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Charts for estimating rock mass shear strength parameters

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Abstract. Charts are used extensively in slope practical application to meet the need of quick assessment of rock slope design. However, Charts for estimating the shear strength of the rock mass of a slope are considerably limited. In this paper, based on the Hoek-Brown (HB) criterion which is widely used in rock slope engineering, we present charts which can be used to estimate the Mohr-Coulomb (MC) parameters angle of friction ϕ and cohesion c for given slopes. In order to present the proposed charts, we firstly present the derivation of the theoretical relationships between the MC parameters and $\sigma_{ci}/(\gamma H)$ which is termed the strength ratio (SR). It is found that the values of c/σ_{ci} and ϕ of a slope depend only on the magnitude of SR, regardless of the magnitude of the individual parameters σ_{ci} (uniaxial compressive strength), γ (unit weight) and H (slope height). Based on the relationships between the MC parameters and SR, charts are plotted to show the relations between the MC parameters and HB parameters. Using the proposed charts can make a rapid estimation of shear strength of rock masses directly from the HB parameters, slope geometry and rock mass properties for a given slope.

Keywords: shear strength parameters; strength ratio; Hoek-Brown; charts assessment

1. Introduction

The Mohr-Coulomb (MC) shear strength parameters cohesion c and angle of friction ϕ are the most representative parameters of the mechanical behavior of rock materials. It is widely used in numerical modeling for the analysis of stress distribution to characterize the rock material behavior. In rock slope engineering, determination of reliable shear strength of rock materials is critical for slope design because small changes in shear strength values can result in significant changes in the slope safe height or slope angle (Wyllie and Mah 2004).

For a given rock slope, there are two models widely used for estimating the shear strength of the rock materials. One is the discontinuity model (Barton 2013) which represents the sliding surface of a slope consist of a single plane over the area of the surface. The other is the rock mass model (Hoek *et al*. 2002) which can be used to represent the global shear strength of jointed rock mass contains both discontinuities and intact rocks.

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Commonly used methods to estimate MC shear strength parameters includes: *in-situ* block shear tests, laboratory tests, and prediction by empirical models. *In-situ* block shear tests can provide direct information on the discontinuity shear strength of a slope in which failure occurs along a plane similar to the laboratory sample being tested.

However, such test results could not be used directly for estimating rock mass shear strength that makes up the full scale slope (Wyllie and Mah 2004). Laboratory tests on limited size rock samples containing discontinuities cannot measure reliably values of shear strength due to the limitation of size of the testing equipment. It is also difficult and expensive to carry out laboratory test for large fractured rock sample.

Due mainly to the above mentioned difficulties encountered in *in-situ* block shear and laboratory tests, the estimation of MC shear strength parameters values using empirical models are commonly accepted approach among rock engineers. Until now, the Hoek-Brown (HB) empirical model is the only rock mass model that directly links rock mass shear strength to slope input parameters, such as uniaxial compressive strength σ_{ci}, unit weight γ, slope height H and the Hoek-Brown parameters which can be used to categorize the rock mass in terms of both the intact rock strength and the characteristics of the discontinuities (Zuo *et al*. 2015).

Although, numerical analysis has been widely used for engineering projects (Azari *et al*. 2015, Chen *et al*. 2013, Fan *et al*. 2013, Zhang *et al*. 2015), having the merit of quick assessment of preliminary slope design, charts (Michalowski 2010, Gao *et al*. 2013, Shen *et al*. 2013, Eid 2014, Yang and Pan 2015, Zheng *et al*. 2015) have still been extensively used in practical applications. Based on our literature review, charts for the estimation of shear strength parameters directly from the HB criterion is still a under research area. Therefore, in this paper, based on the HB model, charts are proposed for estimating shear strength of the rock mass for slope design. The proposed charts are straightforward to use and can be adopted as useful tools for the design of rock slopes.

2. Rock mass shear strength model based on the HB criterion

2.1The Hoek-Brown criterion

The HB criterion has been widely used for the analysis and design of underground excavations in hard rock. It (see Eq. (1)) requires two intact rock properties, namely, the uniaxial compressive strength (UCS) of the intact rock σ_{ci} and a constant of the intact rock m_i which depends upon the frictional characteristics of the component minerals in the intact rock.

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_i \frac{\sigma_3}{\sigma_{ci}} + 1\right)^{0.5} \tag{1}$$

where σ_1 and σ_3 are the maximum and minimum principal stresses. Further, it was extended to estimate the fractured rock mass strength by using the Geological Strength Index (GSI) and a disturbance factor D to reduce intact rock properties.

The latest version of the HB criterion was presented by Hoek *et al*. (2002) and it can be expressed as the following equations

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^{a} \tag{2}$$

$$m_b = m_i e^{\left(\frac{GSI-100}{28-14D}\right)} \tag{3}$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \tag{4}$$

$$a = 0.5 + \frac{e^{\left(\frac{-GSI}{15}\right)} - e^{\left(\frac{-20}{3}\right)}}{6} \tag{5}$$

where, m_b, s and a are the input parameters which can be estimated from the GSI, D and intact rock constant m_i.

For the intact rock properties, the parameter m_i and σ_{ci} can be estimated from the regression method using triaxial tests data. In the absence of triaxial tests data, m_i can be estimated from Guidelines, *R* index and rock type-based methods, and σ_{ci} can be estimated from uniaxial compressive tests using cylindrical specimens or point load tests using unprepared rock cores (Shen and Karakus 2014, Li and Tao 2015). GSI values can be estimated according to the description of structure and block surface conditions of the rock mass (Marinos *et al*. 2005). The value of D varies from zero for undisturbed in situ rock masses to one for very disturbed rock masses, and the selection of D values for engineering application depends on the blast damage as well as stress relief result in disturbance of the rock mass (Hoek 2012).

2.2 Shear strength of the rock mass

The HB criterion Eq. (2) is expressed by the relationship between maximum and minimum principal stresses. However, it can also be expressed in terms of normal stress and shear stress on the failure plane as shown in Fig. 1. There are two methods available to estimate MC shear strength parameter c and ϕ from the HB parameters. One is by locating the tangent of the HB envelope under a given value of normal stress, which gives an upper bound value of rock mass shear strength and may generate optimistic shear strength in stability calculations. Therefore, Hoek *et al*. (2002) proposed an alternative method to estimate shear strength parameters of rock masses that is using a linear MC envelope to represent the non-linear HB envelope, as shown in Fig. 1.

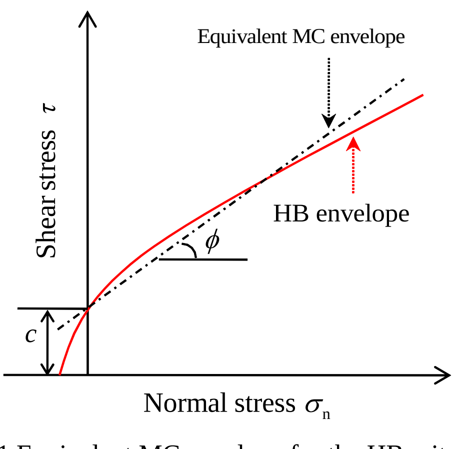


Fig. 1 Equivalent MC envelope for the HB criterion

Eqs. (6)-(10) can be used for calculating c and ϕ for slope stability were presented by Hoek *et al*. (2002) by analysing the stability of a wide range of slope with various rock mass properties and slope geometries using Bishop's circular failure analysis.

$$\phi = \sin^{-1}\left[\frac{6am_b(s+m_b\sigma_{3n})^{a-1}}{2(1+a)(2+a)+6am_b(s+m_b\sigma_{3n})^{a-1}}\right] \tag{6}$$

$$c = \frac{\sigma_{ci}[(1+2a)s+(1-a)m_b\sigma_{3n}](s+m_b\sigma_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1+(6am_b(s+m_b\sigma_{3n})^{a-1})/((1+a)(2+a))}} \tag{7}$$

$$\sigma_{3n} = \frac{\sigma_{3max}}{\sigma_{ci}} \tag{8}$$

$$\sigma_{3max} = 0.72\sigma_{cm}\left(\frac{\sigma_{cm}}{\gamma H}\right)^{-0.91} \tag{9}$$

$$\sigma_{cm} = \frac{\sigma_{ci}\left(m_b+4s-a(m_b-8s)\right)\left(s+\frac{m_b}{4}\right)^{a-1}}{2(1+a)(2+a)} \tag{10}$$

By considering m_b, s and a can be calculated from the GSI, D and m_i, therefore, development of charts for estimating MC shear strength parameters based on the HB criterion is not easy since there are six input parameters (GSI, m_i, D, σ_{ci}, γ, H) involved to calculate the c and ϕ values. Based on literature review, such charts are not available in the literature.

3. Theoretical relationship between SR and MC parameters

The strength ratio (SR) $\sigma_{ci}/(\gamma H)$, the ratio of σ_{ci} to γH which can represent the vertical stress of the rock slope, was used in this study. In this section, we firstly presented the derivation of the theoretical relationships between the SR and MC parameters for a given slope. In the next stage, based on the relationships between the SR and MC parameters, charts for calculating the MC parameters from the HB parameters when D = 0, 0.7 and 1.0 were proposed.

The value of global rock mass strength σ_{cm} (see Eq. (10)) depends on the UCS of intact rock and input parameters m_b, s and a, therefore, Eq. (10) can be transformed into Eq. (11), and the intermediate parameter σ_{3n} in Eq. (8) can also be expressed as Eq. (12).

$$\sigma_{cm} = \frac{\sigma_{ci}\left(m_b+4s-a(m_b-8s)\right)\left(s+\frac{m_b}{4}\right)^{a-1}}{2(1+a)(2+a)} = \sigma_{ci}f_1(m_b,s,a) \tag{11}$$

$$\sigma_{3n} = 0.72\sigma_{cm}\left(\frac{\sigma_{cm}}{\gamma H}\right)^{-0.91} = 0.72f_1\left(\frac{\sigma_{ci}}{\gamma H}f_1\right)^{-0.91} = f_2\left(\frac{\sigma_{ci}}{\gamma H},m_b,s,a\right) \tag{12}$$

From Eqs. (6)-(7), we can see that the value of c/σ_{ci} and ϕ depend on m_b, s and a and σ_{3n} which in turn depends on the strength ratio $\sigma_{ci}/(\gamma H)$. Therefore, Eqs. (6) -(7) can be transformed into Eqs. (13)-(14) as follows

$$\phi = \sin^{-1}\left[\frac{6am_b(s+m_b\sigma_{3n})^{a-1}}{2(1+a)(2+a)+6am_b(s+m_b\sigma_{3n})^{a-1}}\right] = f_3\left(\frac{\sigma_{ci}}{\gamma H}, m_b, s, a\right) \quad (13)$$

$$\frac{c}{\sigma_{ci}} = \frac{[(1+2a)s+(1-a)m_b\sigma_{3n}](s+m_b\sigma_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1+\left(6am_b(s+m_b\sigma_{3n})^{a-1}\right)/\left((1+a)(2+a)\right)}} = f_4\left(\frac{\sigma_{ci}}{\gamma H}, m_b, s, a\right) \quad (14)$$

The parameters m_b, s and a in Eqs. (13)-(14) can be calculated from Eqs. (3)-(5), respectively. Finally, the MC shear strength parameters can be expressed as Eqs. (15)-(16).

$$\phi = f_3\left(\frac{\sigma_{ci}}{\gamma H}, m_b, s, a\right) = f_5(\mathrm{SR}, \mathrm{GSI}, m_i, D) \quad (15)$$

$$\frac{c}{\sigma_{ci}} = f_6(\mathrm{SR}, \mathrm{GSI}, m_i, D) \quad (16)$$

Eqs. (15)-(16) indicate that for a given slope with known of the Hoek-Brown parameters GSI, m_i and D, the values of c/σ_{ci} and ϕ are depend on the dimensionless parameter SR regardless of the values of individual parameters σ_{ci}, γ and H.

Table 1 shows four different groups of σ_{ci}, γ and H associated with the same SR value for a slope that has the same values of GSI, m_i, and D. The results in Table 1 illustrate that the values of the c/σ_{ci} and ϕ of all groups are the same. Based on the relationships between the shear strength parameters and SR, the number of independent parameters for calculating the c/σ_{ci} and ϕ can be

Table 1 Comparison of MC parameters values of slopes with the same value of SR

Input parameters	Group 1	Group 2	Group 3	Group 4
GSI	50	50	50	50
m_i	15	15	15	15
D	0	0	0	0
σ_{ci}, MPa	12.5	0.25	2.5	1
γ, kN/m^3	25	25	25	20
H, m	500	10	100	50
SR	1	1	1	1
Calculated parameters				
ϕ°	26.53	26.53	26.53	26.53
c/σ_{ci}	0.10	0.10	0.10	0.10

reduced to four (SR, GSI, m_i and D). In the next stage, we will propose charts based on the SR and the Hoek-Brown parameters GSI and m_i for D = 0, 0.7 and 1.0.

4. Proposed charts for fractured rock mass slopes

We presented charts which can be used to estimate the MC shear strength parameters of homogenous rock masses from the Hoek-Brown parameters (GSI, m_i and D), slope geometry (H) and rock mass properties (σ_{ci} and γ) for the design of rock slopes.

4.1 Charts based on D = 0

Firstly, the proposed charts for the current study were based on the HB criterion from a range of SR, GSI and m_i, but with a specified disturbance factor D = 0 as shown in Fig. 2.

Fig. 2 indicates that SR has a considerable effect on the values of c/σ_{ci} and ϕ, especially, under the state of low SR values. The values of c/σ_{ci} decrease and ϕ increase when the values of SR

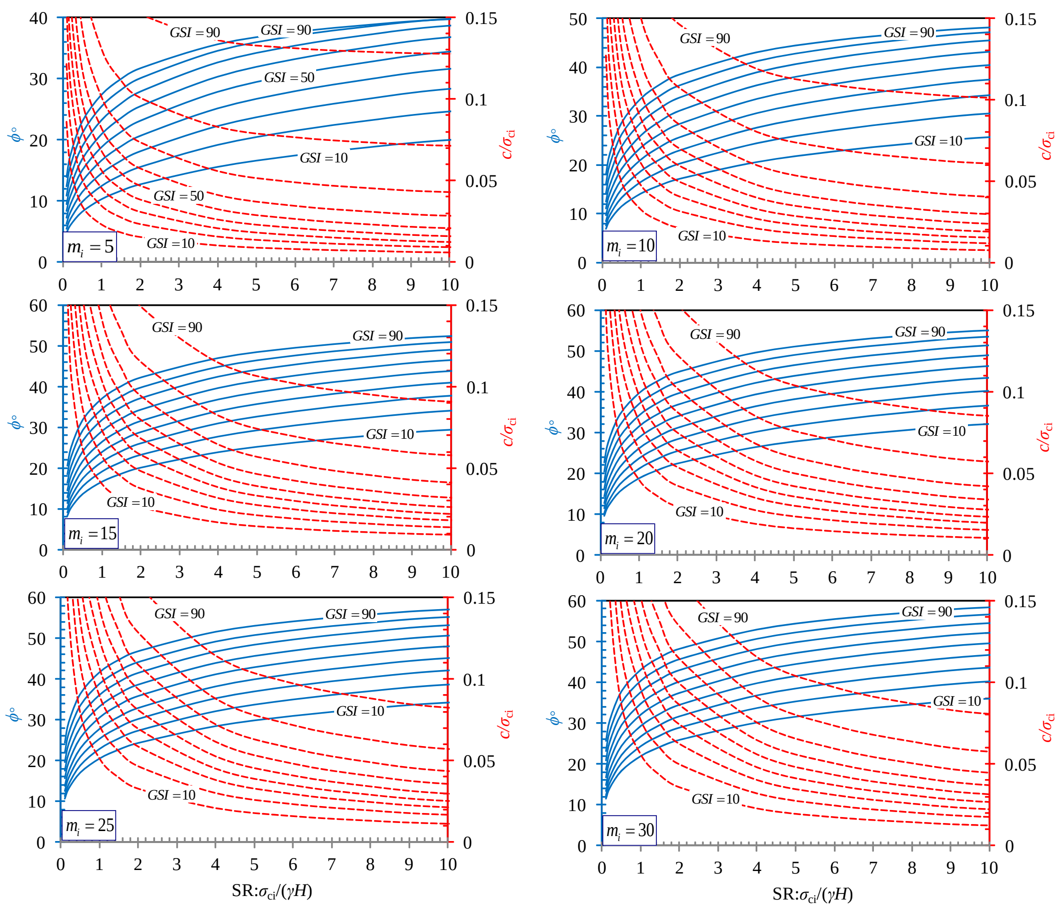
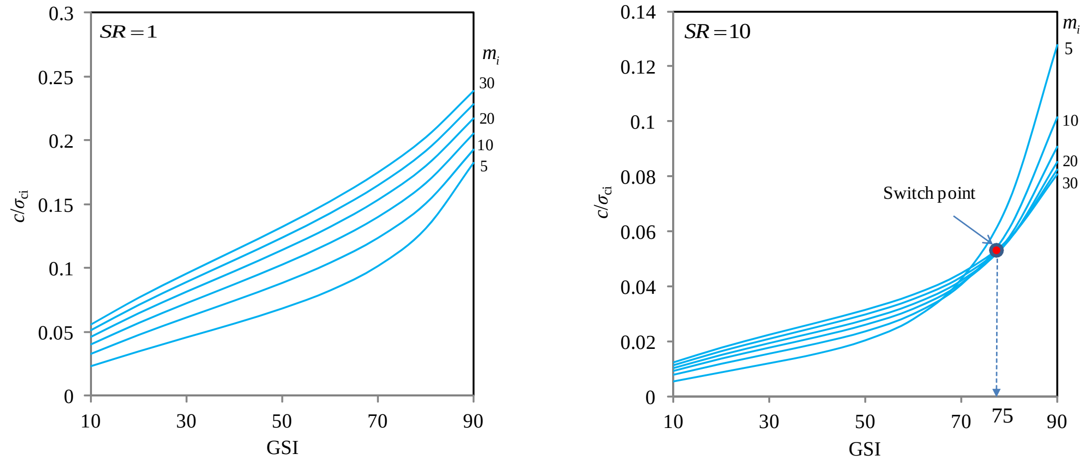


Fig. 2 Proposed charts for estimating shear strength parameters (D = 0)



(a) SR = 1 (b) SR = 10

Fig. 3 Alternative charts for estimating c/σ_{ci}

increase. For example, increasing SR values from 1 to 2 when GSI = 50 and m_i = 5, the values of c/σ_{ci} decrease from 0.068 to 0.046, and the values of ϕ increase from 18.71° to 23.04°.

It is also found that there is a clear trend of increasing of c/σ_{ci} and ϕ with the increase of GSI. For instance, increasing GSI values from 10 to 90 when SR = 4 and m_i = 5, the values of c/σ_{ci} increase from 0.010 to 0.136, and the values of ϕ increase from 15.56° to 35.62°.

It should be noted that in many slopes and shallow tunnels, the value of γH is much smaller than σ_{ci}. For example, when SR > 10 the effect of SR on the values of MC parameters are not significant compared with conditions with small SR values, as shown in Fig. 2.

Alternative form of chart is shown in Fig. 3 which illustrates the relationship between the c/σ_{ci} and HB parameters GSI and m_i values for specific SR values. We can see that the values of c/σ_{ci} increase with the increase of m_i values when SR = 1 as shown in Fig. 3(a). However, at the state of higher GSI when SR = 10, there is a switch point at around GSI = 75 where the values of c/σ_{ci} decrease with the increase of m_i values. For example, the values of c/σ_{ci} decrease from 0.12 for m_i = 5 to 0.08 for m_i = 30 when GSI = 90.

We investigated the switch point values under different SR values, such results are presented in

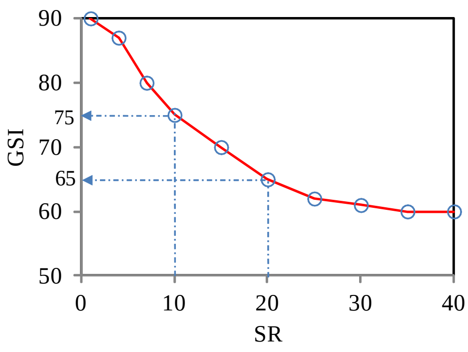
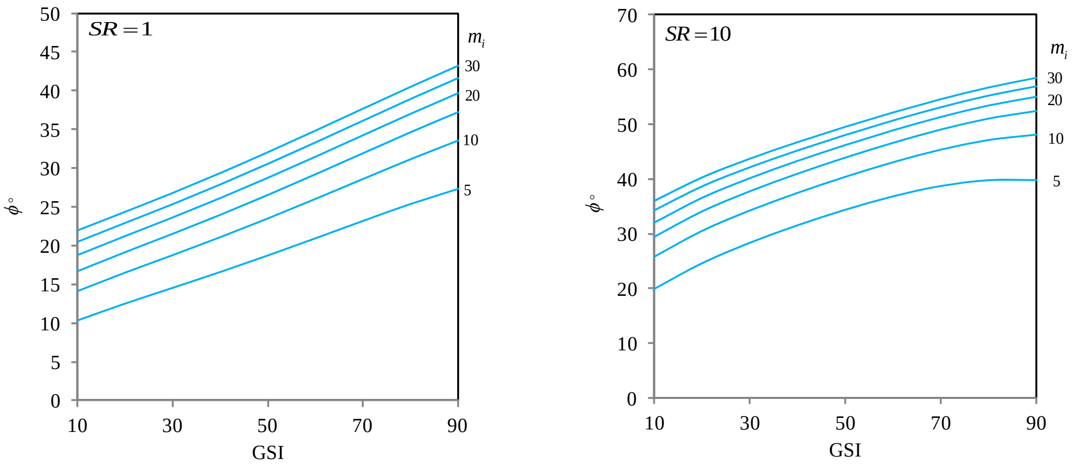


Fig. 4 Switch point locations for estimating c/σ_{ci} under different SR values

Fig. 4. It is found that the location of switch points will change with the change of SR. For example, the switch point decreases from GSI = 75 to 65 when SR increases from 10 to 20, which means when 0 < GSI < 65, the values of c/σ_{ci} increase with the increase of m_i values, however, c/σ_{ci}



(a) SR = 1 (b) SR = 10

Fig. 5 Alternative charts for estimating ϕ values

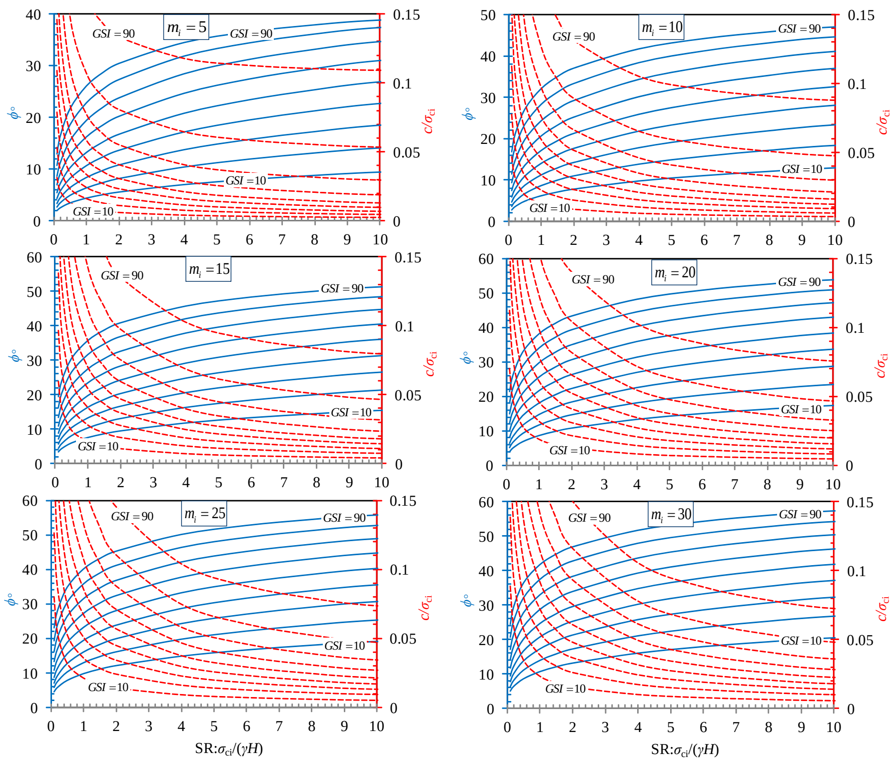


Fig. 6 Proposed charts for estimating shear strength parameters (D = 0.7)

decreases with the increase of m_i value in the state of GSI > 65.

Fig. 5 shows alternative form of chart for estimating ϕ values using a range of GSI and m_i values for different SR values. Such switch point in Fig. 3(b) was not found in Fig. 5.

4.2 Chart based on D = 0.7 and 1.0

By considering practical experience in the design of open pit slopes demonstrate that the selection of D = 0 for estimating rock mass properties are too optimistic, Hoek *et al*. (2002) introduced the disturbance factor D to get reasonable estimation of the behavior of blast damaged rock slopes.

However, it is not easy to determine the exact value of D as various factors can influence the degree of disturbance in the rock mass, such as the design of blasting and rock mass properties. Therefore, based on a number of case studies, Hoek *et al*. (2002) and Hoek (2012) provided some guidance on the selection of appropriate D values for practical application.

In civil engineering, D = 0.7 and 1.0 were recommended under good blasting and poor blasting for small scale rock slope blasting results in modest rock mass damage, respectively. In mining engineering, D = 0.7 and 1.0 were recommended under mechanical excavation and production blasting for large open pit mine slopes which suffer significant disturbance due to blasting and stress relief from overburden removal. Therefore, similar charts based on the D = 0.7 and 1.0 were presented, as shown in Figs. 6-7, which can be used to estimate the shear strength parameters of the rock mass under different blasting damage conditions.

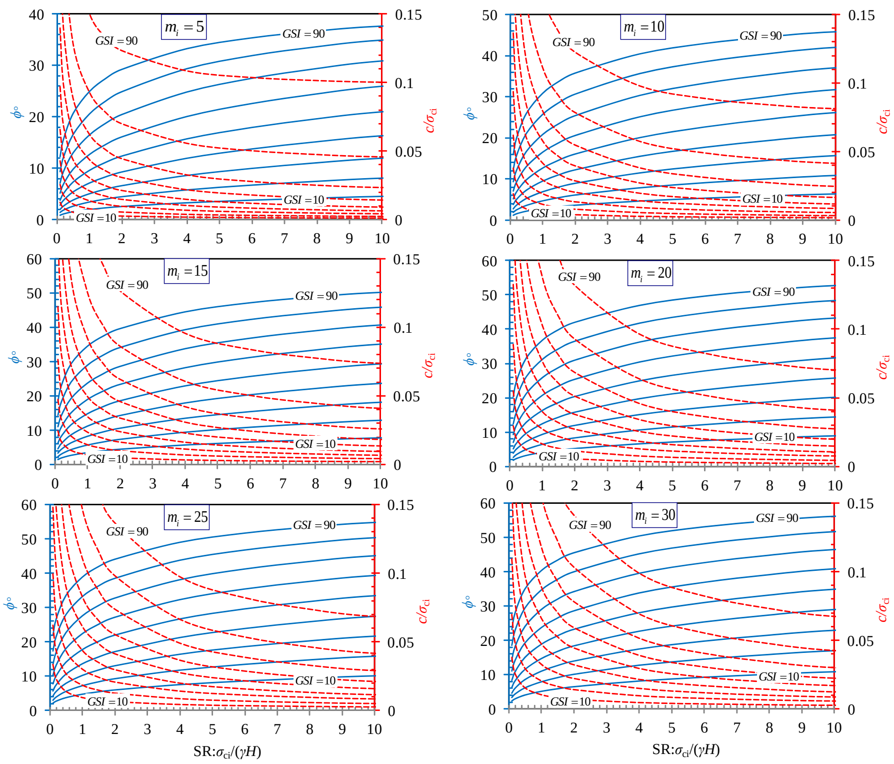
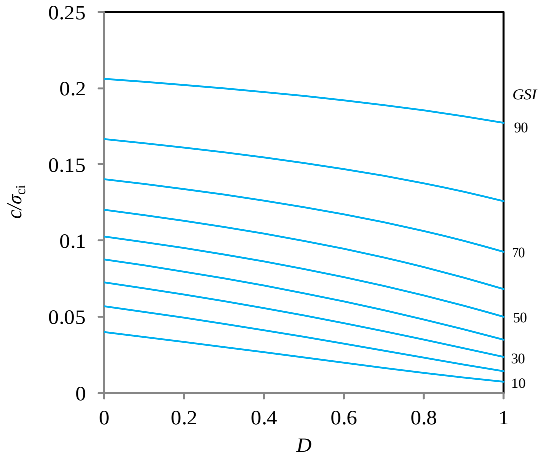
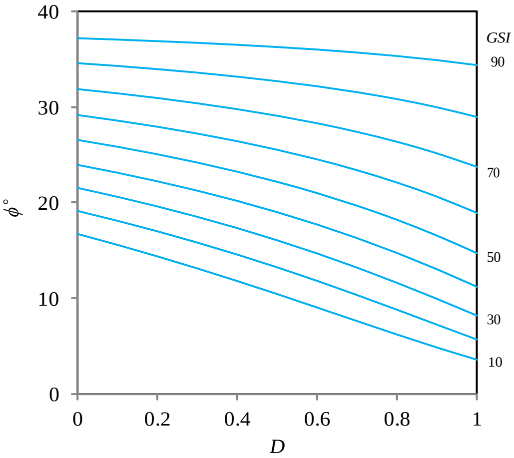


Fig. 7 Proposed charts for estimating shear strength parameters (D = 1.0)



(a) The influence of D on c/σ_{ci}



(b) The influence of D on ϕ

Fig. 8 The influence of the disturbance factor D on shear strength parameters when SR = 1 and m_i = 15

While D values have great influence on the shear strength of rock masses, it is, nevertheless, difficult to determine its exact value. Therefore, a sensitivity analysis of D is probably more important in judging the acceptability of a slope design than the results from a single D analysis. Such analysis was presented in Fig. 8 which illustrates the influence of D on the rock mass shear strength parameters. D values range between 0 or undisturbed rock and 1 for highly disturbed rock mass.

5. Conclusions

Charts for estimating rock mass shear strength parameters for rock slopes satisfying with the Hoek-Brown criterion have been proposed. Firstly, the theoretical relationships between the strength ratio (SR), $\sigma_{ci}/(\gamma H)$ and MC parameters (c/σ_{ci} and ϕ) have been presented. It is found that the values of c/σ_{ci} and ϕ of a slope depend only on the magnitude of SR when the values of GSI, m_i and D are the same. Therefore, the number of independent parameters for calculating the c/σ_{ci} and ϕ can be reduced to four (SR, GSI, m_i and D). Based on the relationships between the SR and MC parameters, charts as shown in Figs. 2 and 6-7 for calculating the MC parameters from the HB parameters for D = 0, 0.7 and 1.0, respectively, have been proposed. The proposed charts, which are quite simple and straightforward to use, can be reliably used to estimate the shear strength of the rock mass of a slope directly from the HB parameters (GSI, m_i and D), slope geometry and rock mass properties (H, σ_{ci} and γ).

Acknowledgments

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