Geomechanics and Engineering, *Vol. 8, No. 5 (2015) 631-647*
DOI: http://dx.doi.org/10.12989/gae.2015.8.5.631

An optimal classification method for risk assessment of water inrush in karst tunnels based on grey system theory

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(*Received September 18, 2013, Revised April 11, 2014, Accepted September 17, 2014*)

Abstract. Engineers may encounter unpredictable cavities, sinkholes and karst conduits while tunneling in karst area, and water inrush disaster frequently occurs and endanger the construction safety, resulting in huge casualties and economic loss. Therefore, an optimal classification method based on grey system theory (GST) is established and applied to accurately predict the occurrence probability of water inrush. Considering the weights of evaluation indices, an improved formula is applied to calculate the grey relational grade. Two evaluation indices systems are proposed for risk assessment of water inrush in design stage and construction stage, respectively, and the evaluation indices are quantitatively graded according to four risk grades. To verify the accuracy and feasibility of optimal classification method, comparisons of the evaluation results derived from the aforementioned method and attribute synthetic evaluation system are made. Furthermore, evaluation of engineering practice is carried through with the Xiakou Tunnel as a case study, and the evaluation result is generally in good agreement with the field-observed result. This risk assessment methodology provides a powerful tool with which engineers can systematically evaluate the risk of water inrush in karst tunnels.

Keywords: optimal classification method; GST; risk assessment; water inrush; engineering application

1. Introduction

Engineers would encounter unpredictable cavities, sinkholes and karst conduits while tunneling in karst area, and water inrush disasters frequently occurred and endangered the construction safety, resulting in huge casualties and economic loss. Therefore, it is essential to accurately predict the occurrence probability of water inrush and take some effective countermeasures to assure the safety of tunnel construction (Li *et al*. 2013b). Typical water inrush disasters that have occurred in China are presented in Table 1.

In recent years, a large number of researches have been devoted to the problem, and some classical analytical methods (Kong 2011, El Tani 2003), numerical models (Meiri 1985, Yao *et al*. 2012) and conceptual models (Berkowitz 2002, Li *et al*. 2009) have been proposed. Analytical methods can estimate the groundwater inflow into tunnels, and numerical models can analyze tunnel water inflow problems in various complicated geological conditions (Hwang and Lu 2007).

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http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

Table 1 Typical water inrush disasters in China

Tunnel (Project)	Water inrush	Year of occurrence
Maluqing (Yiwan R.)	Large-scale water inrush occurred 19 times during the construction period, resulting in 16 deaths and a schedule delay of more than two and a half years.	5 times in 2006 (Jan. 21 (11 deaths), Jan. 24, Jul. 23, Aug. 13 and Aug. 18) and 14 times from Apr. to Aug. 2008 (Apr. 11 (5 deaths), Apr. 19, May. 10, May. 23, May. 27, Jun. 12, Jun. 18, Jun. 20, Jun. 21, Jul. 4, Jul. 5, Jul. 22, Aug. 15 and Aug. 28)
Yesanguan (Yiwan R.)	Once especially large-scale water inrush occurred on Aug. 5, 2007. Only in 90 minutes, the total water inrush quantity is approximately 15.1×10^4 m^3, and the mud and stone quantity is 5.35×10^4 m^3, resulting in 10 deaths and a schedule delay of half a year.	Aug. 5, 2007
Yuanliangshan (Yuhuai R.)	Large-scale water inrush occurred 71 times during the construction period, and the momentary mud inrush quantity is approximately 4200 m^3, and the maxi- mum water inrush rate is approximately 7.2×10^4 m^3/h, resulting in 9 deaths.	From 2001 to 2004 (Construction period). The most serious water inrush occurred on Sept. 11, 2002
Longtan (Hurongxi E.)	Large-scale mud inrush occurred 3 times, and the total mud quantity is approxima- tely 19,000 m^3, resulting in a schedule delay of one and a half years.	Nov. 8, 2006, Mar. 28, 2007 and Jul. 3, 2009
Wulong (Yuhuai R.)	Large-scale water inrush occurred 5 times from May. 2002 to Aug. 2003, resulting in huge economic losses of more than 20 million Yuan.	3 times in 2002 (May. 13, Jun. 20 and Aug. 12) and 2 times in 2003 (May. 21 and Jun. 25)
Guanjiao (Qinghai-Tibet R.)	The total water inrush rate is approxi- mately 32×10^4 m^3/d, and the maximum water inrush rate is approximately 10,000 m^3/h.	From Nov. 2007 to Apr 2014 (Construction period)
Baiyun (Nanguang R.)	The momentary mud inrush quantity is larger than 2,500 m^3, resulting in 6 deaths.	Jan. 4, 2010
Tongyu (Chenqian H.)	Large-scale water inrush occurred 7 times during the construction period, resulting in 5 deaths and huge economic losses.	3 times in 2002 (Jul. 9, Aug. 1 and Sept. 21), 3 times in 2003 (Jul. 2, Aug. 15 and Sept. 1) and 1 time in 2004 (Feb. 22 (5 deaths))
Tianchi (ChongqingS103 E.)	Large-scale water inrush occurred 5 times during the construction period, and once occurred in the operational period, resulting in 3 deaths and 4 injured.	5 times form Apr. 29, 2007 to Jun. 12, 2007 and 1 time on Jun. 9, 2010
Baojiashan (Xiaokang E.)	Water inrush occurred 146 times, including 8 times large-scale water inrush, and the maximum water inrush rate is approximately 3,000 m^3/h.	From Apr. 2006 to Jan. 2009 (Construction period). Three serious water inrush occurred in August, September and October, 2008, separately.
Zhongjiashan (Jilian E.)	Water inrush occurred 8 times during the construction period, and the total mud quantity is approximately 27,900 m^3, and the total water inrush quantity is approximately 20,000 m^3, resulting in a schedule delay of more than one year.	From Jul. 2 to Aug. 19, 2012 (Jul. 2, Jul. 3 (2 times), Jul. 15, Jul. 24, Aug. 13, Aug. 15 and Aug. 19)

* P.S.: R.-Railway; E.-Expressway; H.-Highway

Additionally, various methods and methodologies have been proposed to evaluate the risk of underground engineering, such as the Delphi Method (a structured communication technique which relies on a panel of experts), Analytic Hierarchy Process (a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology), Fault Tree Analysis (a deductive failure analysis using Boolean logic to combine a series of lower-level events), Event Tree Analysis (a logical modeling technique for exploring responses through a single initiating event and assessing probabilities of an undesired state of a system), Fuzzy Mathematics Method (a branch of mathematics related to fuzzy set theory and fuzzy logic which can manage fuzziness problems), Attribute Mathematics Method (a structured technique that can commendably solve the comprehensive evaluation problem with multiple fuzzy attributes), Bayesian Network(a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph) and so on (Saaty 1980, Corotis *et al*. 1981, Einstein 1996, Choi *et al*. 2004, Beard 2010, Sousa and Einstein 2012, Brown 2012), and these methods can be divided into two categories: qualitative analysis method and quantitative analysis method.

Several methods and methodologies have been applied to evaluate water inrush risk in karst tunnels, such as Analytic Hierarchy Process (Xu *et al*. 2011), Fuzzy Mathematics Method (Li *et al*. 2011), Fuzzy Wavelet Neural Network (Li *et al*. 2012), Attribute Mathematics Method (Zhou *et al*. 2013) and some other risk reduction systems (Marinos 2001, Zhang *et al*. 2011, Song *et al*. 2012). However, the above-mentioned methods have their limitations: they either do not account for quantitative data or they are complicated. Therefore, there is a need to propose an accurate and feasible method for risk assessment of water inrush in karst tunnels.

The grey system theory, as initiated by professor Deng (1989), is identified as an emerging multiple attribute decision-making technique. The theory requires a limited knowledge and understanding of an unascertained system to resolve the problem that circumstantial information obtained may be partially unknown, uncertain or incomplete (Yin 2013). In this paper, an optimal classification method based on GST is established and applied to evaluate the risk of water inrush in karst tunnel.

2. Optimal classification method based on GST

2.1 Grey relational analysis

The central idea in grey relational analysis is to analyze the uncertain relationship between two sequences. In grey relational space, there is a referential sequence with m entities

$$r = \{r_1 \mid i = 1, 2, ..., m\} \tag{1}$$

where r is the benchmark sequence; and m is equal to the quantity of evaluation indices.

There are m data sequences available for comparison, with k entities

$$s_j = \{s_j(i) \mid i = 1, 2, ..., m; \quad j = 1, 2, ..., k\} \tag{2}$$

where $s_j(i)$ is the comparative sequence.

The grey relational coefficients between the comparative sequence and the referential sequence are used to look at the relationships in both independent and interrelating data sequences, and can be computed by the following formula (Deng 1989, Wong and Lai 2000, Li *et al*. 2008)

$$\xi_i(r,\ s_j) = \frac{\min\limits_{\forall i}\min\limits_{\forall j}\Delta_j(i) + \rho\max\limits_{\forall i}\max\limits_{\forall j}\Delta_j(i)}{\Delta_j(i) + \rho\max\limits_{\forall i}\max\limits_{\forall j}\Delta_j(i)} \tag{3}$$

where $i = 1, 2,\ldots, m$; $j = 1, 2,\ldots, k$; r is the reference value; and s_j is the comparative value; ρ is the distinguishing coefficient with a value between 0 and 1, typically taken as 0.5. Furthermore, $\Delta_j(i)$ is the absolute value of the difference between $\boldsymbol{r}$ and s_j; and $\Delta_j(i)$ - $|r_i - s_j(i)|$.

The grey relational grade represents the degree of correlation between two sequences and is generally defined as the average of their respective grey relational coefficients, i.e.

$$r_j(r,\ s_j) = \frac{1}{m}\sum_{i=1}^{m}\xi_i(r,\ s_j) \tag{4}$$

However, Eq. (4) does not consider the weights of evaluation indices. A new formula was put forward by Jiang *et al*. (2004), which can be expressed in the following form

$$r_j(r,\ s_j) = \sum_{i=1}^{m}W_i\xi_i(r,\ s_j) \tag{5}$$

where W_i are the weights of evaluation indices.

2.2 Primary sequence normalization

Evaluation indices system consists of *m* indicators, and the standard data format of evaluation indices are presented in Table 2. Because the values of evaluation indices generally has different dimensions and quantitative levels, the effect of the sequence of small numerical value is easily covered up by the sequence of big numerical values. Therefore, to assure the equivalence of all indices, primary sequence data must be normalized depending on three types of pre-processing before calculating the relational coefficients. In the test, the following ways are adopted to deal with primary sequence data.

Table 2 Standard data format of evaluation indices

Evaluation indices	Risk grade			
	V_1	V_2	…	V_k
U_1	$y_{11} - y_{12}$	$y_{12} - y_{13}$	…	$y_{1k} - y_{1,k+1}$
U_2	$y_{21} - y_{22}$	$y_{22} - y_{23}$	…	$y_{2k} - y_{2,k+1}$
⋮	⋮	⋮	⋮	⋮
U_m	$y_{m1} - y_{m2}$	$y_{m2} - y_{m3}$	…	$y_{mk} - y_{m,k+1}$

(1) The referential sequence can be normalized by the following formula

$$r_i = [r_1, r_2, ..., r_m]^T = \frac{x_i - y_{i,k+1}}{y_{i1} - y_{i,k+1}} \tag{6}$$

in which

$$x_i = [x_1, x_2, ..., x_m]^T \tag{7}$$

where r_i is the normalized data of evaluation indices; and x_i is the primary data of evaluation indices.

(2) The normalized data of comparative sequences can be expressed as follows

$$s_{ij} = \frac{y_{ij} - y_{i,k+1}}{y_{i1} - y_{i,k+1}} \tag{8}$$

The comparative sequences can constitute a comparative matrix, and can be expressed in the following form

$$S_{m\times k} = [s_{ij}] = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1k} \\ s_{21} & s_{22} & \cdots & s_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mk} \end{bmatrix} \tag{9}$$

For some evaluation indices, the boundaries which are not specified, such as y_{i1} and $y_{i,k+1}$, can be computed by the following formulas

$$y_{i1} = y_{i2} - (y_{i3} - y_{i2}) \tag{10}$$

$$y_{i,k+1} = y_{ik} + (y_{ik} - y_{i,k-1}) \tag{11}$$

The risk assessment can come down to a mathematical problem: according to the comparative matrix, make the optimal classification of evaluation object.

2.3 Grey relational ranking procedure

Having calculated the grey relational grades, the sequences can be ranked by using a so-called grey relational ranking procedure, which can be carried out as follows:

(1) Compute the Euler distance between the referential sequence and the comparative sequence by the following formula

$$D(r, s_j) = u_j \sqrt{\sum_{i=1}^{m} \left[W_i \xi_i (r, s_j)\right]^2} \tag{12}$$

in which

$$\sum_{j=1}^{k} u_j = 1 \tag{13}$$

(2) Propose the objective function (Eq. (14)), and compute the minimum under the conditions expressed in Eq. (15).

$$\min\{F(u_j)\} = \min\left\{\sqrt{\sum_{j=1}^{k} u_j^2 \left[\sum_{i=1}^{m} \left[W_i \xi_i (r,\ s_j)\right]^2\right]}\right\} \tag{14}$$

$$\sum_{j=1}^{k} u_j = 1, \qquad 0 \le u_j \le 1 \tag{15}$$

(3) Construct the Lagrange Function (Eq. (16)), and set its partial derivatives for variables u_j and λ to zero (Eqs. (17) and (18))

$$L(u_j, \lambda) = \sum_{j=1}^{k} u_j^2 \left[\sum_{i=1}^{m} \left[W_i \xi_i (r,\ s_j)\right]^2\right] - \lambda\left(\sum_{j=1}^{k} u_j - 1\right) \tag{16}$$

$$\frac{\partial L(u_j,\ \lambda)}{\partial u_j} = 2u_j \sum_{i=1}^{m} \left[W_i \xi_i (r,\ s_j)\right]^2 - \lambda = 0 \tag{17}$$

$$\frac{\partial L(u_j, \lambda)}{\partial \lambda} = \sum_{j=1}^{k} u_j - 1 = 0 \tag{18}$$

where λ is Lagrange multiplier.

(4) Compute the relative membership degree u_j by the following formula

$$u_j = \frac{1}{\sum_{i=1}^{m} \left[W_i \xi_i (r,\ s_j)\right]^2 \sum_{j=1}^{k} \frac{1}{\sum_{i=1}^{m} \left[W_i \xi_i (r,\ s_j)\right]^2}} \tag{19}$$

Having calculated the relative membership degree, a sequence with k entities can be obtained, and the corresponding j of the minimum in Eq. (20) represents the risk grade of evaluation object.

$$\mu_j = (\mu_1, \mu_2, ..., \mu_k) \tag{20}$$

3. Evaluation indices systems of water inrush in karst tunnels

More than one hundred water inrush cases in karst tunnels are collected and analyzed, and several influence factors are selected as risk evaluation indices of water inrush based on the statistical information (Li *et al*. 2011, Xu *et al*. 2011, Zhou *et al*. 2013, Li *et al*. 2013b). In the design stage, evaluation indices system consists of seven indices, including formation lithology (the solubility of rock mass), unfavorable geological conditions (the sources and pathways of water inrush), groundwater level (the distance between groundwater level and tunnel floor), proportion of negative landform area (the catchment area of underground karst system), modified strata inclination (attitude of rock formation), contact zones of dissolvable and insoluble rock, and layer and interlayer fissures. More information and clear explanation on each evaluation index can be accessed by referring to Li *et al*. (2013b).

According to the total inrush rates and economic losses caused by water inrush disasters, water inrush risks are divided into four grades, and the order of corresponding occurrence probability is Level I > Level II > Level III > Level IV. Based on the statistical information of water inrush accidents, the relationship between the evaluation indices and the occurrence probability of water inrush and its perniciousness have been systematically analyzed, and the grading standards of evaluation indices have been put forward by Li *et al*. (2011, 2013b) and Xu *et al*. (2011). Therefore, evaluation indices are quantitatively graded according to four risk grades (Li *et al*. 2013b). The grading standards of evaluation indices are shown in Table 3. In addition, the unfavorable geological conditions can be divided into three secondary indices for risk assessment of water inrush in construction stage (Table 4). These indices can be ascertained according to the actual situation in construction stage, whereas they are difficult to ascertain in prospect stage.

However, it is unreasonable to divide the unfavorable geological conditions into three secondary indices in some cases that water inrush is caused by water-bearing structure without water source recharge, rather than fault fracture zone. Therefore, the evaluation indices system proposed by Li *et al*. (2013b) for risk assessment of water inrush in construction stage need be modified as appropriate.

Table 3 Grading standards of evaluation indices

Evaluation index	Risk gradation indices	Risk grade of water inrush			
		V_1 (IV)	V_2 (III)	V_3 (II)	V_4 (I)
	Total water inrush rate (1,000 m^3/d)	< 0.5	0.5-3	3-10	> 10
	Economic losses (1,000,000 *RMB*)	< 1	1-5	5-10	> 10
U_1	Formation lithology t	> 85	85-70	70-60	< 60
U_2	Unfavorable geological conditions	> 85	85-70	70-60	< 60
U_3	Groundwater level (m)	0-10	10-30	30-60	> 60
U_4	Proportion of Negative landform area (%)	0-20	20-40	40-60	> 60
U_5	Modified strata inclination (°)	$0 \le \varphi_1' < 10$	$0 \le \varphi_2' < 10$	$10 \le \varphi' < 25$	$25 \le \varphi' \le 45$
U_6	Contact zones of dissolvable and insoluble rock	> 85	85-70	70-60	< 60
U_7	Layer and interlayer fissures	> 85	85-70	70-60	< 60

Table 4 Grading standards of secondary evaluation indices

Evaluation index		Risk grade of water inrush			
		V_1 (IV)	V_2 (III)	V_3 (II)	V_4 (I)
$U_{2\text{-}1}$	Water-bearing structure	> 85	85-70	70-60	< 60
$U_{2\text{-}2}$	Catchments area of karst water system S (km^2)	0-5	5-7.5	7.5-10	> 10
$U_{2\text{-}3}$	Width of fault fracture zone (m)	0-0.2	0.2-0.5	0.5-1.0	> 1.0

4. Accuracy and feasibility analysis of optimal classification method

In order to verify the accuracy and feasibility of the optimal classification method, the engineering case of water inrush described in Li *et al*. (2013b) was studied in this paper, and the comparison of the results derived from the aforementioned method and attribute synthetic evaluation system was made.

Jigongling tunnel is 4.5 km long, with a maximum overburden thickness of 338m as shown in Fig. 1. The main geologic formation crossed by the Jigongling tunnel is shale, marl and dolomitic limestone at the section located between 19.240 km and 20.180 km (K19+240-K20+180). The main aquifers passed through by the Jigongling tunnel include a weak karst aquifer from K19+450 to K19+760 and a strong karst aquifer from K19+760 to K20+180.

Based on the grading standards of evaluation indices shown in Table 3 and Table 4, the comparative matrix of evaluation indices can be derived by using Eqs. (8)-(11). The computational process can be carried out as follows:

(1) Compute the boundaries of evaluation indices by using Eqs. (10) and (11). For evaluation indices U_1, U_2, U_6, U_7 and $U_{2\text{-}1}$, Eqs. (10) and (11) are used to obtain the values of y_{i1} and y_{i5}. Eq. (11) is applied to obtain the boundary value of y_{i1} for evaluation indices U_3, U_4, $U_{2\text{-}2}$ and $U_{2\text{-}3}$. The results were present in Table 5.

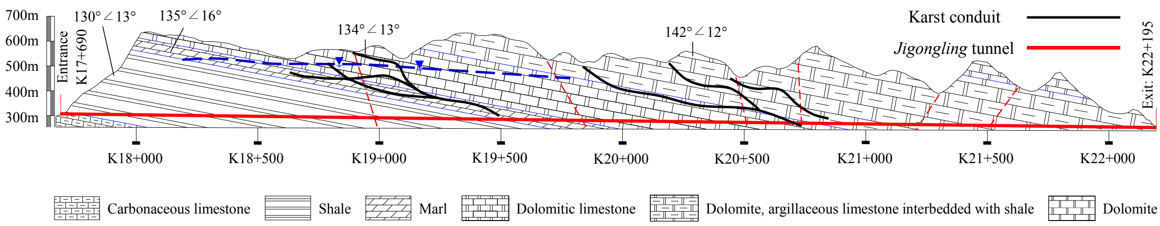

Fig. 1 Engineering geology of Jigongling tunnel (Li *et al*. 2013b)

Table 5 Boundary values of evaluation indices

Evaluation index	y_{i1}	y_{i2}	y_{i3}	y_{i4}	y_{i5}
U_1	100	85	70	60	50
U_2	100	85	70	60	50
U_3	0	10	30	60	90
U_4	0	20	40	60	80
U_5	0	10	/	25	45
	/	0	10		
U_6	100	85	70	60	50
U_7	100	85	70	60	50
$U_{2\text{-}1}$	100	85	70	60	50
$U_{2\text{-}2}$	0	5	7.5	10	12.5
$U_{2\text{-}3}$	0	0.2	0.5	1.0	1.5

(2) Normalize the data of comparative sequences by using Eq. (8), and the normalization results constitute the comparative matrix of evaluation indices by using Eq. (9).

$$S_{7\times4}=[s_{ij}]=\begin{bmatrix}1.000 & 0.700 & 0.400 & 0.200\\ 1.000 & 0.700 & 0.400 & 0.200\\ 1.000 & 0.889 & 0.667 & 0.333\\ 1.000 & 0.750 & 0.500 & 0.250\\ 1.000 & 1.000 & 0.778 & 0.444\\ 1.000 & 0.700 & 0.400 & 0.200\\ 1.000 & 0.700 & 0.400 & 0.200\end{bmatrix} \tag{21}$$

$$S_{9\times4}=[s_{ij}]=\begin{bmatrix}1.000 & 0.700 & 0.400 & 0.200\\ 1.000 & 0.700 & 0.400 & 0.200\\ 1.000 & 0.600 & 0.400 & 0.200\\ 1.000 & 0.87 & 0.667 & 0.333\\ 1.000 & 0.889 & 0.667 & 0.333\\ 1.000 & 0.750 & 0.500 & 0.250\\ 1.000 & 1.000 & 0.778 & 0.444\\ 1.000 & 0.700 & 0.400 & 0.200\\ 1.000 & 0.700 & 0.400 & 0.200\end{bmatrix} \tag{22}$$

where $S_{7\times4}$ is the comparative matrix of evaluation indices used in the design stage; and $S_{9\times4}$ is the comparative matrix of evaluation indices applied in the construction stage.

Based on the grading standards shown in Table 3-4, the referential sequence of evaluation indices can be obtained and normalized by using Eq. (6)-(7)

$$\begin{aligned} r_7 &= [r_1, r_2, r_3, r_4, r_5, r_6, r_7]^T \\ &= \left[\frac{x_1-50}{50}, \frac{x_2-50}{50}, \frac{90-x_3}{50}, \frac{80-x_4}{50}, \frac{45-x_5}{50}, \frac{x_2-50}{50}, \frac{x_2-50}{50}\right]^T \end{aligned} \tag{23}$$

$$\begin{aligned} r_9 &= [r_1, r_{2-1}, r_{2-2}, r_{2-3}, r_3, r_4, r_5, r_6, r_7]^T \\ &= \left[\frac{x_1-50}{50}, \frac{x_{2-1}-50}{50}, \frac{12.5-x_{2-2}}{50}, \frac{1.5-x_{2-3}}{50}, \frac{90-x_3}{90}, \frac{80-x_4}{80}, \frac{45-x_5}{45}, \frac{x_2-50}{50}, \frac{x_2-50}{50}\right]^T \end{aligned} \tag{24}$$

where r_7 is the referential sequence used in the design stage; and r_9 is the referential sequence applied in the construction stage.

4.1 Risk assessment of water inrush in design stage

The evaluation object was divided into five regions, and the values of evaluation indices are presented in Table 5. According to the primary data of evaluation indices which have been given by Li *et al*. (2013b), the referential sequences are derived by using Eq. (23) (Table 6).

The weights of evaluation indices used in the design stage was derived by using a comprehensive assignment method (Li *et al*. 2013b), and the weights of evaluation indices consist of the subjective and objective weights.

The objective weights are derived from Frequency Statistical Method. More than one hundred water inrush cases are collected, and the evaluation indices of water inrush and their appeared frequencies are analyzed and counted (Li *et al*. 2011, Xu *et al*. 2011, Zhou *et al*. 2013, Li *et al*. 2013b). The objective weights can be expressed in the following form

$$w_o = (0.155, 0.349, 0.173, 0.095, 0.039, 0.130, 0.058) \tag{26}$$

The subjective weights can be derived from Analytic Hierarchy Process. A judgment matrix (Li *et al*. 2013b) can be obtained by using the 1-9 scale method suggested by Saaty (1990). The subjective weights of evaluation indices are presented in Eq. (26). One obtains

$$w_s = (0.178, 0.350, 0.178, 0.098, 0.058, 0.098, 0.038) \tag{27}$$

Comprehensive weights can be expressed in the following form

$$\begin{aligned} W_7 &= 0.5w_0 + 0.5w_s \\ &= (0.167, 0.350, 0.176, 0.097, 0.049, 0.113, 0.048) \end{aligned} \tag{28}$$

The grey relational coefficients and grey relational grades of the above evaluation regions can be computed by using Eqs. (3), (5) and (25). The calculated results are presented in Tables 7-11.

According to the calculated results of the grey relational coefficients and grey relational grades presented in Tables 7-11, the relative membership degrees of all levels can be computed by using

Table 6 Primary data and the referential sequences

Evaluation regions	Primary data x_i	The referential sequences r_i
K19+240-K19+450	$x_7=[90, 75, 90, 20, 13, 85, 80]^T$	$r_7=[0.800, 0.500, 0.000, 0.750, 0.711 ,0.700, 0.600]^T$
K19+450-K19+500	$x_7=[80, 60, 90, 40, 16, 70, 75]^T$	$r_7=[0.600, 0.200, 0.000, 0.500, 0.644 ,0.400, 0.500]^T$
K19+500-K19+760	$x_7=[75, 60, 90, 40, 16, 70, 65]^T$	$r_7=[0.500, 0.200, 0.000, 0.500, 0.644 ,0.400, 0.300]^T$
K19+760-K19+800	$x_7=[60, 60, 90, 40, 13, 70, 65]^T$	$r_7=[0.200, 0.200, 0.000, 0.500, 0.711 ,0.400, 0.300]^T$
K19+800-K20+180	$x_7=[55, 65, 90, 30, 13, 80, 70]^T$	$r_7=[0.100, 0.300, 0.000, 0.625, 0.711 ,0.600, 0.400]^T$

Table 7 Grey relational coefficients and grey relational grades of K19+240-K19+450

$\xi_i(r, s_j)$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	r_j
$j=1$	0.714	0.500	0.333	0.667	0.634	0.625	0.556	0.546
$j=2$	0.833	0.714	0.360	1.000	0.634	1.000	0.833	0.734
$j=3$	0.556	0.833	0.428	0.667	0.882	0.625	0.714	0.673
$j=4$	0.455	0.625	0.600	0.556	0.652	0.500	0.556	0.569

Table 8 Grey relational coefficients and grey relational grades of K19+450-K19+500

$\xi_i\,(r, s_j)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	r_j
$j = 1$	0.556	0.385	0.333	0.500	0.584	0.455	0.500	0.439
$j = 2$	0.833	0.500	0.360	0.667	0.584	0.625	0.714	0.576
$j = 3$	0.714	0.714	0.428	1.000	0.789	1.000	0.833	0.733
$j = 4$	0.556	1.000	0.600	0.769	0.714	0.714	0.625	0.769

Table 9 Grey relational coefficients and grey relational grades of K19+500-K19+760

$\xi_i\,(r, s_j)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	r_j
$j = 1$	0.500	0.385	0.333	0.500	0.584	0.455	0.417	0.425
$j = 2$	0.714	0.500	0.360	0.667	0.584	0.625	0.556	0.548
$j = 3$	0.833	0.714	0.428	1.000	0.789	1.000	0.833	0.753
$j = 4$	0.625	1.000	0.600	0.769	0.714	0.714	0.833	0.790

Table 10 Grey relational coefficients and grey relational grades of K19+760-K19+800

$\xi_i\,(r, s_j)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	r_j
$j = 1$	0.385	0.385	0.333	0.500	0.634	0.455	0.417	0.408
$j = 2$	0.500	0.500	0.360	0.667	0.634	0.625	0.556	0.515
$j = 3$	0.714	0.714	0.428	1.000	0.882	1.000	0.833	0.738
$j = 4$	1.000	1.000	0.600	0.769	0.652	0.714	0.833	0.850

Table 11 Grey relational coefficients and grey relational grades of K19+800 ~ K20+180

$\xi_i\,(r, s_j)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	r_j
$j = 1$	0.357	0.417	0.333	0.571	0.634	0.556	0.455	0.435
$j = 2$	0.455	0.556	0.360	0.800	0.634	0.833	0.625	0.567
$j = 3$	0.625	0.833	0.428	0.800	0.882	0.714	1.000	0.721
$j = 4$	0.833	0.833	0.600	0.645	0.652	0.556	0.714	0.728

Table 12 Relative membership degrees and risk grades of evaluation regions

Evaluation regions	u_1	u_2	u_3	u_4	Risk grade	
					Optimal classification method based on GST	Attribute synthetic evaluation system
K19+240-K19+450	0.319	0.177	0.210	0.294	III	III
K19+450-K19+500	0.442	0.256	0.158	0.144	I	I
K19+500-K19+760	0.452	0.272	0.144	0.131	I	I
K19+760-K19+800	0.462	0.290	0.141	0.107	I	I
K19+800-K20+180	0.433	0.255	0.158	0.155	I	I

Eq. (19) and the results are presented in Table 12. The comparisons of the risk grades derived from the aforementioned method and attribute synthetic evaluation system are made, and the results of caparisons are also presented in Table 12.

4.2 Risk assessment of water inrush in construction stage

Based on the practical geological and hydrogeological conditions, the risk of water inrush at K19+509−K19+539 in the construction stage is evaluated, and the primary values of evaluation indices are thereby updated according to the actual situation. The primary data and its referential sequence of K19+509−K19+539 can be expressed in the following forms

$$r_9 = [75, 62, 7.5, 1.0, 90, 40, 13, 72, 65]^T \tag{29}$$

$$r_9 = [0.500, 0.240, 0.400, 0.333, 0.000, 0.500, 0.711, 0.440, 0.300]^T \tag{30}$$

The unfavorable geological conditions were divided into three secondary indices in the construction stage. Their weights can be derived from Analytic Hierarchy Process, and a judgment matrix was established by Li *et al*. (2013b) by using the 1-9 scale method. The sum of the secondary indices weights should be equal to the weight of the unfavorable geological conditions used in the design stage. Therefore, the weights of evaluation indices applied in the construction stage can be expressed in the following form

$$W_9 = (0.167, 0.189,, 0.104, 0.057, 0.176, 0.097, 0.049, 0.113, 0.048) \tag{31}$$

The grey relational coefficients and grey relational grade at K19+509-K19+539 can be calculated by using Eqs. (3), (5) and (31). The computed results are presented in Table 13.

According to the computed results presented in Table 13, the relative membership degrees of all levels can be computed by using Eq. (19) and its sequence can be expressed in the following form

$$\mu_j = (0.409, 0.268, 0.152, 0.171) \tag{32}$$

According to the relative membership degrees of all levels, the risk grade of water inrush at K19+509−K19+539 in the construction stage is assumed to be Level II, which agree well with the results derived from attribute synthetic evaluation system.

The comparison indicated that the results derived from the aforementioned method agree well with the results obtained from the attribute synthetic evaluation system, and it is accurate and

Table 13 Grey relational coefficients and grey relational grade of K19+509−K19+539

$\xi_i(r, s_j)$	$i=1$	$i=2\text{-}1$	$i=2\text{-}2$	$i=2\text{-}3$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	r_j
$j=1$	0.500	0.397	0.455	0.428	0.333	0.500	0.634	0.472	0.417	0.442
$j=2$	0.714	0.521	0.455	0.366	0.360	0.667	0.634	0.658	0.556	0.546
$j=3$	0.833	0.758	0.455	0.600	0.428	1.000	0.882	0.926	0.833	0.724
$j=4$	0.625	0.926	0.556	0.484	0.600	0.667	0.652	0.676	0.833	0.683

feasible to assess the risk of water inrush in karst tunnels through the optimal classification method based on GST.

5. Engineering application

5.1 Engineering background

Xiakou tunnel is approximately 6.5 km long, with a maximum overburden thickness of 1,500 m. The main geologic formation crossed by Xiakou tunnel is limestone, dolomite and quartz sandstone, and the karst developed mainly at the section located between 106.400 km and 108.850 km (YK106+400−YK108+850). One ventilation inclined shaft was designed to satisfy the ventilation requirements of Xiakou tunnel, and the ventilation inclined shaft is approximate 780 m long, with an inclination of 24.5°. The engineering geology of Xiakou tunnel is presented in Fig. 2, and the numbers in Fig. 2 represent the geologic conditions of Xiakou tunnel. The geologic conditions are presented in detail in Table 14. In this study, the optimal classification method was used to evaluate the water inrush risk at XJK0+110−XJK0+060.

5.2 Risk evaluation progress

According to the practical geological and hydrogeological conditions, the evaluation indices system used in construction stage proposed by Li *et al*. (2013a) is unreasonable to assess the water inrush risk at XJK0+110−XJK0+060. Therefore, the evaluation indices system consisting of seven indicators used in design stage was applied for risk assessment of water inrush at XJK0+110−XJK0+060. Based on the practical geological and hydrogeological conditions shown in Fig. 2, the primary values of evaluation indices are definite by using expert investigation method according to the actual situation, which have been given by Li *et al*. (2013a). The primary data and its referential sequence at XJK0+110−XJK0+060 can be expressed in the

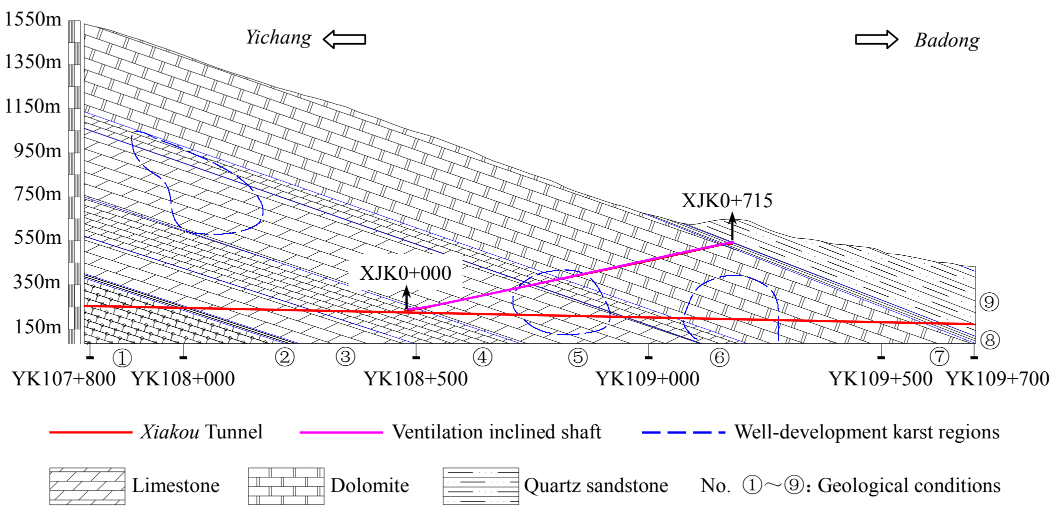

Fig. 2 Engineering geological profile of Xiakou Tunnel (Li *et al*. 2013a)

Table 14 Geologic conditions of Xiakou tunnel

No.	System	Group	Code	Lithology
①	Permian	*Qixia*	P_1q	Carbonaceous asphaltenes limestone, flint limestone intercalated with carbonaceous shale
②	Permian	*Maokou*	P_1m	Silicomanganese shale, carbonaceous shale
③	Permian	*Maokou*	P_1m	Gray massive limestone and chert banded limestone
④	Triassic	*Daye*	T_1d^1	Thin layer of limestone intercalated with shale
⑤	Triassic	*Daye*	T_1d^2	Medium thick layer of limestone, thin-medium thick layer of limestone
⑥	Triassic	*Daye*	T_1d^3	Thin-bedded argillaceous limestone
⑦	Triassic	*Jialing*	$T_{1\text{-}2}j$	Plate-dolomite, dolomite breccia, gray massive limestone, medium thick argillaceous limestone
⑧	Triassic	*Badong*	T_2b^1	Shale interbedded with fine sandstone, purple siltstone interbedded with shale
⑨	Triassic	*Shaximiao*	T_3s	Thin-bedded siltstone, clay stone intercalated with carbonaceous shale and coal seams

Table 15 Grey relational coefficients and grey relational grade of XJK0+110-XJK0+060

$\xi_i(r, s_j)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	r_j
$j = 1$	0.450	0.386	0.360	0.645	0.389	0.474	0.403	0.428
$j = 2$	0.600	0.491	0.389	0.919	0.389	0.643	0.519	0.546
$j = 3$	0.900	0.675	0.463	0.815	0.463	1.000	0.730	0.718
$j = 4$	0.900	0.900	0.648	0.665	0.648	0.818	1.000	0.816

following forms

$$x_7 = [65, 55, 90, 27, 40, 68, 58]^T \tag{33}$$

$$r_7 = [0.300, 0.100, 0.000, 0.6625, 0.111, 0.360, 0.160]^T \tag{34}$$

The grey relational coefficients and grey relational grade at XJK0+110 ~ XJK0+060 were calculated by using Eqs. (3), (5) and (28) and presented in Table 15. The relative membership degrees of all levels are computed by using Eq. (19) and expressed in the following form

$$\mu_j = (0.446, 0.274, 0.158, 0.123) \tag{35}$$

According to the relative membership degrees presented in Eq. (35), the risk of water inrush at XJK0+110−XJK0+060 is assumed to be Level I.

(a) Water inrush at XJK0+101 (b) The cavity at XJK0+093 (c) Mud burst at XJK0+093

Fig. 13 Verification by excavation results

5.3 Practical situation at XJK0+110−XJK0+060

During the drilling of boreholes at XJK0+101, water inrush with a certain pressure and a total water inrush rate of approximate 64 m³/h from the boreholes at left on August 7, 2011 (see Fig. 3(a)). The water in rushed from the boreholes was pumped out until August 30, and the total pump discharge was approximate 28,000 m³.

The water inrush rate increased because of strong rainfall, and the construction was shut down at XJK0+097 on September 15. The maximum water inrush rate reached 220 m³/h from October 1 to 4, and the water was pumped out until November 10.

Mud burst occurred at XJK0+093 on November 28, and the mud quantity was approximately 1,200 m³. A cavity filled with mud was exposed at left, and the cavity is 70 m in length, 40 m in width and 28 m in height (see Fig. 3(b)). A mud burst occurred again on December 5, and the momentary mud inrush quantity was approximately 4,500 m³ (see Fig. 3(c)).

Due to the rainfall beginning on March 21, 2012, water inrush occurred at the left side wall of XJK0+086 ~ XJK0+098 on March 22, and the total water inrush rate was approximately 7.2 m³/min. According to the ranking standard of water inrush risk expressed in Table 3, the risk of water inrush at XJK0+086 ~ XJK0+098 was assumed to be Level I, which agree well with the evaluation result derived from optimal classification method based on GST.

6. Conclusions

An optimal classification method based on grey system theory (GST) for risk assessment of water

a inrush is established and applied to accurately predict the occurrence probability of water inrush disasters. The method consists of three parts: grey relational analysis, primary sequence normalization and grey relational ranking procedure. The traditional formula used to calculate the grey relational grade does not consider the weights of evaluation indices. Therefore, an improved equation is applied in this paper.

Two evaluation indices systems re proposed for risk assessment of water inrush in design stage and construction stage, respectively, and the evaluation indices are quantitatively graded according to four risk grades, and the risk evaluate progress of water inrush by using optimal classification method based on GST is described in detail. In addition, comparisons of the evaluation results derived from the aforementioned method and attribute synthetic evaluation system are made, and the comparison results indicated that it is accurate and feasible to evaluate water inrush risk through the above-mentioned method based on GST. Evaluation of engineering practice is carried out with Xiakou tunnel at XJK0+110-XJK0+060 as a case study, and the evaluation result is generally in a good agreement with the field-observed result.

This risk assessment methodology provides a powerful tool with which engineers can systematically evaluate the risk of water inrush in karst tunnels. Compared with other analysis methods, the optimal classification method has a small amount of calculation, computes easily and requires no classic distribution of large stylebook of data. However, the aforementioned method also has its limitations: because of the complexity and uncertainty of underground engineering geological conditions, some primary values of evaluation indices and their primary values are derived from expert evaluation method with a certain subjective bias.

Acknowledgments

Much of the work presented in this paper was supported by the State Key Development Program for Basic Research of China (Grant No. 2013CB036000) and the National Natural Science Foundation of China (Grant No. 51479106). The authors would like to express appreciation to the reviewers for their valuable comments and suggestions that helped to improve the quality of the paper.

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