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Nonexistence and non-decoupling of the dissipative potential for geo-materials

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Abstract. Two fundamental issues exist in the damage theory of geo-material based on the concept of thermodynamics: existence or nonexistence of the dissipation potential, and whether the dissipation potential could be decoupled into a damage potential and a plastic one or not. Thermodynamics theory of elastoplastic damage assumes the existence of dissipation potential, but the presence of dissipation potential is conditional. Based on the dissipation inequality in accord with the second law of thermodynamics, the sufficient and necessary conditions are given for the existence of the dissipation potential separately in total and incremental forms firstly, and proved strictly in theory. With taking advantage of the basic mechanical properties of geo-materials, the nonexistence of the dissipative potential is verified. The sufficient and necessary conditions are also given and proved for the decoupling of the dissipation potential of geo-materials in total and incremental forms. Similarly, the non-decoupling of the dissipation potential has also been proved, which indicates the dissipation potential of geo-materials in total or incremental forms could not be decoupled into a dissipative potential for plasticity and that for damage respectively. The research results for the fundamental issues in the thermodynamics theory of damage will help establish and improve the theoretic basis of elastoplastic damage constitutive model for geo-materials.

Keywords: geo-material; constitutive relation; thermodynamics; damage; dissipative potential; decoupling

1. Introduction

Damage mechanics is an important branch of solid mechanics, and the deterioration process of geo-materials is always described by the constitutive model involving damage (Shojaei *et al*. 2014, Lai *et al*. 2009, Mortazavi and Molladavoodi 2012, Zhou *et al*. 2013, Zhu *et al*. 2010). The phenomenological method which integrates the irreversible thermodynamics with continuum theory is used to describe the deterioration process of the material (Krajcinovic 1985, 1989, Lemaitre 1985, Lemaitre and Chaboche 1990, Voyiadjis and Kattan 1990, Guo *et al*. 2008, Zhou *et al*. 2013). A distinctive property of the theory is that the constitutive relation could be derived from two specific functions, the free energy function and the dissipation potential function. Of particular importance is the problem of constructing the dissipation potential function, because both plasticity and damage mechanisms involve the energy dissipation.

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Currently, the dissipation potential function is dealt with along two principal approaches. One approach is that the unique dissipation potential function is used for plasticity and damage which is deemed that these two dissipation processes occur simultaneously and correlate with each other (Voyiadjis and Kattan 1992, Faria *et al.* 1998, Loredana and Massimo 2002, Guo *et al.* 2008, Zhou *et al.* 2013). Another approach is that the energy dissipated due to plasticity and damage are independent of each other (Nguyen and Houlsby 2004, Nguyen 2005, Salari *et al.* 2004, Shao *et al.* 2005, 2006, Mortazavi and Molladavoodi 2012).

On account of the complexity and speciality of the geo-material damage, a lot of constitutive models are built up considering the influence of different conditions. The damage theory for geo-materials is not so perfect as classical plastic theory. The main reason may be that some fundamental theoretical problems have not been solved in damage mechanics for geo-materials. For example, the existence or nonexistence of the dissipative potential, and whether the dissipative potential could be decomposed into a damage potential and a plastic one or not, namely, whether these two mechanisms can be decoupled or not. In this paper, sufficient and necessary conditions are given for the existence of dissipation potential in total relation and incremental relation for geo-material, and the sufficient and necessary conditions are provided for the decoupling of dissipation potential in total and incremental forms for geo-materials also.

2. Thermodynamic foundation

2.1 First law of thermodynamics

Although the energy has various different forms and can transit from one form to another, from one object to another, energy conversion should be satisfied in these processes. The thermodynamics framework can be obtained in interrelated literature (Voyiadjis *et al.* 2011, Huang 2003). Generally, in any nonlinear dynamic system there will be an internal energy (U) and a kinetic energy (*K*).

$$U = \int_v \rho u dv, \quad K = \frac{1}{2}\int_v \rho VV dv \tag{1}$$

where ρ is the mass density, u is the internal energy density, V is velocity vector, v is the body occupied spatial domain.

Similarly, dW is the mechanical work increment and dQ is the thermal increment, and can be written as

$$dW = \int_v f\mathrm{V}\mathrm{d}v + \oint_{\partial v} p\mathrm{V}\mathrm{d}A \tag{2}$$

$$dQ = \int_v \rho r\mathrm{d}v - \oint_{\partial v} qn\mathrm{d}A \tag{3}$$

where f is a body force vector, p is the surface force vector, ∂v is the boundary, dA is the infinitesimal area, r is a body thermal vector, q is the surface thermal vector, and n is the unit vector of outward normal.

The first law of thermodynamics (energy conservation) in the incremental form can be written as

$$\mathrm{d}K + \mathrm{d}U = \mathrm{d}W + \mathrm{d}Q \tag{4}$$

For arbitrarily part of v, the local form of the energy conservation would be

$$\rho \mathrm{d}u - \rho r + \mathrm{div}q - \sigma \mathrm{d}\varepsilon = 0 \tag{5}$$

where σ, $d\varepsilon$ are stress and strain increment respectively.

2.2 Second law of thermodynamics

The second law of thermodynamics, which determines the direction of energy transition, states that the entropy of a thermodynamic system cannot decrease. It can be stated as

$$\mathrm{d}S - Q_T \geq 0, \qquad Q_T = \int_v \frac{\rho r}{\mathrm{T}} \mathrm{d}v - \oint_{\partial v} \frac{q}{T} \cdot n \mathrm{d}A \tag{6}$$

where S is entropy, and T is absolute temperature.

Applying the divergence theorem to the heat flux across the boundary of a unit volume

$$\oint_{\partial v} \frac{q}{T} n \mathrm{d}A = \int_v \mathrm{div}(\frac{q}{T}) \mathrm{d}v = \int_v (\frac{1}{T} \mathrm{div}q - \frac{q}{T^2} \mathrm{grad}T) \mathrm{d}v \tag{7}$$

where S is entropy, and T is absolute temperature.

Applying the divergence theorem to the heat flux across the boundary of a unit volume

$$dS - (\rho r - divq + \frac{q}{T} \mathrm{grad}T) / T \geq 0 \tag{8}$$

Invoking Eq. (5) into Eq. (8), the local form is obtained for the Clausius-Duhem inequality

$$\sigma \mathrm{d}\varepsilon - \rho \mathrm{d}u + T\mathrm{d}S - \frac{q}{T} \mathrm{grad}T \geq 0 \tag{9}$$

The factor ρ simply appears as a multiplier throughout the analysis. If the extensive quantities are all converted to a per unit volume, rather than per unit mass, then this factor disappears (Houlsby and Puzrin 2000).

$$\sigma \mathrm{d}\varepsilon - \mathrm{d}u + T\mathrm{d}S - \frac{q}{T} \mathrm{grad}T \geq 0 \tag{10}$$

2.3 Thermodynamics potential and dissipative inequality

While the heat exchanged could be neglected, the free energy ψ would be

$$\psi = u - TS \tag{11}$$

The incremental change in free energy can be written as

$$d\psi = du - TdS - dTS \tag{12}$$

On the use of Eqs. (11) and (12), the following relation is derived

$$\sigma \mathrm{d}\varepsilon - (\mathrm{d}\psi + S\mathrm{d}T) - \frac{1}{T} q \,\mathrm{grad} T \geq 0 \tag{13}$$

On the other hand, ψ can be written as

$$\psi = \psi(\varepsilon^e, k, D, T) \tag{14}$$

where ε^e, k, D are elastic strain, plastic internal variable, and damage variable respectively.

Then the chain rule yields

$$\mathrm{d}\psi = \frac{\partial \psi}{\partial \varepsilon^e}\mathrm{d}\varepsilon^e + \frac{\partial \psi}{\partial k}\mathrm{d}k + \frac{\partial \psi}{\partial D}\mathrm{d}D + \frac{\partial \psi}{\partial T}\mathrm{d}T \tag{15}$$

So Eq. (15) would take the form

$$(\sigma - \frac{\partial \psi}{\partial \varepsilon^e})\mathrm{d}\varepsilon^e + \sigma \mathrm{d}\varepsilon^p - \frac{\partial \psi}{\partial k}\mathrm{d}k - \frac{\partial \psi}{\partial D}\mathrm{d}D - (S + \frac{\partial \psi}{\partial T})\mathrm{d}T + qg \geq 0 \tag{16}$$

where $d\varepsilon^p = d\varepsilon - d\varepsilon^e$, is the plastic strain increment, $g = \frac{1}{T}\mathrm{grad}T$.

Eq. (16) holds for arbitrarily chosen part of volume, there will be

$$\sigma = \frac{\partial \psi}{\partial \varepsilon^e}, \quad S = -\frac{\partial \psi}{\partial T} \tag{17}$$

Defining the thermodynamics force K, Y as

$$K = -\frac{\partial \psi}{\partial k}, \quad Y = -\frac{\partial \psi}{\partial D} \tag{18}$$

On using Eq. (16), the energy equation and local entropy production inequality take following forms

$$\sigma \mathrm{d}\varepsilon^p + K\mathrm{d}k + Y\mathrm{d}D + qg \geq 0 \tag{19}$$

It is assumed there is no local heat source, which means, the thermal dissipation is zero. Then Eq. (19) takes the form

$$\sigma \mathrm{d}\varepsilon^p + K\mathrm{d}k + Y\mathrm{d}D \geq 0 \tag{20}$$

3. Dissipative potential

Except the free energy function, the dissipative function is also needed for establishing of constitutive model based on thermodynamics. However, the misunderstanding, which takes the dissipative function as the result of the second law of thermodynamics, exists in the determining of dissipative function. In fact, only the dissipative inequality Eq. (20) could be obtained according to the second law of thermodynamics. The dissipative potential would be developed under a set of additional hypothesis.

3.1 Total relation

The total relation for constitutive model indicates that the arbitrary value of state variables corresponding to a certain value of the corresponding thermodynamics forces, and the dissipative potential Ω for total relation expression is defined as function of state variables

$$\Omega=\Omega(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D) \tag{21}$$

where ε_v^p, ε_s^p, θ_{ε^p} are the plastic volumetric strain, generalized plastic shear strain, and Lode angle of plastic strain respectively.

Together with the differential operation and Eq. (20)

$$\mathrm{d}\Omega=p\mathrm{d}\varepsilon_v^p+q\mathrm{d}\varepsilon_s^p+q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p}+K\cdot\mathrm{d}k+Y\cdot\mathrm{d}D\geq 0 \tag{22}$$

where p, q, θ_σ are the hydrostatic stress, generalized shear stress, and Lode angle of stress respectively.

$$\mathrm{d}\Omega(\varepsilon_v^p,\varepsilon_s^p,k,D)=\frac{\partial\Omega}{\partial\varepsilon_v^p}\mathrm{d}\varepsilon_v^p+\frac{\partial\Omega}{\partial\varepsilon_s^p}\mathrm{d}\varepsilon_s^p+\frac{\partial\Omega}{\partial\theta_{\varepsilon^p}}\mathrm{d}\theta_{\varepsilon^p}+\frac{\partial\Omega}{\partial k}\mathrm{d}k+\frac{\partial\Omega}{\partial D}\mathrm{d}D \tag{23}$$

In analogy to Eq. (22)

$$p=\frac{\partial\Omega}{\partial\varepsilon_v^p},\quad q=\frac{\partial\Omega}{\partial\varepsilon_s^p},\quad q\theta_\sigma=\frac{\partial\Omega}{\partial\theta_{\varepsilon^p}},\quad K=\frac{\partial\Omega}{\partial k},\quad Y=\frac{\partial\Omega}{\partial D} \tag{24}$$

Ω^* is given via the Legendre transform of Ωd

$$\Omega^*=\Omega^*(p,q,\theta_\sigma,K,Y) \tag{25}$$

Then the total relation would be yielded for constitutive model

$$\varepsilon_v^p=\frac{\partial\Omega^*}{\partial p},\quad \varepsilon_s^p=\frac{\partial\Omega^*}{\partial q},\quad \theta_{\varepsilon^p}=\frac{\partial\Omega^*}{q\partial\theta_\sigma},\quad k=\frac{\partial\Omega^*}{\partial K},\quad D=\frac{\partial\Omega^*}{\partial Y} \tag{26}$$

Eq. (26) expresses the constitutive relation in which the thermodynamic strain depends upon the dissipative potential leading to a total relation between thermodynamics forces and the state variables. Once the dissipative potential $\Omega=\Omega(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)$ is specified, the evolution of the state variable can be obtained by Eq. (26). A key point to observe here is that such a constitutive equation will not be allowed under the nonexistence of dissipative potential. Thus, we have to find the additional requirement for the existence of dissipative potential of geo-materials.

Theorem 1: There exists a dissipative potential $\Omega=\Omega(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)$, such that equations $\frac{\partial p}{\partial\varepsilon_s^p}=\frac{\partial q}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial\theta_{\varepsilon^p}}=\frac{q\partial\theta_\sigma}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial k}=\frac{\partial K}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial D}=\frac{\partial Y}{\partial\varepsilon_v^p}$, $\frac{\partial q}{\partial\theta_{\varepsilon^p}}=\frac{q\partial\theta_\sigma}{\partial\varepsilon_s^p}$, $\frac{\partial q}{\partial k}=\frac{\partial K}{\partial\varepsilon_s^p}$, $\frac{\partial q}{\partial D}=\frac{\partial Y}{\partial\varepsilon_s^p}$, $\frac{q\partial\theta_\sigma}{\partial k}=\frac{\partial K}{\partial\theta_{\varepsilon^p}}$, $\frac{q\partial\theta_\sigma}{\partial D}=\frac{\partial Y}{\partial\theta_{\varepsilon^p}}$, $\frac{\partial K}{\partial D}=\frac{\partial Y}{\partial k}$ holds in the thermodynamics strain field.

Necessary condition:
If there exists a dissipative potential $\Omega=\Omega(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)$, then we have

$$\mathrm{d}\Omega=p\mathrm{d}\varepsilon_v^p+q\mathrm{d}\varepsilon_s^p+q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p}+K\mathrm{d}k+Y\mathrm{d}D \tag{27}$$

Using Eq. (23), yield

$$\frac{\partial\Omega}{\partial\varepsilon_v^p}=p,\quad \frac{\partial\Omega}{\partial\varepsilon_s^p}=q,\quad \frac{\partial\Omega}{\partial\theta_{\varepsilon^p}}=q\theta_\sigma,\quad \frac{\partial\Omega}{\partial k}=K,\quad \frac{\partial\Omega}{\partial D}=Y \tag{28}$$

Note that from

$$\frac{\partial\Omega}{\partial\varepsilon_v^p}=p,\quad \frac{\partial\Omega}{\partial\varepsilon_s^p}=q \tag{00}$$

We can get

$$\frac{\partial^2\Omega}{\partial\varepsilon_s^p\partial\varepsilon_v^p}=\frac{\partial p}{\partial\varepsilon_s^p},\quad \frac{\partial^2\Omega}{\partial\varepsilon_v^p\partial\varepsilon_s^p}=\frac{\partial q}{\partial\varepsilon_v^p},\quad \frac{\partial^2\Omega}{\partial\varepsilon_s^p\partial\varepsilon_v^p}=\frac{\partial^2\Omega}{\partial\varepsilon_v^p\partial\varepsilon_s^p} \tag{29}$$

Namely

$$\frac{\partial p}{\partial\varepsilon_s^p}=\frac{\partial q}{\partial\varepsilon_v^p} \tag{30}$$

With the same process, the following equation can be derived

$$\begin{gathered}\frac{\partial p}{\partial\theta_{\varepsilon^p}}=\frac{q\partial\theta_\sigma}{\partial\varepsilon_v^p},\quad \frac{\partial p}{\partial k}=\frac{\partial K}{\partial\varepsilon_v^p},\quad \frac{\partial p}{\partial D}=\frac{\partial Y}{\partial\varepsilon_v^p},\quad \frac{\partial q}{\partial\theta_{\varepsilon^p}}=\frac{q\partial\theta_\sigma}{\partial\varepsilon_s^p},\quad \frac{\partial q}{\partial k}=\frac{\partial K}{\partial\varepsilon_s^p},\\ \frac{\partial q}{\partial D}=\frac{\partial Y}{\partial\varepsilon_s^p},\quad \frac{q\partial\theta_\sigma}{\partial k}=\frac{\partial K}{\partial\theta_{\varepsilon^p}},\quad \frac{q\partial\theta_\sigma}{\partial D}=\frac{\partial Y}{\partial\theta_{\varepsilon^p}},\quad \frac{\partial K}{\partial D}=\frac{\partial Y}{\partial k}\end{gathered} \tag{31}$$

Sufficient condition

Let $\frac{\partial p}{\partial\varepsilon_s^p}=\frac{\partial q}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial\theta_{\varepsilon^p}}=\frac{q\partial\theta_\sigma}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial k}=\frac{\partial K}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial D}=\frac{\partial Y}{\partial\varepsilon_v^p}$, $\frac{\partial q}{\partial\theta_{\varepsilon^p}}=\frac{q\partial\theta_\sigma}{\partial\varepsilon_s^p}$, $\frac{\partial q}{\partial k}=\frac{\partial K}{\partial\varepsilon_s^p}$, $\frac{\partial q}{\partial D}=\frac{\partial Y}{\partial\varepsilon_s^p}$, $\frac{q\partial\theta_\sigma}{\partial k}=\frac{\partial K}{\partial\theta_{\varepsilon^p}}$, $\frac{q\partial\theta_\sigma}{\partial D}=\frac{\partial Y}{\partial\theta_{\varepsilon^p}}$, $\frac{\partial K}{\partial D}=\frac{\partial Y}{\partial k}$, and we know that $\int_L p\mathrm{d}\varepsilon_v^p+q\mathrm{d}\varepsilon_s^p+q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p}+K\mathrm{d}k$ $+Y\mathrm{d}D$ is independent of the path taken.

The dissipative energy from point 0 to point $(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)$ can be expressed

$$\int_0^{(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)} p\mathrm{d}\varepsilon_v^p+q\mathrm{d}\varepsilon_s^p+q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p}+K\mathrm{d}k+Y\mathrm{d}D \tag{32}$$

The quantity of Eq. (32) is determined by the end point $M(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)$, so Eq. (32) is the function of $\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k$ and D, and we have

$$\Phi=\int_0^{(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)} p\mathrm{d}\varepsilon_v^p+q\mathrm{d}\varepsilon_s^p+q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p}+K\mathrm{d}k+Y\mathrm{d}D \tag{33}$$

Because of

$$\begin{aligned}\Phi &= \int_0^{(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)} p\mathrm{d}\varepsilon_v^p + q\mathrm{d}\varepsilon_s^p + q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p} + K\mathrm{d}k + Y\mathrm{d}D \\ &= \int_0^{\varepsilon_v^p} p\mathrm{d}\varepsilon_v^p + \int_0^{\varepsilon_s^p} q\mathrm{d}\varepsilon_s^p + \int_0^{\varepsilon_{\theta_\sigma}^p} q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p} + \int_0^k K\mathrm{d}k + \int_0^D Y\mathrm{d}D\end{aligned} \tag{34}$$

Hence

$$\frac{\partial\Phi}{\partial\varepsilon_v^p} = \frac{\partial}{\partial\varepsilon_v^p}\int_0^{\varepsilon_v^p} p\mathrm{d}\varepsilon_v^p + \frac{\partial}{\partial\varepsilon_v^p}\int_0^{\varepsilon_s^p} q\mathrm{d}\varepsilon_s^p + \frac{\partial}{\partial\varepsilon_v^p}\int_0^{\theta_{\varepsilon^p}} q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p} + \frac{\partial}{\partial\varepsilon_v^p}\int_0^k K\mathrm{d}k + \frac{\partial}{\partial\varepsilon_v^p}\int_0^D Y\mathrm{d}D = p \tag{35}$$

Similarly

$$\frac{\partial\Phi}{\partial\varepsilon_s^p} = q,\quad \frac{\partial\Phi}{\partial\theta_{\varepsilon^p}} = q\theta_\sigma,\quad \frac{\partial\Phi}{\partial k} = K,\quad \frac{\partial\Phi}{\partial D} = Y \tag{36}$$

So

$$\mathrm{d}\Phi(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D) = \frac{\partial\Phi}{\partial\varepsilon_v^p}\mathrm{d}\varepsilon_v^p + \frac{\partial\Phi}{\partial\varepsilon_s^p}\mathrm{d}\varepsilon_s^p + \frac{\partial\Phi}{\partial\theta_{\varepsilon^p}}\mathrm{d}\theta_{\varepsilon^p} + \frac{\partial\Phi}{\partial k}\mathrm{d}k + \frac{\partial\Phi}{\partial D}\mathrm{d}D \tag{37}$$

We get the dissipative potential

$$\Omega = \Phi = \int_0^{(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)} p\mathrm{d}\varepsilon_v^p + q\mathrm{d}\varepsilon_s^p + q\theta_\sigma\mathrm{d}\theta_{\varepsilon^p} + K\mathrm{d}k + Y\mathrm{d}D \tag{38}$$

Proof finished.

A point to note for the analysis is that the equations $\frac{\partial p}{\partial\varepsilon_s^p} = \frac{\partial q}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial\theta_{\varepsilon^p}} = \frac{q\partial\theta_\sigma}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial k} = \frac{\partial K}{\partial\varepsilon_v^p}$, $\frac{\partial p}{\partial D} = \frac{\partial Y}{\partial\varepsilon_v^p}$, $\frac{\partial q}{\partial\theta_{\varepsilon^p}} = \frac{q\partial\theta_\sigma}{\partial\varepsilon_s^p}$, $\frac{\partial q}{\partial k} = \frac{\partial K}{\partial\varepsilon_s^p}$, $\frac{\partial q}{\partial D} = \frac{\partial Y}{\partial\varepsilon_s^p}$, $\frac{q\partial\theta_\sigma}{\partial k} = \frac{\partial K}{\partial\theta_{\varepsilon^p}}$, $\frac{q\partial\theta_\sigma}{\partial D} = \frac{\partial Y}{\partial\theta_{\varepsilon^p}}$, $\frac{\partial K}{\partial D} = \frac{\partial Y}{\partial k}$ have to hold in the thermodynamics strain field, if Ω is the dissipative potential.

The basic mechanical characteristics of geo-materials are used to verify whether the dissipative potential exists or not. For geo-materials, the plastic strain could be described as

$$\begin{cases} d\varepsilon_v^p = Adp + Bdq + Ld\theta_\sigma \\ d\varepsilon_s^p = Cdp + Ddq + Md\theta_\sigma \\ d\theta_{\varepsilon^p} = Edp + Fdq + Nd\theta_\sigma \end{cases} \tag{39}$$

where A, B, C, D, E, F, L, M, N are the corresponding deformation coefficients respectively.

In the pseudo triaxial condition, the three principal stresses satisfy $\sigma_1 > \sigma_2 = \sigma_3$, and the stress Lode angle (θ_σ) keeps constant, so the computation of plastic strain could be simplified as

$$\begin{cases} d\varepsilon_v^p = Adp + Bdq \\ d\varepsilon_s^p = Cdp + Ddq \end{cases} \tag{40}$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ for geo-materials here, is different from that for the metal material with $B = C = 0$. Generally $A > 0, D > 0$ (Liu *et al.* 2009). Under the negative dilatancy deformation stage (shear leads to volume shrinking) of geo-materials, $B > 0$ and $C > 0$ can be derived, i.e., $B \neq C$. Hence, the matrix should not be a singular one, and the following transformation exists

$$\begin{Bmatrix} dp \\ dq \end{Bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{Bmatrix} d\varepsilon_v^p \\ d\varepsilon_s^p \end{Bmatrix} \tag{41}$$

Namely

$$\begin{Bmatrix} dp \\ dq \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial p}{\partial \varepsilon_v^p} & \dfrac{\partial p}{\partial \varepsilon_s^p} \\ \dfrac{\partial q}{\partial \varepsilon_v^p} & \dfrac{\partial q}{\partial \varepsilon_s^p} \end{bmatrix} \begin{Bmatrix} d\varepsilon_v^p \\ d\varepsilon_s^p \end{Bmatrix} \tag{42}$$

For the negative dilatancy deformation stage of geo-materials, $B \neq C$. So

$$\frac{\partial p}{\partial \varepsilon_s^p} = \frac{-B}{AD - BC} \neq \frac{-C}{AD - BC} = \frac{\partial q}{\partial \varepsilon_v^p} \tag{43}$$

The main import of the above result is that equation $\frac{\partial p}{\partial \varepsilon_s^p} = \frac{\partial q}{\partial \varepsilon_v^p}$ cannot hold at all the deformation stage of geo-materials. According to the necessary and sufficient condition for the dissipation potential of geo-materials, it could be concluded that there is no dissipative potential with total relation expression for geo-materials, and the total relation for constitutive model cannot be constructed by the dissipative potential.

3.2 Incremental relation

Different from the total relation for constitutive model, the incremental relation indicates that arbitrary increment of state variables corresponding to a certain increment of the thermodynamics forces, and the dissipative potential Ω with incremental expression is defined as function of state variable increment

$$\Omega = \Omega(\mathrm{d}\varepsilon_v^p, \mathrm{d}\varepsilon_s^p, \mathrm{d}\theta_{\varepsilon^p}, \mathrm{d}k, \mathrm{d}D) \tag{44}$$

where $\mathrm{d}\varepsilon_v^p$, $\mathrm{d}\varepsilon_s^p$, $\mathrm{d}\theta_{\varepsilon^p}$, $\mathrm{d}k$ and $\mathrm{d}D$ are the increments for plastic volumetric strain, generalized plastic shear strain, Lode angle of plastic strain, plastic internal variable, and damage variable respectively.

Ω^* is given via the Legendre transform of Ω

$$\Omega^* = \Omega^*(p, q, \theta_\sigma, K, Y) \tag{45}$$

Then the incremental relation for constitutive model would be yielded

$$\mathrm{d}\varepsilon_v^p=\frac{\partial Q^*}{\partial p},\quad \mathrm{d}\varepsilon_s^p=\frac{\partial Q^*}{\partial q},\quad \mathrm{d}\theta_{\varepsilon^p}=\frac{\partial Q^*}{q\partial\theta_\sigma},\quad \mathrm{d}k=\frac{\partial Q^*}{\partial K},\quad \mathrm{d}D=\frac{\partial Q^*}{\partial Y} \tag{46}$$

Eq. (46) expresses the constitutive relation where the thermodynamic strain increments depend upon the dissipative potential leading to an incremental relation between thermodynamics forces and the state variables. Once the dissipative potential $\Omega=\Omega(\mathrm{d}\varepsilon_v^p,\mathrm{d}\varepsilon_s^p,\mathrm{d}\varepsilon_{\theta_\sigma}^p,\mathrm{d}k,\mathrm{d}D)$ is specified, the evolution of the state variable can be obtained by Eq. (46). Obviously, such a constitutive equation will not be allowed under the nonexistence of dissipative potential with incremental expression. In the following, the additional requirements are provided for the existence of dissipative potential of geo-materials with incremental expression.

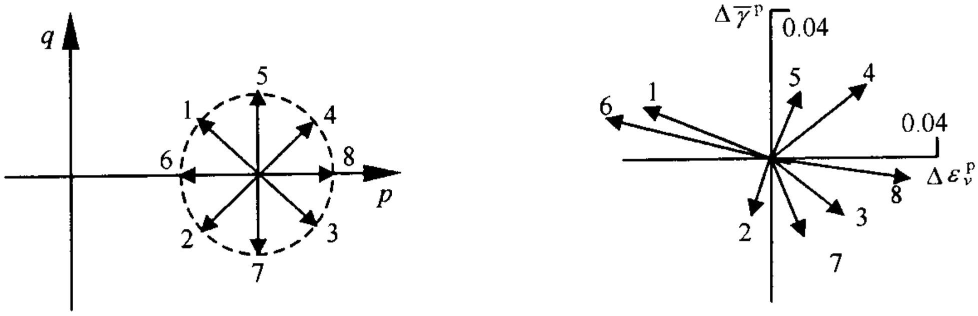
Theorem 2: There exists a dissipative potential $Q=Q(\mathrm{d}\varepsilon_v^p,\mathrm{d}\varepsilon_s^p,\mathrm{d}\theta_{\varepsilon^p},\mathrm{d}k,\mathrm{d}D)$, whose necessary and sufficient conditionis that equations $\frac{\partial p}{\partial(\mathrm{d}\varepsilon_s^p)}=\frac{\partial q}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}\theta_{\varepsilon^p})}=\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}k)}=\frac{\partial K}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}D)}=\frac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial q}{\partial(\mathrm{d}\theta_{\varepsilon^p})}=\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{\partial q}{\partial(\mathrm{d}k)}=\frac{\partial K}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{\partial q}{\partial(\mathrm{d}D)}=\frac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}k)}=\frac{\partial K}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)}=\frac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\frac{\partial K}{\partial(\mathrm{d}D)}=\frac{\partial Y}{\partial(\mathrm{d}k)}$ hold in the field of thermodynamics strain increment.

The proof of theorem 2 is the same as theorem 1, so there is no need to give unnecessary details here. A point to note for the analysis is that these equations have to hold in the field of thermodynamics strain increment.

The basic mechanical characteristics of geo-materials are made use of to verify whether the dissipative potential function with incremental expression exists or not.

The experimental data are reported in Figs. 1(a)-(b) for the pseudo triaxial test of soil specimen (Anandarajah *et al.* 1995), from which the experimental result is observed.

$\frac{\partial q}{\partial(d\varepsilon_v^p)}=0$ (q keeps constant with $d\varepsilon_v^p$ increasing), while $\frac{\partial p}{\partial(d\varepsilon_s^p)}<0$ (p increasing with $d\varepsilon_s^p$ ($\Delta\bar{\gamma}^p$) decreasing) under the action of No. 8 stress increment path, i.e., $\frac{\partial q}{\partial(d\varepsilon_v^p)}\neq\frac{\partial p}{\partial(d\varepsilon_s^p)}$.



(a) Stress increment (b) Strain increment (%)

Fig. 1 The strain increment influenced by stress increment (Anandarajah *et al.* 1995)

The main import of the above result is that equation $\frac{\partial q}{\partial(d\varepsilon_v^p)}=\frac{\partial p}{\partial(d\varepsilon_s^p)}$ could not be satisfied. According to theorem 2, it could be obtained that there is no dissipative potential with incremental expression for geo-materials, and the incremental relation for constitutive model cannot be constructed by the dissipative potential with incremental expression.

4. Decoupling

In order to avoid the difficulty of constructing the complete dissipative potential, another attempt is to establish the dissipative potentials for plasticity and damage separately. Its basic assumption is that plastic dissipation and damage dissipation are independent. In this form decoupling is required for these two mechanisms. But there has been a lot of controversy on this method. Some people think this is the thermodynamic limit which is slightly stronger than the second law of thermodynamics, and materials generally meet it (Nguyen and Houlsby 2004, Nguyen 2005, Salari *et al.* 2004). Although some other people speculate that the dissipation potential can't be decoupled (Voyiadjis and Kattan 1992, Faria *et al.* 1998), but they can't give convincing arguments. The qualification for the decoupling of dissipative potential of geo-materials is presented and that geo-materials can't meet the dissipative decoupling requirements is verified in the following.

4.1 Decoupling for the dissipative potential with total relation expression

In this part, the qualification is provided for the dissipative potential with total relation expression $(\Omega=\Omega(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D))$ decoupled into the dissipative potentials for plasticity $(\Omega_p=\Omega_p(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k))$ and damage $(\Omega_d=\Omega_d(D))$.

Theorem 3: There exists a dissipative potential with total relation expression which could be decoupled into a plastic dissipative potential Ω_p and a damage one Ω_d, whose necessary and sufficient condition is that equations $\frac{\partial p}{\partial D}=0$, $\frac{\partial q}{\partial D}=0$, $\frac{q\partial\theta_\sigma}{\partial D}=0$, $\frac{\partial K}{\partial D}=0$, $\frac{\partial Y}{\partial\varepsilon_v^p}=0$, $\frac{\partial Y}{\partial\varepsilon_s^p}=0$, $\frac{\partial Y}{\partial\theta_{\varepsilon^p}}=0$, $\frac{\partial Y}{\partial k}=0$ holds in the thermodynamics strain field.

Necessary condition

$$\frac{\partial p}{\partial D}=\frac{\partial Y}{\partial\varepsilon_v^p}=\frac{\partial^2\Omega}{\partial\varepsilon_v^p\partial D}=\frac{\partial^2(\Omega_p+\Omega_d)}{\partial\varepsilon_v^p\partial D}=\frac{\partial^2\Omega_p}{\partial\varepsilon_v^p\partial D}+\frac{\partial^2\Omega_d}{\partial\varepsilon_v^p\partial D}=0 \tag{47}$$

So on.

Namely, $\frac{\partial p}{\partial D}=0$, $\frac{\partial q}{\partial D}=0$, $\frac{q\partial\theta_\sigma}{\partial D}=0$, $\frac{\partial K}{\partial D}=0$, $\frac{\partial Y}{\partial\varepsilon_v^p}=0$, $\frac{\partial Y}{\partial\varepsilon_s^p}=0$, $\frac{\partial Y}{\partial\varepsilon_{\theta_\sigma}^p}=0$ and $\frac{\partial Y}{\partial k}=0$ hold in the thermodynamics strain space.

Sufficient condition: If $\frac{\partial p}{\partial(\mathrm{d}\varepsilon_s^p)}=\frac{\partial q}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}\theta_{\varepsilon^p})}=\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}k)}=\frac{\partial K}{\partial(\mathrm{d}\varepsilon_v^p)}$,

$\dfrac{\partial p}{\partial(\mathrm{d}D)}=\dfrac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\dfrac{\partial q}{\partial(\mathrm{d}\theta_{\varepsilon^p})}=\dfrac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\dfrac{\partial q}{\partial(\mathrm{d}k)}=\dfrac{\partial K}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\dfrac{\partial q}{\partial(\mathrm{d}D)}=\dfrac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\dfrac{q\partial\theta_\sigma}{\partial(\mathrm{d}k)}=\dfrac{\partial K}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\dfrac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)}=\dfrac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\dfrac{\partial K}{\partial(\mathrm{d}D)}=\dfrac{\partial Y}{\partial(\mathrm{d}k)}$, $\dfrac{\partial p}{\partial D}=0$, $\dfrac{\partial q}{\partial D}=0$, $\dfrac{q\partial\theta_\sigma}{\partial D}=0$, $\dfrac{\partial K}{\partial D}=0$, $\dfrac{\partial Y}{\partial\varepsilon_v^p}=0$, $\dfrac{\partial Y}{\partial\varepsilon_s^p}=0$, $\dfrac{\partial Y}{\partial\varepsilon_{\theta_\sigma}^p}=0$, $\dfrac{\partial Y}{\partial k}=0$ hold in the thermodynamics strain space, and then the thermodynamic dissipative potential $\Omega(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k,D)$ could be expressed as $\Omega=\Omega_p(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k)+\Omega_d(D)$.

As for $\dfrac{\partial p}{\partial D}=\dfrac{\partial Y}{\partial\varepsilon_v^p}=0$, $p=\dfrac{\partial\Omega}{\partial\varepsilon_v^p}$, $Y=\dfrac{\partial\Omega}{\partial D}$, we have

$$\frac{\partial^2\Omega}{\partial\varepsilon_v^p\partial D}=0 \tag{48}$$

Eq. (48) is a simple second-order homogeneous partial differential equation, so we have

$$\Omega=\Omega_1(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k)+\Omega_2(\varepsilon_s^p,\theta_{\varepsilon^p},k,D) \tag{49}$$

As for $\dfrac{\partial q}{\partial D}=\dfrac{\partial Y}{\partial\varepsilon_s^p}=0$, we arrive at

$$\frac{\partial^2\Omega}{\partial\varepsilon_s^p\partial D}=0 \tag{50}$$

Combining with Eq. (49), we obtain

$$\Omega=\Omega_1(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k)+\Omega_2(\theta_{\varepsilon^p},k,D) \tag{51}$$

As for $\dfrac{q\partial\theta_\sigma}{\partial D}=\dfrac{\partial Y}{\partial\theta_{\varepsilon^p}}=0$, we arrive at

$$\frac{\partial^2\Omega}{\partial\theta_{\varepsilon^p}\partial D}=0 \tag{52}$$

Combining with Eq. (51), we have

$$\Omega=\Omega_1(\varepsilon_v^p,\varepsilon_s^p,\theta_{\varepsilon^p},k)+\Omega_2(k,D) \tag{53}$$

At last $\dfrac{\partial K}{\partial D}=\dfrac{\partial Y}{\partial k}=0$, we have

$$\frac{\partial^2\Omega}{\partial k\partial D}=0 \tag{54}$$

Combining with Eq. (53), we arrive at

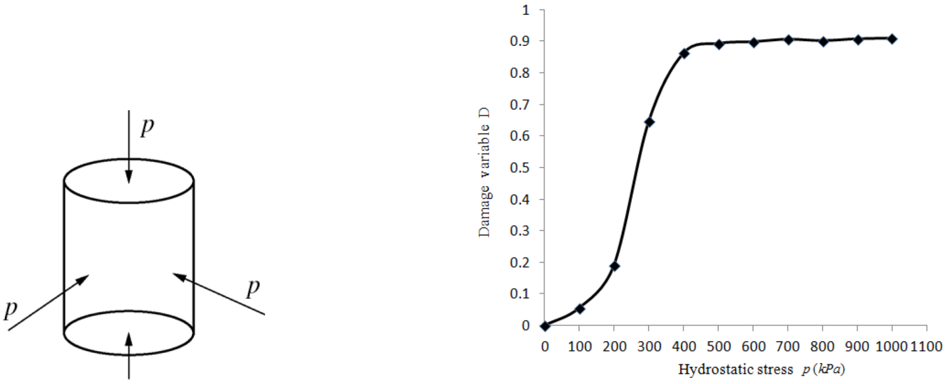
$$\Omega = \Omega_1(\varepsilon_v^p, \varepsilon_s^p, \theta_{\varepsilon^p}, k) + \Omega_2(D) \tag{55}$$

Marking $\Omega_p(\varepsilon_v^p, \varepsilon_s^p, \theta_{\varepsilon^p}, k) = \Omega_1(\varepsilon_v^p, \varepsilon_s^p, \theta_{\varepsilon^p}, k)$, $\Omega_d(D) = \Omega_2(D)$

$$\Omega(\varepsilon_v^p, \varepsilon_s^p, \theta_{\varepsilon^p}, k, D) = \Omega_p(\varepsilon_v^p, \varepsilon_s^p, \theta_{\varepsilon^p}, k) + \Omega_d(D) \tag{56}$$

Proof finished.

Thus, qualifications for the decoupling of dissipative potential of geo-materials are $\frac{\partial p}{\partial(\mathrm{d}\varepsilon_s^p)} = \frac{\partial q}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}\theta_{\varepsilon^p})} = \frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}k)} = \frac{\partial K}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial q}{\partial(\mathrm{d}\theta_{\varepsilon^p})} = \frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{\partial q}{\partial(\mathrm{d}k)} = \frac{\partial K}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{\partial q}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}k)} = \frac{\partial K}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\frac{\partial K}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}k)}$, $\frac{\partial p}{\partial D} = 0$, $\frac{\partial q}{\partial D} = 0$, $\frac{q\partial\theta_\sigma}{\partial D} = 0$, $\frac{\partial K}{\partial D} = 0$, $\frac{\partial Y}{\partial \varepsilon_v^p} = 0$, $\frac{\partial Y}{\partial \varepsilon_s^p} = 0$, $\frac{\partial Y}{\partial \varepsilon_{\theta_\sigma}^p} = 0$ and $\frac{\partial Y}{\partial k} = 0$. That is to say eight new requirements should be satisfied $\left(\frac{\partial p}{\partial D} = 0, \frac{\partial q}{\partial D} = 0, \frac{q\partial\theta_\sigma}{\partial D} = 0, \frac{\partial K}{\partial D} = 0, \frac{\partial Y}{\partial \varepsilon_v^p} = 0, \frac{\partial Y}{\partial \varepsilon_s^p} = 0, \frac{\partial Y}{\partial \varepsilon_{\theta_\sigma}^p} = 0 \text{ and } \frac{\partial Y}{\partial k} = 0\right)$. That in part 2.1 has shown that equation $\frac{\partial p}{\partial \varepsilon_s^p} = \frac{\partial q}{\partial \varepsilon_v^p}$ cannot hold at all the deformation stage of geo-materials. So the dissipative potential for geo-materials with total relation expression could not be decoupled into the dissipative potentials for plasticity and damage separately. In addition, we can also take the structured soil for instance. When the structural unit has been completely damaged, the structural soil would transit to remolded soil without structure property. (Lagioia and Nova 1995, Kavvadas and Amorosi 2000, Zhou and Liu 2007). Based on the concept of continuum mechanics, Zhou and Liu (2007) put forward a composite damage theory and established an isotropic damage constitutive model for geo-materials. In order to verify the model, the relationship of the hydrostatic stress p and volumetric strain were also given in the literature. Through these data, we can get the relationship curve of the damage variable D and the



(a) (b)

Fig. 2 Damage evolvement of structured soils under isotropic pressure (Zhou and Liu 2007) hydrostatic stress p under isotropic damage condition, as shown in Fig. 2(b). In the stress path of isotropic pressure condition with p varing from 0 to 1000 kPa (Fig. 2(a)), the properties of structured soils get gradually close to the remolded soil with the increase of surrounding pressure p, and the damage sustains to develop, as shown in Fig. 2(b). Via Fig. 2, we can obtain that $\frac{\partial p}{\partial D} > 0$, which doesn't conform to the requirement on $\frac{\partial p}{\partial D} = 0$ for the dissipative potential with total relation expression being decoupled into the dissipative potentials for plasticity and damage separately.

4.2 Decoupling for the dissipative potential with incremental expression

The qualification would be presented for the dissipative potential with incremental expression ($Q = Q(\mathrm{d}\varepsilon_v^p, \mathrm{d}\varepsilon_s^p, \mathrm{d}\theta_{\varepsilon^p}, \mathrm{d}k, \mathrm{d}D)$) decoupled into the dissipative potentials for plasticity ($Q_p = Q_p(\mathrm{d}\varepsilon_v^p, \mathrm{d}\varepsilon_s^p, \mathrm{d}\theta_{\varepsilon^p}, \mathrm{d}k)$) and damage ($\Omega_d(dD)$).

Theorem 4: There exists a dissipative potential with incremental expression which could be decoupled into the dissipative potentials for plasticity (Q_p) and damage Q_d, whose necessary and sufficient condition is that equations $\frac{\partial p}{\partial(\mathrm{d}D)} = 0$, $\frac{\partial q}{\partial(\mathrm{d}D)} = 0$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^\mathrm{p}})} = 0$, $\frac{\partial K}{\partial(\mathrm{d}D)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}k)} = 0$ holds in the field of thermodynamics strain increment.

The proof of theorem 4 is the same as theorem 3. It is unnecessary to present the details here. We should know qualifications for the decoupling of dissipative potential of geo-materials with incremental expression are $\frac{\partial p}{\partial(\mathrm{d}\varepsilon_s^p)} = \frac{\partial q}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}\theta_{\varepsilon^p})} = \frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}k)} = \frac{\partial K}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial p}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)}$, $\frac{\partial q}{\partial(\mathrm{d}\theta_{\varepsilon^p})} = \frac{q\partial\theta_\sigma}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{\partial q}{\partial(\mathrm{d}k)} = \frac{\partial K}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{\partial q}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)}$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}k)} = \frac{\partial K}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^p})}$, $\frac{\partial K}{\partial(\mathrm{d}D)} = \frac{\partial Y}{\partial(\mathrm{d}k)}$, $\frac{\partial p}{\partial(\mathrm{d}D)} = 0$, $\frac{\partial q}{\partial(\mathrm{d}D)} = 0$, $\frac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)} = 0$, $\frac{\partial K}{\partial(\mathrm{d}D)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)} = 0$, $\frac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^\mathrm{p}})} = 0$ and $\frac{\partial Y}{\partial(\mathrm{d}k)} = 0$. Thus eight new requirements should be satisfied $\left(\frac{\partial p}{\partial(\mathrm{d}D)} = 0, \frac{\partial q}{\partial(\mathrm{d}D)} = 0, \frac{q\partial\theta_\sigma}{\partial(\mathrm{d}D)} = 0, \frac{\partial K}{\partial(\mathrm{d}D)} = 0, \frac{\partial Y}{\partial(\mathrm{d}\varepsilon_v^p)} = 0, \frac{\partial Y}{\partial(\mathrm{d}\varepsilon_s^p)} = 0, \frac{\partial Y}{\partial(\mathrm{d}\theta_{\varepsilon^\mathrm{p}})} = 0 \text{ and } \frac{\partial Y}{\partial(\mathrm{d}k)} = 0\right)$. In part 2.2, we know equation $\frac{\partial q}{\partial(d\varepsilon_v^p)} = \frac{\partial p}{\partial(d\varepsilon_s^p)}$ could not be satisfied for geo-materials. Also, from Fig. 2(b), we can obtain that: the damage begins to intensify from the initial un-damaged state as p increases, which results in the increase of damage variable D at different growth rate. So, with the increase of stress p, $\frac{\partial p}{\partial(\mathrm{d}D)} = 0$ can't be always satisfied. So the dissipative potential with incremental expression could not be decoupled into the dissipative potentials for plasticity and damage respectively.

5. Conclusions

- There is no dissipative potential with total relation expression for geo-materials, and the total relation for constitutive model cannot be constructed by the dissipative potential with total relation expression.
- There is no dissipative potential with incremental expression for geo-materials, and the incremental relation for constitutive model cannot be constructed by the dissipative potential with incremental expression.
- The dissipative potential for geo-materials with total relation expression could not be decoupled into the dissipative potentials for plasticity and damage separately.
- The dissipative potential with incremental expression could not be decoupled into a dissipative potential for plasticity and that for damage respectively.

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