gamma_{clay}D = 10$, respectively, at H/D equal to 4 and 5 without ($D_t/D = 0$) and with granular column ($D_t/D = 1$). The dark red color in diagrams indicates the plastic failure state. It has been observed in Figs. 7(a) and 7(d) that the anchor behaves as a deep anchor in soft clay with $c_0/\gamma_{clay}D = 1$ for H/D equals to 4 and 5 in absence of the granular column. The shear failure has been localized only around the anchor plate in case of deep anchor. However the plastic failure extends up to the ground surface while granular column being used in the same clay deposit with $c_0/\gamma_{clay}D = 1$ [refer Figs. 7(b)-(c) and 7(e)-(f)]. In this case a more prominent and clear plastic failure zone has been observed along the interface between the granular column and saturated clay deposit for $H/D = 4$

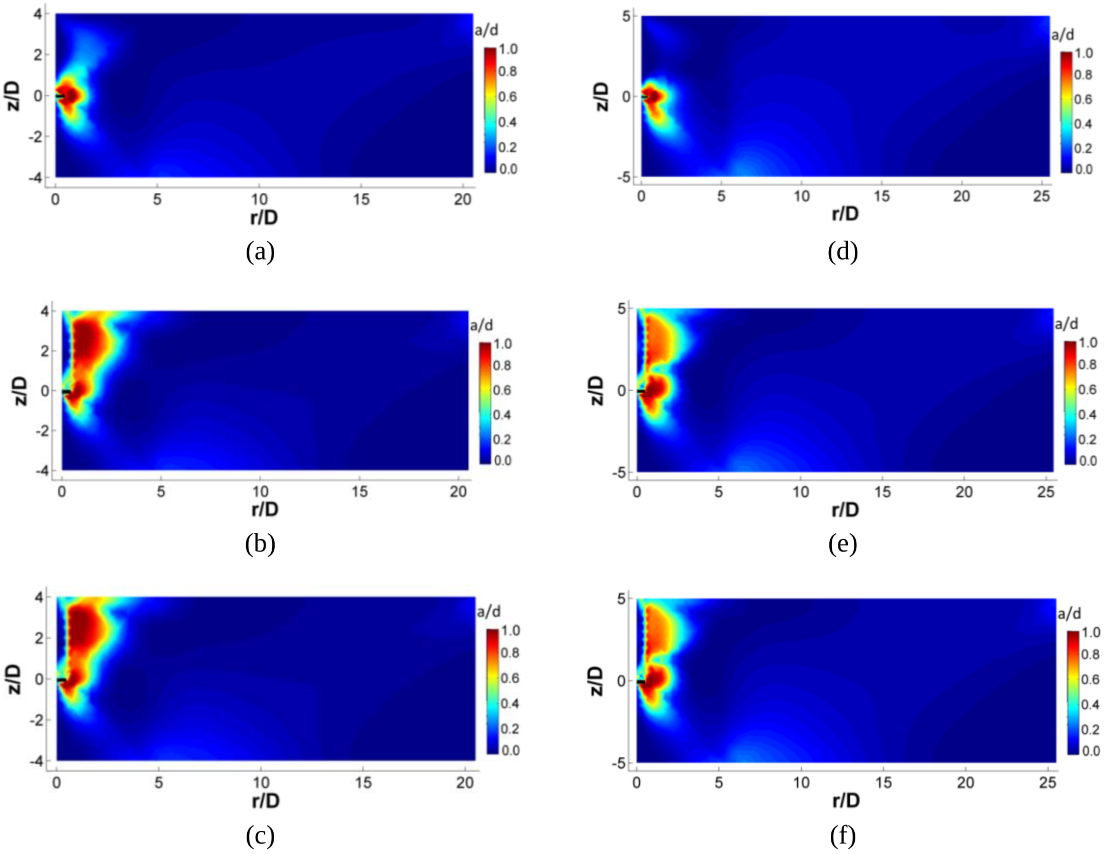


Fig. 7 Proximity of the stress state to failure with $\gamma_{sand}/\gamma_{clay} = 1$ and $c_0/\gamma_{clay}D = 1$ for;
(a) $H/D = 4$ and $D_t/D = 0$; (b) $H/D = 4$, $\phi = 40°$ and $D_t/D = 1$;
(c) $H/D = 4$, $\phi = 45°$ and $D_t/D = 1$; (d) $H/D = 5$ and $D_t/D = 0$;
(e) $H/D = 5$, $\phi = 40°$ and $D_t/D = 1$; and (f) $H/D = 5$, $\phi = 45°$ and $D_t/D = 1$

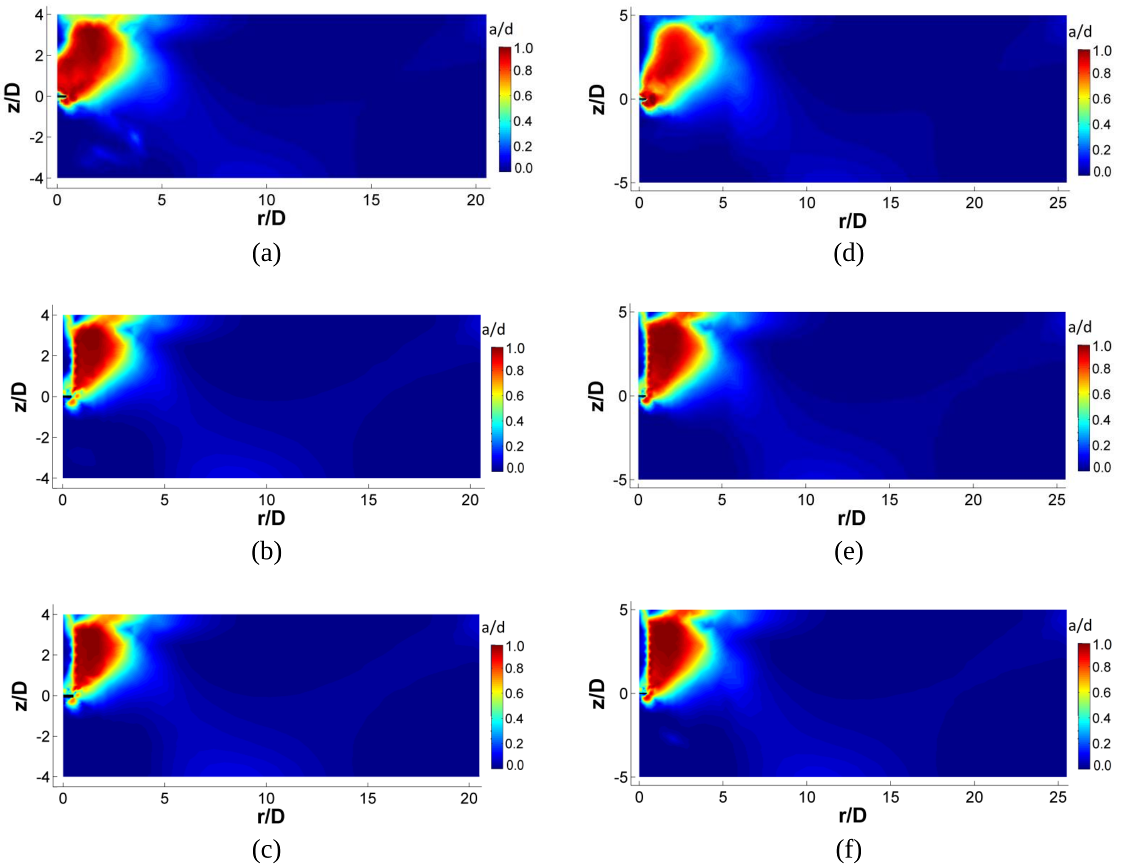


Fig. 8 Proximity of the stress state to failure with $\gamma_{sand}/\gamma_{clay}$ = 10 and $c_0/\gamma_{clay}D$ = 10 for; (a) H/D = 4 and D_t/D = 0; (b) H/D = 4, ϕ = 40° and D_t/D = 1; (c) H/D = 4, ϕ = 45° and D_t/D = 1; (d) H/D = 5 and D_t/D = 0; (e) H/D = 5, ϕ = 40° and D_t/D = 1; and (f) H/D = 5, ϕ = 45° and D_t/D = 1

with granular column compared to the plastic failure zone for H/D = 5 with granular column. It may be happened due to the fact that for lower embedment depth in soft clay failure occurs along the periphery of the granular column [refer Figs. 7(b)-(c)], whereas for higher embedment depth in soft clay the failure may occur due to bulging as noticed in Figs. 7(e)-(f). Figs. 8(a)-(f) illustrate the failure patterns around the anchor plates embedded at H/D = 4 and 5 in saturated clay with $c_0/\gamma_{clay}D$ = 10 without (D_t/D = 0) and with (D_t/D = 1) granular column. In Figs. 8(a) and (d), failure zone is found to be extended from the edge of the anchor plate to the ground surface in absence of the granular column. However, it is observed in Figs. 8(b)-(c) and 8(e)-(f) that in presence of granular column (say D_t/D = 1) the failure patterns extend from the edge of the anchor plate to the ground surface along the interface between the granular column and clay surface. No plastic failure has been occurred inside the granular column in all the cases. The presence of the granular column has attributed the stress distribution over a larger clay domain; thereby mobilizing greater capacity. There is no change in the failure patterns for the usage of different granular materials with different ϕ values.

4.4 Comparison of present work for pullout load in clay reinforced by granular column with available works in literature

A comparison between present lower bound solution of ultimate pullout load ($P_{u,clay\ with\ granular\ column}$) and the results provided by O'Kelly *et al*. (2014) by conducting field tests and numerical works on granular anchors is presented in Table 1. O'Kelly *et al*. (2014) had performed the pullout tests of granular anchors installed in Dublin Boulder Clay (DBC) which is very stiff, comprised of slightly sandy slightly gravely clay/silt layers and heavily over-consolidated with over-consolidation ratio 15-30. The bulk density of the DBC was reported equal to 22 kN/m^3. From the *in-situ* cone penetration test (CPT) and the unconsolidated undrained triaxial tests of the remolded samples it was approximated that the remolded undrained shear strength of the Dublin Boulder Clay at ground surface (c_0) was 64 kN/m^2 which was increasing uniformly with depth at a rate of 12.5 kN/m^2 per metre depth. The gravel column used for field and numerical tests to form the granular column had a density of 20 kN/m^3, peak friction angle 42° and dilatancy angle of 10^o. Using these parameters in the present numerical analysis, the uplift loads have been obtained and presented in Table 1. O'Kelly *et al*. (2014) carried out finite element analysis using PLAXIS 2D 2010 with K_o= 1.5 and Mohr-Coulomb model was used for over-consolidated Dublin clay. In the present numerical analysis, the approximate lower bound solution of failure load (Sloan 2013) has also been found out considering the effect of dilatancy of granular material by finding out the reduced shear strength parameters ϕ^* for granular material following Eq. (8) proposed by Davis (1968).

$$\tan\phi^* = \eta\tan\phi; \qquad \eta = \frac{\cos\psi\cos\phi}{1-\sin\psi\sin\phi} \tag{8}$$

where ψ is the dilatancy angle for granular material.

Table 1 Comparison of present results with experimental and numerical results reported by O'Kelly *et al*. (2014)

Sl. No.	D (m)	L (m)	L/D	Experimentally measured load [1] (kN)	Theoretical load [2] (kN)	Present analysis [3] (kN)
GA1	0.219	1.2	5.5	51	61.8	59.3[#] (57.5)[*]
GA2	0.219	0.96	4.4	43	42.0	45.2(43.2)
GA3	0.2	0.5	2.5	19.1	-	18.6(17.4)
GA4	0.219	1.00	4.6	47	-	47.8 (46)
GA5	0.168	1.47	8.7	42.5	-	57.7 (55.8)
GA6	0.168	0.8	4.8	33	-	28.9(27.7)
GA7	0.15	0.45	3.0	12.8	11.9	12.5 (11.4)
GA8	0.168	1.62	9.6	42	47.2	62.8 (60.2)

[1] Full scale field trials conducted on granular anchors installed in Brown Dublin Boulder clay (O'Kelly *et al*. 2014)
[2] Finite element analysis in PLAXIS 2D 2010 (O'Kelly *et al*. 2014)
[3] Numerical lower bound limit analysis with finite elements and linear programing
[#] Pullout load value following an associated flow rule up to one decimal place
[*] Approximate pullout load value up to one decimal place with dilatancy angle ψ = 10° following Sloan (2013)

The present solution matches well with experimental and theoretical solutions provided by O'Kelly *et al*. (2014) for $L/D \leq 8$. The difference between present numerical results and the theoretical solution provided by O'Kelly *et al*. (2014) has been found to vary 1-7%. However a slightly higher difference has been noticed between the present lower bound solution and the experimental results for $L/D \geq 8$. This may be due to inherent variability and material heterogeneity in the field soil. Moreover the *in-situ* Dublin Boulder Clay is formed in layers by deposition of sandy or gravely clay/silt whereas in the numerical investigation the unconsolidated undrained strength of the remolded soil is used with an approximate rate of increment of shear strength with depth. In the analysis done by O'Kelly *et al*. (2014), the effects of end bulging become more pronounced with increase in L/D ratio for the columns and full shaft capacity cannot be mobilized. Hence the difference between present results and ultimate pullout load estimated by O'Kelly *et al*. (2014) becomes higher at higher slenderness ratio, say $L/D > 8$. However, the failure loads of the present model compares well with the theoretical failure load predicted by O'Kelly *et al*. (2014).

5. Conclusions

The ultimate uplift capacity offered by a circular anchor plate embedded in undrained clay at different embedment depth without and with granular column has been determined numerically. The numerical analysis has been performed by using lower bound theorem of limit analysis with finite elements. The results are presented in terms of efficiency factor (ξ) which is defined as ratio of the ultimate uplift capacity of anchor plate embedded in soft clay with granular column to the same without granular column. It has been found that the efficiency factor (ξ) increases with an increase in diameter of the granular column. The efficiency factor has been found to increase slightly due to the usage of granular material having higher frictional angle. The efficiency factor (ξ) can be reduced in the less soft clay (i.e., $c_0/\gamma_{clay}D \geq 5$) keeping other parameters H/D, ϕ, D_t/D constant. It has also been noticed that efficiency factor (ξ) decreases with an increase in the rate of increase of cohesion of the clay with depth (r_c).

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